Analysis of Unbalance Due to Asymmetrical Loads

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***Abstract*—The paper analyses current unbalance caused by loads connected either to line or phase voltages. Follow-up voltage unbalance is calculated for receiving end of HV line. Different definitions of balanced system of phasors are discussed and illustrated by examples. Current unbalance factors for all possible load connections are described and analysed in detail and the ranges of their possible values are determined. Also formulae for these factors are shown for two single-phase loads.**

***Index Terms*—Asymmetrical load, symmetrical components, unbalance factor.**

I. Introduction

When evaluating quality of electricity today, the unbalance is considered according to the value of voltage unbalance coefficient [1]. Voltage unbalance arises mainly as a consequence of unbalanced currents in points of common coupling drawn by asymmetrical loads. The other elements of power system which are symmetrical (or can be considered as symmetrical with relatively high accuracy) are participating on voltage unbalance origin only as causa remota.

The term balanced multi-phase system expresses system, whose phasors are of the same amplitude and displaced from each other by the same angle. If any of these conditions is not met, the system is considered as unbalanced. The unbalanced system can further be counterweighted (the sum of phasors which create the system is equal to zero) or non-counterweighted (the sum of phasors is not equal to zero).

For three-phase systems, the phasors are generally considered as balanced, if their amplitudes are equal and the displacement from each other in complex plane is 120^o ("classical" definition of balanced system, given also by standards, e.g. IEC 50 (161), [2]). This is valid equally for voltages as well as for currents. In case, when system of voltages and currents is analyzed, another condition should be added - the same phase sequence for voltages and currents.

large loads in power system are mostly three-phase symmetrical ones. A great number of existing small single-phase loads does not exceedingly distort the symmetry, as these loads are distributed almost evenly to all three phases and statistical variation in powers drawn by each phase is usually insignificant. Unbalance is mainly caused by big single-phase loads, as single-phase electric railways, big single-phase electric furnaces (induction or arc ones) and others. Unbalanced currents drawn by these loads are flowing via respective elements of power system, unbalanced voltage drops are created and thus the voltage system formerly balanced becomes also unbalanced. Power system then feeds all loads by unbalanced voltage system and even symmetrical loads are loaded by unbalanced currents.

The unbalance can also occur under fault conditions in power system (short-circuits), as a result of incorrect operation of switching devices, in case of conductor abruptions, etc. All these cases usually have a short duration (excluding single-line-to-ground faults in systems with neutral not grounded, or grounded via impedance). Unbalance is also caused by voltage and current harmonics.

Voltage unbalance in HV and UHV systems does not almost cause problems, as even asymmetrical loads do not originate voltage unbalance, whose values (evaluated according to the voltage unbalance factor ρ_u) are exceeding the limit levels. This is mainly due to the value of network impedance in points of common coupling, where the asymmetrical loads are connected. The values of that impedance is relatively small in most cases, as the short-circuit powers of network in these points are high enough. Current unbalance is not yet generally (with few exceptions) considered a quality parameter of electricity according to the valid standards. Thus some authors dedicate it only a minor concern [3]-[5]. Research of voltage unbalance is of course very important particularly for its influence on symmetrical three-phase loads, which are loaded by unbalanced currents when connected to the unbalanced voltages. It reduces their performance and operating life. Considerably affected are mainly electric rotating machines [6]-[8]. Current unbalance can reach large values, but even highly asymmetrical small loads have no further negative influence on the power system, however big asymmetrical loads can distort formerly balanced voltages [9].

Fortes cue method of symmetrical components is used in calculations in this paper

$$\begin{Vmatrix} X_0 \\ X_+ \\ X_- \end{Vmatrix} = \frac{1}{3} \cdot \begin{Vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{Vmatrix} \cdot \begin{Vmatrix} X_A \\ X_B \\ X_C \end{Vmatrix} \quad . \tag{1}$$

where X_A, X_B, X_C are the phasors of unbalanced phasor system, X_0, X_+, X_- are the phasors of symmetrical components (zero, positive and negative sequence, respectively) and $a = 1\angle 120°$ is unit complex operator.

The level of unbalance is described by current unbalance factor (more precisely negative sequence current unbalance factor) ρ_i , which is given as the modulus of ratio of negative to positive sequence currents (same for the negative sequence voltage unbalance factor ρ_v)

Manuscript received June 8, 2004; revised November 25, 2004.

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Publisher Item Identifier S 1682-0053(05)0286

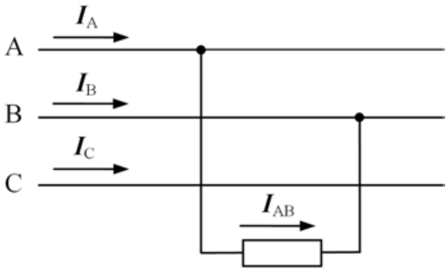

Fig. 1. One single-phase load connected to line voltage.

$$\rho_i = \frac{I_-}{I_+} \times 100\ (\%)\ ,\quad \rho_v = \frac{V_-}{V_+} \times 100\ (\%)\ . \qquad (2)$$

Moreover, in non-counterweighted systems the zero sequence current (voltage) unbalance factor ε_i (ε_v) is defined as the modulus of ratio of zero to positive sequence currents (voltages)

$$\varepsilon_i = \frac{I_0}{I_+} \times 100\,(\%)\ ,\quad \varepsilon_v = \frac{V_0}{V_+} \times 100\,(\%)\ . \qquad (3)$$

The latter factor determines the size of node of unbalanced phasor system shift towards the balanced one.

Contrary of the mentioned unbalance factors definition according to the IEC (International Electrotechnical Commission), NEMA (National Electrical Manufacturers Association) in standard [10] presents voltage unbalance factor only as the ratio of maximum deviation from the average line voltage to the average of three voltages. This definition, where phase asymmetry is not included can primarily be used for voltages investigation, where substantive differences among voltage magnitudes, or/and phase shifts from symmetrical positions are not frequently observed [5]. Various definitions of voltage unbalance factor are analysed e.g. in [11], [12]. Computations of current unbalance factors (especially for single- or two-phase loads) are to be carried according to the IEC definition.

The article presents detailed analysis and information about current unbalance factor values for all single-, two- and three-phase loads connections. Previous works were usually not aimed to determination of current unbalance factor values, some accomplished analysis for particular loads, e.g. [13], which is dedicated to power supply for electric railways.

II. Unbalance Caused by Loads Connected to Line Voltages

A. One Single-Phase Load Connected to Line voltage

Let us consider one load (e.g. traction transformer supplied by 110 kV line) connected as to line voltage V_{AB} as shown in Fig. 1. It is easy to prove that the value of negative sequence current unbalance factor ρ_i is always 100 %, regardless of the value of power drawn from the line, and power factor. Zero sequence current is not created, thus zero sequence current unbalance factor ε_i is always zero.

Determination of voltage unbalance caused by single-phase load is not that easy. Transmission equations for long lines, routine used for electric energy transmission calculations are valid for balanced loads only.

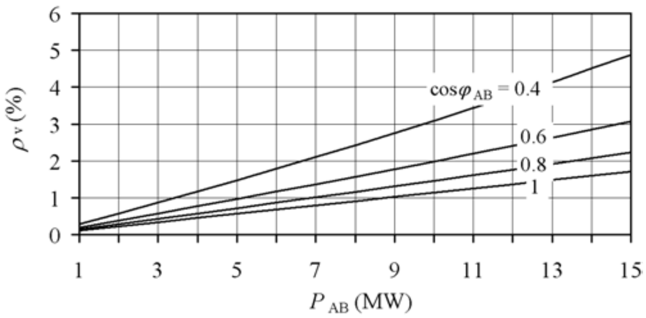

Fig. 2. Negative sequence voltage unbalance factor for single-phase load connected to line voltage.

Transmission for unbalanced loads can therefore be solved using comprehensive three-phase line model or symmetrical components.

Distribution or transmission HV and UHV lines can be regarded as symmetrical for unbalanced voltage drops calculations, and positive and negative sequence components of their impedances are equal [14]. Transmission equations thus can be solved separately for positive and negative sequence components. Actual values of currents and voltages are then obtained by combining two sets of results together. As the voltage at load's terminals depends on voltage drop along the line (and thus on value of line current as well), an iterative technique has to be chosen for all electrical quantities at the receiving end of the line calculation. The values of negative sequence voltage unbalance factor were calculated for load connected according to Fig. 1 for different values of power factor and variable real power drawn by load (Fig. 2). It was assumed that voltage unbalance arose only from 110 kV line. The line for the above calculations has the following parameters: $R_k = 0.1\ \Omega \cdot \text{km}^{-1}$, $L_k = 1.2\ \text{mH} \cdot \text{km}^{-1}$, $C_k = 9.0\ \text{nF} \cdot \text{km}^{-1}$, $G_k = 50\ \text{nS} \cdot \text{km}^{-1}$, $l = 35\ \text{km}$, $U_1 = 110\ \text{kV}$, $f = 50\ \text{Hz}$.

Note 1: The results of calculations of current unbalance factor for unsymmetrical load considering also voltage unbalance caused by the load itself at the feeding line are not substantially different from the results, where symmetrical voltages at the point of common coupling are taking into account [15].

Note 2: The unbalance will be affected by the harmonic components contained in currents drawn by the asymmetrical load. Phasors of harmonic order $h = 3k + 1$ $(k = 0, 1, 2...)$ will rotate in the positive sequence, phasors of harmonic order $h = 3k + 2$ will rotate in the negative sequence and phasors of harmonic order $h = 3k$ will be in phase. The influence of current harmonics will be different for balanced and unbalanced harmonic distortion and also for different phase shifts of individual harmonics. Calculations accomplished by some authors showed relatively small impact of current harmonic distortion created by non-linear loads on the values of current unbalance factor [16].

B. Two Single-Phase Loads Connected to Line Voltages

Two single-phase loads (e.g. two traction transformers supplied by 110 kV line) are connected as in Fig. 3 to two different line voltages V_{AB} and V_{BC}. The currents in line's conductors according to that figure are

$$I_A = I_{AB}\ ,\ \ I_B = I_{BC} - I_{AB}\ ,\ \ I_C = -I_{BC}\ . \qquad (4)$$

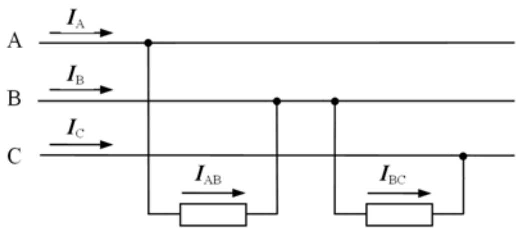

Fig. 3. Two single-phase loads connected to line voltages.

Phasors of symmetrical components can be determined using (1)

$$\begin{aligned} I_0 &= \frac{1}{3}(I_A + I_B + I_C) = \frac{1}{3}(I_{AB} + (I_{BC} - I_{AB}) - I_{BC}) = 0 \\ I_+ &= \frac{1}{3}(I_A + a\, I_B + a^2 I_C) = \frac{1}{3}(1-a)(I_{AB} + a\, I_{BC}) \\ I_- &= \frac{1}{3}(I_A + a^2 I_B + a\, I_C) = \frac{1}{3}(1-a)((1+a) I_{AB} - a I_{BC}) \end{aligned} \quad . (5)$$

Again, zero sequence current is not created in this case, thus zero sequence current unbalance factor ε_i has zero value. Negative sequence current unbalance factor

$$\rho_i = \frac{I_-}{I_+} \times 100 = \frac{|(1+a)\, I_{AB} - a I_{BC}|}{|I_{AB} + a\, I_{BC}|} \times 100 \,. \quad (6)$$

Using denomination k for the ratio of current's amplitudes and δ for the difference of phase shifts of both loads, which are respectively

$$k = \frac{I_{\mathrm{BC}}}{I_{\mathrm{AB}}}\,,\ \delta = \varphi_{\mathrm{BC}} - \varphi_{\mathrm{AB}}\,, \quad (7)$$

one can (after some manipulation) derive the following formula

$$\rho_i = \sqrt{\frac{1 + k^2 - 2 \cdot k \cdot \cos(\delta + 60°)}{1 + k^2 + 2 \cdot k \cdot \cos\delta}} \times 100 \,. \quad (8)$$

The values of negative sequence current unbalance factor for two single-phase loads, calculated from (8) are displayed in Fig. 4 [17] for the inductive character of loads, i.e. for the phase shifts of both loads $\varphi_{\mathrm{AB}} \in (0; 90^{\mathrm{o}})$ and $\varphi_{\mathrm{BC}} \in (0; 90^{\mathrm{o}})$. The values of the quantity δ are then $\delta \in (-90^{\mathrm{o}}; 90^{\mathrm{o}})$.

Some typical cases can be specified from (8) and Fig. 4:

(i) For two identical loads, $k = 1$, $\delta = 0$: the value of negative sequence current unbalance factor $\rho_{\mathrm{i}} = 50\ \%$. This value is often compared with the situation for one load.

(ii) In case of one load substantially higher then the other, $k >> 1$: the value of negative sequence current unbalance factor $\rho_{\mathrm{i}} = 100\ \%$, regardless of the value of δ. This case is close the situation when only one load is connected.

(iii) For $\delta = 60^{\mathrm{o}}$ the value of negative sequence current unbalance factor is always $\rho_{\mathrm{i}} = 100\ \%$, regardless of the values of loads (k). This point can be clearly seen in Fig. 4, but it has no special meaning from the operational point of view.

(iv) There also exists such combination of loads, where the value of negative sequence current unbalance factor $\rho_{\mathrm{i}} = 0\ \%$. This state can be determined from (8), setting the numerator equal to zero

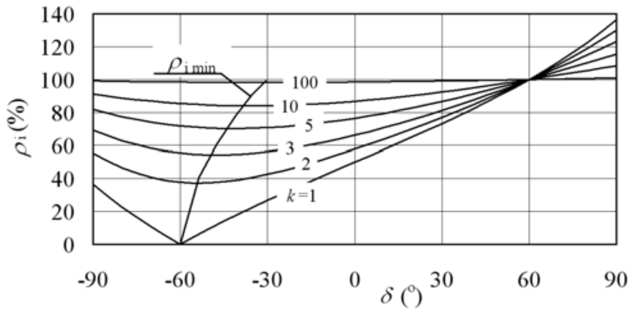

Fig. 4. Negative sequence current unbalance factor for two single-phase loads connected to line voltages.

$$1 + k^2 - 2 \cdot k \cdot \cos(\delta + 60°) = 0 \,. \quad (9)$$

As ρ_{i} is a real number, the determinant of quadratic equation (9) has to be nonnegative

$$4 \cdot \cos^2(\delta + 60^{\mathrm{o}}) - 4 \geq 0 \,. \quad (10)$$

This unequality on the interval $(-90^{\mathrm{o}}; 90^{\mathrm{o}})$ can only be satisfied for one value $\delta = -60^{\mathrm{o}}$. For this value the (9) gives the value $k = 1$. So for two loads with equal current amplitudes a zero unbalance can be achieved for number of combinations of phase shifts, e.g. $\cos\varphi_{\mathrm{BC}} = 1$ together with $\cos\varphi_{\mathrm{AB}} = 0.5$. Simultaneous meeting of both mentioned conditions has a low probability under common operating conditions.

(v) Minimal values of negative sequence current unbalance factor for two considered loads could be determined by finding the minimum of (8). After some manipulations, an equation for the value of $\delta_{\min}$ can be obtained. Negative sequence current unbalance factor for $\delta_{\min}$ and for certain ratio of currents k has the lowest value.

$$\delta_{\min} = 60^{\mathrm{o}} - \mathrm{acos}\left(\frac{-k}{1+k^2}\right). \quad (11)$$

Negative sequence current unbalance factor for $\delta_{\min}$ and for certain ratio of currents k has the lowest value (see Fig. 4).

(vi) Maximal values of negative sequence current unbalance factor are at the value $\delta = 90^{\mathrm{o}}$, considering interval $\delta \in (-90^{\mathrm{o}}; 90^{\mathrm{o}})$. If also loads of capacitive character are taken into account, when δ can reach values from interval $\delta \in (-180^{\mathrm{o}}; 180^{\mathrm{o}})$, then for $k = 1$ and $\delta = \pm 180^{o}$ the negative sequence current unbalance factor $\rho_{\mathrm{i}} \to \infty$, i.e. current has only negative sequence component and no positive sequence one. This can be achieved by connecting inductor and capacitor.

Generally, it is possible to state for two single-phase loads connected to line voltages that negative sequence current unbalance factor can reach values from 0 to ∞. Zero unbalance is for two currents of the same amplitude and for -60^{o} of phase shift difference, infinitely high unbalance occurs when only inductor and capacitor with equal impedances are connected. For two equal loads, negative sequence current unbalance factor has the value of 50 %.

In some cases, the results of calculations or/and measurements might have values not obvious at the first sight. For example, if the two loads from Fig. 3 have the following data $I_{\mathrm{AB}} = 100\ \mathrm{A}$, $\cos\varphi_{\mathrm{AB}} = 0.5$, $I_{\mathrm{BC}} = 100\ \mathrm{A}$

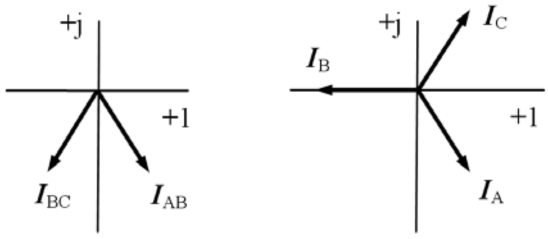

Fig. 5. Example of current phasor diagram for two loads from Fig. 3 (a) load currents, and (b) line currents.

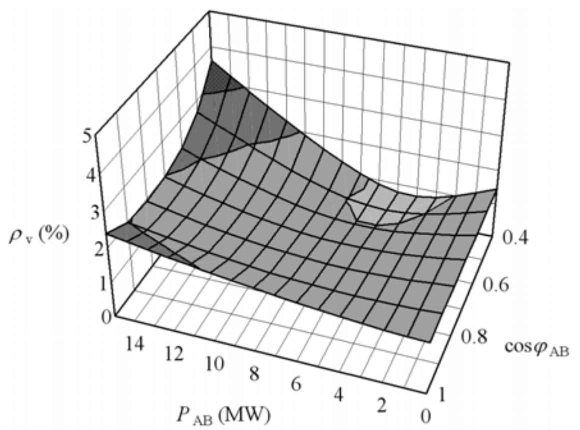

Fig. 6. Negative sequence voltage unbalance factor for two single-phase loads connected to line voltages.

following data $I_{AB} = 100\,\mathrm{A}$, $\cos\varphi_{AB} = 0.5$, $I_{BC} = 100\,\mathrm{A}$ and $\cos\varphi_{BC} = 1$, it is obvious that load currents do not comply with the classical definition of balanced system (Fig. 5(a)). Even though, negative sequence current unbalance factor, when calculating from load currents as well from line currents has value $\rho_i = 0\,\%$. Currents in the line are balanced (Fig. 5(b)) and the set of two loads loads the line evenly. Zero sequence current unbalance factor has the value $\varepsilon_i = 0$, when calculating from line currents, but value $\varepsilon_i = 100\ \%$ when calculating from load currents.

Determination of formulae for voltage unbalance factors for unbalance caused by two single-phase loads for different load parameters is basically not possible, similarly as for one single-phase load. Computation can again be accomplished using iterative technique separately for every case. Fig. 6 shows the results of negative sequence voltage unbalance factor calculations for unbalance arisen only from 110 kV line, whose parameters were mentioned at the end of subsection. II.A. One load has variable parameters (and $\cos\varphi_{AB}$), the second one has stable parameters $P_{BC} = 10\,\mathrm{MW}$ and $\cos\varphi_{BC} = 0.8$.

C. Three Single-Phase Loads Connected to Line Voltages

Three different single-phase loads connected to line voltages $\mathbf{V}_{AB}, \mathbf{V}_{BC}$, and $\mathbf{V}_{CA}$ can create unbalanced three-phase load. Unbalance can also originate during failures in formerly symmetrical delta-connected three-phase load. Current and voltage unbalance factors for unbalance caused by mentioned loads are to be calculated separately for each case.

Again, zero sequence current is not created in case of three single-phase loads connected to line voltages or asymmetrical delta-connected three-phase load, hence zero sequence current unbalance factor ε_i has zero value.

TABLE I
EXAMPLES OF CURRENT UNBALANCE FACTORS FOR LOADS CONNECTED TO LINE VOLTAGES

Example	i	ii	iii	iv	v
I_{AB} (A)	100	100	100	80	80
I_{BC} (A)	100	100	100	100	100
I_{CA} (A)	100	100	100	100	100
$\cos\varphi_{AB}$	0.7	0.7	0.7	0.8	0.7
$\cos\varphi_{BC}$	0.7	0.7	0.95	0.8	0.95
$\cos\varphi_{CA}$	0.7	0.9	0.8	0.8	0.8
ρ_i (%)	14.3	18.9	15.1	32.8	43.9

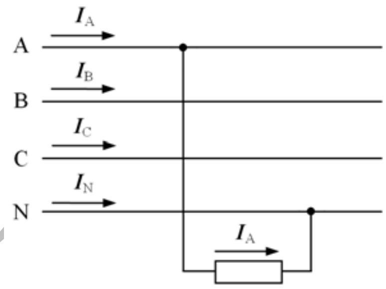

Fig. 7. One single-phase load connected to phase voltage.

To illustrate values of current unbalance factor some numerical examples were calculated for loads connected to all three phases: the load with one different current magnitude and equal power factors (i) or with one power factor changed (ii), the load with equal current magnitudes but different power factors (iii), the load with different current magnitudes and equal power factors (iv) and eventually the load with different current magnitudes and different power factors (v). The influence of these unsymmetrical loads on the value of current unbalance factor is shown in Table I. Values of current unbalance factor can vary in relatively wide range in real systems.

III. UNBALANCE CAUSED BY LOADS CONNECTED TO PHASE VOLTAGES

A. One Single-Phase Load Connected to Phase Voltage

Let us consider one load connected as in Fig. 7 to phase voltage $\mathbf{V}_A$. The currents in line's conductors according to that figure are

$$\mathbf{I}_A,\quad \mathbf{I}_B = 0,\quad \mathbf{I}_C = 0, \tag{12}$$

and phasors of symmetrical components can be determined using (1)

$$\begin{aligned} \mathbf{I}_0 &= \frac{1}{3}\cdot\left(\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C\right) = \frac{1}{3}\cdot\mathbf{I}_A \\ \mathbf{I}_+ &= \frac{1}{3}\cdot\left(\mathbf{I}_A + \mathbf{a}\cdot\mathbf{I}_B + \mathbf{a}^2\cdot\mathbf{I}_C\right) = \frac{1}{3}\cdot\mathbf{I}_A \\ \mathbf{I}_- &= \frac{1}{3}\cdot\left(\mathbf{I}_A + \mathbf{a}^2\cdot\mathbf{I}_B + \mathbf{a}\cdot\mathbf{I}_C\right) = \frac{1}{3}\cdot\mathbf{I}_A \end{aligned} \tag{13}$$

Neutral conductor is loaded by current

$$\mathbf{I}_N = -3\cdot\mathbf{I}_0 = -\mathbf{I}_A\,. \tag{14}$$

As seen from (13), zero sequence components are created in the line for this load and phasors of all symmetrical components have equal amplitudes. It can be further derived that one single-phase load connected to

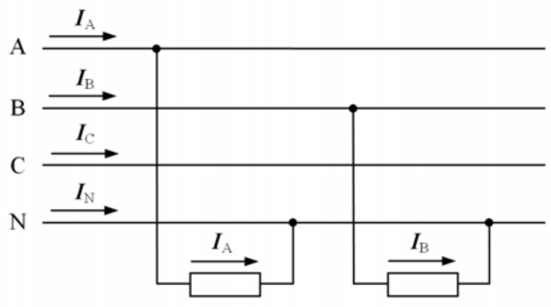

Fig. 8. Two single-phase loads connected to phase voltages.

phase voltage causes unbalance with values of negative/zero sequence current unbalance factors always being $\rho_{\text{i}} = 100$ % and $\varepsilon_{\text{i}} = 100$ % respectively, regardless of the value of power drawn from the line and power factor.

Unbalanced current flowing in one phase conductor and neutral conductor creates voltage drops and balanced voltage system is distorted.

B. Two Single-Phase Loads Connected to Phase Voltages

Two single-phase loads are connected as on Fig. 8 to two different phase voltages $\mathbf{V}_A$ and $\mathbf{V}_B$. The currents in line's conductors according to that figure are

$$\mathbf{I}_A,\quad \mathbf{I}_B,\quad \mathbf{I}_C = 0, \tag{15}$$

Phasors of symmetrical components can again be determined using (1)

$$\begin{aligned} \mathbf{I}_0 &= \frac{1}{3}\cdot(\mathbf{I}_{\text{A}}+\mathbf{I}_{\text{B}}+\mathbf{I}_{\text{C}}) = \frac{1}{3}\cdot(\mathbf{I}_{\text{A}}+\mathbf{I}_{\text{B}}) \\ \mathbf{I}_+ &= \frac{1}{3}\cdot\left(\mathbf{I}_{\text{A}}+\mathbf{a}\cdot\mathbf{I}_{\text{B}}+\mathbf{a}^2\cdot\mathbf{I}_{\text{C}}\right) = \frac{1}{3}\cdot\left(\mathbf{I}_{\text{A}}+\mathbf{a}\cdot\mathbf{I}_{\text{B}}\right) \\ \mathbf{I}_- &= \frac{1}{3}\cdot\left(\mathbf{I}_{\text{A}}+\mathbf{a}^2\cdot\mathbf{I}_{\text{B}}+\mathbf{a}\cdot\mathbf{I}_{\text{C}}\right) = \frac{1}{3}\cdot\left(\mathbf{I}_{\text{A}}+\mathbf{a}^2\cdot\mathbf{I}_{\text{B}}\right) \end{aligned} \quad . \tag{16}$$

Also for two loads connected this way zero sequence components are created.

Neutral conductor is loaded by current

$$\mathbf{I}_N = -3.\mathbf{I}_0 = -\mathbf{I}_A - \mathbf{I}_B\,. \tag{17}$$

Formula for negative sequence current unbalance factor calculated from currents determined by (16) is after some manipulations

$$\rho_{\text{i}} = \sqrt{\frac{1+k^2-2\cdot k\cdot\cos(\delta+60°)}{1+k^2+2\cdot k\cdot\cos\delta}}\times 100\,, \tag{18}$$

where the meaning of k and δ is now (respective)

$$\text{k} = \frac{I_{\text{B}}}{I_{\text{A}}}\,, \delta = \varphi_{\text{B}} - \varphi_{\text{A}}\,. \tag{19}$$

Formula (18) for determination of negative sequence current unbalance factor is formally identical to (8), which stands for two loads connected to line voltages. The only difference is the meaning of quantities k and δ, now given by (19). Regarding to that formal equality, the values of negative sequence current unbalance factor in Fig. 4, which were calculated for two loads connected to line voltages, are also valid for case of two loads connected to phase voltages (including typical values analysed in subsection II.B). The value of negative sequence current

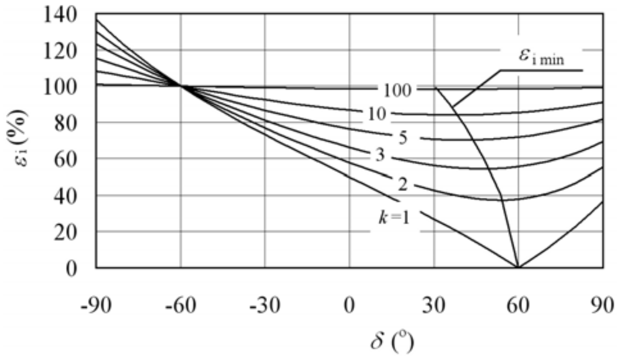

Fig. 9. Zero sequence current unbalance factor for two single-phase loads connected to phase voltages.

unbalance factor is $\rho_{\text{i}} = 50$ % for two identical loads and the other characteristic values correspond to the situation effective for loads connected to line voltages.

Zero sequence current unbalance factor determined according to (3) can be after some manipulations written as

$$\varepsilon_{\text{i}} = \sqrt{\frac{1+k^2-2\cdot k\cdot\cos(\delta-60°)}{1+k^2+2\cdot k\cdot\cos\delta}}\times 100\,, \tag{20}$$

where the values k and δ have the meaning given by (19).

Minimal values of zero sequence current unbalance factor for given ratio of current amplitudes k are at values of δ_{min}

$$\delta_{\text{min}} = \text{acos}\left(\frac{-k}{1+k^2}\right) - 60^{\text{o}}\,. \tag{21}$$

Values of zero sequence current unbalance factor (corresponding with (20) and (21)) are in the graph in Fig. 9 for the inductive character of loads. For that connection of two single-phase loads zero sequence current unbalance factor reaches significantly high values and can properly describe an asymmetrical load.

In case of zero sequence current unbalance factor there also exists such a combination of two single-phase loads, where its value is equal to zero. This situation can only occur in one special case, namely for $k = 1$ and $\delta = 60^{\text{o}}$ simultaneously (although the combination of two single-phase loads satisfying both mentioned conditions has a very low probability). This paradox again confirms the fact that quantities $_{\text{i}}$ and ε_{i} do not guarantee by their values alone the achievement of balanced state of set of phasors according to the classical definition.

For some combinations of loads, e.g. for $I_{\text{A}} = 100\,\text{A}$, $\cos\varphi_{\text{A}} = 0.5$, $I_{\text{B}} = 100\,\text{A}$ and $\cos\varphi_{\text{B}} = 1$, calculated or measured negative sequence current unbalance factor has value $\rho_{\text{i}} = 0$, but phasors do not satisfy classical definition of balanced system. Line conductors are unevenly loaded, neutral conductor is loaded by current of 173 A and $\varepsilon_{\text{i}} = 100$ %.

Here again the balanced voltage system is distorted due to uneven voltage drops along the line.

C. Three Single-Phase Loads Connected to Phase Voltages

Three single-phase loads connected to phase voltages (Fig. 10) can often be found in LV networks in residential areas. Three-phase way-connected loads are being

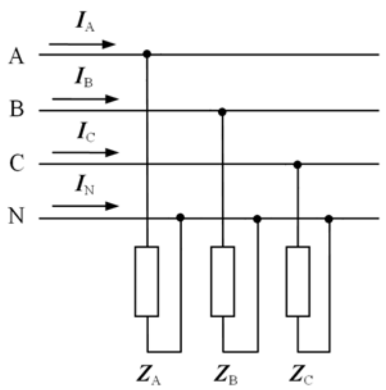

Fig. 10. Asymmetrical three-phase load composed of three single-phase loads connected to phase voltages.

as symmetrical ones, and only in case of failure can be in operation (for a certain period of time) as asymmetrical ones. The three-phase load's neutral could either be connected with transformer's neutral or not.

In case of three different single-phase loads from Fig. 10 and in case of an asymmetrical three-phase load with its neutral connected to neutral conductor the negative and zero sequence current unbalance factors ρ_i and ε_i have non-zero values. When such load is connected to source with sufficiently firm voltages, line voltages will stay unchanged. The positions of phase voltage phasors will be shifted due to voltage drop on neutral conductor, which is loaded by current $\boldsymbol{I}_{\mathrm{N}} = -3 \cdot \boldsymbol{I}_0$. When unbalanced voltage drops in phase conductors of the line are also taken into account, the line voltages in the point of common coupling will be unbalanced too.

Again, similar examples as in section 2.3 were calculated in Table II (loads in these examples have the same current magnitudes and power factors as in Table I, but are connected to phase voltages instead of line voltages). Negative sequence current unbalance factor ρ_i has the same values as in Table I, zero sequence current unbalance factor ε_{i} has non-zero values in this case.

In case of an asymmetrical three-phase load with its neutral not connected to neutral conductor, the phase voltages in load are unbalanced. Non-zero values of negative and zero sequence voltage unbalance factors ρ_{v} and ε_{v} for that load can be calculated or measured, even when a „firm“ source is assumed in the point of common coupling. After more detailed analysis, for this kind of load connected to the „firm“ source, the following can be found out:

- negative sequence voltage unbalance factor ρ_{v} is always equal to zero, load's voltages do not create a „classical" balanced system, phase voltages on load have different amplitudes and also different mutual phase shifts, their node is shifted against the node of source (or against the node of balanced system) by $\boldsymbol{V}_0$,
- zero sequence current unbalance factor ε_{i} is always equal to zero, as zero sequence components of current are not created for this case.

IV. Conclusions

Voltage unbalance is not a key point in power quality assessment today. But a high values of current unbalance can occur in some points of common coupling as a result of rising number and individual powers of single-phase loads as well as a result of current harmonics, which contribute to unbalance. Following suggestions can be used for calculations and measurements in unbalanced systems (see also [18]).

TABLE II
EXAMPLES OF CURRENT UNBALANCE FACTORS FOR LOADS, CONNECTED TO PHASE VOLTAGES

Example	i	ii	iii	iv	v
ρ_i (%)	14.3	18.9	15.1	32.8	43.9
ε_i (%)	14.3	18.9	13.4	32.8	24.1

Zero value of negative sequence unbalance factor can be reached when negative sequence component is zero. It is possible to prove that three phasors will not create negative sequence components when their end points are placed in the cusps of equilateral triangle and have the right phase sequence [19]. These phasors may not necessary fulfil the "classical" definition of balanced system. For instance, line currents can always be calculated from currents of delta-connected load but not vice-versa (except the symmetrical load). Various combinations of load's currents can create the same line currents. The same negative sequence current unbalance factor will be obtained when calculated (or measured) from line currents as well as from load's currents.

Zero value of zero sequence unbalance factor can be reached when the zero sequence component has zero value, i.e. when sum of phasors is zero. This condition is identical to the condition for the node of phasors, which has to be placed in the centroid of triangle, whose cusps are in the end points of these phasors. For instance, for wye-connected asymmetrical load with phase currents $I_{\mathrm{A}} = 70$ A, $\cos\varphi_{\mathrm{A}} = 1$, $I_{\mathrm{B}} = 150$ A, $\cos\varphi_{\mathrm{B}} = 0.94$ (ind.), $I_{\mathrm{C}} = 106.4$ A, $\cos\varphi_{\mathrm{C}} = 0.57$ (cap.) zero sequence current unbalance factor reaches the value $\varepsilon_{\mathrm{i}} = 0$ ($\rho_{\mathrm{i}} = 49.7$ %).

Generally, the following range of values can be expected in three-phase system:

- negative sequence unbalance factor reaches values from 0 to 100 % for only "amplitude" unbalance (phase shifts between the phasors are equal),
- negative sequence unbalance factor reaches values from 0 to $(1+\sqrt{3}) \cdot 100 = 273.205$ % for only „phase" unbalance (amplitudes of phasors are equal),
- negative sequence unbalance factor reaches values from 0 to ∞ for „mixed“ unbalance („amplitude" unbalance along with "phase" unbalance).

The phase sequence in unbalanced system is usually considered according to the theorem, assuming positive sequence components having the same phase sequence as authentic unbalanced system. However, this is not valid in all cases. For instance, in an unbalanced system with only "phase" unbalance and:

- $\rho_{\mathrm{i}} < 100$ %: the phase sequence of an authentic unbalanced system is equal to phase sequence of positive sequence components,
- $\rho_{\mathrm{i}} > 100$ %: the phase sequence of an authentic unbalanced system is opposite to phase sequence of positive sequence components.

Aforementioned consideration implies a better way for phase sequence definition. The phase sequence of voltage system can be taken as the basis, since much lower voltage unbalance can be expected, compared to current unbalance. Thus positive sequence current components have the same phase sequence as authentic (unbalanced or balanced) voltage system that is coupled with current system under investigation.

Theoretically, also such loads exist, for which positive sequence current components are not developed. Two inductors and one capacitor connected to line voltages can for instance create such load, where negative sequence current unbalance factor may reach value $\rho_i \to \infty$. That kind of load does not usually exist in regular operation but can be represented by some compensating installation being in operation with compensated load disconnected.

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