Electromagnetic Penetration Through Three Different Dielectric Regions Separated by Two Parallel Planes Perforated with Multiple Apertures

Karim Y. Kabalan, Ali El-Hajj, and Asaad Rayes

Abstract—**The electromagnetic coupling of incident plane wave through rectangular apertures perforating two parallel infinite conducting planes is analyzed using the characteristic modes theory based on the application of image theory and the equivalence principle. First, the integral equations for the parallel planes problem are established. Then, the integral equations are discretized into matrix equivalence. Next, the characteristic modes theory is applied to solve the matrix equations and to obtain the equivalent magnetic currents over the apertures surfaces in the planes. Finally, numerical results for the equivalent magnetic currents and for the output radiation pattern are given for the case of three apertures distributed over the two planes.**

Index Terms—**Aperture problems, characteristic mode theory, electromagnetic scattering and radiation, rectangular aperture.**

I. INTRODUCTION

THE COUPLING The coupling of electromagnetic fields between two or more isolated regions coupled with one or more apertures have been analyzed by many investigators [1]-[3]. In electromagnetic shielding, the problem of electromagnetic noise generated from electromagnetic equipment and microelectronic circuits with different power and frequency levels is becoming more critical. The treatment of parallel screens with multiple apertures is of practical interest to determine the reflection and transmission properties of infinite apertured screens. This analysis is useful for validation of screening effect where relevant simulation parameters may be modified to optimize simulation accuracy.

The method of moments [4] is one of the most developed techniques used in solving such electromagnetic problems. On the other hand, different numerical methods such as finite difference frequency domain (FD-FD) techniques [5], the control region with suitable absorbing boundary conditions (ABC) are said to be able to solve 2-D aperture coupling problems [6], [7], the trans-finite element method [8], and others.

The central theme of this paper is to apply the characteristic modes theory [9] to solve integral equations formulated in terms of the aperture surface electric fields

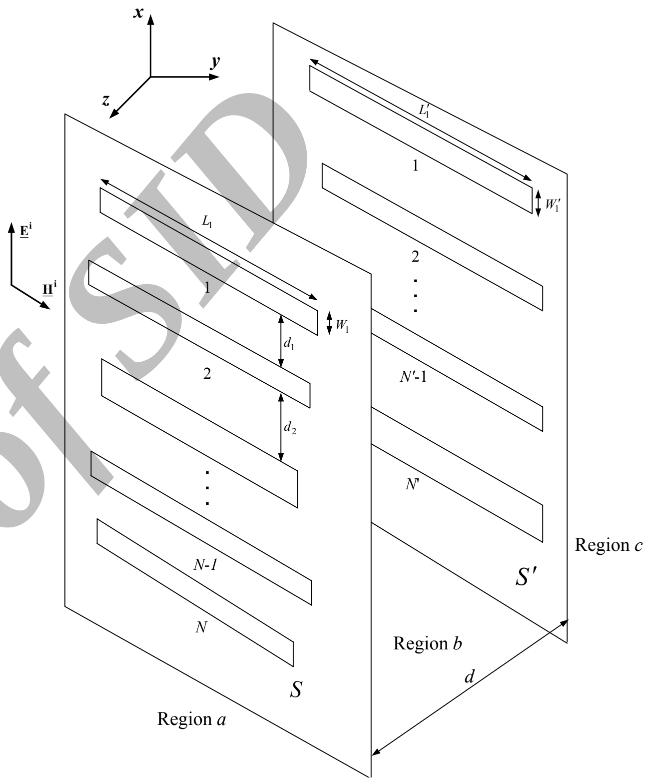

Fig. 1. The Geometry of the problem.

and the geometry of the problem. The equivalent apertures magnetic currents are expanded in terms of a set of orthogonal expansion functions. The moment Galerkin's method is applied to transform the integral functional equations into scalar matrix equations. The admittance matrices of 3-D apertures perforating two parallel infinite conducting planes are combined for all equivalent magnetic currents on every side of the closed apertures. The characteristic modes theory, which is applicable to simulate acoustic and electromagnetic wave propagations and to solve a large class of 3-D aperture problems [10], [11], is attempted to solve this problem. The objective is to obtain the self and mutual admittance of the apertures from which coupling is calculated. These fields are used to obtain radiation Pattern.

II. PROBLEM FORMULATION

Fig. 1 shows the geometry of the problem, in which a plane wave is incident on a conducting plane containing an array of N apertures coupling the outside region, region a, to region b which is formed by two parallel conducting

Manuscript received March 3, 2002; revised Novembber 19,2002.
The authors are with the Electrical and Computer Engineering Department, American University of Beirut, P. O. Box: 11-0236, Riad El Solh 1107 2020, Beirut, Lebanon. (e-mail: kabalan@aub.edu.lb, elhajj@aub.edu.lb, ar16@aub.edu.lb).
Publisher Item Identifier S 1682-0053(03)0143

planes. Region b is also coupled to the exterior region, region c, through another array of apertures in the conducting plane separating region b from region c. The three regions media are linear, homogeneous, isotropic, and dissipation free and are therefore characterized respectively by their real scalar permitivities ε_a, ε_b, and ε_c. The infinite conducting plane separating region a from region b is assumed to be infinitely thin and is called S. The infinite conducting plane separating region b from region c is also assumed to be infinitely thin and is denoted by S'. The width of the i-th aperture in S is W_i and the length is L_i, and the width of the j-th aperture in S' is W'_j and the length is L'_j. The distance between the apertures in S is $d_i (i = 1,2,3,...,N-1)$, and the distance between the apertures in S' is $d'_j (j = 1,2,3,...,N'-1)$. The distance between the two planes is denoted by d.

The equivalence principle [12] allows the use of equivalent magnetic current sheets over the first array of apertures and divide the problem of coupling region a to region b into two uncoupled parts. In region a, the excitation is transverse electric (TE) to the arrays axis and the exciting field has only a y-component of magnetic field given by:

$$\mathbf{H}_y^i = e^{-j\kappa_a (x\cos\theta + z\sin\theta)} \tag{1}$$

where $\kappa_a = \omega\sqrt{\varepsilon_a \mu_a}$ is the wavenumber of the medium of region a and θ is the angle that the propagation vector makes with the x-axis. The total field, incident and scattered, must have a zero electric field component tangential to the screen and continuous electric field components across the apertures surfaces. Another boundary condition enforces magnetic field continuity across the array of apertures.

The magnetic current sheets in region $\underline{M}^{(i)}$ are placed on the apertures areas in S just external to the closed plane conductors. The total magnetic field in region a is equal to the incident field in the presence of a complete conductor on S plus the field produced by the equivalent magnetic currents sheets $\underline{M}^{(i)}$ with the apertures covered by perfect electric conductors

$$\underline{M}^{(i)} = \underline{n} \times \underline{E}_a^{(i)} \qquad \mathrm{i} = 1,2,..., \mathrm{N} \tag{2}$$

where $\underline{E}_a^{(i)}$ is the total electric field in the i-th aperture defined for $z = 0$, $y_i - L_i/2 < y < y_i + L_i/2$, and $x_i - W_i/2 < x < x_i + W_i/2$. Also, $\underline{n}$ is the outward unit vector normal to the S plane, i.e., the i-th aperture. To ensure electric field continuity, $\underline{M}^{(i)}$ exists only on the i-th aperture to compensate for the aperture's electric field and vanishes over the rest of the surface of the conducting plane.

In region b, the equivalent problem of the conducting plane is obtained by placing magnetic currents sheet $-\underline{M}^{(i)}$ on each of the i-th aperture array in the S plane. As such, the excitation is then specified by the electric field due to $-\underline{M}^{(i)}$. Similarly, the magnetic currents $\underline{M}'^{(j)}$ are placed on the array of apertures areas in the S' plane just external to the closed plane conductors. The total magnetic field in region b is equal to the magnetic field due to $-\underline{M}^{(i)}$ plus the magnetic fields produced by the equivalent magnetic currents $\underline{M}'^{(j)}$ with the apertures in S and S' are closed by electric conductors.

$$\underline{M}'^{(j)} = \underline{n}' \times \underline{E}_b^{(j)} \qquad \mathrm{j} = 1,2,..., \mathrm{N}' \tag{3}$$

In (3), $\underline{E}_b^{(j)}$ is the electric field in the j-th aperture of S' defined for $z = s$, $y_j - L'_j/2 < y < y_j + L'_j/2$, and $x_j - W'_j/2 < x < x_j + W'_j/2$, and $\underline{n}'$ is the unit vector normal to the S' plane.

The equivalence of region c is given by attaching thin sheets of magnetic current $-\underline{M}'^{(j)}$ on every j-th aperture and by short circuiting the apertures.

Let $\underline{H}^a(\underline{M}^{(i)})$ denotes the electromagnetic field in region a due to the magnetic currents $\underline{M}^{(i)}$ and $\underline{H}^{sc}$ refers to the short-circuited incident magnetic field after closing the aperture array by perfect electric conductors. Hence, the total magnetic field in region a is given by:

$$\underline{H}^a = \underline{H}^{sc} + \sum_{i=1}^{N} \underline{H}_a(\underline{M}^{(i)}) . \tag{4}$$

Similarly, $\underline{H}^b(-\underline{M}^{(i)})$ and $\underline{H}^b(\underline{M}'^{(j)})$ denote the electromagnetic fields in region b due to $-\underline{M}^{(i)}$ and $\underline{M}'^{(j)}$, respectively. The total magnetic field in region b is then given by:

$$\begin{aligned} \underline{H}^b &= \sum_{i=1}^{N} \underline{H}_b(-\underline{M}^{(i)}) + \sum_{j=1}^{N'} \underline{H}_b(\underline{M}'^{(j)}) \\ &= -\sum_{i=1}^{N} \underline{H}_b(\underline{M}^{(i)}) + \sum_{j=1}^{N'} \underline{H}_b(\underline{M}'^{(j)}) \end{aligned} \tag{5}$$

In a similar manner, $\underline{H}^c(-\underline{M}'^{(j)})$ denotes the electromagnetic fields in region c due to $(-\underline{M}'^{(j)})$. The total magnetic field in region c is:

$$\underline{H}^c = \sum_{j=1}^{N'} \underline{H}_c(-\underline{M}'^{(j)}) = -\sum_{j=1}^{N'} \underline{H}_c(\underline{M}'^{(j)}) . \tag{6}$$

To satisfy the boundary condition on tangential H across the apertures region, equate the tangential components of $\underline{H}^a$ to those of $\underline{H}^b$ over the aperture regions (A_ℓ) of the aperture array in S and equate the tangential components of $\underline{H}^b$ to those of $\underline{H}^c$ over the aperture regions $(A'_{\ell'})$ of the aperture array in S'. This leads to equations (7) and (8) where the subscripts a, b, or c designates the media and subscript t refers to tangential components:

$$\begin{aligned} -\sum_{i=1}^{N} \underline{H}_{at}^{(\ell)}(\underline{M}^{(i)}) - \sum_{i=1}^{N} \underline{H}_{bt}^{(\ell)}(\underline{M}^{(i)}) \\ + \sum_{j=1}^{N'} \underline{H}_{bt}^{(\ell)}(\underline{M}'^{(j)}) = \underline{H}_t^{sc(\ell)} \quad \text{over } A_\ell \quad \ell = 1,2,...,N \end{aligned} \tag{7}$$

$$\begin{aligned} -\sum_{i=1}^{N} \underline{H}_{bt}^{(\ell')}(\underline{M}^{(i)}) + \sum_{j=1}^{N'} \underline{H}_{bt}^{(\ell')}(\underline{M}'^{(j)}) + \\ \sum_{j=1}^{N'} \underline{H}_{ct}^{(\ell')}(\underline{M}'^{(j)}) = 0 \quad \text{over } A'_{\ell'} \quad \ell' = 1,2,...,N' \end{aligned} \tag{8}$$

III. Characteristic Currents for the Apertures

The eigenvalue equations (7), (8) can be put in a linear admittance operator form:

$$\underline{Y}(\underline{M}) = \underline{I} \tag{9}$$

where

$$\underline{Y}(.) = \bigcup_{i=1}^{2} \sum_{j=1}^{2} \underline{Y}_{ij}(.) \tag{10}$$

with $\underline{Y}_{11}(.) = \bigcup_{\ell=1}^{N} \left(-\underline{H}_{at}^{(\ell)}(.) - \underline{H}_{bt}^{(\ell)}(.)\right)$,

$\underline{Y}_{12}(.) = \bigcup_{l=1}^{N} (\underline{H}_{bt}^{(l)}(.))$, $\underline{Y}_{21}(.) = \bigcup_{\ell'=1}^{N'} (\underline{H}_{bt}^{(\ell')}(.))$, and

$\underline{Y}_{22}(.) = \bigcup_{\ell'=1}^{N'} (-\underline{H}_{bt}^{(\ell')}(.) - \underline{H}_{ct}^{(\ell')}(.))$.

and

$$\underline{I} = \bigcup_{j=1}^{2} \underline{I}_j \tag{11}$$

with $\underline{I}_1 = \bigcup_{\ell=1}^{N} H_t^{sc(\ell)} = \bigcup_{\ell=1}^{N} I^{(\ell)}$ and $\underline{I}_2 = \bigcup_{\ell'=1}^{N'} 0 = 0$. In (9), define $\underline{G} = (\underline{Y} + \underline{Y}^*)/2$ and $\underline{B} = (\underline{Y} - \underline{Y}^*)/2j$, then

$$\underline{Y}(\underline{M}) = \underline{G}(\underline{M}) + j\underline{B}(\underline{M}) \tag{12}$$

where the asterisk denotes the complex conjugate of the operator. Following Harrington and Mautz [9], the characteristic currents for the apertures are defined to be the eigenfunctions M_n of the eigenfunction equation.

$$\underline{Y}(M_n) = y_n \underline{G}(M_n) \tag{13}$$

where the eigenfunctions M_n are defined as:

$$M_n = \left\{\bigcup_{i=1}^{N} M_n^{(i)}\right\} \bigcup \left\{\bigcup_{j=1}^{N'} M_n'^{(j)}\right\} \tag{14}$$

Put the eigenvalues y_n as:

$$y_n = 1 + jb_n \tag{15}$$

then

$$\underline{B}(M_n) = b_n \underline{G}(M_n) \tag{16}$$

It is to note that region a admittances are assumed to be dominant, so the effect of regions b and c is small compared to that of region a admittances. The operators $\underline{G}$ and $\underline{B}$ are self adjoint, whereas $\underline{G}$ is positive definite as well. It then follows that b_n and hence M_n are real and are chosen to satisfy the orthogonality relationships:

$$\begin{aligned} \langle M_m, G(M_n)\rangle &= \delta_{mn} \\ \langle M_m, B(M_n)\rangle &= b_n \delta_{mn} \\ \langle M_m, Y(M_n)\rangle &= y_n \delta_{mn} \end{aligned} \tag{17}$$

where δ_{mn} is the Kronecker delta function (0 if $m \neq n$ and 1 if $m = n$), and $\langle .,. \rangle$ denotes the inner product

$$\langle C, D\rangle = \sum_{k=1}^{N} \iint_{A_k} C^{(k)*} D^{(k)} ds + \sum_{l=1}^{N'} \iint_{A_l'} C^{(l)*} D^{(l)} ds \tag{18}$$

where $C = C^{(k)}$ and $D = D^{(k)}$ on the k-th aperture in S, $C = C^{(l)}$ and $D = D^{(l)}$ on the l-th aperture in S'. All the currents in the apertures are required to radiate some power, however, the characteristic currents corresponding to large b_n are basically non-radiating.

IV. Numerical Solution

An exact solution of the eigenvalue equation (16) for the eigenfunctions is rather difficult. Therefore, an approximate solution of the eigenvalue problem is attempted using Galerkin's method. The apertures of rectangular shapes are of considerable interest in such problems.

In this problem, each i-th aperture in S is subdivided uniformly into $L^{(i)}.N^{(i)}$ subareas of dimensions $\Delta x \Delta y$ where $N^{(i)}$ represents the number of discretizations of the aperture along the x direction and $L^{(i)}$ represents the number of discretizations of the aperture along the y direction. Similarly, each j-th aperture in S' is subdivided uniformly into $L'^{(j)}.N'^{(j)}$ subareas of dimensions $\Delta x \Delta y$ where $N'^{(j)}$ represents the number of discretizations of the aperture along the x direction and $L'^{(j)}$ represents the number of discretizations of the aperture along the y direction. The index i values range between 1 and N, and the index j values ranges between 1 and N'. Accordingly, the characteristic currents are split into two components along the surface of each aperture:

$$\begin{aligned} \underline{\mathrm{M}}_{\mathrm{n}} = &\left\{\bigcup_{\mathrm{k}=1}^{\mathrm{N}} \left(\sum_{\mathrm{i}=1}^{\mathrm{N}^{(\mathrm{k})}\mathrm{L}^{(\mathrm{k})}} \mathrm{U}_{\mathrm{nk}}^{(\mathrm{i})} \mathrm{f}_{\mathrm{k}}^{(\mathrm{i})} \underline{\mathrm{u}}_{\mathrm{x}} + \sum_{\mathrm{l}=1}^{\mathrm{L}^{(\mathrm{k})}\mathrm{N}^{(\mathrm{k})}} \mathrm{L}_{\mathrm{nk}}^{(\mathrm{l})} \mathrm{g}_{\mathrm{k}}^{(\mathrm{l})} \underline{\mathrm{u}}_{\mathrm{y}} \right)\right\} \\ &\bigcup \left\{\bigcup_{\mathrm{k}'=1}^{\mathrm{N}'} \left(\sum_{\mathrm{i}=1}^{\mathrm{N}'^{(\mathrm{k}')}\mathrm{L}'^{(\mathrm{k}')}} \mathrm{U}_{\mathrm{nk}'}'^{(\mathrm{i})} \mathrm{f}_{\mathrm{k}'}'^{(\mathrm{i})} \underline{\mathrm{u}}_{\mathrm{x}} + \sum_{\mathrm{l}=1}^{\mathrm{L}'^{(\mathrm{k}')}\mathrm{N}'^{(\mathrm{k}')}} \mathrm{L}_{\mathrm{nk}'}'^{(\mathrm{l})} \mathrm{g}_{\mathrm{k}'}'^{(\mathrm{l})} \underline{\mathrm{u}}_{\mathrm{y}} \right)\right\} \end{aligned} \tag{19}$$

In (19), the indices k and k' represent the number of apertures in S and S', respectively, $U_{mk}^{(i)}, L_{mk}^{(l)}, U_{mk'}'^{(i)}$ and $U_{mk'}'^{(l)}$ are the coefficients of the characteristic currents where the primes refers to the currents in the apertures in S'. Substituting (19) into (17), and using the linearity of the $\underline{\mathrm{G}}$ and $\underline{\mathrm{B}}$ operators together with the symmetric product, the integral equations are converted into matrix equations. A suitable choice is to pick testing functions identically the same as basis functions as well as satisfying the edge conditions on the apertures. The characteristic currents are approximated by a linear combination of the following functions:

$$\begin{aligned} f^{(k)}(x,y) &= T_u(x)P_v(y) \\ g^{(k)}(x,y) &= T_u(x)P_v(y) \\ f'^{(l)}(x,y) &= P_{u'}(x)T_{v'}(y) \\ g'^{(l)}(x,y) &= P_{u'}(x)T_{v'}(y) \end{aligned} \tag{20}$$

where $T(.)$ represents a triangular function, $P(.)$ represents a pulse function, u is the index of discretization along x-axis in S, v is the index of discretization along y-axis in S, u' is the index of discretization along x-axis in S', and v' is the index of discretization along y-axis in S'. The testing functions, W_n, are chosen as follows:

$$W_n = \left\{\bigcup_{k=1}^{N} \left(f_k^{(i)} u_x + g_k^{(l)} u_y\right)\right\} \bigcup \left\{\bigcup_{k'=1}^{N'} \left(f_{k'}'^{(i)} u_x + g_{k'}'^{(l)} u_y\right)\right\} \tag{21}$$

To construct a numerical solution, apply the symmetric product using the basis and testing functions. In order to arrive at a consistent set of linear equation, apply the following identity when there are differential operators in

TABLE I
THE CONVERGENCE OF THE CHARACTERISTIC VALUES FOR EXAMPLE 1

N	$\lvert B_0 \rvert$	$\lvert B_1 \rvert$	$\lvert B_2 \rvert$	$\lvert B_3 \rvert$	…	$\lvert B_N \rvert$
8	1.381583	1.882777	2.120752	2.542718	…	58609.152239
10	1.334005	1.833745	2.044572	2.474918	…	14243.777775
12	1.313263	1.812392	2.006496	2.445773	…	11233.049174
14	1.304449	1.803449	1.979432	2.433939	…	114590.595915

the kernel of integral equations.

$$\iint A \cdot \nabla_s B \mathrm{d}s = -\iint B \nabla \cdot A \mathrm{d}s \tag{22}$$

It should be noted that at the points where the observation point coincides with the source point, the kernel of the integral equation exhibits singular behavior. Consequently, analytical solutions must be exercised in evaluating integrals in the neighborhood of the integrable singularities. Substituting (19), (20), and (21) into (17) results in the matrix eigenvalue equation

$$[B] = b_n [G] \tag{23}$$

where

$$\vec{U}_n = \begin{bmatrix} \vec{U}_{n1} & \vec{L}_{n1} & \cdot & \cdot & \vec{U}_{nN} & \vec{L}_{nN} & \vec{U}'_{n1} & \vec{L}'_{n1} & \cdot & \cdot & \vec{U}'_{nN'} & \vec{L}'_{nN'} \end{bmatrix}^T \tag{24}$$

and

$$[B] = \begin{bmatrix}
[BUU_{11}] & [BUL_{11}] & \cdot\;\cdot & [BUU_{1N}] & [BUL_{1N}] & [BUU'_{11}] & [BUL'_{11}] & \cdot\;\cdot & [BUU'_{1N'}] & [BUL'_{1N'}] \\
[BLU_{11}] & [BLL_{11}] & \cdot\;\cdot & [BLU_{1N}] & [BLL_{1N}] & [BLU'_{11}] & [BLL'_{11}] & \cdot\;\cdot & [BUU'_{1N'}] & [BLL'_{1N'}] \\
\cdot & \cdot & \cdot\;\cdot & \cdot & \cdot & \cdot & \cdot & \cdot\;\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot\;\cdot & \cdot & \cdot & \cdot & \cdot & \cdot\;\cdot & \cdot & \cdot \\
[BUU_{N1}] & [BUL_{N1}] & \cdot\;\cdot & [BUU_{NN}] & [BUL_{NN}] & [BUU'_{N1}] & [BUL'_{N1}] & \cdot\;\cdot & [BUU'_{NN'}] & [BUL'_{NN'}] \\
[BLU_{N1}] & [BLL_{N1}] & \cdot\;\cdot & [BLU_{NN}] & [BLL_{NN}] & [BLU'_{N1}] & [BLL'_{N1}] & \cdot\;\cdot & [BUU'_{NN'}] & [BLL'_{NN'}] \\
[BU'U_{11}] & [BU'L_{11}] & \cdot\;\cdot & [BU'U_{1N}] & [BU'L_{1N}] & [BU'U'_{11}] & [BU'L'_{11}] & \cdot\;\cdot & [BU'U'_{1N'}] & [BU'L'_{1N'}] \\
[BL'U_{11}] & [BL'L_{11}] & \cdot\;\cdot & [BL'U_{1N}] & [BL'L_{1N}] & [BL'U'_{11}] & [BL'L'_{11}] & \cdot\;\cdot & [BL'U'_{1N'}] & [BL'L'_{1N'}] \\
\cdot & \cdot & \cdot\;\cdot & \cdot & \cdot & \cdot & \cdot & \cdot\;\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot\;\cdot & \cdot & \cdot & \cdot & \cdot & \cdot\;\cdot & \cdot & \cdot \\
[BU'U_{N'1}] & [BU'L_{N'1}] & \cdot\;\cdot & [BU'U_{N'N}] & [BU'L_{N'N}] & [BU'U'_{N'1}] & [BU'L'_{N'1}] & \cdot\;\cdot & [BU'U'_{N'N'}] & [BU'L'_{N'N'}] \\
[BL'U_{N'1}] & [BL'L_{N'1}] & \cdot\;\cdot & [BL'U_{N'N}] & [BL'L_{N'N}] & [BL'U'_{N'1}] & [BL'L'_{N'1}] & \cdot\;\cdot & [BL'U'_{N'N'}] & [BL'L'_{N'N'}]
\end{bmatrix} \tag{25}$$

In (25), U, U', L, L' correspond to discretization along, x-axis in S, x-axis in S', y-axis in S, and y-axis in S', respectively. For example, $[BUL'_{ij}]$ corresponds to a matrix of $N^{(i)} \times L'^{(j)}$ elements and the indices i and j represent respectively the location of the aperture in S for the x-axis discretization and the location of the aperture in S' for the y-axis discretization. A similar matrix representation may be obtained for $[G]$. Similarly, the vector matrix $\vec{U}_n$ elements has a similar explanation, for example the vector L_{ni} corresponds to a vector of $L^{(i)}$ elements.

V. EXAMPLES

Some examples are treated with different apertures lengths and widths. A computer program is developed to solve a problem where the number of apertures in S, $N = 1$ and the number of apertures in S', $N' = 2$ and to treat different cases where the apertures dimensions are varied as well as the locations of the apertures and the separation between the two parallel planes. In these examples, numerical results are presented to show the convergence of the algorithm and the characteristic modes with their corresponding equivalent magnetic currents over the apertures surfaces. Representative numerical results of the transmitted power as function of different geometrical parameters are also shown.

In these examples, the characteristic modes theory is used to express the eigenvalues table, the normalized characteristic modes and their corresponding equivalent magnetic currents in the apertures normalized to a maximum value and the resulting power transmitted. The apertures are assumed equal; the length of each aperture $L_1 = L'_1 = L'_2 = 0.6\lambda$ and the width of each aperture is $W_1 = W'_1 = W'_2 = 0.04\lambda$. The constitutive parameters are maximum value and the resulting power transmitted. The with their corresponding equivalent magnetic currents over apertures are assumed equal; the length of each aperture $L_1 = L'_1 = L'_2 = 0.6\lambda$ and the width of each aperture is $W_1 = W'_1 = W'_2 = 0.04\lambda$. The constitutive parameters are assumed to be real: permitivity of media a and c are $\varepsilon_a = \varepsilon_c = \varepsilon_0$, the permitivity of medium b is $\varepsilon_b = 2.56\varepsilon_0$, and the pemeabilities of media a, b, and c are $\mu_a = \mu_b = \mu_c = \mu_0$. The discretization along x-axis or y-axis for each aperture are $N^{(1)} = N'^{(1)} = N'^{(2)} = 10$ and $L^{(1)} = L'^{(1)} = L'^{(2)} = 1$.

Example 1: The first aperture is centered at $(0.22\lambda, 0.3\lambda, 0)$. The distance between the two planes is 0.4λ and the second and third apertures are located at $(0.22\lambda, 0.3\lambda, 0.4\lambda)$ and $(0.62\lambda, 0.3\lambda, 0.4\lambda)$, respectively.

Example 2: The first aperture is centered at $(0.22\lambda, 0.3\lambda, 0)$. The distance between the two planes is 0.4λ and the second and third apertures are located at $(0.62\lambda, 0.3\lambda, 0.4\lambda)$ and $(0.42\lambda, 0.3\lambda, 0.4\lambda)$, respectively.

Example 3: The first aperture is centered at $(0.22\lambda, 0.3\lambda, 0)$. The distance between the two planes is d and the second and third apertures are located at $(0.42\lambda, 0.3\lambda, d)$ and $(0.22\lambda, 0.3\lambda, d)$, respectively.

The tabulated eigenvalues of Example 1 are shown in Table I. The characteristic modes for each aperture in this

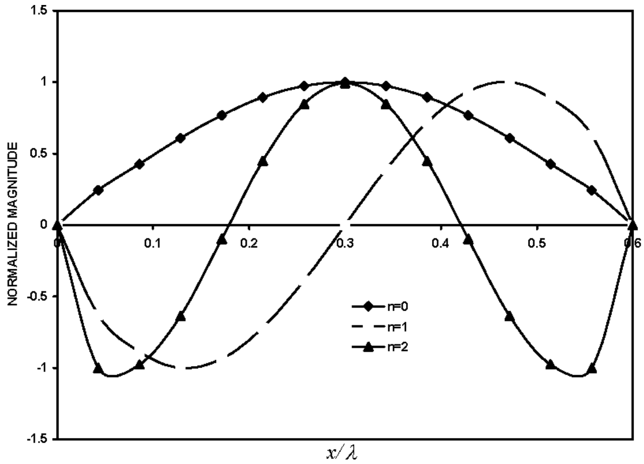

Fig. 2. The normalized characteristic modes for the aperture A_1 whose dimensions are $0.6\lambda \times 0.04\lambda$ centered at $(0.22\lambda, 0.3\lambda, 0)$ and the separation between the 2 planes is 0.4λ.

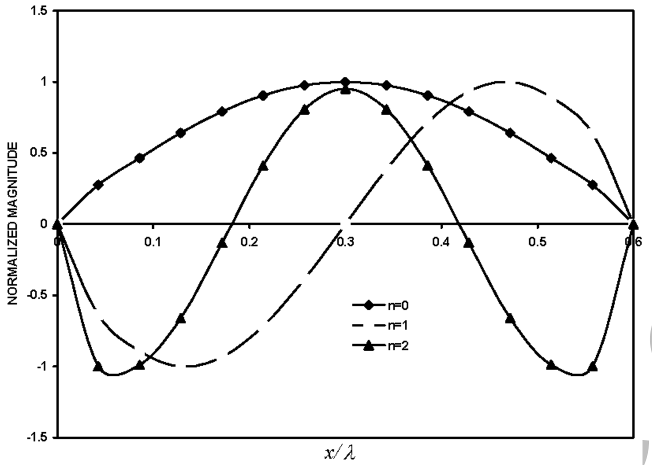

Fig. 3. The normalized characteristic modes for the aperture A_1' whose dimensions are $0.6\lambda \times 0.04\lambda$ centered at $(0.22\lambda, 0.3\lambda, 0.4\lambda)$.

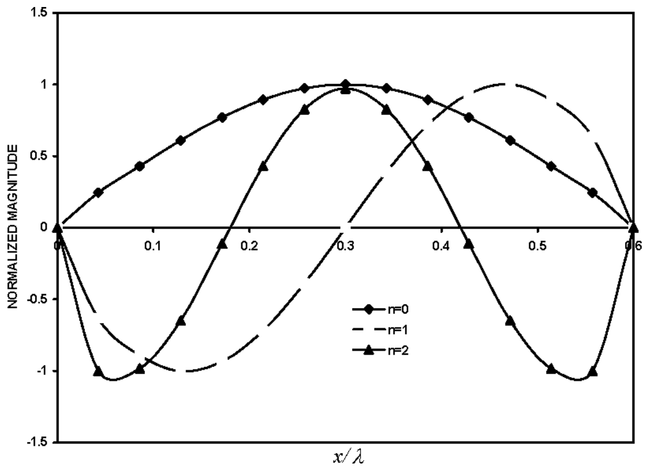

Fig. 4. The normalized characteristic modes for the aperture A_2' whose dimensions are $0.6\lambda \times 0.04\lambda$ centered at $(0.62\lambda, 0.3\lambda, 0.4\lambda)$.

example are shown in Figs. 2 to 4. For Example 2, the real and imaginary parts for the equivalent magnetic currents in each aperture are shown in Figs. 5 to 7. The transmitted power for Example 3, as a function of the distance between the two planes, is shown in Fig. 8.

It is clear from Table I, that the results show a trend toward convergence and that the convergence is faster for lower order modes approximating the solution of the problem. Usually, 7 modes show that an acceptable

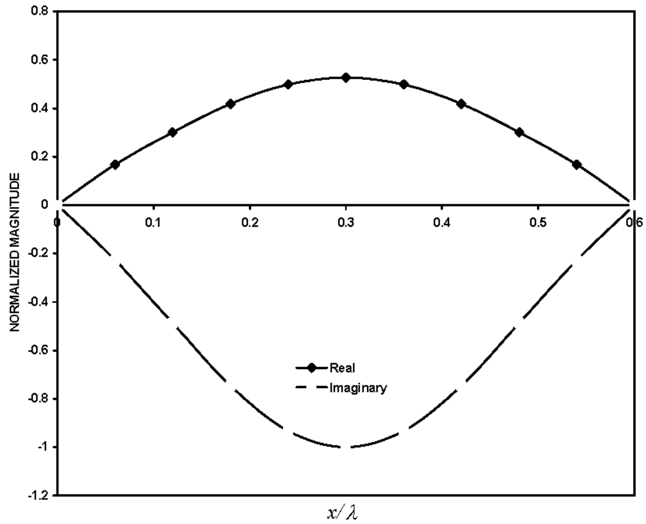

Fig. 5. The normalized real and imaginary parts of the equivalent magnetic current for the aperture A_1 whose dimensions are $0.6\lambda \times 0.04\lambda$ centered at $(0.22\lambda, 0.3\lambda, 0)$ and the separation between the 2 planes is 0.4λ.

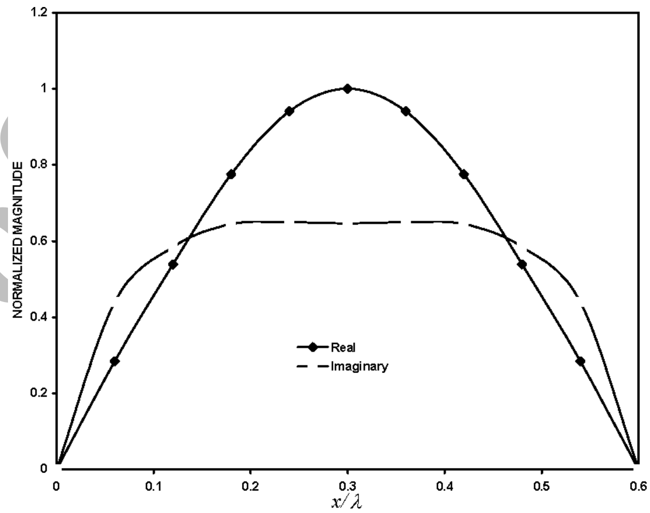

Fig. 6. The normalized real and imaginary parts of the equivalent magnetic current for the aperture A_1' whose dimensions are $0.6\lambda \times 0.04\lambda$ centered at $(0.62\lambda, 0.3\lambda, 0.4\lambda)$.

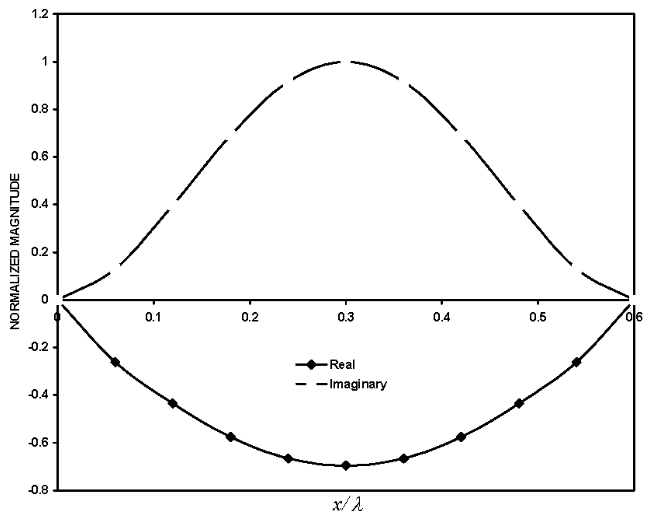

Fig. 7. The normalized real and imaginary parts of the equivalent magnetic current for the aperture A_2' whose dimensions are $0.6\lambda \times 0.04\lambda$ centered at $(0.42\lambda, 0.3\lambda, 0.4\lambda)$.

accurate solution may be reached. The eigenvalues of higher order modes have less contribution to the transmitted power than the lower order modes. The characteristic modes exhibit same characteristics due to the symmetry of the problem and the fact that the modes are independent of the excitation.

VI. Conclusion

A method based on the characteristic modes theory has been applied to solve the problem of electromagnetic radiation through three regions separated by two infinite parallel conducting planes and coupled through a number of apertures in each plane. The method can handle problems with different geometrical parameters. The analysis of the rectangular aperture of finite dimensions is similar to the situation of rectangular aperture coupling two regions separated by a perforated infinite conducting plane. The algorithms are applied efficiently to typical case situations. The convergence is proven for different geometrical parameters.

References

[1] H. A. Bethe, "Theory of diffraction by small holes," *Phys. Rev.*, vol. 66, pp. 163-182, Oct. 1944.

[2] C. J. Bouwkamp, "Theoretical and numerical treatment of diffraction through a circular aperture," *IEEE Trans. Antennas Propag.*, vol. 18, no. 2, pp. 152-176, Mar. 1970.

[3] C. M. Butler and D. R. Wilton, "General analysis of narrow strips and slots," *IEEE Trans. Antennas Propag.,* vol. 28, no. 1, pp. 42-48, Jan. 1980.

[4] R. F. Harrington, *Field Computation by Moment Methods*, Macmillan, New York, 1968. (Reprinted by Krieger, Melbourne, Fla., 1982)

[5] R. T. Ling, "A finite-difference frequency-domain approach to electromagnetic scattering problems," in *Proc. URSI Radio Sci. Meeting,* p. 250, 1987.

[6] G. Meltz, J. McCartin, and L. J. Bahrmasel, "Application of the control region approximation to electromagnetic scattering," in *Proc. URSI Radio Sci. Meeting*, p. 185, 1987.

[7] B. J. McCartin, G. Meltz, R. Mittra, and L. J. Bahrmasel, "Application of control region approximation in conjunction with absorbing boundary conditions to the direct solution of electromagnetic scattering problems," in *Proc. URSI Radio Sci. Meeting*, p. 14, 1988.

[8] J. F. Lee and Z. J. Cendes, "The transfinite element method for computing electromagnetic scattering from arbitrary lossy cylinders," *IEEE Antennas and Propagat. Soc. Int. Symp. Dig.*, pp. 99-102, 1987.

[9] R. F. Harrington and J. R. Mautz, "Characteristic modes for aperture problems", *IEEE Trans. Microwave Theory Tech.*, vol. 33, no. 6, pp. 500-505, Jun. 1985.

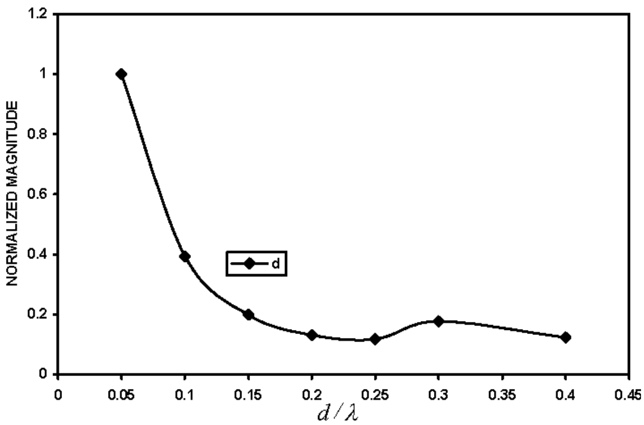

Fig. 8. The normalized transmitted power in region c of Example 3 due to a normal plane wave illuminating region a.

[10] A. El-Hajj, K. Y. Kabalan, and A. Rayes, "Characteristic Mode Formulation of Multiple Rectangular Aperture in a Conducting Plane with a Dielectric Filled Cavity," *IEEE Trans. Electromagnetic Compatibility*, vol. 40, no. 2, pp. 89-93, May 1998.

[11] A. El-Hajj, K. Y. Kabalan, and A. Rayes, "Electromagnetic Scattering and Penetration into a Filled Cavity Through an Aperture in a Conducting Plane," *Radio Science*, vol. 33, no. 5, pp. 1267-1275, Sep./Oct. 1998.

[12] R. F. Harrington, *Time Harmonic Electromagnetic Fields*, McGraw-Hill, New York, 1961.

Karim Y. Kabalan was born in Jbeil, Lebanon. He received the B.S. degree in Physics from the Lebanese University in 1979, and the M.S. and Ph.D. degrees in Electrical and Computer Engineering from Syracuse University, in 1983 and 1985, respectively. During the 1986 fall semester, he was a visiting Assistant Professor of Electrical and Computer Engineering at Syracuse University. Currently, he is a Professor of Electrical and Computer Engineering with the Electrical and Computer Engineering Department, Faculty of Engineering and Architecture, American University of Beirut. His research interests are numerical solution of electromagnetic field problems and software development.

Ali El-Hajj was born in Aramta, Lebanon, 1959. He received the Lisense degree in Physics from the Lebanese University, Lebanon in 1979, the degree of “Ingenieur” from L’Ecole Superieure d’Electricite, France in 1981, and the “Docteur Ingenieur” degree from the University of Rennes I, France in 1983. From 1983 to 1987, he was with the Electrical Engineering Department at the Lebanese University. In 1987, he joined the American University of Beirut where he is currently a Professor of Electrical and Computer Engineering. His research interests are numerical solution of electromagnetic field problems and engineering education.

Asaad Rayes was born in Himlaya, Lebanon, 1967. He received the B.E. and the M.E. degrees in Electrical Engineering from the American University of Beirut, Lebanon in 1989 and 1994, respectively. Since 1994, he has been working at the American University of Beirut as a Research Assistant. His research interests are numerical solution of electromagnetic field problems and engineering education.