# Design of New Models for Tight Upper Bound Approximation of Cell Loss Ratio in ATM Networks

A. T. Haghighat, K. Faez, S. Khorsandi, and M. Dehghan

Abstract—ATM as a high-speed cell switching technology can support multiple classes of traffic with different quality of service (QoS) requirements and diverse traffic characteristics. A main QoS requirement is the cell loss ratio (CLR). We need an expression for the CLR calculation in ATM networks where the statistical multiplexing is an important factor. The existing analytical methods for the CLR estimation are mostly based on fluid-flow and stationary approximation models. In this paper, we first evaluate these methods against the results obtained through simulation. The simulation is done at the cell level that provides very accurate results with buffer size as a variant. It is shown that the CLR estimation based on existing analytical models are widely overestimated. We have, then, proposed two new approaches that yield significant improvement in the accuracy of the CLR approximation. First, we have found global correction coefficients to compensate for the error of the current analytical methods. Second, we have proposed a new upper bound based on exact modeling of system behavior in the finite buffer case. This is a novel approach that combines fluid-flow and stationary approximation models and outperforms all the previous ones. The accuracy of the proposed model is verified by simulation.

# Index Terms—ATM, QoS, CLR approximation.

# I. INTRODUCTION

SYNCHRONOUS transfer mode (ATM) is a cell Aswitching technology that can handle multiple types of traffic with different quality of service (QoS) requirements and diverse traffic characteristics. To facilitate the coexistence of multiple traffic classes, virtual path (VP) subnetworks within the ATM network have been proposed. Therefore, a VP is a single logical direct link between two nodes that can be shared by many virtual circuits (VCs) similar bandwidth characteristics and OoS with requirements. The VP concept simplifies traffic control and resource management. As a consequence, processing requirement for call establishment decreases and routing becomes more flexible. Statistical multiplexing of VCs enables efficient use of transmission capacity for bursty sources.

We are interested in finding accurate practical expressions for the cell loss ratio in VP-based ATM networks. This expression will be used in the call

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admission control (CAC) mechanism and routing algorithms of ATM networks. The call admission control scheme determines whether a call can be routed on a VP without violating the guaranteed QoS requirements of the existing calls. The essential issue of ATM call admission control is exact estimation of network performance through real-time calculations. Finite-buffer fluid-flow model [1], [2] and the equivalent capacity (effective bandwidth) concept proposed by Guerin *et al.* [3] are the foundations of many CAC algorithms [4]-[8]. The simplifying assumption of ( $\beta = 1$ ) used in [3] results in ignoring the effect of statistical multiplexing. Therefore, the equivalent capacity and the cell loss probability expressions, which are obtained using this assumption are not accurate.

In this work, we first study four expressions for approximating CLR based on stationary and fluid-flow models. We have built an accurate numerical (simulation) model for a finite buffer system at the cell level. We use the results of the simulations for evaluation of the analytical models and it is shown that the existing models are overly loose. It is also shown that these models are complementary and their combination through a minimum operator provides a tighter upper bound for the CLR. We have, then, proposed two new approaches that yield significant improvements in the accuracy of CLR approximation. First, we have found global correction coefficients to compensate for the error of the analytical methods while preserve their upper bound property. Second, we have proposed a new upper bound based on exact modeling of system behavior in the finite buffer case. This is a novel approach that combines fluid-flow and stationary approximation models and outperforms all the previous ones. The accuracy of the proposed model is verified by simulation.

The remainder of this paper is organized as follows: Section II gives an introduction to the traffic model of sources. The equivalent capacity and existing analytical cell loss probability expressions are discussed in Section III. In Section IV, we propose an accurate numerical model for finding the cell loss probability for the finite buffer case. Also, the simulation results and evaluations of the existing models for CLR estimation are given in this section. In Section V, we propose two new models for CLR approximation in ATM networks. Section VI contains the main conclusions and related discussions.

### II. TRAFFIC MODEL

In this paper, we consider statistically independent twostate On-Off fluid-flow [1] sources. Such a source in an On

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period transmits at the peak rate and in an *Off* period does not generate any traffic. The duration of the *On* and the *Off* periods are assumed to be exponentially distributed. The time unit is selected to be the average *On* period, and the average *Off* period is denoted by  $1/\delta$ .

The *On-Off* Markov model is simple and flexible, as it can be used for modeling traffic streams ranging from burst to continuous bit. This model can be used for VBR as well as CBR sources [9]. This model have also been successfully used to characterize the *On-Off* nature of an individual source or source element, like packetized voice and video [10].

In the literature, many studies have been reported on the characterization of ATM statistical multiplexers using twostate *On-Off* model [1], [3], [9]-[17]. These analyses are based on exponential-type tail probabilities for the cell loss distribution. There are also traffic models based on the long-range dependence, or self-similarity, which is shown to be applicable to video traffic and LAN traffic. However, for most traffic streams, and especially for superposition of them, models with exponential-type tails work well for a wide range of buffer sizes of interest (e.g., real-time services) [15].

# III. EQUIVALENT CAPACITY AND CELL LOSS PROBABILITY

Use The equivalent capacity of a set of VCs statistically multiplexed on a VP is defined as the amount of bit rate required to achieve a desired QoS requirement, such as the cell loss probability  $P_{loss}$ . The cell loss probability is equal to the probability of buffer overflow. It is a function of the traffic characteristics of sources and the available network resources such as buffers. Guerin et al. [3] proposed two approximation, one of which is based on the fluid-flow approximation and the other one is relies on the stationary approximation. In the stationary approximation, the distribution of the aggregate stationary bit rate is approximated by a binomial distribution in the case of identical two-state Markov sources and also can be approximated by a Gaussian distribution in general (e.g., heterogeneous sources). The first approximation accurately estimates the equivalent capacity if the impact of individual connection characteristics is critical. The two approaches of the second approximation (Stationary models) are good representatives of bandwidth requirements when the effect of statistical multiplexing is significant [3]. However, because both approximations are conservative and are inaccurate for different ranges of connection characteristics (which will be shown by simulation results), these models complement each other.

In the following, we review existing analytical models and find expressions for the cell loss probability to be used in ATM CAC and routing mechanisms.

### A. Fluid-Flow Approximation

The fluid-flow model for two-state Markov sources is proposed in [1], [3]. In this model, the bit rate generated by a number of statistically multiplexed VCs is represented as a continuous flow of bits with varying intensity according to the state of an underlying continuous-time Markov chain. We first consider the case of a single two-state Markov source described by a triplet  $(r, \rho, b)$ , where r is

TABLE I LIST OF THE NOTATION USED IN THE TEXT

Symbol	Description				
$1/\delta$	Average Off period (average On period is 1)				
r	Peak rate of the source (VC)				
ρ	Fraction of time the source is active				
т	Mean aggregate bit rate				
σ	Standard deviation of the aggregate bit rate				
з	Desired CLR				
x	Buffer capacity				
L	Number of sources (VCs) present in the link (VP)				
С	Link (VP) Capacity				
c'	Equivalent capacity of the source (VC)				
F	Ratio of the link capacity to the source peak rate $(F = C/r)$				

the peak rate,  $\rho$  is the fraction of time the source is active and *b* is the mean of the *On* period. Other parameters of interest, such as the mean *m* and the variance  $\sigma^2$  of the bit rate are identified completely from the source metric vector  $(r, \rho, b)$ . In this case, the distribution of the buffer contents can be derived using standard techniques for either infinite or finite buffer systems. In the case of finite buffer size *x*, the capacity required, *c'*, so that the CLR is limited to  $\varepsilon$  is defined as the equivalent capacity and is found as follows [3]

$$\varepsilon = \beta \exp\left(-\frac{x(c'-r\rho)}{b(1-\rho)(r-c')c'}\right)$$
(1)

where

$$\beta = \frac{(c'-r\rho) + \varepsilon\rho (r-c')}{(1-\rho)c'}$$

The notations used in the text are listed in Table I.

In Section V, we will show that an infinite buffer system satisfies the same equation, only with different value of  $\beta$ . From (1), it can be seen that, even for a single VC, there is no explicit expression for the equivalent capacity, and (1) must be solved numerically. However,  $\beta$  is typically close to (in fact, always smaller than) 1 and approximating  $\beta$  by 1 provides explicit expressions for *c'* and  $\varepsilon$ , which are slightly greater than the exact values.

In the case of multiple heterogeneous superposed sources, the approach is more complex than a single source. In the special case of N multiplexed two-state Markov sources, the VP equivalent capacity is of the form [3]

$$C = \sum_{i=1}^{N} c'_{i} .$$
 (2)

Let *C* be the VP capacity and *L*-*1* is the number of VCs present in the VP. Our objective is to determine the admissibility of the *L*-th call without violating the target cell loss probability ( $\varepsilon$ ). Let *F* be the ratio of the VP capacity *C* to the VC peak rate *r* (*F* = *C*/*r*). In a homogeneous environment, through (2), we have c' = C/L. In other words, the VP capacity is divided up equally among the *L* identical VCs. Therefore, with simplifying assumption  $\beta = 1$ , and after some manipulation (see more details in [11]), the cell loss probability,  $Pl_{loss}(L)$ , is found as follows:

$$\begin{cases} P1_{loss}(L) = \exp\left(-\frac{x}{r}\left(1+\delta-\frac{L\delta}{F}\right)/\left(1-\frac{F}{L}\right)\right) if(L>F) \\ P1_{loss}(L) = 0 \quad if(L\le F) \end{cases}$$
(3)

Nevertheless, the simplifying assumption  $\beta = 1$  results in ignoring the effect of statistical multiplexing on the cell loss probability. Therefore, a modification is needed to accurately estimate the  $P_{loss}$  for cases in which statistical multiplexing is significant. Although we can not find an explicit expression for the equivalent capacity from (1), but we can find an expression for cell loss ratio,  $P2_{loss}(L)$ , without assuming  $\beta = 1$ , as follows:

$$P2_{loss}(L) = \frac{\beta_1 . P1_{loss}(L)}{1 - \beta_2 . P1_{loss}(L)}$$
(4)

where

$$\beta_1 = 1 + \delta \left(1 - \frac{L}{F}\right)$$
 and  $\beta_2 = \delta \left(\frac{L}{F} - 1\right)$ 

### B. Stationary Approximation

In the following, we introduce two equations for  $P_{loss}$  based on the stationary approximation proposed in [3].

# 1) Stationary Approximation Using Binomial Distribution

In the special case of N identical two-state Markov sources, we can consider the stationary bit rate distribution as a binomial distribution. Let C be the VP capacity and B the aggregate bit rate generated by N sources and  $\varepsilon$  the desired cell loss probability, we have:  $\Pr(B > C) \le \varepsilon$ 

This means that, the frequency of overload periods must be less than  $\varepsilon$ . In the case of N identical two-state Markov sources, the probability  $P_k$ , that k out of N sources are active is given by a binomial distribution [3]

 $P_{k} = \binom{N}{k} \rho^{k} (1-\rho)^{N-k} .$ 

The value of C, i.e., the smallest VP capacity needed to satisfy the desired cell loss probability, is then obtained by finding the smallest integer k' such that:

$$\sum_{k=k'+1}^{N} P_k \leq$$

The stationary approximation then gives:

$$C = k'r \tag{5}$$

where r is the peak rate of each source. We need to find  $P3_{loss}(L)$ , which is the cell loss probability computed from stationary approximation using binomial distribution. From (5), we obtain:

$$F = k' = \left[\frac{C}{r}\right], P_k = {\binom{L}{k}} \rho^k (1-\rho)^{L-k}$$
$$p3_{loss} (L) = \sum_{k=F+1}^{L} P_k$$
(6)

# 2) Stationary Approximation Using Gaussian distribution

In a general case (e.g., non-homogeneous sources), the computation of C is more complex than the special case

discussed earlier. However, in most cases when the effect of statistical multiplexing is of significance, the distribution of the stationary bit rate can be rather accurately approximated by a Gaussian distribution [3]. A good approximation is given by:

$$C = m + \alpha'\sigma, \quad \alpha' = \sqrt{-2\ln(\varepsilon) - \ln(2\pi)} \quad (7)$$

where *m* is the mean aggregate bit rate and  $\sigma$  is the standard deviation of the aggregate bit rate, we have:

$$\sigma^{2} = \sum_{i=1}^{N} \sigma_{i}^{2}$$
,  $m = \sum_{i=1}^{N} m_{i}$ 

Now, we should obtain  $P4_{loss}(L)$ , which is the cell loss probability based on stationary approximation using Gaussian distribution. From (7), and after some manipulation (see more details in [11]), we have:

$$P4_{loss}(L) = \exp[\frac{r^2}{L}(-\frac{(F-\rho L)^2}{2L\sigma^2}) - 0.5\ln(2\pi)]$$
(8)

IV. NUMERICAL STUDY AND SIMULATION RESULTS

## A. System Modeling for Simulation

In this section, an accurate numerical model for obtaining the cell loss ratio through simulation will be introduced. Just like the last section, here again, we consider a finite buffer with the capacity of x (Mbits), the FIFO queuing, and two-state Markov (*On-Off*) arrival traffic, such that *On* and *Off* periods have the exponential distribution, with respective means of 1 (s) and  $1/\delta$  (s). The source bit rate is zero during the *Off* periods and r (Mbps) in the *On* periods. The VP capacity is *C* (Mbps) and we have a finite buffer of size x, receiving traffic from *L On-Off* sources and is discharged at the constant rate of *C*. The objective is to find the buffer overflow probability (the loss probability). A discrete event simulation is built in C<sup>++</sup> to obtain the *P*<sub>loss</sub> for different values of *L*, x,  $\delta$ , and *F*.

- The simulation is done at the cell level and the results of the simulations are accurate with a confidence interval of  $P_{loss} \pm 10^{-10}$  and confidence level of 99.9%.
- We compare the results of the simulation with the results of the fluid-flow approximation, stationary approximation using Gaussian distribution and stationary approximation using binomial distribution methods
- We study the applicability of the analytical methods for the P<sub>loss</sub> approximation as a function of L and δ. These results lead to a new expression, based on the combination of the three analytical methods.
- The result of the simulation will help us to determine the minimum, the maximum and the average error of each of the analytical methods. These results lead us to the error correction factors applied on analytical results to compensate for their errors.

Fig. 1 shows the details of the model used in this work.

### B. Numerical Evaluation of Existing Models

In this section, we will present the results of the simulation. We compare  $P1_{loss}$ ,  $P2_{loss}$ ,  $P3_{loss}$  and  $P4_{loss}$  of (3), (4), (6), and (8) against the simulation results.



Fig. 1. Numerical model of finite buffer with On-Off Markov sources.



Fig. 2. Comparison of the  $PN_{loss}$ . with  $P1_{loss}$ ,  $P1_{loss}$ ,  $P3_{loss}$  and  $P4_{loss}$ , when  $\delta = 0.125$ , F = 50, and x = 24.



Fig. 3. Comparison of the  $PN_{loss}$  with  $P1_{loss}$ ,  $P1_{loss}$ ,  $P3_{loss}$  and  $P4_{loss}$  when  $\delta = 1.0$ , F = 50, and x = 24.

System parameters are assumed to be C = 150 Mbps, r = 3 Mbps, F = 50, and x = 24 Mbps. Figs 2-4 compare the  $P1_{loss}$ ,  $P2_{loss}$ ,  $P3_{loss}$  and  $P4_{loss}$  expressions with the  $PN_{loss}$  obtained from simulation for different values of  $\delta$ , 0.125, 1.0, and 5.0, respectively. Note that in Fig. 4, there is a singularity at L = 60 for  $P2_{loss}$ , because at this point  $\beta_1$  is equal to zero. CLR can be calculated at this point using linear interpolation:

$$P2_{loss}(L) = (P2_{loss}(L-1) + P2_{loss}(L+1))/2$$
(9)

Figs. 2-4 demonstrate that  $P4_{loss}$  is very much different from  $P1_{loss}$ ,  $P2_{loss}$ ,  $P3_{loss}$  and  $PN_{loss}$ . Therefore, the stationary model using Gaussian distribution is not a good approximation for CLR in the case of two-state Markov sources. This conclusion was not unpredictable, because this model is a general approximation, but the other two analytical models are specialized for two-state Markov sources.

Also, it is shown that  $P1_{loss}$ ,  $P2_{loss}$ ,  $P3_{loss}$ , and  $P4_{loss}$  are all overly loose at least in some practical range of the CLR. We have repeated the study for a wide range of F and x and have found the similar results.

# V.NEW ACCURATE CLR APPROXIMATION TECHNIQUES

The results of the simulation show that the existing analytical models are not accurate. In this section two new techniques are proposed. We have shown that they yield significant improvement in CLR approximation. First, we find global correction coefficients to compensate for the



Fig. 4. Comparison of the  $PN_{loss}$ . with  $P1_{loss}$ ,  $P1_{loss}$ ,  $P3_{loss}$  and  $P4_{loss}$ , when  $\delta = 5.0$ , F = 50, and x = 24.

TABLE II
THE MAXIMUM, MINIMUM AND AVERAGE OF THE ERROR CORRECTION
COEFFICIENTS

		PN <sub>loss</sub> / P1 <sub>loss</sub>	PN <sub>loss</sub> / P3 <sub>loss</sub>	PN <sub>loss</sub> / P4 <sub>loss</sub>
δ = 0.125	Maximum	4.48E-03	9.55E-03	2.41E-03
	Minimum	2.51E-06	4.15E-05	1.89E-07
	Average	9.47E-04	2.58E-03	3.77E-04
	Maximum	1.04E-02	5.70E-03	3.36E-03
$\delta = 1.0$ $\delta = 5.0$	Minimum	2.11E-04	5.85E-06	1.93E-07
	Average	3.31E-03	1.11E-03	5.29E-04
	Maximum	8.05E-03	6.59E-03	7.37E-03
	Minimum	2.94E-03	1.76E-07	1.01E-07
	Average	5.12E-03	2.24E-03	2.49E-03

error of the analytical methods. We then combined these modified results through a Min operator. Second, we propose a new upper bound based on combination of fluidflow and stationary approximation models

#### A. Error Compensation of the Existing models

Let us define  $\alpha_i$  to be equal to the ratio of actual CLR (estimated through simulation) to that obtained from the analytical model  $Pi_{loss}$  We denote  $\alpha_i$  as the error correction coefficient. Table II shows the minimum, the maximum and the average of the error correction coefficient for the three analytical models in the practical range. Let us define  $\alpha_1$ ,  $\alpha_3$  and  $\alpha_4$  as the maximum values of the error correction coefficients for the three analytical models, respectively. We maintain that new CLR estimations through the following still provide valid upper bounds for CLR:

$$P1_{loss} = \alpha_1 \times P1_{loss} (old)$$
  

$$P3_{loss} = \alpha_3 \times P3_{loss} (old)$$
  

$$P4_{loss} = \alpha_4 \times P4_{loss} (old)$$

where  $\alpha_1 = 1.04 \times 10^{-2}$ ,  $\alpha_3 = 9.55 \times 10^{-3}$ , and  $\alpha_4 = 7.37 \times 10^{-3}$ . Since  $\alpha_i \ll 1$ , new upper bounds are much more accurate than old ones.

Our simulation study in the previous section indicated that the inaccuracy of the analytical models vary for different ranges of connection parameters. Since all of these models are valid upper bounds, the minimum of these values will give us a more accurate estimation in all ranges

$$P_{loss}(L) = \min\{P1_{loss}(L), P3_{loss}(L), P4_{loss}(L)\}$$
(10)

Fig. 5 shows the results of CLR estimation through (10) in comparison with the simulation results. We repeated the simulation for other values of the VP capacity and the buffer size, and we saw that (10) is always a valid upper bound for  $P_{lass}$ .

### B. A New Accurate Model for CLR Approximation

In this section we propose a new more accurate analytical model for CLR approximation. This approach is based on a modified version of the fluid-flow approximation designed for accurate CLR estimation with a finite buffer.

In the infinite-buffer fluid-flow model with N two-state Markov sources, we have the following:

 $F_i(x)$  = equilibrium probability that *i* sources are On and buffer content does not exceed *x*.

The following expression represents the model in a matrix notation:

$$D \frac{d}{dx} F(x) = MF(x), \quad x \ge 0,$$
  

$$D = diag \{-F, 1 - F, 2 - f, ..., N - F\},$$
  

$$M = \begin{bmatrix} -N\delta & 1 \\ N\delta & -\{(N-1)\delta + 1\} & 2 \\ (N-1)\delta & -\{(N-2)\delta + 2\} \end{bmatrix}$$
  

$$M = \begin{bmatrix} 3 & ... \\ \vdots \\ 2\delta & -\{\delta + (N-1)\} & N \\ \delta & -N \end{bmatrix}$$

and

 $G(x) = \Pr(\text{Buffer content} > x) = 1 - I'F(x), \quad x \ge 0.$ 

where I denotes the unity vector and prime denotes transposition. G(x) is referred to as the 'Probability of buffer overflow beyond x', and is obtained as follows [1]

$$G(x) = -\sum_{i=0}^{N-[F]-1} e^{z_i x} a_i (I^{\Phi}_i)$$
(11)

where  $\{z_i\}$  are the eigenvalues of  $D^{-1}M$  and  $\{\Phi_i\}$  are the associated right eigenvectors and  $\{a_i\}$  are the coefficients, which are obtained as follows

$$a_j = -(\frac{\delta}{\delta + 1})^N \prod_{i=0(i \neq j)}^{N-[F]-1} \frac{z_i}{z_i - z_j} , \quad 0 \le j \le N-[F]-1$$

Since the form of G(x) is a sum of exponential terms and the computation of this expression is complex for call admission and routing mechanisms of high speed networks, for simplicity of computation, G(x) is approximated by the term with the largest negative exponent

$$G(x) \cong -a_0 (I'\Phi_0) e^{z_0 x} \tag{12}$$



Fig. 5. Comparison of the  $PN_{loss}$ . with (10) when  $\delta = 5.0, F = 50$ , and x = 24.

where

$$I'\Phi_0 = (\frac{N}{F})^N$$

and  $z_0$  is the largest negative eigenvalue of  $D^{-1}M$  and is obtained as follows

$$z_0 = -\frac{(1+\delta - \frac{N\delta}{F})}{1-\frac{F}{N}},$$

We first address the case of a single two-state Markov source. Assuming N = 1 in the above model, we find the following expression, which is the same as (1), only with a different value for  $\beta$  (Note that the unit of time is selected to be the average *On* period (b = 1) in this model):

$$P_{loss} = \beta \exp\left(-\frac{x(c'-r\rho)}{(1-\rho)(r-c')c'}\right)$$
(13)

where

$$\beta = \frac{r\rho}{c'}$$

In the special case of N multiplexed two-state Markov sources, based on (2), we have c' = C/L and hence the following expression for the CLR is obtained.

$$\begin{cases} P5_{loss}(L) = \frac{L\delta}{F(\delta+1)} \exp\left(-\frac{x}{r}\left(1+\delta-\frac{L\delta}{F}\right)/\left(1-\frac{F}{L}\right)\right) & if(L>F) \\ P5_{loss}(L) = 0 & if(L\leq F) \end{cases}$$
(14)

The CLR expression (14) is not accurate (see Figs. 6-8), mainly because of the following reason. In this model, the buffer is considered infinite and the CLR is assumed to be equal to the probability of the buffer content being greater than the threshold of x. A more accurate assumption is 'the probability that the buffer content exceeds x and the aggregate rate of the On sources exceeds C. In other words, the percentage of the times in which the content of an infinite buffer exceeds the threshold x and the aggregate arrival rate is less than the service rate (the buffer is being emptied) must be eliminated from the cell loss probability. In a finite buffer case, if the aggregate arrival rate is less than the service rate, cells are not lost. In an infinite buffer case, if the buffer content is greater than x and the aggregate arrival rate is less than the service rate, the buffer content will decrease toward x. In this condition, although the buffer content is still more than x and consequently the



Fig. 6. Comparison of the  $PN_{loss}$  with  $P1_{loss}$  to  $P7_{loss}$ , when  $\delta = 0.125, F = 50$ , and x = 24.

above fluid-flow model supposes that cells are being lost, but in practice, this assumption are not correct and cells are not being lost.

We, therefore, propose the following scheme for the CLR estimation

 $P_{loss} = \Pr(\{ aggregate rate of the On sources > C \} \cap \{Buffer content > x \})$ 

In the extreme case of a bufferless system, the CLR is reduced to the probability that the aggregate rate of the Onsources exceeds C. Based on the independence assumption, we have

 $P_{loss} \approx \Pr(\text{Number of } On \text{ sources} > F) \times$ 

 $\Pr(\text{Buffer content} > x)$ .

The first term represents the stationary approximation using binomial distribution and the second term is the buffer overflow probability obtained from the fluid-flow approximation. Therefore, our new expression for the CLR is obtained as follows.:

$$P6_{loss} = P3_{loss} P5_{loss} . \tag{15}$$

In the case of the finite buffer, the same rationale applies and an accurate CLR estimation is obtained as follows

$$P7_{loss} = P3_{loss} P2_{loss} . (16)$$

Figs. 6-8 compare seven CLR expressions ( $P1_{loss}$  to  $P7_{loss}$ ) with the  $PN_{loss}$  (the simulation results) for  $\delta$  equal to 0.125, 1.0, and 5.0, respectively. These figures show that for different traffic, from nearly burst traffic to nearly constant bit rate, P7loss is the most accurate upper bound approximation for CLR. The execution time is of the order of a few microseconds using a typical commercial processor. We can write expression (16) in the following form

$$P7_{loss} = P2_{loss} \sum_{k=F+1}^{L} (\prod_{i=1}^{L-k} \frac{k+i}{i}) \rho^{k} (1-\rho)^{L-k} .$$
 (17)

There are similar ideas but different approaches in [9], [18]. In [9], Thuy and Ha have approximated the overall cell loss ratio to be equal to the probability of cell loss without the buffer multiplies by the probability of the queue exceeding the buffer size x, or the overflow probability  $CLR \approx CLR_{ub}$  Pr(Buffer content > x)

In their paper, the bufferless cell loss ratio has been approximated by the following integral:

Number of VCs (L)



Fig. 7. Comparison of the  $PN_{loss}$ . with  $P1_{loss}$  to  $P7_{loss}$ , when  $\delta = 1.0, F = 50$ , and x = 24.



Fig. 8. Comparison of the  $PN_{loss}$ . with  $P1_{loss}$  to  $P7_{loss}$ , when  $\delta = 5.0, F = 50$ , and x = 24.

$$CLR_{ub} = \frac{E[\lambda(t) - C]^+}{E[\lambda(t)]} = \int_C^\infty \frac{z - C}{m\sigma\sqrt{2\pi}} e^{\frac{(z-m)^2}{2\sigma^2}} dz$$

In this model, the aggregate rate process  $\lambda(t)$  with set of parameters  $(m, \sigma^2)$  has been approximated as a Gaussian process with mean m and variance  $\sigma^2$ . Finally, after solving the above integral, the following expression has been found ( $\Gamma$  denoting the Gamma function)

$$CLR_{ub} = \frac{1}{2\sqrt{2}}(C-m)\{\Gamma(m,\sigma^2)\}$$

Since our simulation results showed that the stationary approximation by Gaussian distribution is a very loose approximation for aggregate bit rate (see  $P4_{loss}$  in Figs 2-4), a more tight upper bound approximation of CLR is presented in (17).

Also, in [18], Yan and Beshai have considered the cell loss ratio as follows

 $\ln(P1_{loss}) = \ln(\beta) + \ln(\eta)$ 

where  $\beta$  is the probability of joining the buffer,  $v = (L/F) \times \rho$  is the link mean cell occupancy, and b is the blocking probability in a bufferless loss system with F = C/r servers and L sources (Erl denoting the Erlang function)

$$b = (1 - \frac{F}{L}) \operatorname{Erl}(L\rho, F)$$
$$\beta = \frac{b}{1 - \upsilon + \upsilon b}$$

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Also  $\eta$  is the conditional cell overflow probability as

$$\ln(\eta) = \frac{x}{r} \frac{Lr}{C} \left(\frac{\rho}{1-\rho} - \frac{C}{Lr-C}\right)$$

Note that  $\eta$  is exactly equal to  $Pl_{loss}$ , which has been found from (1) with the simplifying assumption ( $\beta = 1$ ). Since our simulation results showed that  $P2_{loss}$  is more accurate than  $Pl_{loss}$  (see Figs. 2-4), a tighter upper bound for the CLR is presented in (17).

### VI. CONCLUSION

In this paper, first we discussed three analytical approximation models for cell loss ratio in the finite buffer system. Second, we provided an accurate numerical model for simulation of a buffer with the buffer size as a variant. We used the simulation results to evaluate the analytical models and we showed that the existing analytical models are all overly loose at least in some practical range of the CLR. Then we proposed two new approaches to increase the accuracy of CLR approximation. First, we found global correction coefficients to compensate for the error of the analytical methods. Second, we proposed a new tight upper bound based on exact modeling of system behavior in the finite buffer case. We combined the fluid-flow and the stationary approximation models and we showed that this novel approach outperforms all the previous ones. The simulation results verified the accuracy of the proposed model.

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