

Volterra Filters: A Promising Tool for Wideband and Narrowband Interference Suppression in DS-SS Communication Systems

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Abstract—The direct-sequence spread-spectrum (DS-SS) transmission system offers a promising solution to an overcrowded frequency spectrum amid growing demand for mobile and personal communication systems. Spread-spectrum system by its very nature is an interference-tolerant modulation. However, there are situations where the processing gain is inadequate and interference suppression techniques must be employed. In this paper, it will be shown how linear or non-linear estimators inserted into DS-SS receivers can be applied for interference suppression. In our consideration, Wiener filters (WF) and Volterra filters (VF) will be used as the estimators. In order to demonstrate the performance properties of discussed DS-SS receivers, a number of computer experiments have been done. The results of the experiments will show that in the case of wideband, narrowband or joined wideband and narrowband interference the VF application in DS-SS receivers can outperform the WF application or DS-SS receivers based on simple application of matched filters (MF).

Index Terms—Direct-sequence spread-spectrum transmission system, interference suppression, Wiener filter, Volterra filter

I. INTRODUCTION

THE GROWTH in wireless communications necessitates more efficient utilization of the spectrum. The growing demand for wireless communications will reach the capacity limits of existing systems soon. Hence, only the introduction of new concepts and systems will increase data transmission capacities since the frequency spectrum is limited. Here, DS-SS techniques can be exploited to increase the communication system capacity [1]-[3]. The increased sharing of the spectrum translates into a higher likelihood of users interfering with one another. Therefore, interference rejection technique applications allow increased capacity of users within the available spectrum.

A bit more comprehensive analysis of the interference sources (e.g. [1]-[3]) has shown that with respect to frequency bandwidth occupied by DS-SS signals and interference, there is narrowband interference, wideband (broadband) interference or joined (combined) narrowband and wideband interference [1]-[8]. It is well-known, that the DS-SS signals have non-Gaussian distribution. Besides, multiple access interference in the case of CDMA transmission systems possesses non-Gaussian character. Similarly, the suppression of the wideband interference due

to a non-spread spectrum digital transmission system or the task of narrowband interference rejection in DS-SS signals represents the non-Gaussian signal processing problem. With regard to these facts, linear methods for interference suppression in DS-SS transmission systems are no longer optimal and non-linear methods could be applied for that purpose with success.

The excellent overview of the advanced interference rejection techniques for DS-SS communication systems can be found in [1]. As the examples of non-linear techniques for interference suppression can be given: non-linear predictors [1], non-linear multi-user detectors [1], neural networks [1], [8], VF [4]-[6], [9], [10], extended Kalman filters [7], non-linear prediction in combination with interpolating filters structures [7], decision feedback filters [1], fractionally spaced decision feedback filters [1], etc.

It has been shown in [4], [5], that the VF based non-linear multi-user detectors are able to provide good results in CDMA interference suppression. Motivated by such conclusions, it has been presented in [6] that in the case of DS-SS communication systems the VFs are very efficient in suppressing the co-channel wideband non-spread spectrum BPSK interference. Apart from that, we proposed in [9] the application of VFs for narrowband interference suppression in DS-SS transmission systems.

Following these results, this paper is devoted to VF applications for interference suppression in DS-SS communication systems. Here, the DS-SS receiver based on VF is described. In this receiver, the VF is inserted in the conventional receiver structure between the demodulator and despreading stage ([2], [9], [10]). The properties of the proposed DS-SS receiver are analyzed by a number of computer experiments under conditions of wideband, narrowband or joined wideband and narrowband interference. The results expressed by bit-error-rate (*BER*) vs. signal to interference ratio (*SIR*) show that the VFs clearly outperform the receivers based on MF or WF application. These results refer also to a new fact, namely, that the VFs can also be applied very effectively for joined wideband and narrowband interference suppression. Following the works [4]-[6], [9], [10], and taking into account the new results presented in this paper it can be concluded that VF could be a real promising tool for wideband and narrowband interference suppression in DS-SS receivers.

II. INPUT SIGNAL MODEL

The signal that appears at the input of the receiver consists of three components. They are the BPSK DS-SS signal distorted by a linear transmission channel ($x(t)$),

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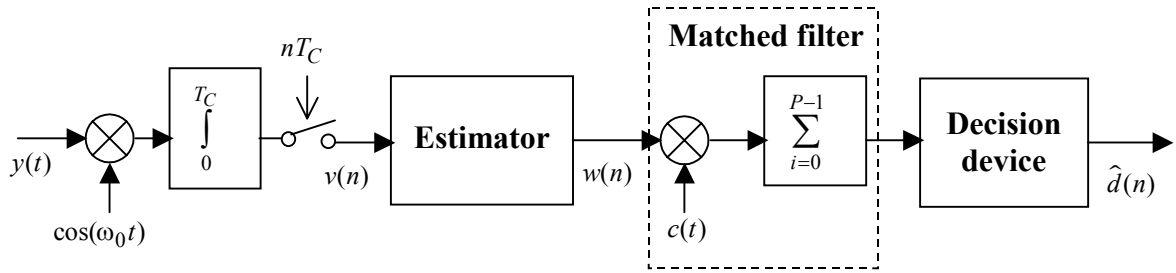


Fig. 1. The DS-SS receiver structure based on estimator application.

wideband, narrowband or joined wideband and narrowband interference $i(t)$ and additive white Gaussian noise $n(t)$ (AWGN) with power spectral density at the receiver input N_0 . The AWGN level will be expressed by the ratio E_b / N_0 (information signal energy per bit to noise power spectral density). All three-signal components are supposed to be independent and stationary signals. Then, the input signal to the receiver is given by

$$y(t) = x(t) + i(t) + n(t) \quad (1)$$

The BPSK DS-SS signal $s(t)$ can be modeled as

$$s(t) = U c(t) d(t) \cos(\omega_0 t) \quad (2)$$

where U , ω_0 , $c(t)$ and $d(t) \in \{+1, -1\}$ represent amplitude and angular frequency of the carrier, the spreading code of chip duration T_C and transmitted information base-band signal of bit duration T , respectively. The BPSK DS-SS signal distorted by a linear transmission channel with an impulse response $h(t)$ is given by

$$x(t) = h(t) * s(t) \quad (3)$$

where $*$ denotes the convolution operation.

The interference $i(t)$ is defined as $i(t) = i_W(t)$ for wideband interference, $i(t) = i_N(t)$ for narrowband interference and $i(t) = i_W(t) + i_N(t)$ for joined wideband and narrowband interference. The wideband interference $i_W(t)$ is modeled as a wideband BPSK signal expressed as

$$i_W(t) = U_S d_S(t + \tau) \cos(\omega_0 t + 2\pi f_1^W t + \theta) \quad (4)$$

where U_S , f_1^W , τ , $d_S(t) \in \{+1, -1\}$ and θ are interference amplitude and offset carrier frequency, random data bit delay, interference data and initial carrier phase, respectively. It is assumed that $\tau \in \langle 0, T \rangle$ and $\theta \in \langle 0, 2\pi \rangle$ is uniformly distributed.

The discrete representation of narrowband interference $i_N(t) = i_N(nT_S)$ is modeled by

$$i_N(n) + \sum_{k=1}^p a(k) i_N(n-k) = e(n) \quad (5)$$

where T_S is a sampling period of $i_N(t)$, and $e(n)$ is a white Gaussian process with variance σ_e^2 . A process $i_N(t)$ generated by the above model is known as the p -th order autoregressive process ($AR(p)$) [11].

III. RECEIVER STRUCTURE

The simplest well-known DS-SS receiver (so called the MF receiver) is based on the MF filter application [2], [12]. It follows from the MF theory, that a MF is optimal when a filtered signal is impaired by AWGN. However, this

assumption is usually not valid in the case of wideband or narrowband interference in DS-SS communication systems. Therefore, the receiver structure based on the MF application cannot generally be considered as optimal.

In order to improve the receiver performance, the modified structures of DS-SS receiver equipped by an estimator can be applied. Fig. 1 shows how the estimator fits into the DS-SS receiver structure. The estimator given in the Fig. 1 extracts the demodulated signal from noise. This signal pre-processing operation can improve signal to noise ratio before signal despreading. It can result in the improvement of BER in a significant way. The estimation operation is performed at the base-band, and perfect carrier and chip synchronization is assumed. The estimator is followed by conventional a MF (despreading stage) and a decision device generating the receiver output.

The only difference between the MF receiver and the receiver given in the Fig. 1 is the fact that the estimator is not included into the simple MF receiver structure. As the estimator applied in the DS-SS receiver given in the Fig. 1 a number of conventional or advanced digital filters can be used. In our considerations, VFs and WFs will be proposed for that purpose.

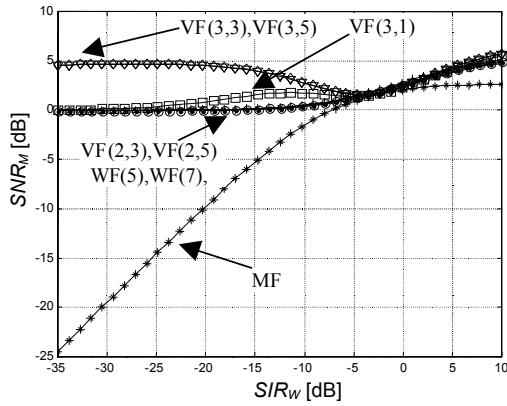
IV. VOLTERRA AND WIENER FILTERS

The VF is a minimum mean-square non-linear estimators mathematical model, which is represented by a truncated discrete Volterra series [13], [14]. The mathematical model of the M -th order VF memory span of which is $N = N_1 + N_2 + 1$ samples long (VF(M, N)) is given by

$$w(n) = h_0 + \sum_{i=1}^M \sum_{k_1=-N_1}^{N_2} \sum_{k_2=-N_1}^{N_2} \dots \dots \sum_{k_i=-N_1}^{N_2} h_{i k_1 k_2 \dots k_i} v(n-k_1) v(n-k_2) \dots v(n-k_i) \quad (6)$$

In this expression $v(n)$ and $w(n)$ are the input signal and the filter response, respectively. The i -dimensional sequence $h_{i k_1 k_2 \dots k_i}$ is called the Volterra kernel of the i -th order. The order M of the VF is defined by the highest order of the Volterra kernel which can be found in (6). The length of the VF memory span is given by the number of the mutually different samples of the input signal, which can be applied in the VF response computation. Under the condition that $N_1 > 0$, the VF(M, N) possesses a two-sided transversal structure. In this paper, we will deal only with the two-sided VF(M, N) for $N_1 = N_2 = N_0$ and $N_0 > 0$.

With regard to (6), the well-known WF of the N -th order (WF(N)) can be defined as the first order VF, i.e.


 Fig. 2. SNR_M vs. SIR_W .

$VF(N)=VF(1, N)$. The time-invariant VF design procedure is described in the Appendix of this paper. The others details concerning the design and performance properties of time-invariant and adaptive VFs and WFs can be found e.g. in [13], [14].

V. EXPERIMENTAL RESULTS

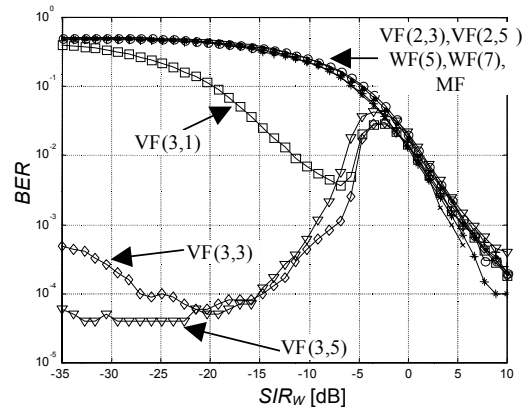
In this section, a comparison of performance properties of the BPSK DS-SS receiver based on a simple MF and its modified versions (Fig. 1) is presented. For that purpose, a number of computer experiments were carried out. In all experiments, the transmission model described in Section 2 was used.

The parameters of the BPSK DS-SS signal $s(t)$ were: $U=1$ and $\omega_0=2\pi F_S/4$, where F_S stands for sampling frequency. As the spreading code of chip duration $T_C=4/F_S$, the Gold sequence with the period $P=7$ chips was applied. The bit duration of information baseband signal was set to $T=28/F_S$.

As the AWGN channel model, the linear time-invariant system represented by the FIR filter of the 15-th order was used. The filter pass-band was centralized at the carrier frequency ω_0 and its bandwidth was set to $1.2B_M$ where B_M is the minimum bandwidth for the BPSK DS-SS signal transmission. The power spectral density of AWGN at the receiver input was set in such a way as $E_b/N_0=13\text{ dB}$.

In the receiver, time-invariant WF and VF were applied. For their design the method described in the appendix was used. The details concerning WF and VF design can be found in [13], [14], too. In order to estimate the correlation and cross-correlation functions necessary for the WF and VF design, the training sequence consisting of 200 information bits was transmitted before each information date sequence transmission (i.e. the parameter L took value $200 \times 28 = 5600$ samples). The original training sequence was available at the receivers for the purpose of filter design. In all experiments, perfect synchronization of the BPSK modulator and demodulator and the DS-SS modulator and MF is assumed. For the purpose of receiver performance property evaluation, the date stream consisting of 10^6 information bits (not including a training sequence) was used.

As the performance indices of the tested DS-SS receivers, signal to noise ratio at the MF input (SNR_M) vs. SIR due to wideband interference (SIR_W), SNR_M vs.


 Fig. 3. BER vs. SIR_W .

SIR due to narrowband interference (SIR_N), BER vs. SIR_W and BER vs. SIR_N were used, where

$$SNR_M = 20 \cdot \log \frac{E[\hat{s}^2(n)]}{E[(w(n) - \hat{s}(n))^2]} \quad (7)$$

$$\hat{s}(n) = \hat{s}(nT_C) = U c(nT_C) d(nT_C) \quad (8)$$

$$SIR_W = 20 \cdot \log \frac{E[s^2(t)]}{E[i_W^2(t)]} \quad (9)$$

$$SIR_N = 20 \cdot \log \frac{E[s^2(t)]}{E[i_N^2(t)]} \quad (10)$$

$w(n)$ applied in (7) represents the input signal to the MF (Fig. 1). The specific conditions concerning the particular experiments as well as the obtained results are described in the next subsections.

A. Wideband Interference

In the first set of experiments, the interference $i(t)=i_W(t)$ was synchronized with the BPSK DS-SS signal. Thus, the parameters of $i_W(t)$ were: $U_S=1$, $f_1^W=0$, $\tau=0$ and $\theta=0$. The bit duration of the baseband interference signal was set to $T_I=T_C$. Therefore, the BPSK DS-SS signal bandwidth (B_S) to interference signal bandwidth (B_I) ratio was $B_S/B_I=1$.

The results given by SNR_M vs. SIR_W and BER vs. SIR_W can be found in Figs. 2 and 3, for the MF, $WF(N)$ ($N=5,7$), $VF(2, N)$ ($N=2,5$) and $VF(3, N)$ ($N=1,3,5$). It can be seen from Fig. 2 that the level of SNR_M can be improved by the application of $VF(3, N)$ for $N=1,2,3$. Based on this improvement, the best results expressed by BER are provided by the $VF(3, N)$ receivers. It can be seen from these figures that for $SIR_W \in <-35\text{ dB}, -3\text{ dB}>$ the $VF(3,3)$ and $VF(3,5)$ provide the best and similar results.

It can be seen from Fig. 3 that BER become better in $VF(3,3)$ and $VF(3,5)$ as SIR_W become smaller (for $SIR_W \in <-20\text{ dB}, -3\text{ dB}>$). The same effect was observed in [6]. It follows from the fact that the similar effect can be observed for SNR_M (SNR_M become better in $VF(3,3)$ and $VF(3,5)$ as SIR_W become smaller for $SIR_W \in <-20\text{ dB}, -3\text{ dB}>$). Because the behavior of

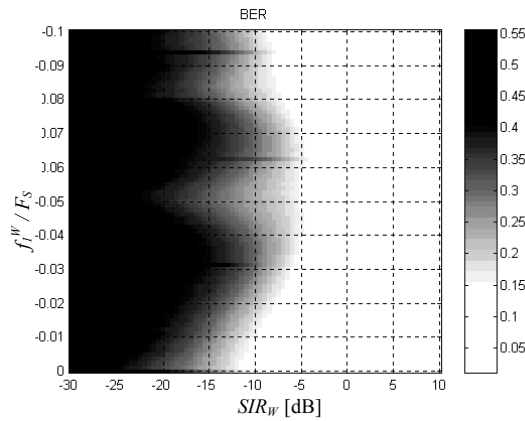


Fig. 4. BER vs. SIR_W and f_1^W . No estimator is applied.

SIR_W and SNR_M are similar, the both effects have the origin in VF properties. This behavior of the VF can be explained as follows.

The noise at the VF input consists of two components: the AWGN and the wideband interference. In the experiment, the level of the AWGN is constant ($E_b/N_0 = 13\text{ dB}$). If the power of the wideband interference is much higher than the AWGN level (for $SIR_W \in \langle -35\text{ dB}, -20\text{ dB} \rangle$), then the dominated component of the noise is given by the wideband interference. This kind of interference can be suppressed by the VF(3, N) with success and SNR_M is approximately constant. On the other hand, as SIR_W is increased at the constant level of AWGN, the wideband interference is no longer dominated component of the noise. The AWGN cannot be suppressed by a higher-order Volterra kernel application. However, the influence of AWGN to SNR_M can be decreased by pass-band filtering of the input signal of the VF. This is done by the linear part of the VF. Therefore, decreasing SNR_M (increasing BER) for the $SIR_W \in \langle -20\text{ dB}, -3\text{ dB} \rangle$ can be explained as the field of change of the dominated mechanism of noise suppression. As the received wideband interference is almost completely suppressed by the VF, receiver performance is almost independent of the input interference power. Therefore, for $SIR_W \in \langle -3\text{ dB}, 10\text{ dB} \rangle$ all tested receivers provide almost the same results.

These results expressed by BER , show that the VF(3, N) clearly outperform MF, WF and VF(2, N) in wideband interference suppression process, thus enabling the DS-SS signal reception even in the presence of the strong co-channel interference. This gives the opportunity of co-existence of classical wideband communication systems and DS-SS systems in the same frequency band.

It follows from the interference model (4) that BER will be also dependent on the offset carrier frequency f_1^W . In order to follow the dependence BER on SIR_W and f_1^W , the above described experiment was repeated for the receiver based on the MF and VF(3,3) application and for different values of $f_1^W \in \langle -0.1F_S, 0 \rangle$. The results of this set of computer experiments are presented in Figs. 4 and 5. It can be seen from these figures, that also in this case the VF(3,3) receiver clearly outperforms the MF receiver the MF receiver.

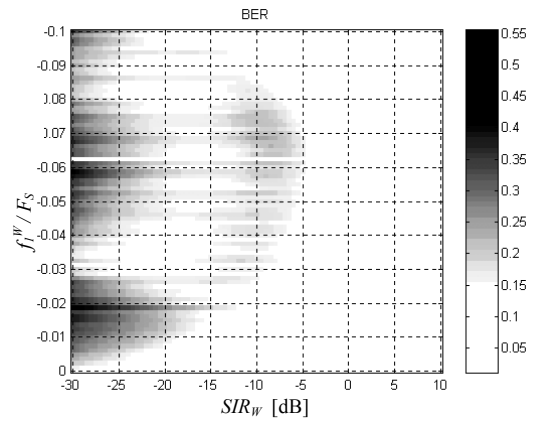


Fig. 5. BER vs. SIR_W and f_1^W . VF(3,3) is applied.

B. Narrowband Interference

In the experiments illustrating the narrowband interference suppression, the interference $i(t) = i_N(t)$ was set to the AR(2) given by (5). The magnitudes of complex conjugate poles p_1 and p_2 of the AR(2) filter were set to 0.99. Their arguments (resonance frequencies) were $\pm\omega_0 \pm 2\pi f_1^N$, respectively.

The experiment results for f_1^N represented by the performance indices SNR_M vs. SIR_N and BER vs. SIR_N can be found in Figs. 6 and 7, for the MF, WF(N) ($N=5,7$), VF(2, N) ($N=3,5$) and VF(3, N) ($N=1,3,5$). It can be seen from these figures, that for $SIR_N \in \langle -35\text{ dB}, 3\text{ dB} \rangle$ the best results (BER) are provided by the VF(3,5) application. On the other hand, for $SIR_N \in \langle 3\text{ dB}, 10\text{ dB} \rangle$ the best results especially with respect to BER vs. SIR_N are provided by the MF receiver.

The behavior of the DS-SS receiver can be explained by the fact that the interference level due to the AWGN at $SIR_N \in \langle 3\text{ dB}, 10\text{ dB} \rangle$ is much higher than that of SIR_N and hence, the interfering signals are almost Gaussian. Then, the performance of the conventional MF receiver is almost optimal and it can outperform non-linear receivers.

As was mentioned in the previous subsection, BER will be dependent on the offset carrier frequency f_1^N . In order to follow BER vs. SIR_N and f_1^N , the above described experiment was repeated for the MF and VF(3,3) application and for different values of $f_1^N \in \langle -0.1F_S, 0 \rangle$. The results of these computer experiments are presented in Figs. 8 and 9. It can be seen from these figures, that similar to the previous subsection, the receiver based on the VF(3,3) application can provide better results than that of the MF receiver.

C. Joined Wideband and Narrowband Interference

This subsection illustrates results obtained for the joined wideband and narrowband interference suppression. Now, the interference $i(t)$ has been set to $i_W(t) + i_N(t)$, where the interferences $i_W(t)$ and $i_N(t)$ were described in the previous subsections. The computer experiments have also been arranged in accordance with the previous subsections. The results of this computer experiment for the MF and VF(3,3) receivers represented by BER vs. SIR_W and SIR_N at zero offsets of interference carrier/resonance frequencies can be found in Figs. 10 and 11. It can be

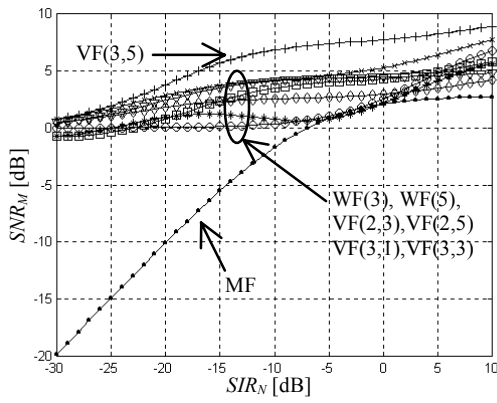


Fig. 6. SNR_M vs. SIR_N

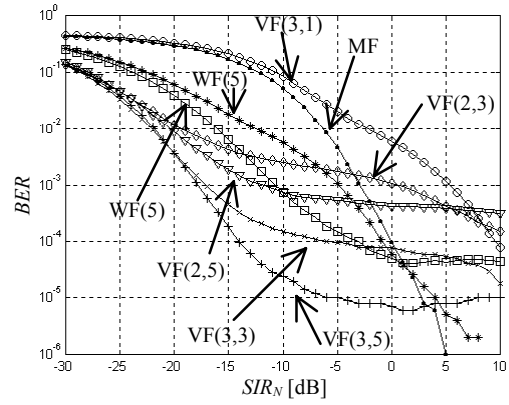


Fig. 7. BER vs. SIR_N

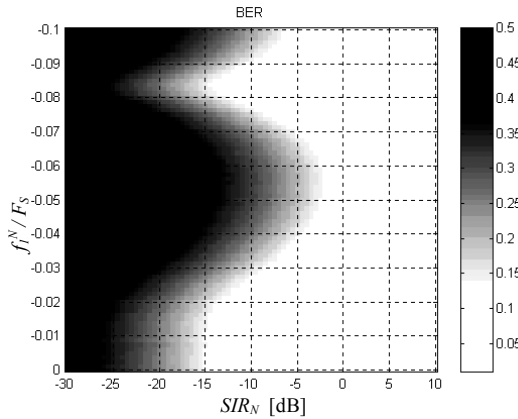


Fig. 8. BER vs. SIR_N and f_1^N . No estimator is applied.

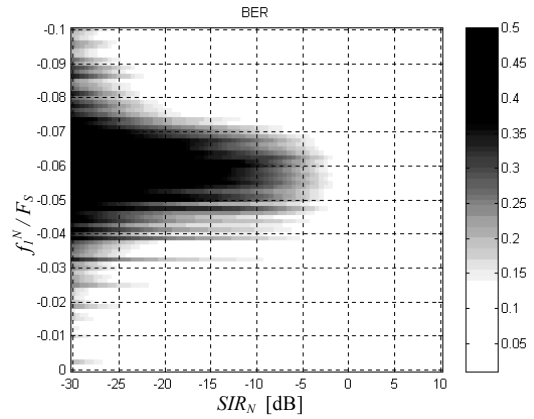


Fig. 9. BER vs. SIR_N and f_1^N . VF(3,3) is applied.

observed from these figures that for the tested levels of SIR_W and SIR_N , the VF(3,3) receiver clearly outperforms the MF based receivers.

The behavior of the DS-SS receiver can be explained by the fact that the total level of joined interference through this region is much more higher than that of AWGN, and hence, the interfering signals are non-Gaussian. Then, the MF performance is no longer optimal for interference suppression and non-linear the VF(3,3) based receiver is able to provide better performance.

As was mentioned in the previous subsections, BER depends on the interference carrier frequency offsets. In order to illustrate the receiver performance properties at the different values of the frequency offsets, the previous computer experiments were carried out for $f_1^W \in \langle -0.1F_S, 0 \rangle$, and $f_1^N \in \langle -0.1F_S, 0 \rangle$ at $SIR_W = SIR_N = -25\text{ dB}$.

The results of these computer experiments are presented in Figs 12 and 13. These results illustrate that the VF(3,3) receiver possesses a higher capability to suppress the joined interference than that of the MF receiver.

VI. CONCLUSIONS

In this paper, the BPSK DS-SS receivers based on MF, WF and VF have been described. The results obtained by the VF(3, N) receiver were compared against the MF, WF(N) and VF(2, N) receivers. It follows from these results that under conditions of a high level of interference (wideband, narrowband or joined wideband and narrowband), the receiver based on VF(3, N) performed better than that based on the MF or WF application. This behavior can be explained by the fact, that the high-level

interference signals are highly structured and essentially non-Gaussian. Then, the MF receiver is no longer optimal and non-linear VF(3, N) receiver can provide better performance properties. Here, we would like to stress that this successful application of the VF has the origin not in the channel properties but in the character of the interference and DS-SS signals.

It follows from the VF theory that the higher order correlation functions (up to the 6-th order) and cross-correlation functions (up to the 4-th order) are applied for the VF(3, N) design [15]. On the other hand, the second order correlation and cross-correlation functions are applied at the WF design. It follows from these facts that the improvement of SNR_M and BER obtained in the case of VF(3, N) applications is based on “information” available in the 6-th order correlation and the 4-th order cross-correlation functions of signals to be processed. Of course, the above-mentioned improvements of SNR_M and BER are reached at the cost of higher computational complexity of VF(3, N) in comparison with that of the MF, VF(2, N) or WF.

The obtained results have also shown that for the purpose of the interference suppression the only VF of the odd order can be applied with success. On the other hand, the 2-nd order VFs are not able to provide any meaningful results. The explanation of this effect can be found in the analysis of the 4-th order correlation function of DS-SS signals [4].

It is well known that the high computational complexity of the VF due to its high memory span is usually the most frequent obstacle in many VF applications. However, it can be seen from the presented computer experiments that the

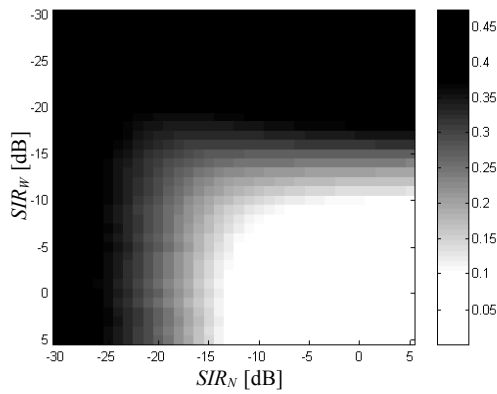


Fig. 10. BER vs. SIR_W and SIR_N . No estimator is applied.

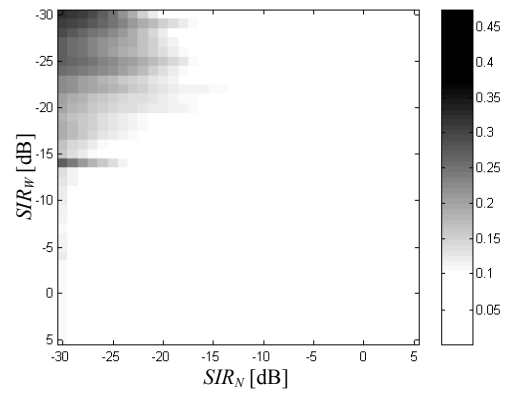


Fig. 11. BER vs. SIR_W and SIR_N . VF(3,3) is applied.

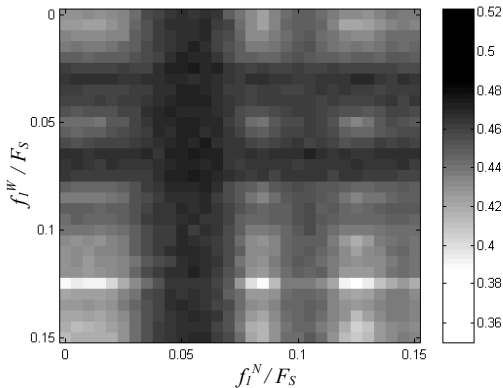


Fig. 12. BER vs. SIR_W and SIR_N . $SIR_W = -25$ dB, $SIR_N = -25$ dB. No estimator is applied.

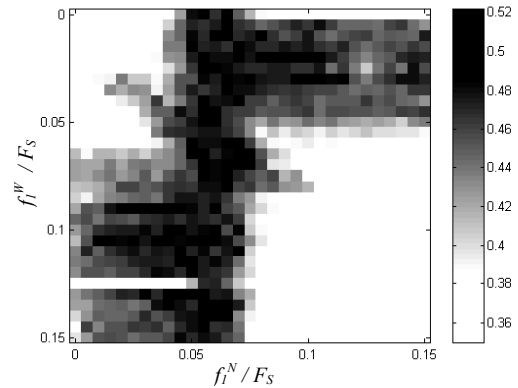


Fig. 13. BER vs. SIR_W and SIR_N . $SIR_W = -25$ dB, $SIR_N = -25$ dB. VF(3,3) is applied.

proposed application of the VFs does not require their very high memory span (e.g. only $N = 1, 3$). Therefore, the complexity of the DS-SS receiver equipped by the VF can be acceptable. Taking into account this fact and the results presented in Section 5, it can be concluded that the VFs are the promising tool for the wideband and narrowband interference suppression in DS-SS communication systems.

The computer experiments presented in this paper should demonstrate the ability of the VF to suppress interference of different kinds. Interference rejection techniques often need to be adaptive because of the dynamic nature of the interference or the channel. Therefore, the above-mentioned VFs should be applied in adaptive form and by methods providing the computational complexity reduction. Here, we would like to stress that for the computer experiments presented in this paper, the application of the adaptive VFs will not provide any improvement in the field of SNR_M and BER . It follows from the fact that the VFs applied in the experiment are the optimum once.

In the adaptive VF theory, a number of adaptive algorithms for adaptive VFs can be found (e.g. [14]). The well-known LMS algorithm is not suitable for the adaptive VFs training. The reason is the fact that the LMS algorithm application results in very slow rate of convergence due to a large eigenvalue spread of the autocorrelation matrix of the input signal of the adaptive VF. On the other hand, the RLS algorithm or fast RLS [14] can provide very fast convergence reached at the cost of extremely high computational complexity in comparison with that of the LMS algorithm. Therefore, it is necessary to look for an adaptive algorithm providing acceptable rate of

convergence at the acceptable computational complexity. Here, the XLS algorithm based on line search principle [16] or block conjugate gradient algorithm (BCG) [17] could be good candidate for that application. The discussion of the XLS or BCG algorithm properties is beyond this paper. However, we believe that their application in DS-SS receiver is a topic for the next research.

APPENDIX

TIME-INVARIANT VOLTERRA FILTER DESIGN

In this appendix, an algorithm of the time-invariant VF design will be derived. Firstly, the modified mathematical model of the VF based on vector approach will be introduced. Then, by using the developed model, the procedure of the time-invariant VF design will be proposed. At the end of this section, a summary of the time-invariant VF design procedure with the application for the wideband and narrowband interference suppression in DS-SS communication systems will be given.

A. Modified Mathematical Model of Volterra Filters

The mathematical model of the VF of the M -th order memory span of which is $N_0 = 2N + 1$ samples long is given by

$$w(n) = h_0 + \sum_{i=1}^M \sum_{k_1=-N}^N \sum_{k_2=-N}^N \dots \sum_{k_i=-N}^N h_{i,k_1 k_2 \dots k_i} v(n-k_1) v(n-k_2) \dots v(n-k_i) \quad . \quad (A.1)$$

In this expression $v(n)$ and $w(n)$ are the input signal and

the filter response, respectively.

Let \mathbf{H}_i is a vector containing elements of the Volterra kernel of the i -th order arranged in accordance with their indices k_1, k_2, \dots, k_i in a lexicographical way. E.g. \mathbf{H}_2 for $N=1$ is given by

$$\mathbf{H}_2 = [h_{2_{-1-1}} \quad h_{2_{-1-0}} \quad h_{2_{0-1}} \quad h_{2_{00}} \quad h_{2_{01}} \quad h_{2_{10}} \quad h_{2_{11}}]^T$$

Next, let us define a vector $\mathbf{V}_i(n)$ containing input signal sample products of the i -th order of the VF(M, N) defined by

$$v(n, i, k_1, k_2, \dots, k_i) = v(n-k_1)v(n-k_2)\dots v(n-k_i) \quad (\text{A.2})$$

arranged in accordance with their indices k_1, k_2, \dots, k_i in a lexicographical way. E.g. $\mathbf{V}_2(n)$ for $N=1$ is given by

$$\mathbf{V}_2(n) = [x^2(n+1) \quad x(n+1)x(n) \quad x(n)x(n+1) \quad x^2(n) \\ x(n)x(n-1) \quad x(n-1)x(n) \quad x^2(n-1)]^T$$

Then, by using the vectors \mathbf{H}_i and $\mathbf{V}_i(n)$ for $i=1, 2, \dots, M$ defined above, the expression (A.1) can be rewritten as

$$w(n) = h_0 + [\mathbf{H}_1^T \quad \mathbf{H}_2^T \quad \dots \quad \mathbf{H}_M^T]^T [\mathbf{V}_1^T(n) \quad \mathbf{V}_2^T(n) \quad \dots \quad \mathbf{V}_M^T(n)] \quad (\text{A.3})$$

The vectors \mathbf{H}_i and $\mathbf{V}_i(n)$ for $i=1, 2, \dots, M$ and the constant terms 1 and h_0 can be joined into the block vectors \mathbf{H} and $\mathbf{V}(n)$ as follows

$$\mathbf{H} = [h_0 \quad \mathbf{H}_1^T \quad \mathbf{H}_2^T \quad \dots \quad \mathbf{H}_M^T]^T \quad (\text{A.4})$$

$$\mathbf{V}(n) = [1 \quad \mathbf{V}_1^T(n) \quad \mathbf{V}_2^T(n) \quad \dots \quad \mathbf{V}_M^T(n)]^T \quad (\text{A.5})$$

Now, we can see from the definitions \mathbf{H}_i , $\mathbf{V}_i(n)$, \mathbf{H} and $\mathbf{V}(n)$ and by using (A.1)-(A.5), that the VF(M, N) output can be given by

$$w(n) = \mathbf{H}^T \mathbf{V}(n) = \mathbf{V}^T(n) \mathbf{H} \quad (\text{A.6})$$

The expression (A.6) represents the modified mathematical model of the VF(M, N) based on the vector approach. It can be seen from (A.6) that the response of the VF is still a linear function with respect to the VF coefficients although the VF is a nonlinear filter. With regard to that fact, a linear system theory can be used for the optimization and analysis of the VFs.

B. Time-Invariant Volterra Filter Design

Let us assume, that the input signal $v(n)$ and the desired signal $w_d(n)$ of the VF are stationary random processes. As was mentioned above, the VF is minimum mean-square non-linear estimators. Therefore, at the design of the optimum VF, the coefficients of the optimum VF minimizing the mean-square error (MSE) between the desired signal $w_d(n)$ and the VF output $w(n)$ have to be found. Then, the optimum VF coefficients are obtained as the solution that minimizes

$$MSE = E[e^2(n)] = E[(w_d(n) - w(n))^2], \quad (\text{A.7})$$

where $e(n)$ is the estimation error given by

$$e(n) = w_d(n) - w(n) \quad (\text{A.8})$$

By using (A.6) and (A.7), the MSE can be expressed in the following forms:

$$MSE = E\left[\left[w_d(n) - \mathbf{H}^T \mathbf{V}(n)\right]^2\right] = E\left[w_d^2(n)\right] - 2\mathbf{H}^T E[w(n)\mathbf{V}(n)] + \mathbf{H}^T E[\mathbf{V}(n)\mathbf{V}^T(n)] \mathbf{H} \quad (\text{A.9})$$

i.e.

$$MSE = \sigma_w^2 - 2\mathbf{H}^T \mathbf{P} + \mathbf{H}^T \mathbf{R} \mathbf{H} \quad (\text{A.10})$$

where the symbols σ_w^2 , \mathbf{P} and \mathbf{R} have the following meanings.

σ_w^2 is the mean square value of $w_d(n)$ defined as

$$\sigma_w^2 = E[w_d^2(n)] \quad (\text{A.11})$$

\mathbf{P} is cross-correlation vector given by expression

$$\mathbf{P} = E[w_d(n)\mathbf{V}(n)] \quad (\text{A.12})$$

Its elements are represented by the samples of the cross-correlation functions of the desired signal $w_d(n)$ and input signal sample product given by (A.2). The samples of the cross-correlation functions are defined as follows

$$P(n, k_1, k_2, \dots, k_i) = E[w_d(n)v(n, i, k_1, k_2, \dots, k_i)] = E[w_d(n)v(n-k_1)v(n-k_2)\dots v(n-k_i)] \quad (\text{A.13})$$

for $i=1, 2, \dots, M$ and $k_i = -N, -N+1, \dots, N-1, N$. The cross-correlation functions given by (A.13) for $i > 1$ are known as the higher-order cross-correlation function [15]. If the signals $w_d(n)$ and $v(n)$ are stationary, it can be shown that the samples of the cross-correlation functions can be estimated by using the following formula

$$P(n, k_1, k_2, \dots, k_i) = P(k_1, k_2, \dots, k_i) = \frac{1}{L} \sum_{i=L_1}^{L_2} w_d(i)v(i-k_1)v(i-k_2)\dots v(i-k_i) \quad (\text{A.14})$$

where $L = L_2 - L_1 + 1$ is the number of the samples $w_d(n)$ and $v(n)$ applied for $P(k_1, k_2, \dots, k_i)$ estimation.

\mathbf{R} is correlation matrix given by the expression

$$\mathbf{R} = E[\mathbf{V}(n)\mathbf{V}^T(n)] \quad (\text{A.15})$$

The matrix \mathbf{R} is symmetric and semi-definite. Its elements are given by the samples of the correlation functions of the input signal sample products $v(n, i, k_1, k_2, \dots, k_i)$ given by (A.2). These correlation functions are defined as follows:

$$R(n, k_1, \dots, k_i, l_1, \dots, l_j) = E[v(n, i, k_1, \dots, k_i)v(n, j, l_1, \dots, l_j)] = E[v(n-k_1)\dots v(n-k_i)v(n-l_1)\dots v(n-l_j)] \quad (\text{A.16})$$

for $i=1, 2, \dots, M$, $k_i = -N, -N+1, \dots, N-1, N$ and $l_j = -N, -N+1, \dots, N-1, N$. The correlation functions given by (A.16) for $i > 1$ and $j > 1$ are known as the higher-order correlation function [15]. If the signal $v(n)$ is stationary, it can be shown that the samples of the correlation functions can be estimated by using the

following formula

$$R(n, k_1, \dots, k_i, l_1, \dots, l_j) = R(k_1, \dots, k_i, l_1, \dots, l_j) = \frac{1}{L} \sum_{i=L_1}^{L_2} v(i-k_1)v(i-k_2)\dots v(i-k_i)v(i-l_1)v(i-l_2)\dots v(i-l_j) \quad (\text{A.17})$$

where $L = L_2 - L_1 + 1$ is the number of the samples $v(n)$ applied for $R(k_1, \dots, k_i, l_1, \dots, l_j)$ estimation.

It is well-known, that the more samples of $w_d(n)$ and $v(n)$ will be used for the estimation of $P(k_1, k_2, \dots, k_i)$ and $R(k_1, \dots, k_i, l_1, \dots, l_j)$ the better estimation will be obtained.

Now, the problem of an optimum VF design under consideration is that of finding an optimum vector \mathbf{H}^* minimizing the cost function MSE given by (A.10). Because \mathbf{R} is a semi-definite matrix, MSE is a quadratic and a convex function of the VF coefficients. Therefore, MSE has only one extreme corresponding to a global minimum. Then, the optimum vector \mathbf{H}^* is a solution of the equation

$$\text{grad}_{\mathbf{H}} MSE = -2\mathbf{P} + 2\mathbf{R}\mathbf{H}^* = 0 \quad (\text{A.18})$$

The optimum vector \mathbf{H}^* has to satisfy the condition

$$\mathbf{R}\mathbf{H}^* = \mathbf{P} \quad (\text{A.19})$$

The expression (A.19) represents the system of the linear algebraic equations. Under the condition that the matrix \mathbf{R} is regular, the coefficient vector \mathbf{H}^* of the optimum VF can be computed by using the expression

$$\mathbf{H}^* = \mathbf{R}^{-1}\mathbf{P} \quad (\text{A.20})$$

The minimum mean-square error corresponding to the optimum VF (MSE_{OPT}) can be obtained by the substitution of (A.20) into (A.10) in this form

$$MSE_{OPT} = \sigma_w^2 - \mathbf{P}^T \mathbf{R}^{-1} \mathbf{P} \quad (\text{A.21})$$

C. Summary of Time-Invariant VF Design Procedure

In order to design the optimum time-invariant VF with the application for the wideband and narrowband interference suppression in DS-SS communication systems the following steps have to be done.

1) $P(k_1, k_2, \dots, k_i)$ and $R(k_1, \dots, k_i, l_1, \dots, l_j)$

Estimation

In order to estimate the cross-correlation and correlation functions, the training sequence is transmitted before each information data sequence transmission. The signal $w_d(n)$ representing the sampled version of the training sequence is transmitted from the transmitter through transmission channel to the receiver. The signal $v(n)$ is the sampled version of the channel output to the signal $w_d(n)$. $w_d(n)$ and $v(n)$ are available in the receiver. The number of the samples of these signals L has to satisfy the condition $L \gg 2N + 1$. Then, $P(k_1, k_2, \dots, k_i)$ and $R(k_1, \dots, k_i, l_1, \dots, l_j)$ will be estimated by using (A.14) and (A.17).

2) *Composing of the Equation Set (A.19)*

By using (A.12), (A.15), $P(k_1, k_2, \dots, k_i)$ and $R(k_1, \dots, k_i, l_1, \dots, l_j)$ the equation set (A.19) will be composed.

3) *Solution of the Equation Set (A.19)*

For that purpose, a suitable method of solution of the linear algebraic equation set can be used.

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REFERENCES

- [1] J. D. Laster and J. R. Reed, "Interference rejection in digital wireless communications," *IEEE Signal Processing Magazine*, vol. 14, no. 3, pp. 37-62, May 1997.
- [2] S. Glisic and B. Vucetic, *Spread Spectrum CDMA Systems for Wireless Communication*. Artech House, Inc., Noewood, 1997.
- [3] R. S. Dickson, *Spread Spectrum Systems with Commercial Applications*. John Wiley & Sons, 1994.
- [4] R. Tanner and D. G. M. Cruickshank, "Volterra filter structure for DS-CDMA," *The 4th IMA Conference*, 1996.
- [5] R. Tanner and D. G. M. Cruickshank, "Volterra based receivers for DS-CDMA," in *Proc. 8th IEEE Int. Symp. on Personal, Indoor and Mobile Radio Communications PIMRC '97*, vol. 3, pp. 1166-1170, Helsinki, Finland, Sep. 1997.
- [6] Z. Dobrosavljevic, M. Dukic and V. Zaljkovic, "Wideband interference suppression in DSSS receiver by Volterra filter," in *Proc. of IEEE ISSSTA '98*, vol. 1/3, pp. 127-129, South Africa, 1998.
- [7] A. L. Rusch and V. H. Poor, "Narrowband interference suppression in CDMA spread spectrum communications," *IEEE Trans. on Communications*, vol. 42, no. 4, pp. 1969-1979, Apr. 1994.
- [8] R. Tanner and D. G. M. Cruickshank, "RBF based receivers for DS-CDMA with reduced complexity," in *Proc. IEEE ISSSTA '98*, vol. 1/3, South Africa, 1998.
- [9] D. Kocur and R. Zetik, "Volterra filter application in DS-SS receiver for narrowband interference suppression," in *Proc. DSP-MCOM 2001, 5th Int. Scientific Conf. on Digital Signal Processing and Multimedia Communications*, pp. 34-37, Košice, Slovakia, Nov. 27-29, 2001, Mercury Smékal Publishing House, Košice, 2001.
- [10] R. Zetik and D. Kocur, "Non-linear filter applications in DS-SS spread spectrum receivers," in *Proc. 6th Int. Conf. on Engineering of Modern Electric Systems-EMES'01*, Oradea - Felix Spa, Romania, May 24-26, 2001.
- [11] N. Kalouptsidis and S. Theodoridis, Eds. *Adaptive System Identification and Signal Processing Algorithms*. Prentice-Hall International (UK) Limited, Hemel Hempstead, 1993.
- [12] S. Verdu, *Multiuser Detection*, Cambridge University Press, 1998.
- [13] D. Kocur, "Adaptive Volterra filters – a tutorial review," in *Proc. Workshop on DSP and Communications*, Oradea, 1995, pp. 1-33.
- [14] D. Kocur, *Adaptive Volterra Digital Filters* (in Slovak). Elfa s.r.o. Košice, 2001.
- [15] L. Nikias and P. Peropulu, *Higher-order Spectra Analysis: A Non-linear Signal Processing Framework*. PTR Prentice-Hall, Inc., 1993.
- [16] D. Kocur, "Line Search algorithm for the adaptive Volterra digital filters," in *Proc. of Int. Conf. on Digital Signal Processing' 93*, pp. 104-109, Herl'any, Slovakia, 1993.
- [17] D. Kocur and I. Hendel, "BCG adaptive algorithms for microstatistic Volterra filters," in *Proc. of 4th Int. TEMPUS TELECOMNET Workshop*, pp. 86-93, Barcelona, Spain, Jul. 7, 1998.

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