The Use of Fractal Analysis for Characterizing Nonlinear Load Harmonics

Kingsley C. Umeh, Azah Mohamed, Aini Hussain, and Ramizi Mohamed

Abstract—**One of the power quality concerns that have received most attention is the problem of harmonics which are generated by widely dispersed single phase nonlinear loads. Such loads cause harmonic distortion of voltages and currents in a power distribution system. In order to fully understand the problem of harmonic distortion, an effective means of identifying the harmonic patterns generated by different types of nonlinear loads is considered. This paper presents the application of fractal analysis for analyzing various harmonic current waveforms generated by typical nonlinear loads such as personal computer, fluorescent lights and uninterruptible power supply. The fractal technique provides both time and spectral information of the nonlinear load harmonic patterns. The analysis results shows that the various harmonic current waveforms can be easily identified from the characteristics of the fractal features. This investigation proves that the fractal technique is a useful tool for identifying harmonic current waveforms and forms a basis towards the development of the harmonic load recognition system.**

Index Terms—**Power Quality, harmonics, nonlinear loads, fractal analysis.**

I. Introduction

POWER quality problems are increasing with the proliferation of non-linear loads which draw non-sinusoidal current waveforms when supplied by a sinusoidal voltage source. When these loads are present in a power distribution system, they cause harmonic distortion of voltages and currents. Individually, single-phase non-linear loads may not pose many serious harmonic problems but large concentrations of these loads have the potential to raise harmonic voltages and currents to unacceptable high levels which may result in increased neutral currents, over heating of system components and mechanical oscillations in generators and motors. Other undesirable effects are capacitor and insulation failure due to harmonic resonance, unpredictable behavior of installed protection devices, rapid voltage fluctuations and over heating of transformer [1]. To solve the harmonic problems, one of the early task that needs to be performed is to identify the various captured harmonic waveform in accordance to nonlinear load types. In an attempt to study the various waveforms generated by the nonlinear loads, laboratory test and measurements were performed on the single-phase nonlinear loads such as personal computers, fluorescent lights of the electronic ballast and magnetic ballast types and uninterruptible power supply.

The most widely applied signal processing technique used to extract the harmonic contents of voltage and current waveforms is the fast Fourier transform (FFT). However, the FFT technique is effective in computing the harmonic components of the current waveform but it is not very suitable for identifying the load harmonic patterns due to the presence of noise in terms of spike in real current waveforms and also its inefficiency in tracking the signal dynamics [2]. The wavelet transform technique has been used to overcome the problems of Fourier analysis but the efficiency and accuracy of the wavelet transform depends on the choice of the mother wavelet. The Daubechies' wavelet has been proven to be the most suitable wavelet for accurately detecting fast transient and short time information signals [3]. However, the wavelet transform is not widely applied for identifying harmonic current and voltage waveforms. In addition, the disadvantage of the wavelet transform is that it is highly dependent on factors such as the number of data points and sampling frequency.

In recent years, fractal techniques have attracted increased attention as a tool in image and signal processing. The method has been used in many applications as an alternative for analyzing time-varying signals where other techniques have not achieved the desired results [4]. Fractal theory is an extension of the classical geometry that can be used to make precise models of physical structures [5]. The applications of fractal geometry extend to biological modeling, geography, coastlines, computer graphics, images and so on. Different concept of fractal geometry has been applied for quantization of nonlinear systems in recent years [6]. Fractal techniques have been successfully applied in the analysis of chaotic systems [7], in biomedical signal analysis [8], in radar weak target detection [9] and image processing [10]. In the field of power engineering, fractal based techniques has been used for the classification of impulse faults [11], analysis of chaotic properties of high impedance faults [12], pattern recognition of partial discharge [13] and characterization of power quality disturbances including harmonic voltage [14]. In this paper, an attempt has been made using the fractal technique in identifying and representing the nonlinear load harmonic current waveforms in terms of fractal numbers. The technique provides both time and spectral information which is useful in solving the frequency localization problem encountered in the wavelet transform. The effectiveness of fractals as a measure to quantify chaotic systems makes it very feasible to monitor changes in harmonic distortion levels [14].

II. Harmonic Measurements Of Nonlinear Loads

This section presents the experimental setup in conducting the harmonic measurement on the nonlinear

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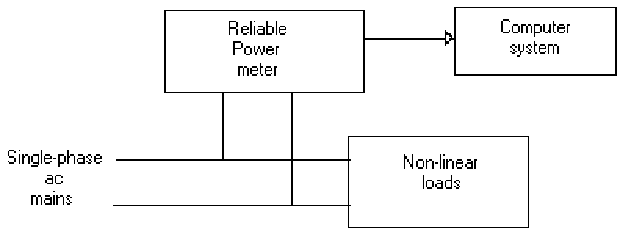

Fig. 1. Experimental set up for nonlinear load harmonic measurement.

loads. The purpose of the measurement is to characterize the behavior of harmonic generating loads and also to provide preliminary data for the development of the real-time harmonic signature recognition system. The harmonic measurements being conducted is described in terms of a schematic block diagram as shown in Fig. 1. A power recorder (RPM) compatible with a personal computer is placed between the single-phase AC mains and the non-linear load to be measured. The readings on the recorder are stored with the load energized. These readings are then accessed and down loaded to the computer for report making, printouts and analysis of readings. A windows-compatible software package available with the power recorder reads the recorded data files generated by the meter and performs the analysis and summary of the information stored. The harmonic information in terms of individual harmonic (up to sixty-third including DC and fundamental), total harmonic distortion, frequency, rms amplitude, current crest factor and phase angle are recorded.

Harmonic current measurements were obtained by means of the RPM and the single-phase nonlinear loads that have been considered in the measurements are listed as follows:

- Personal Computer: AC 100-230V 3.0/1.5A 50/60 Hz
- Fluorescent Lamp (Electronic Ballast): AC 220/240,0.1A, 50/60Hz
- Fluorescent Lamp (Magnetic Ballast): AC 220/240, 0.42A, 50Hz
- Uninterruptible Power Supply: AC 230V, 2.6A, 600VA, 50Hz

III. Fractal Theory

Mendelbrot's observation on the existence of geometric nature led to the concept of fractal dimension [15]. Since then, it has evolved from mere intuitive ideas to a more elaborate algorithm in recent years. According to Barnsley, the fractal dimension of a set is that number which quantifies how densely the set occupies the metric space in which it lies [5]. Fractal dimensions provide an empirical means for comparing fractals between closely related objects through numbers associated with them. A fractal space is a complete metric space in which all functions defined in such space can be easily measured by fractal dimensions.

Consider a complete metric space (X,d). For each $\varepsilon > 0$, let $N(A,\varepsilon)$ denotes the smallest number of closed balls of radius ε needed to cover the set of A. If this assumption holds, then there exists the fractal dimension D given by:

$$D_B = \lim_{\varepsilon\to 0}\left(\frac{\log(N(\varepsilon))}{\log(1/\varepsilon)}\right) \tag{1}$$

The capacity fractal dimension, D_B provides a quantitative approach of describing a set. Several other definitions of fractal dimension as measures of chaotic system behavior exist. Some of the commonly used ones are the information and correlation fractal dimensions. The information dimension is a generalization of the capacity function having N_B replaced with the average information function $I(\varepsilon)$:

$$I(\varepsilon) = \sum_{i=1}^{N(\varepsilon)} -P(\varepsilon,i)\log(P(\varepsilon,i)) \tag{2}$$

where $P(\varepsilon,i)$ is the probability that a point of set S is in the i-th box of size ε. Thus, the information dimension can be defined as,

$$D_I = \lim_{\varepsilon\to 0}\left(\frac{\log I(\varepsilon)}{\log(1/\varepsilon)}\right). \tag{3}$$

The correlation dimension is given as,

$$D_C = \lim_{r\to 0}\left\{\frac{\log C(r)}{\log r}\right\} \tag{4}$$

where $C(r)$ is the correlation integral defined as,

$$C(r) = \lim_{N\to\infty}\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=i+1}^{N} I\left(\left\|x_j - x_i\right\| \le r\right) \tag{5}$$

where I is an indicator function.

In general, the three dimensions are related by the inequality [10]:

$$D_c \le D_I \le D_B . \tag{6}$$

A. Computation of Fractal Number

In recent years, the estimation of fractal dimensions has been widely applied in the studies involving time varying signals. The algorithm has been well tested on data sets ranging from a few hundreds to tens of thousands of data points [12]. In terms of computability, the correlation dimension is much easier to compute when compared to the capacity dimension. For a collection of N points in a trajectory either through simulations or measurements, the task of finding the correlation dimension simply reduces to finding the number, N_p of pairs of data points. Considering a given S set of N_p pairs of data points (x_i, x_j) in a metric space, the correlation dimension can be calculated using the distances between each pair of the points in the set which is given as,

$$\left|x_i - x_j\right| = \left(\sum_{j=1}^{N}\left|x_i - x_j\right|^2\right)^{\frac{1}{2}} \tag{7}$$

such that

$$\left\|x_i - x_j\right\| < \varepsilon . \tag{8}$$

The equation for the correlation dimension is reduced to,

$$D_C = \lim_{N\to\infty}\frac{1}{N^2}\left[N_p\right]. \tag{9}$$

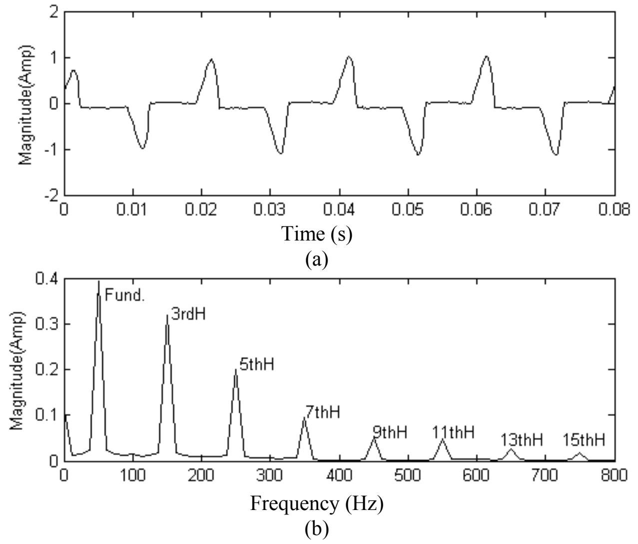

Fig. 2. Personal computer load (a) current waveform, and (b) harmonic spectrum.

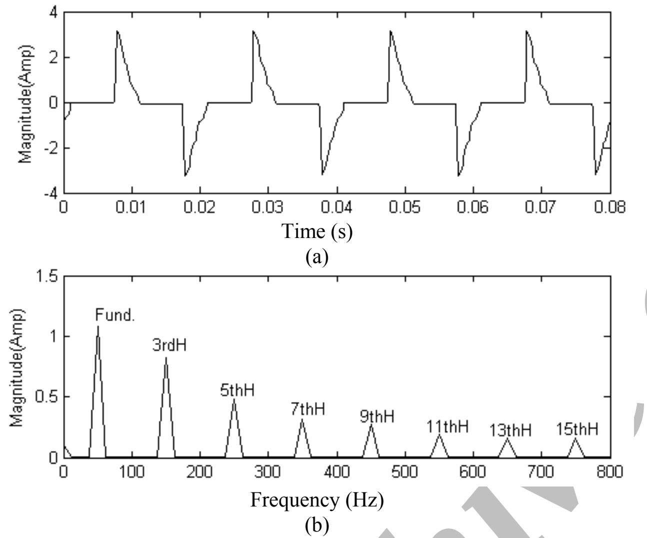

Fig. 3. Fluorescent lamp with electronic ballast (a) current waveform, and (b) harmonic spectrum.

In this study, the current values are normalized and partitioned into subsets with each data sets confined to a strict range of [0,1] having the Euclidian metric scaled down by the factor of 100. The process is quantified in terms of the derived fractal numbers using,

$$F = \left[\frac{\sqrt{\sum_{j=1}^{N-l} \left\| x_{j+k} - x_j \right\|^2}}{\sqrt{\sum_{j=1}^{N-l} \left\| x_{j+l} - x_j \right\|^2}} \right] \times 100 \qquad (10)$$

where, F is the fractal number of m-th data set S_m containing N data points, k and l are small integer sampling steps size which specifies the time interval between close data points, such that l is greater than k. For the effectiveness of the equation, different norms can be selected, however for this particular work, the Euclidean norm is used due to its simplicity [14]. The fractal number computation turned out to be more efficient in this case if the ratio l to k is not an integer. The data points have to be properly classified such that data points belonging to k-step sampled subsets do not fall into the l-step subsets.

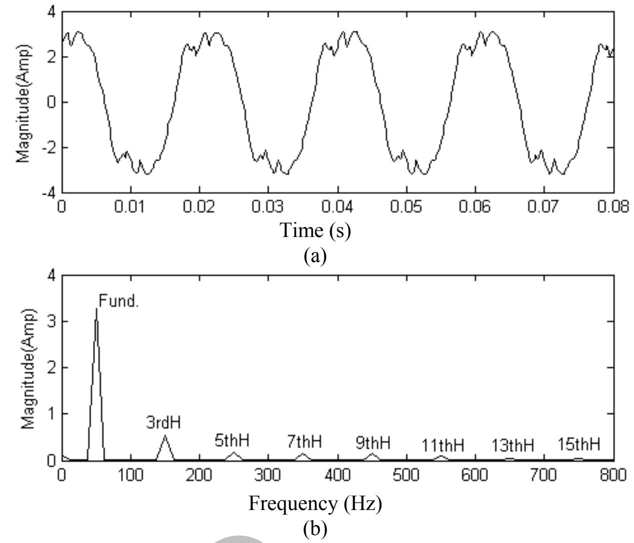

Fig. 4. Fluorescent lamp with magnetic ballast (a) current waveform, and (b) harmonic spectrum.

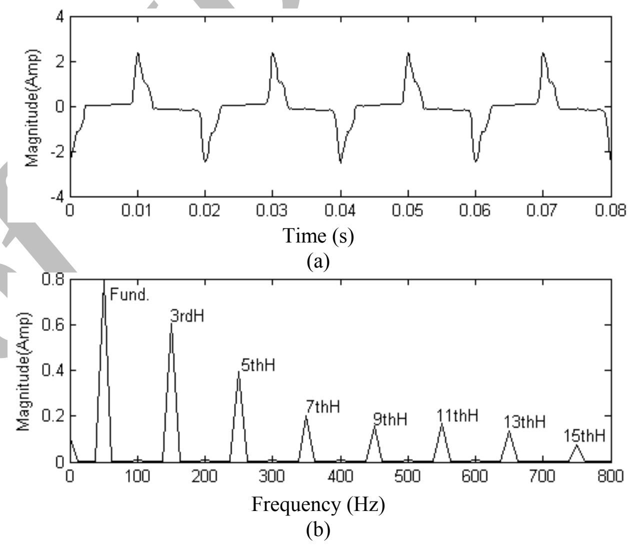

Fig. 5. UPS load (a) current waveform, and (b) harmonic spectrum.

IV. Results and Discussion

The measured harmonic current waveforms obtained from the single-phase nonlinear loads are first presented followed by the fractal analysis carried out on these waveforms.

A. Nonlinear Load Harmonics

The personal computer (PC) utilizes the switched mode power supply which has a capacitor and a diode bridge as basic components in its input stage. Fig. 2(a) shows the non-sinusoidal and highly distorted harmonic current drawn by a PC. The harmonic spectrum for this current is shown in Fig. 2(b). The harmonics generated by the current are significantly high in amplitude and from the harmonic spectra it can be observed that the current is particularly rich in odd harmonics namely 3rd, 5th and 7th harmonics. There is a significant amount of the 9th and 11th harmonic current components as well. The total harmonic distortion (THD) for the current is 98.4%. This value is excessively higher than the limits imposed by IEEE 519-1992.

Fig. 3(a) shows the current drawn by 12 fluorescent lamps with electronic ballasts and Fig. 3(b) shows its harmonic spectrum with a recorded THD current of

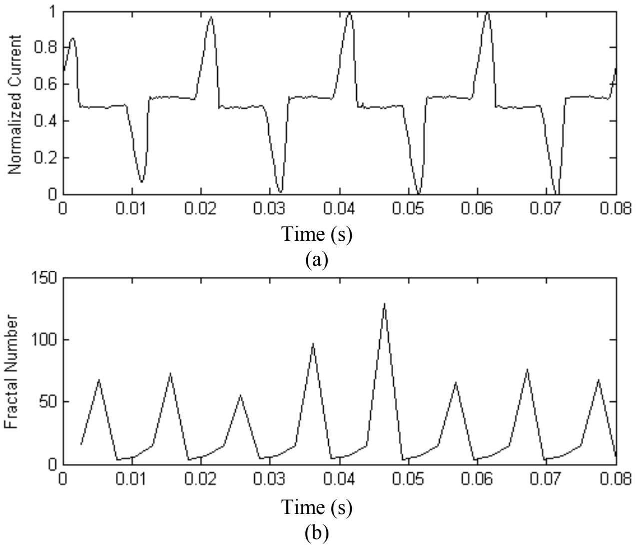

Fig. 6. PC load (a) current waveform, and (b) fractals.

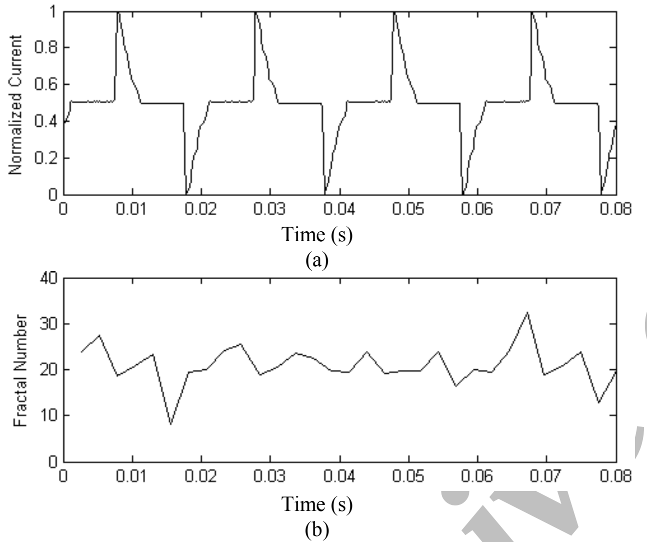

Fig. 7. Fluorescent lamp with electronics ballast (a) current waveform, and (b) fractals.

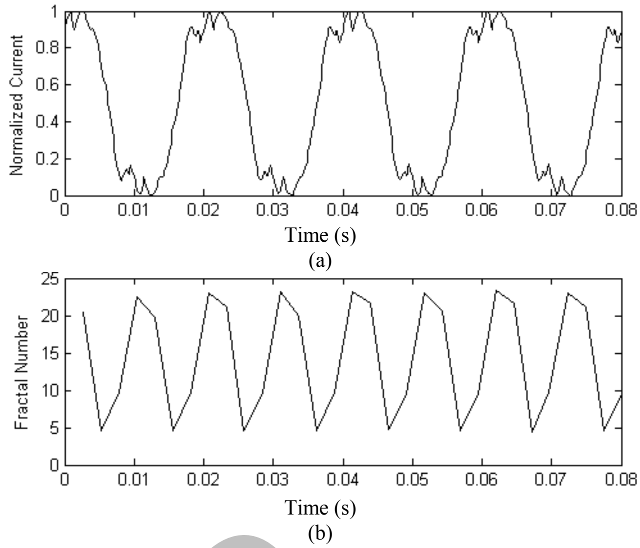

Fig. 8. Fluorescent lamp with magnetic ballast (a) current waveform, and (b) fractals.

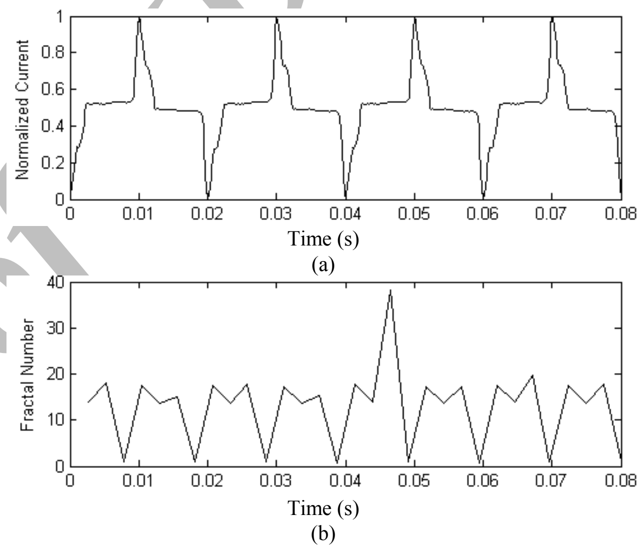

Fig. 9. UPS load (a) current waveform, and (b) fractals.

102.8%. In this case, the major harmonic components contributing to this high distortion level are the 3rd, 5th, 7th, 9th, 11th, 13th, and even the 15th harmonic components. Fig. 4(a) shows the current from a lighting panel comprising of 12 fluorescent lamps with magnetic ballast. The nonlinear nature of this current is as a result of the nonlinearity between the ballast and the lamp arc itself. The harmonic spectrum for 12 light loads is shown in Fig. 4(b) with a current THD value of 22.26% including a significant amount of even harmonics (3.85%) and dc harmonic component. The fluorescent lamp with magnetic ballast draws input current which is less distorted when compared to the fluorescent lamp with electronic ballast.

The recorded current drawn by the UPS is shown in Fig. 5(a). The current is highly non-sinusoidal, indicating that the current drawn by the UPS is a major contributor to the overall harmonic pollution. The input stage of the UPS comprises of a smoothing inductor and a rectifier and hence the input current drawn when energized is very rich in harmonic ranging from the 3rd up to the 15th harmonic component as can be seen in Fig. 5(b). The current THD for this load is 101.6%.

B. Fractal Features of Nonlinear Loads Currents

A sampling rate of 6.4 kHz was in recording the current waveforms of the monitored nonlinear loads. From the recorded waveform, the data are sampled such that the data sets consist of 512 data points divided into 31 subsets with each subset containing 16 data points. Fractal number computation was then performed for a set of the collected data. The fractal analysis has been tested on harmonic current waveforms of various nonlinear loads. From the analysis, the fractal features of the nonlinear loads in terms of fractal numbers were obtained. Fig. 6 shows the current waveform and fractals of the PC load where it can be observed that every half cycle of the current, yielded repetitive fractal patterns with sharp peaks. Applying the fractal analysis to waveforms obtained from other PC loads produced patterns that are similar.

Fig. 7 depicts the current waveform and the fractals of the fluorescent lamp with electronic ballast. The current THD for the lamp with electronic ballast is high, that is 102.8%, thus yielding a high level of chaotic behavior in its fractal pattern. Fig. 8 shows the current waveform and fractals of the fluorescent lamp with magnetic ballast. It is noted in Fig. 8(b) that the less chaotic and uniformly

TABLE I
FRACTAL NUMBERS FOR DIFFERENT LOAD HARMONIC PATTERNS

Fractal Division	Fractal Numbers			
	Personal Computer	Lamp with Electronic Ballast	Lamp with Magnetic Ballast	UPS
1	16.083	23.855	20.454	13.947
2	67.631	27.333	4.6048	18.057
3	4.137	18.646	9.8149	0.9465
4	6.1804	20.873	22.427	17.468
5	15.248	23.261	19.672	13.698
6	73.4	8.046	4.5884	15.093
7	3.4727	19.553	9.7131	0.82815
8	6.3133	19.873	22.981	17.477
9	14.857	24.228	21.106	13.704
10	55.324	25.507	4.598	17.738
11	4.8431	18.884	9.7325	0.9471
12	6.2649	20.476	23.136	17.275
13	14.776	23.627	19.907	13.611
14	96.894	22.533	4.6157	15.426
15	4.568	19.796	9.6823	0.63478
16	6.326	19.54	23.213	17.759
17	14.673	23.85	21.636	13.942
18	129.21	19.263	4.6677	38.298
19	3.4185	19.672	9.6403	0.86655
20	6.3021	19.678	23.03	17.215
21	14.531	23.758	20.556	13.598
22	66.154	16.403	4.6132	17.276
23	3.9492	20.038	9.6508	0.79892
24	6.3045	19.529	23.342	17.59
25	14.732	24.075	21.661	13.98
26	76.202	32.509	4.442	19.629
27	3.7339	18.909	9.7337	0.8838
28	6.2605	20.791	22.986	17.43
29	14.522	23.75	21.102	13.658
30	67.831	12.732	4.6105	17.805
31	3.6163	19.779	9.6006	0.97547

distributed fractal patterns with slashed peaks is due to the lower current THD value of the lamp with magnetic ballast as compared to the THD value of the lamp with electronic ballast.

The highly harmonically distorted current waveform of the UPS yielded a current THD of 101.6% (Fig. 9(a)). Due to the high distortion of the current waveform, a chaotic but repetitive double-peaked fractal pattern is observed as shown in Fig. 9(b).

Table I shows the fractal numbers obtained for the different load harmonic patterns. These fractal numbers can represent the fractal features of the four types of non-linear loads.

V. CONCLUSION

A fractal-based method for the characterization of harmonic waveforms produced from single-phase non-linear loads has been proposed. The measured harmonic current waveforms obtained from the four types of non-linear loads, namely, personal computer, fluorescent lamp with electronic ballast, fluorescent lamp with magnetic ballast and uninterruptible power supply were processed using fractal analysis. The harmonic current waveforms analyzed using fractal analysis were represented by fractal numbers. Thus, the fractal numbers can characterize and extract features of a particular type of load. The proposed method proves the usefulness of fractal geometry in pattern recognition of the harmonic current waveforms of the various non-linear loads tested. The fractal features obtained will be used as input features into an expert system towards the development of an automatic harmonic load recognition system.

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