

# A Sufficient Condition for Decentralized Stabilization

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**Abstract**—This paper considers the problem of stabilizing a class of linear time-invariant large-scale systems composed of a number of subsystems using several local dynamic output feedback controllers. For this problem, a sufficient condition on each closed-loop individual subsystem is derived under which the decentralized controller composed of the local controllers designed for individual subsystems, achieves stability for the overall system. This condition is used to convert the decentralized stabilization problem to a set of the  $H_\infty$  disturbance rejection subproblems.

**Index Terms**—Decentralized control,  $H_\infty$  control, large-scale systems, linear matrix inequalities.

## I. INTRODUCTION

THERE has been continuing interest in the study of large-scale systems consisting of a number of interconnected subsystems [1]–[4]. The reason for this interest follows since many control problems of modern industrial society are associated with the control of complex interconnected systems, e.g., electric power systems, transportation systems, chemical process control systems, socioeconomic systems, network flow problems, etc. In the study of such large-scale systems, an important issue is decentralized control [5]–[8]. In decentralized control, large-scale system has several local controllers such that each local controller observes only local subsystem outputs and controls only local inputs; all of the local controllers are involved, however, in controlling the same large-scale system.

A decentralized control system exhibits several advantages over a centralized control system, i.e., a single controller which observes all outputs of the system to control all inputs of the system. In the ideal case these advantages include: simplified design, simplified tuning, flexibility in operation, and failure tolerance [9]. The requirement that the control system be decentralized introduces the overall stability problem, i.e.; when the decentralized controller is applied to the whole system, the stability of the closed-loop system are not preserved. As a result, the stability achieved with the block diagonal system is not guaranteed, and the overall stability is lost in most cases. This illustrates the need for a sufficient condition to examine the overall stability and alternative ways to design the decentralized controllers while they guarantee the overall stability.

There are two main classes of available approaches to the overall stability analysis: the Lyapunov methods [10] and the input-output methods [11]. In the Lyapunov

methods, the overall stability condition is in terms of the individual Lyapunov functions and bounds on interconnections. Behavior of each isolated subsystem is characterized by its own Lyapunov function and the characterization does not require the knowledge of other subsystems. However, these methods are successful only when the coupling among subsystems is weak. In the input-output methods, the overall stability analysis is based on the Small Gain Theorem [12] and the properties of the Metzler matrix. In comparing the two classes of methods, the input-output methods are often less conservative and easier to apply than the Lyapunov methods [13].

The approaches proposed in [14] and [15] are two sample of the input-output methods that are more relevant to our approach. In [15], the structured singular value interaction measure was proposed as a tool for the design of decentralized control. This approach provides a sufficient condition for the overall stability in terms of the subsystem design constraints, under which an aggregation of stable subsystem designs yields an overall stable design. However in [15], it is assumed that the initial system is square and it also requires very complicated computations when the dimensionality of the initial system is high. The proposed approach in [14] provides a sufficient condition for the overall stability in terms of the  $H_\infty$  norm of the closed-loop block diagonal transfer function matrix and the structured singular value of the interaction matrix. In addition in [14], by a simple example, it is shown that the proposed stability condition is less conservative than the one proposed in [15].

In this paper, a combination of the Lyapunov methods and input-output methods is used to obtain an overall stability condition. This condition is stated in terms of the  $H_\infty$  norm of a transfer function matrix of each closed-loop individual subsystem and the Hermitian part of the interaction matrix. Our stability condition is both straightforward to examine and less conservative than the ones proposed in [14] and [15]. In addition, this condition is used to convert the decentralized stabilization problem to a set of the  $H_\infty$  disturbance rejection subproblems.

The remainder of this paper is organized as follows. Section II is devoted to the formulation of control problem and statement of preliminary definitions used throughout the paper. Section III gives the overall stability condition. In Section IV, a comparison example is presented. Section V is devoted to the  $H_\infty$  formulation for the decentralized stabilization problem. Finally, Section VI concludes the paper.

## II. PROBLEM STATEMENT

Consider an input-output decentralized large-scale system  $S$ , with state-space equations

Manuscript received January 12, 2004; revised June 30, 2004.

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Publisher Item Identifier S 1682-0053(04)0260

$$S: \begin{cases} \dot{x} = Ax + Bu, \\ y = Cx, \end{cases} \quad (1)$$

composed of  $N$  subsystems  $S_i$ , described by

$$S_i: \begin{cases} \dot{x}_i = A_{ii}x_i + \sum_{j=1, j \neq i}^N A_{ij}x_j + B_{ii}u_i, \\ y_i = C_{ii}x_i, \quad i = 1, 2, \dots, N, \end{cases} \quad (2)$$

where  $x_i \in \mathbf{R}^{n_i}$  is the state,  $u_i \in \mathbf{R}^{m_i}$  is the control input,  $y_i \in \mathbf{R}^{p_i}$  is the measured output of the  $i$ -th subsystem. The matrices  $A_{ii}$ ,  $B_{ii}$ , and  $C_{ii}$  are real, constant, and of appropriate dimensions, which represent the  $i$ -th subsystem. The subsystems interact each other through the interconnections  $A_{ij}x_j$ 's, where  $A_{ij}$ 's are real constant matrices. In this note, we assume that the triple  $(A_{ii}, B_{ii}, C_{ii})$  is stabilizable and detectable.

We consider the set of local dynamic output feedback controllers  $K_{ii}(s)$ 's, described by

$$\begin{cases} \dot{x}_{ki} = A_{ki}x_{ki} + B_{ki}y_i, \\ u_i = C_{ki}x_{ki} + D_{ki}y_i, \quad i = 1, 2, \dots, N, \end{cases} \quad (3)$$

where  $x_{ki} \in \mathbf{R}^{n_{ki}}$  is the state of the  $i$ -th local controller.  $A_{ki}$ ,  $B_{ki}$ ,  $C_{ki}$ , and  $D_{ki}$  are constant matrices of appropriate dimensions to be determined. The resulting decentralized controller  $K(s)$  is given by

$$K(s) = \text{block-diag}\{K_{ii}(s)\}, \quad i = 1, 2, \dots, N, \quad (4)$$

with state-space equations

$$\begin{cases} \dot{x}_k = A_k x_k + B_k y \\ u = C_k x_k + D_k y \end{cases} \quad (5)$$

The problem is to find the decentralized controller  $K(s)$  composed of  $N$  local dynamic output feedback controllers  $K_{ii}(s)$  in order to stabilize the large-scale system in (1), as shown in Fig. 1.

When the decentralized controller (5) is applied to system (1), the closed-loop system is described by

$$\begin{cases} \dot{x}_c = \begin{bmatrix} A_d + BD_k C & BC_k \\ B_k C & A_k \end{bmatrix} x_c + \begin{bmatrix} H & 0 \\ 0 & 0 \end{bmatrix} x_c \\ y = [C \ 0] x_c, \end{cases} \quad (6)$$

where

$$A_d = \text{block-diag}\{A_{ii}\}, \quad H = A - A_d, \quad x_c = \begin{bmatrix} x \\ x_k \end{bmatrix} \quad (7)$$

The matrix  $H$  is also called the interaction matrix.

Now, we define the matrix

$$\tilde{K} = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix}, \quad (8)$$

which collects the representation for  $K(s)$  into one matrix. It is simple to show that the closed-loop state-space equations (6) can be represented in terms of the controller matrix  $\tilde{K}$  as

$$\begin{cases} \dot{x}_c = \tilde{A}_c x_c \\ y = [C \ 0] x_c \end{cases} \quad (9)$$

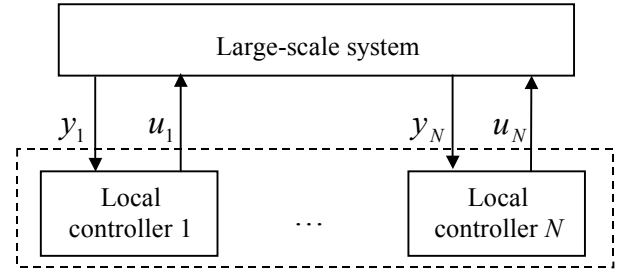


Fig. 1. Decentralized control structure.

where  $\tilde{A}_c = \tilde{A}_d + \tilde{H} + \tilde{B}\tilde{K}\tilde{C}$  and

$$\tilde{A}_d = \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0 & B \\ I & 0 \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} 0 & I \\ C & 0 \end{bmatrix}, \quad \tilde{H} = \begin{bmatrix} H & 0 \\ 0 & 0 \end{bmatrix} \quad (10)$$

Therefore, the problem of achieving stability for system (1) by a dynamic output feedback controller (5), is reduced to the problem of achieving stability for the system with state-space realization  $[\tilde{A}_d + \tilde{H}, \tilde{B}, \tilde{C}, 0]$  by a static output feedback controller (8), i.e., all the eigenvalues of  $\tilde{A}_c$  have negative real-part.

### III. OVERALL STABILITY CONDITION

The following preliminary results are required to state the main theorem of this paper.

**Lemma 1:** For a square matrix  $M \in \mathbf{R}^{n \times n}$ , we have

$$\text{Re}[\lambda_i(M)] \leq \lambda_{\max}[\text{Herm}(M)], \quad i = 1, 2, \dots, n, \quad (11)$$

where  $\lambda_i(M)$  is the  $i$ -th eigenvalue of  $M$  and  $\text{Herm}(\cdot)$  denotes the Hermitian part of  $(\cdot)$ .

**Proof:** Assuming that  $v_i$  is the  $i$ -th eigenvector of  $M$ , then we have

$$v_i^H \text{Herm}(M) v_i = \text{Re}[\lambda_i(M)] v_i^H v_i, \quad i = 1, 2, \dots, n. \quad (12)$$

Since for a symmetric matrix for example  $\text{Herm}(M)$  and every vector  $x$ , we have [16]

$$x^H \text{Herm}(M) x \leq \lambda_{\max}[\text{Herm}(M)] x^H x \quad (13)$$

therefore

$$\text{Re}[\lambda_i(M)] \leq \lambda_{\max}[\text{Herm}(M)], \quad i = 1, 2, \dots, n. \quad (14)$$

**Lemma 2:** The linear matrix inequality

$$\begin{bmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{bmatrix} < 0, \quad (15)$$

where  $Q(x) = Q(x)^T$ ,  $R(x) = R(x)^T > 0$ , and  $S(x)$  depend affinely on  $x$ , is equivalent to

$$Q(x) - S(x)R(x)^{-1}S(x)^T < 0. \quad (16)$$

**Proof:** See [17] for the proof of this lemma.

**Lemma 3:** Consider a continuous-time transfer function  $T(s)$  with state-space realization  $[A, B, C, D]$ . The following statements are equivalent:

- (i) The matrix  $A$  is Hurwitz and  $\|T(s)\|_{\infty} < \gamma$ .
- (ii) There exists a symmetric positive definite matrix  $P$  such that

$$\begin{bmatrix} A^T P + PA & PB & C^T \\ B^T P & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0. \quad (17)$$

(iii) There exists a symmetric positive definite matrix  $P$  such that

$$\begin{bmatrix} A^T P + PA + C^T C & PB + C^T D \\ B^T P + D^T C & D^T D - \gamma^2 I \end{bmatrix} < 0. \quad (18)$$

Proof: See [18] for the proof of this lemma.

*Definition 1:* The  $H_\infty$  norm of a continuous-time transfer function  $T(s)$  is defined as

$$\|T(s)\|_\infty := \sup_{\omega \in \mathbf{R}} \sigma_{\max}[T(j\omega)], \quad (19)$$

where  $\sigma_{\max}[T(j\omega)]$  denotes the maximum singular value of  $T(j\omega)$  [12].

*Property 1:* Let  $A$  and  $B$  are any matrices with appropriate dimensions. Then [12]

$$\sigma_{\max}(AB) \leq \sigma_{\max}(A)\sigma_{\max}(B). \quad (20)$$

The following theorem presents the overall stability condition.

*Theorem 1:* The decentralized controller  $K(s)$  composed of  $N$  local controllers  $K_{ii}(s)$  with the state-space representation  $\tilde{K}_i$  defined as

$$\tilde{K}_i = \begin{bmatrix} A_{ki} & B_{ki} \\ C_{ki} & D_{ki} \end{bmatrix}, \quad (21)$$

stabilizes the large-scale system (1), if  $\tilde{K}_i$  stabilizes the  $i$ -th augmented subsystem with the state-space realization  $[\tilde{A}_{ii}, \tilde{B}_{ii}, \tilde{C}_{ii}, 0]$  and

$$\alpha_i < \rho, \quad (22)$$

where

$$\tilde{A}_{ii} = \begin{bmatrix} A_{ii} & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{B}_{ii} = \begin{bmatrix} 0 & B_{ii} \\ I & 0 \end{bmatrix}, \quad \tilde{C}_{ii} = \begin{bmatrix} 0 & I \\ C_{ii} & 0 \end{bmatrix}, \quad (23)$$

$$\tilde{A}_{cd_i} = \tilde{A}_{ii} + \tilde{B}_{ii} \tilde{K}_i \tilde{C}_{ii}, \quad \rho = \left( \|\text{Herm}(\tilde{H})\|_2 \right)^{-1} \quad (24)$$

$$\alpha_i = \left\| [sI - \text{Herm}(\tilde{A}_{cd_i})]^{-1} \right\|_\infty.$$

*Proof:* Let  $\tilde{K}_i$ 's are stabilizer local controllers that achieve the condition in (22). It can easily be concluded that

$$\alpha < \rho, \quad (25)$$

where

$$\alpha = \left\| [sI - \text{Herm}(\tilde{A}_{cd})]^{-1} \right\|_\infty, \quad \tilde{A}_{cd} = \tilde{A}_d + \tilde{B} \tilde{K} \tilde{C}. \quad (26)$$

Note that

$$\alpha = \max_i \{\alpha_i\}, \quad i = 1, 2, \dots, N. \quad (27)$$

From the definitions of the  $H_\infty$  norm and 2-norm, the condition in (25) can be written as

$$\sigma_{\max}[\text{Herm}(\tilde{H})] \times \sup_{\omega \in \mathbf{R}} \sigma_{\max}\{[j\omega - \text{Herm}(\tilde{A}_{cd})]^{-1}\} < 1. \quad (28)$$

By Property 1, we have

$$\sup_{\omega \in \mathbf{R}} \sigma_{\max}\{\text{Herm}(\tilde{H})[j\omega - \text{Herm}(\tilde{A}_{cd})]^{-1}\} < 1. \quad (29)$$

From Definition 1, this can be written as

$$\left\| \text{Herm}(\tilde{H})[sI - \text{Herm}(\tilde{A}_{cd})]^{-1} \right\|_\infty < 1. \quad (30)$$

According to Lemma 3, the above inequality can be expressed as the following LMI

$$\begin{bmatrix} \text{Herm}(\tilde{A}_{cd})P + P\text{Herm}(\tilde{A}_{cd}) + [\text{Herm}(\tilde{H})]^2 & P \\ P & -I \end{bmatrix} < 0 \quad (31)$$

where  $P$  is a symmetric positive-definite matrix. By Lemma 2, this is equivalent to

$$\text{Herm}(\tilde{A}_{cd})P + P\text{Herm}(\tilde{A}_{cd}) + [\text{Herm}(\tilde{H})]^2 + PP < 0 \quad (32)$$

Since for any matrix for example  $[\text{Herm}(\tilde{H}) - P]$  we have

$$[\text{Herm}(\tilde{H}) - P]^T [\text{Herm}(\tilde{H}) - P] \geq 0 \quad (33)$$

or equivalently,

$$[\text{Herm}(\tilde{H})]^2 + PP \geq \text{Herm}(\tilde{H})P + P\text{Herm}(\tilde{H}), \quad (34)$$

one can easily see that if (32) holds then the following inequality holds

$$\begin{aligned} & \text{Herm}(\tilde{A}_{cd})P + P\text{Herm}(\tilde{A}_{cd}) + \\ & \text{Herm}(\tilde{H})P + P\text{Herm}(\tilde{H}) < 0. \end{aligned} \quad (35)$$

Now, by substituting  $\tilde{A}_{cd} = \tilde{A}_c - \tilde{H}$  into this, we obtain

$$\text{Herm}(\tilde{A}_c)P + P\text{Herm}(\tilde{A}_c) < 0. \quad (36)$$

Finally, by invoking the Lyapunov stability theorem and Lemma 1, it can be concluded that all the eigenvalues of  $\tilde{A}_c$  have negative real-part, i.e., the overall closed-loop system (9) is stable.

Note that the stability condition in (22) is straightforward to examine; since  $\tilde{H}$  is a constant matrix,  $\rho$  is easily computable. Also,  $\alpha_i$  is the  $H_\infty$  norm of a transfer function matrix of each closed-loop individual subsystem using its local controller. In addition, this condition is less conservative than the ones proposed in [14], [15], as illustrated in [19].

#### IV. COMPARISON EXAMPLE

In this section, the stability condition proposed in the previous section is compared with the conditions of [14], [15].

In [15], the stability of the overall system  $G(s)$ , by a decentralized controller is guaranteed if

$$\bar{\sigma}[\tilde{T}(j\omega)] < \mu^{-1}[E(j\omega)], \quad \forall \omega \in \mathbf{R} \quad (37)$$

where

$$E(j\omega) = [G(j\omega) - \tilde{G}(j\omega)]\tilde{G}^{-1}(j\omega). \quad (38)$$

$\tilde{T}(j\omega)$  represents the complementary sensitivity function of the block diagonal system  $\tilde{G}(j\omega)$  and  $\bar{\sigma}[\tilde{T}(j\omega)]$  denotes the maximum singular value of  $\tilde{T}(j\omega)$ .

In [14], it is proposed that the decentralized controller  $K(s)$  stabilizes the overall system  $G(s)$ , if  $K(s)$  stabilizes the block diagonal system  $\tilde{G}(s)$  and

$$\rho_{\max} < \mu^{-1}(H) \quad (39)$$

where

$$\rho_{\max} = \max_i \left\{ \left\| (sI - A_{ii} + B_{ii}K_{ii}(s)C_{ii})^{-1} \right\|_{\infty} \right\} \quad (40)$$

and

$$\mu(H) = (\min_{\Delta} \{ \overline{\sigma}(\Delta) \mid \det(I + \Delta H) = 0 \})^{-1}. \quad (41)$$

Note that  $\mu(H) = 0$ , if no structured  $\Delta$  exists such that  $\det(I + \Delta H) = 0$ .

The following example illustrates that the proposed stability condition is less conservative than the stability conditions of [14], [15]. Consider the system  $(A, B, C)$  and the decentralized controller  $K(s)$  where

$$A = \begin{bmatrix} 1 & -4 \\ 4 & 1 \end{bmatrix}, \quad B = C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad (42)$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad K(s) = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}.$$

It is simple to show that

$$\overline{\sigma}[\tilde{T}(j0)] = \frac{3}{7}, \quad \mu^{-1}[E(j0)] = \frac{1}{3} \quad (43)$$

$$\rho_{\max} = \frac{1}{2}, \quad \mu^{-1}(H) = \frac{1}{4}. \quad (44)$$

Since  $\overline{\sigma}[\tilde{T}(j0)] > \mu^{-1}[E(j0)]$  and  $\rho_{\max} > \mu^{-1}(H)$ , the conditions in (37) and (39), respectively, fail to be satisfied for this example. Also, it is straightforward to obtain that

$$\alpha = \frac{1}{2}, \quad \rho = \infty. \quad (45)$$

It is clear that the stability condition in (22) is satisfied for this case. Therefore, it is concluded that the decentralized controller stabilizes the overall system.

#### V. $H_{\infty}$ FORMULATION OF PROBLEM

In this section, we show how the decentralized stabilization problem can be converted to a set of the  $H_{\infty}$  disturbance rejection subproblems.

Let us consider the new subsystems  $\hat{S}_i$ 's, described by

$$\hat{S}_i : \begin{cases} \dot{\hat{x}}_i = \hat{A}_i \hat{x}_i + \hat{B}_{1i} \hat{w}_i + \hat{B}_{2i} \hat{u}_i, \\ \hat{z}_i = \hat{C}_{1i} \hat{x}_i, \\ \hat{y}_i = \hat{C}_{2i} \hat{x}_i, \end{cases} \quad i = 1, 2, \dots, N, \quad (46)$$

where  $\hat{x}_i$  is the state,  $\hat{w}_i$  is the disturbance input,  $\hat{u}_i$  is the control input,  $\hat{z}_i$  is the controlled output, and  $\hat{y}_i$  is the measured output of the  $i$ -th subsystem. The state-space matrices in (46) are defined as

$$\hat{A}_i = (\tilde{A}_{ii} + \tilde{A}_{ii}^T)/2, \quad \hat{B}_{2i} = [\tilde{B}_{ii} \quad \tilde{C}_{ii}^T],$$

$$\hat{B}_{1i} = I, \quad \hat{C}_{1i} = I, \quad \hat{C}_{2i} = [\tilde{C}_{ii}^T \quad \tilde{B}_{ii}^T]^T.$$

By  $T_{\hat{z}_i \hat{w}_i}(s)$ , we denote the transfer function from  $\hat{w}_i$  to  $\hat{z}_i$  of the  $i$ -th closed-loop subsystem obtained by applying the output feedback control law  $\hat{u}_i = \hat{K}_i \hat{y}_i$  to subsystem (46) where

$$\hat{K}_i = \begin{bmatrix} \tilde{K}_i & 0 \\ 0 & \tilde{K}_i^T \end{bmatrix}. \quad (48)$$

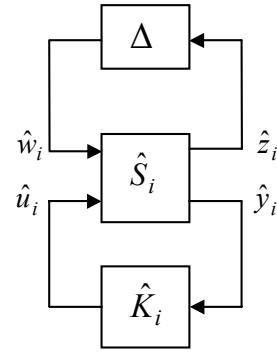


Fig. 2. Standard representation of each closed-loop subsystem.

**Theorem 2:** If there exists  $\hat{K}_i$  such that

$$\|T_{\hat{z}_i \hat{w}_i}(s)\|_{\infty} < \rho, \quad (49)$$

then the decentralized controller  $K(s)$  composed of  $N$  local controllers  $K_{ii}(s)$  with the state-space representation  $\hat{K}_i$  stabilizes the system (1).

*Proof:* With the plant  $\hat{S}_i$  and controller  $\hat{K}_i$  defined as above, the closed-loop system admits the realization

$$\hat{S}_{cl} : \begin{cases} \dot{\hat{x}}_i = \hat{A}_{ci} \hat{x}_i + \hat{B}_{1i} \hat{w}_i, \\ \hat{z}_i = \hat{C}_{1i} \hat{x}_i, \end{cases} \quad i = 1, 2, \dots, N, \quad (50)$$

where

$$\hat{A}_{ci} = \hat{A}_i + \hat{B}_{2i} \hat{K}_i \hat{C}_{2i}. \quad (51)$$

Now, we obtain  $T_{\hat{z}_i \hat{w}_i}(s)$  as

$$T_{\hat{z}_i \hat{w}_i}(s) = \hat{C}_{1i} (sI - \hat{A}_{ci})^{-1} \hat{B}_{1i}. \quad (52)$$

By substituting the state-space matrices from (47) and (51) into this, we have

$$T_{\hat{z}_i \hat{w}_i}(s) = [sI - \text{Herm}(\tilde{A}_{ii} + \tilde{B}_{ii} \tilde{K}_i \tilde{C}_{ii})]^{-1}, \quad (53)$$

or equivalently,

$$T_{\hat{z}_i \hat{w}_i}(s) = [sI - \text{Herm}(\tilde{A}_{cdi})]^{-1}. \quad (54)$$

If  $\hat{K}_i$  be a controller that achieves  $\|T_{\hat{z}_i \hat{w}_i}(s)\|_{\infty} < \rho$ , then we have

$$\| [sI - \text{Herm}(\tilde{A}_{cdi})]^{-1} \|_{\infty} < \rho.$$

Finally, by Theorem 1, we can conclude the decentralized controller  $K(s)$  stabilizes the overall system (1).

In fact, Theorem 2 states that the interconnections between the subsystems of large-scale system (1) can be considered as the model uncertainty for the block diagonal system composed of  $N$  subsystems  $\hat{S}_i$ . Then the local  $H_{\infty}$  controllers  $\hat{K}_i$  can be designed to reduce the effect of this uncertainty. The final decentralized controller is a block diagonal collection of  $\hat{K}_i$ 's. Fig. 2 shows the standard representation of each closed-loop individual subsystem.

**Remark 1:** The subproblems in Theorem 2 are equivalent to a set of standard static output feedback  $H_{\infty}$  disturbance rejection problems with additional structure constraint on the controller. Therefore, any available approaches for solving such these problems that it is possible to choose a desired structure on the controller, can be used to solve the subproblems [20]-[23].

*Remark 2:* Combining Theorem 2 and Lemma 3, the LMI formulation of decentralized stabilization problem can be also obtained. Linear matrix inequalities have emerged as a powerful formulation and design technique for a variety of linear control problems [18]. Since solving the LMI problem is a convex optimization problem, such formulation offer a numerically tractable means of attacking problems that lack an analytical solution. In addition, a variety of efficient algorithms are now available to solve the generic LMI problems [24], [25]. Consequently, reducing a decentralized stabilization problem to an LMI problem, can be considered as a practical solution to this problem.

## VI. CONCLUSIONS

In this paper, a sufficient condition on each closed-loop individual subsystem of a large-scale system has been derived under which, a block diagonal collection of the local controllers designed for individual subsystems, achieves overall stability. This condition is straightforward to examine and is also less conservative than the proposed conditions in previous researches. In addition, this condition has been used to convert the decentralized stabilization problem to a set of the  $H_\infty$  disturbance rejection subproblems.

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