Discrete Sliding Mode Control for Processes with Long Dead-Time

D. H. Sha and V. B. Bajić

Abstract—**A new, input-output based discrete sliding mode control (DSMC) method is proposed for the control of processes with dead-time. However, it is necessary to combine DSMC with the Smith predictor for the dead-time compensation of processes with long dead-time. Because DSMC is sensitive to apparent dead-time in the process to be controlled, two Smith predictors were used in the control schema. One with high order is used to cancel out actual process output signal as good as possible. Another one with low order is used in the identification of the controlled process and in the design of DSMC. When this new DSMC is compared to the optimally tuned PID controller, it shows much better overall characteristics. Simulation experiments are made to illustrate the effectiveness of the new DSMC both for plants with short and with long dead-times.**

Index Terms—**Sliding mode control, process control, Smith predictor.**

I. Introduction

Control of processes with long dead-time is notoriously difficult [1]-[4]. Some authors like Shinskey [1] have an opinion that the dead-time is the greatest problem in process control. Different techniques are proposed to reduce the problem. One of the best known is the Smith predictor based control [5]. For control of processes with long dead-time we will combine the Smith predictor with the new discrete sliding mode control (DSMC) that is developed to cater for such situations.

The Sliding Mode Control (SMC) is characterized by the existence of a specific operation regime, the so-called sliding mode, which occurs on a predetermined sliding surface [6], [7]. The control is always designed to force system trajectory to reach the sliding surface and to slide along it, or to remain in its vicinity. The SMC is generally robust to plant parameter variations [6], [8], [9]. However, there are a number of problems with regard to practical utilization of the SMC. These motivated different developments of the SMC. Steady-state performance of the SMC for continuous systems has been improved by development of an integral variable structure control (IVSC), which comprises an integral controller followed by a variable structure controller (VSC), as proposed in [10]. IVSC for discrete time systems is developed in [11]. Chattering, which is an inherent problem of the basic SMC designs, is effectively reduced by solution in [10], as well as by other approaches such as via the quasi-sliding mode [12] and its variation in [13]. Another effective approach for dealing with chattering is given in [14]. The SMC for discrete time systems received great attention due to increased power of computerized control equipment [9], [11]-[13], [15]-[22]. The SMC, either continuous or discrete, is not always I/O based, and requires state measurements [9], [15], [16]. Practical control engineering applications in such cases generally demand the use of an observer in order to provide assessment of the system states for control implementation. In [17] and [18], an observer-based SMC for continuous multivariable systems is proposed. In [19] a version of an observer-based DSMC is proposed. A number of SMC results that are based on I/O measurements only are developed in [23]-[26], [27]. In [27], discrete-time sliding mode control strategies based on the multirate output feedback and the quasi-sliding mode control was discussed. However, all these studies haven't been applied to the systems with computational time delay and for systems that have dead-time [28].

In this paper we develop a new VSC for discrete time systems. Inspired by the results of [10], [11], [20], [21], [28], [29] and by extending these results further, we propose an I/O based DSMC for single input single output (SISO) discrete process control systems with dead-time. The new controller proposed combines: (a) a new nonlinear output feedback controller designed on the basis of the Lyapunov's direct method to guarantee the existence of a sliding mode, (b) an integral control, and (c) a pole placement procedure, which has been developed for determining the coefficients of the integral controller gain and of the sliding mode plane; this procedure is used to specify the dynamics of controlled system during the sliding regime. The solution deviates from the one proposed in [28] and [29] do not suit the need of control of plants with dead-time. Moreover, the solution presented in this paper removes restriction of the first order plant model as required in [28], and thus relates to the arbitrary order model case. The solution does not require the Smith predictor in the cases when the apparent dead-time is relatively small. However, if the dead-time is significant (as compared to the dominant plant time constant) the use of the Smith predictor is necessary.

The effectiveness of the proposed solution is verified via simulation applied to the control two high order processes with short and long dead-time and subjected to a sudden external disturbance. The comparison is made with the results of the control effects obtained by a PID controller optimally tuned for disturbance rejection [30] and [31]. The results achieved show that the proposed DSMC has much better overall performance than the optimally tuned PID controller. The DSMC shows much better tracking

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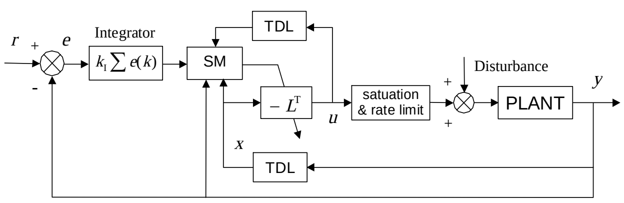

Fig. 1. DSMC block diagram for plant control.

characteristics and retains excellent disturbance rejection property equal to or better than that obtained by the conventional optimally tuned PID controller.

II. Discrete Sliding Mode Control

The proposed configuration of the DSMC, which is based exclusively on the I/O data, is shown in Fig. 1 and consists of three parts: (a) an integral control that is used to enable tracking of an arbitrary input signal to the system and to reduce-eliminate chattering, (b) a nonlinear output feedback control, and (c) an adjustable sliding mode block (SM) to guarantee the existence of sliding motion. TDL in Fig. 1 denotes a tapped delay line whose output vector has as its elements the delayed values of the input signal.

A. System Model in the Controllable Canonical Form

Let us consider a plant model with the transfer function in the discrete time domain given by

$$\frac{Y(z)}{U(z)} = \frac{z^{-d}(b_1 z^{-1} + b_2 z^{-2} + ... + b_m z^{-m})}{1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_n z^{-n}} = \frac{z^{-d-1} B(z^{-1})}{A(z^{-1})} \quad (1)$$

where, Y and U are the z transforms of the model output y and the input u, respectively. Also, a_i $(i = 1,...,n)$ and b_i $(i = 1,...,m)$ are time invariant coefficients and d is the process dead-time (pure time delay). Note at this point that in addition to the actual process dead-time d can also accommodate computational time delay necessary for the implementation of the control algorithm. In (1), A and B are the polynomials in z^{-1}. The controllable canonical form of the state equations for plant model is

$$\begin{aligned} x(k+1) &= Gx(k) + Hu(k) + Qv(k-1), \\ y(k) &= Cx(k) \end{aligned} \quad (2)$$

where the second equation is the plant output equation. Matrices in (2) have the following structures

$$x(k) = \begin{bmatrix} x_1(k) & ... & x_{d+n}(k) \end{bmatrix}^T = \begin{bmatrix} y(k-n-d+1) & ... & y(k-1) & y(k) \end{bmatrix}^T \in R^{(d+n)\times 1},$$

$$G = \begin{bmatrix} 0 & 1 & 0 & ... & 0 & 0 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 & 0 & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 0 & 0 & 0 & ... & 1 \\ 0 & 0 & 0 & ... & 0 & -a_n & -a_{n-1} & ... & -a_1 \end{bmatrix} \in R^{(d+n)\times(d+n)},$$

$$Q = \begin{bmatrix} 0 & ... & 0 \\ 0 & ... & 0 \\ ... & ... & ... \\ 0 & ... & 0 \\ 0 & ... & 0 \\ ... & ... & ... \\ 0 & ... & 0 \\ b_m & ... & b_2 \end{bmatrix} \in R^{(d+n)\times(m-1)},$$

$$v(k-1) = \begin{bmatrix} u(k-m+1) \\ u(k-m+2) \\ ... \\ u(k-2) \\ u(k-1) \end{bmatrix} \in R^{(m-1)\times 1},$$

$$H = \begin{bmatrix} 0 & ... & 0 & b_1 \end{bmatrix}^T \in R^{(d+n)\times 1},$$
$$C = \begin{bmatrix} 0 & ... & 0 & 0 & ... & 1 \end{bmatrix} \in R^{1\times(d+n)}$$

B. Control Law

We will consider the control law given by

$$u(k) = -L^T x(k) \quad (3)$$

We point here that because of the output equation in (2) and the form of C, the vector x is obtained from the process output y. Let us select the switching hyperplane as

$$s_k(x,\zeta) = p^T x(k) - k_I \zeta(k) \quad (4)$$

where $p_i = const$, $i = 1,..., d+n-1$, $p_{d+n} = 1$, and k_I is an integral controller gain, with

$$\zeta(k) = e(k-1) + \zeta(k-1), \; e(k) = r(k) - y(k) \quad (5)$$

A nonlinear control law that will ensure global asymptotic stability of the system is obtained (see Appendix A) by means of the Lyapunov's direct method as

$$\begin{aligned} u(k) &= -L^T(k)x(k) \\ &= -\frac{1}{b_1}\begin{bmatrix} p^T Gx(k) + \frac{\sqrt{\rho}}{d+n}\mathrm{sgn}(b_1) D^T \; \mathrm{sgn}(x)\left|s_k\right| \\ + p^T Qv(k-1) - k_I \zeta(k+1) \end{bmatrix} \end{aligned} \quad (6)$$

where $|D_i| \le 1$, $i = 1,2,...,d+n$, and $|\rho| < 1$.

In (6), $\zeta(k+1)$ is known at the moment k, since here we define $\zeta(k) = e(k-1) + \zeta(k-1)$, instead of $\zeta(k) = e(k) + \zeta(k-1)$ as used in [28]. In [28], to get $\zeta(k+1)$ at the moment k one has to use $\zeta(k)$ to approximate $\zeta(k+1)$. Although this change seems trivial, it results in the completely different set of coefficients of the sliding mode plane and the integral gain (see the next section) and thus has a crucial effect on the quality of the algorithm in the presence of the dead-time in the plant. This is of particular importance as we use the Smith predictor structure to compensate for the significant dead-time; this structure relies on the approximate plant models which introduce errors in the dead-time estimates. Thus the whole structure operates as possessing some small dead-time.

The control law given above is discontinuous and needs to be smoothed for implementation. As usual, we can replace signum nonlinearity by a saturation nonlinearity, which is specified as

$$sat(x/\phi)=\begin{cases}\operatorname{sgn}(x), & \text{if } |x|>\phi\\ x/\phi, & \text{if } |x|\le\phi\end{cases}$$

where ϕ is boundary layer thickness. With this boundary layer, the SMC control law given by (6) becomes

$$u(k)=-\frac{1}{b_1}\left[\begin{array}{l}p^T Gx(k)+\dfrac{\sqrt{\rho}}{d+n}\operatorname{sgn}(b_1)D^T sat(x/\phi)|s_k|\\ +p^T Qv(k-1)-k_I\zeta(k+1)\end{array}\right]$$

C. Integral Control Gain and Coefficients of the Switching Hyperplan

During the sliding motion, $s_k(x,\zeta)=0$, i.e.,

$$\begin{cases}p^T x(k)-k_I\zeta(k)=0,\\ x_{d+n}(k)+\sum_{i=1}^{d+n-1}p_i x_i(k)-k_I\zeta(k)=0\end{cases}$$

Using this and (2) the system equations can be reduced to the following linear equations

$$\begin{cases}x_i(k)=z^{-d-n+i}x_{d+n}(k),\ i=1,\cdots,d+n-1,\\ x_{d+n}(k)=k_I\zeta(k)-\sum_{i=1}^{d+n-1}p_i x_i(k)\end{cases}$$

Thus, the transfer function of the system described by the above equations is obtained as

$$\frac{x_{d+n}(k)}{r(k)}=\frac{k_I z^{-1}}{k_I z^{-1}+(1-z^{-1})(1+\sum_{i=1}^{d+n-1}p_i z^{-d-n+i})}$$
$$=k_I z^{-1}[1+(p_{d+n-1}-1+k_I)z^{-1}+\cdots+(p_i-p_{i+1})z^{-d-n+i}$$
$$+\cdots+(p_d-p_{d+1})z^{-n}+\cdots+(p_1-p_2)z^{-d-n+1}-p_1 z^{-d-n}]^{-1}$$

It is important to note that because this characteristic equation is independent of plant parameters, the control will be robust to the plant parameter variations for motions on the switching hyperplane. System can achieve zero steady-state error and the eigenvalues of its model describing behavior on the switching hyperplane can be set arbitrarily. Let the desired eigenvalues of the system on the switching hyperplane be $\lambda_i, i=1,\cdots,d+n$ or, equivalently, let the desired characteristic equation on the switching hyperplane be $\prod_{i=1}^{d+n}(z+\lambda_i)=0$ i.e. $z^{d+n}+\alpha_1 z^{d+n-1}+\alpha_2 z^{d+n-2}+\cdots+\alpha_{d+n}=0$. Then, comparison of the coefficients of this equation and the coefficients of the characteristic equation of the system, implies that the integral control gain and the switching hyperplane coefficients can be chosen as follows (for derivation see Appendix B)

$$\begin{cases}k_I=1+\sum_{i=1}^{d+n}\alpha_i,\\ p_{d+n}=1,\\ p_{d+n-1}=p_{d+n}+\alpha_1-k_I,\\ p_i=p_{i+1}+\alpha_{d+n-i},\ i=d+n-2,\cdots,2,\\ p_1=-\alpha_{d+n}.\end{cases}$$

III. DSMC AND COMPENSATION OF LONG DEAD-TIME BY THE SMITH PREDICTOR

For process control it is more typical to have process with significant dead-time. A standard way to compensate for the presence of dead-time is the use of Smith predictor [5]. In our simulation, we will utilize a variant of the Smith predictor as depicted in Fig. 2 [32]. Let the process

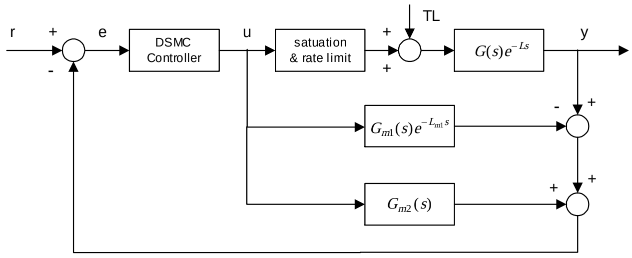

Fig. 2. A variant of the Smith predictor structure.

has the model $G(s)e^{-Ls}$ where $G(s)$ is the rational transfer function and L is the process dead-time. For processes of higher order or more complex dynamic, we would first attempt to obtain as good as possible model $G_{m1}(s)e^{-L_{m1}s}$ to be used for cancellation of the actual process output signal y. This model can be of relatively high order. If the order of $G_{m1}(s)$ greater than two is obtained for good cancellation, then we need the additional identification of the process by a lower order model $G_{m2}(s)e^{-L_{m2}s}$.

The reason for using these two approximations in the Smith predictor schema is the sensitivity of the DSMC to apparent dead-time in the process to be controlled. Combining DSMC with the Smith predictor, the following equations hold $e=r-(y-y_{m1}+y_{m2})$, $y_{m1}=[G_{m1}(s)e^{-L_{m1}s}]u$, $y_{m2}=[G_{m2}(s)]u$, $y=[G(s)e^{-Ls}](u+T_L)$, where u is the control signal and T_L denotes the disturbance that the loop may experience. The transfer function $G_{m1}(s)$ and the estimated dead-time L_{m1} of the process model act to compensate the actual process output y. The other model of a lower order represented by $G_{m2}(s)$ serves as an estimate of the transfer function $G(s)$ of the process without dead-time in the Smith predictor schema. This approach gives very good results when applied to practical situations.

IV. SIMULATION EXPERIMENTS

A. Process with Short Dead-time

The simulation study will be performed to verify the control performance of the new DSMC. The first experiment is based on the sixth order model with short dead-time. The continuous transfer function is of the form

$$\frac{Y(s)}{U(s)}=\frac{64}{[(s+1)(s+2)(s+4)]^2}\mathrm{e}^{-0.05s}.$$

The sampling time is $T_s=0.01$, so the time delay of this system is five steps, i.e. $d=5$. Since the dead-time is relatively small compared to the dominant time constant of the process, we will not use any dead-time compensation technique.

The results for the DSMC will be shown simultaneously with the results for an optimally tuned PID controller based on the step disturbance rejection. To avoid control signal jumps with PID control, an additional slew rate limiter and control magnitude limiter are added to the control system and used with both the DSMC and the PID controller. The slew rate limits are selected as $[-10, 10]$ and the saturation values of the control limiter as $[-3, 3]$. The reference PID controller is optimally tuned to minimize step type disturbances acting at the input of the process with the rate limiter and control signal limiter

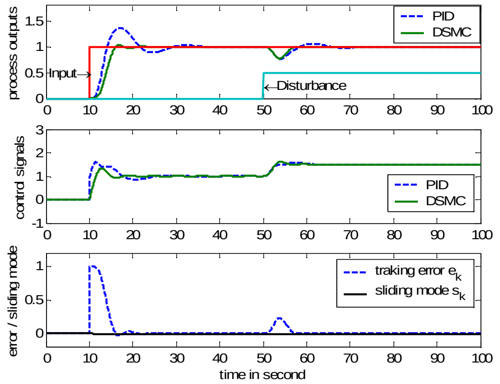

Fig. 3. Process responses, control signals, error and sliding mode motion with short dead-time.

active. The performance index function was taken as the integral (sum) absolute error (IAE) between the achieved process response and the given reference input [30], [31]. The tuned parameter vectors for the DSMC are selected as $\lambda^T = [0.993 \quad 0.99] \in C^n$, $D^T = \begin{bmatrix} 0.5 & -0.5 \end{bmatrix} \in R^n$, and parameters for PID controller are $K_p = 0.9$, $K_i = 0.7$, $K_d = 1.8$, respectively. The control loop responses for the process controlled by the DSMC and by the PID controller are plotted in Fig 3. One can observe the significantly better performance achieved by using DSMC, than by using optimized PID, both for setting point tracking and disturbance rejection properties. It is important to note that the control signal is smooth - no chattering appears in DSMC. The sliding motion is close to zero during entire control process.

B. Process with Long Dead-time

To illustrate the performance of the new DSMC we consider the process of 6th order with the long dead-time. Its transfer function is

$$G(s) = \frac{64}{[(s+1)(s+2)(s+4)]^2} e^{-10s}$$

For this system we have to use dead-time compensation and we will use the one based on the Smith predictor. The model of the fourth order for the cancellation of the signal y in the Smith predictor schema is obtained as

$$G_{m1}(s) = \frac{0.0639s^3 + 0.0348s^2 + 1.4656s + 0.7597}{s^4 + 4.6507s^3 + 5.8618s^2 + 3.6052s + 0.7597} \times e^{-10.7175s}$$

while the second order model in the second parallel feedback branch in the Smith predictor control structure is obtained as

$$G_{m2}(s) = \frac{0.5135}{s^2 + 1.2608s + 0.5135} e^{-11.0217s}.$$

The control signals, both for PID and DSMC, are set under the constraints of the slew rate change to $[-10, 10]$ and the control magnitude range to $[-3, 3]$. The PID controller is optimally tuned for disturbance rejection, where the criterion selected was IAE, and disturbance was of step-type. The values of the controller gains $K_p = 6.8411$, $K_i = 10$, $K_d = 6.8799$, were limited in the range of $[0, 10]$. The reference model for the DSMC is first order system with time constant 1. Two poles are respectively selected as 0.93 and 0.99. The process

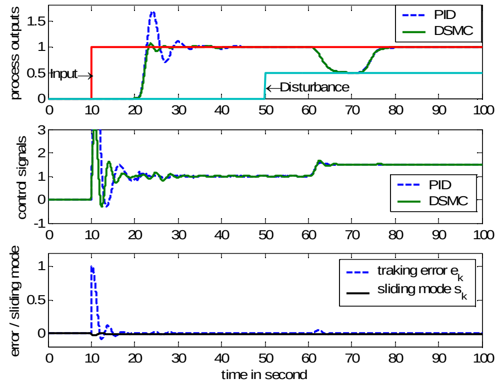

Fig. 4. Process with long dead-time.

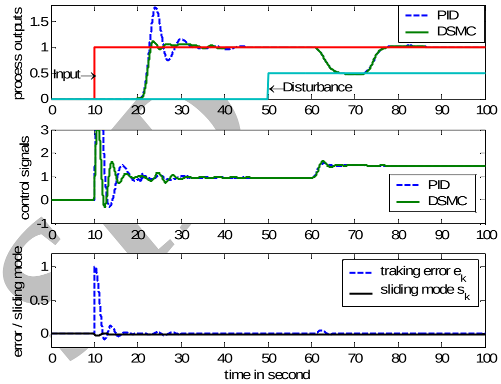

Fig. 5. The gain of process has a perturbation by increasing 5%.

response to the step input and disturbance, and the control signals are plotted in Fig. 4. Simulation results show that DSMC can improve the set-point response significantly while maintaining the disturbance rejection property virtually the same as with the optimally tuned PID controller. The control signal of the DSMC has no chattering.

To show the robustness of DMSC to the parameter perturbation, the gain of process has been increased by 5%. Fig. 5 shows that the DSMC still gives the best results on the setting point tracking although the tracking of setting point changed a little bit both with DSMC and PID control.

PID control gains ($K_p = 4, K_i = 0.7$, $K_d = 1.8$) have been manually adjusted to set up the best result for the setting point tracking which is similar to the result that DSMC achieved. However, DSMC control gives the best result on disturbance rejection (see Fig. 6).

V. Conclusions

The new DSMC based on input-output measurement is proposed and its application in control of systems with long dead-time is presented. Simulation studies reveal that this approach offers an effective solution. Results have shown that the proposed DSMC performs much better than conventional (optimally tuned) PID controller. In the experiment with system having small dead-time, both the set-point tracking and disturbance rejection are better than that with the optimally tuned PID controller. In the experiment with long dead-time, the disturbance rejection remains virtually the same for the DSMC and the PID controller, while the set-point tracking is considerably better with the DSMC. In both cases the control signal

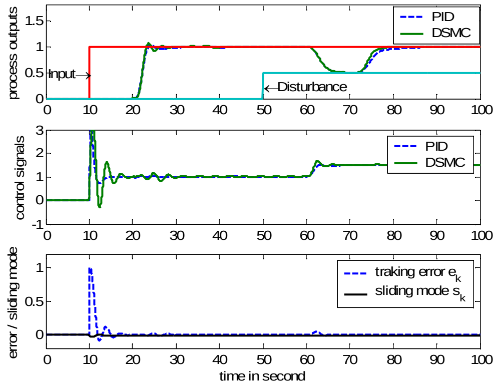

Fig. 6. The optimal PID was tuned based on setting point.

generated with the DSMC does not have chattering.

In summary, the proposed DSMC has the following characteristics: (1) it has excellent tracking properties; (2) the sliding mode motion can be set up arbitrarily via pole placement technique; (3) DSMC can achieve a zero steady state error and can track arbitrary input; (4) DSMC is based only on input and output data and does not need state measurements or estimates; (5) it has good disturbance rejection property; (6) it is able to operate effectively even in the presence of some time delay; (7) unlike general sliding mode control, the control signal has no chattering. However, the Lyapunov's method used in the derivation of the control law essentially implies that this type of controller is most suitable when there is only a small time delay in the process. Thus, in cases when significant time delay occurs, which is a typical situation in process control, the dead-time compensation structures (e.g., Smith predictor) has to be used in combination with DSMC.

APPENDIX

A. Derivation of Discrete Sliding Mode Control Law

The following proof is similar to the method used in [28] and [29]. Let $V_k = s_k^2(x,\zeta)$ be a Lyapunov function. For global asymptotic stability of the switching plane $s_k(x,\zeta)=0$. We need

$$\begin{aligned}\Delta V &= V_{k+1} - V_k \\ &= s_{k+1}^2 - s_k^2 \le (\rho-1)V_k = (\rho-1)s_k^2 \le 0\end{aligned}$$

i.e.,

$$s_{k+1}^2 - \rho s_k^2 \le 0 \text{ where } |\rho| < 1$$

so that $-\sqrt{\rho}\,|s_k| < s_{k+1} < \sqrt{\rho}\,|s_k|$ Using (2), (3), and (4) one gets

$$\begin{aligned}s_{k+1} &= p^T x(k+1) - k_I\zeta(k+1) \\ &= p^T\left[Gx(k) + Hu(k) + Qv(k-1)\right] - k_I\zeta(k+1) \\ &= p^T\left[Gx(k) + H\left(-L^T x(k)\right) + Qv(k-1)\right] - k_I\zeta(k+1)\end{aligned}$$

$$\begin{aligned}s_{k+1} &= p^T\left[Gx(k) - HL^T x(k) + Qv(k-1)\right] - k_I\zeta(k+1) \\ &= p^T(G - HL^T)x(k) + p^T Qv(k-1) - k_I\zeta(k+1)\end{aligned}$$

Let $w(k) = p^T Qv(k-1) - k_I\zeta(k+1)$ then

$$\begin{aligned}s_{k+1} &= p^T\left(G - HL^T\right)x(k) + p^T Qv(k-1) - k_I\zeta(k+1) \\ &= p^T\left(G - HL^T\right)x(k) + w(k)\end{aligned}$$

Note that $p^T H = p_{d+n}b_1 = b_1$ and that $p^T H(p^T H)^{-1} = 1$ and if we define $x^{-1}(k)$ as

$$x^{-1}(k) = \frac{x^T(k)}{x^T(k)x(k)}$$

then also $x^{-1}(k)x(k) = 1$ if $|x(k)| \ne 0$. With this we can write

$$\begin{aligned}s_{k+1} &= p^T\left(G - HL^T\right)x(k) + w(k) \\ &= p^T\left(G - HL^T\right)x(k) + w(k)x^{-1}(k)x(k) \\ &= p^T\left(G - HL^T\right)x(k) + p^T H(p^T H)^{-1}w(k)x^{-1}(k)x(k) \\ &= p^T Gx(k) - p^T HL^T x(k) + p^T H(p^T H)^{-1}w(k)x^{-1}(k)x(k) \\ &= p^T Gx(k) - p^T H\left[L^T x(k) - (p^T H)^{-1}w(k)x^{-1}(k)x(k)\right] \\ &= p^T Gx(k) - p^T H\left[L^T - (p^T H)^{-1}w(k)x^{-1}(k)\right]x(k) \\ &= p^T\left(G - H\left[L^T - (p^T H)^{-1}w(k)x^{-1}(k)\right]\right)x(k)\end{aligned}$$

So, let

$$\bar{L}^T = \begin{bmatrix}\bar{l}_1 & \bar{l}_2 & \cdots & \bar{l}_{d+n}\end{bmatrix} = L^T - (p^T H)^{-1}w(k)x^{-1}(k),$$

$$\begin{aligned}f^T &= \begin{bmatrix}f_1 & f_2 & \cdots & f_{d+n}\end{bmatrix} = (p^T H)^{-1}p^T G = p^T G / b_1 \\ &= \frac{1}{b_1}\big[0 \quad p_1 \quad p_{d-1} \quad p_d - a_n \quad p_{d+1} - a_{n-1} \quad \cdots \\ &\qquad p_{d+i} - a_{n-i} \quad \cdots \quad p_{d+n-1} - a_1\big]\end{aligned}$$

and the expression for s_{k+1} becomes

$$\begin{aligned}s_{k+1} &= p^T(G - H\bar{L}^T)x(k) \\ &= p^T H(p^T H)^{-1}p^T Gx(k) - p^T H\bar{L}^T x(k) \\ &= p^T H\left[(p^T H)^{-1}p^T G - \bar{L}^T\right]x(k) \\ &= p^T H(f^T - \bar{L}^T)x(k) \\ &= b_1(f^T - \bar{L}^T)x(k) = b_1\sum_{i=1}^{d+n}(f_i - \bar{l}_i)x_i(k),\end{aligned}$$

$$\begin{aligned}s_{k+1} &= p^T\left(G - HL^T\right)x(k) + w(k) \\ &= p^T\left(G - HL^T\right)x(k) + w(k)x^{-1}(k)x(k) \\ &= p^T\left(G - HL^T\right)x(k) + p^T H(p^T H)^{-1}w(k)x^{-1}(k)x(k) \\ &= p^T Gx(k) - p^T HL^T x(k) + p^T H(p^T H)^{-1}w(k)x^{-1}(k)x(k) \\ &= p^T Gx(k) - p^T H\left[L^T x(k) - (p^T H)^{-1}w(k)x^{-1}(k)x(k)\right] \\ &= p^T Gx(k) - p^T H\left[L^T - (p^T H)^{-1}w(k)x^{-1}(k)\right]x(k) \\ &= p^T\left(G - H\left[L^T - (p^T H)^{-1}w(k)x^{-1}(k)\right]\right)x(k)\end{aligned}$$

so that if $\bar{l}_i = f_i + D_i\left|\dfrac{\sqrt{\rho}s_k}{(d+n)x_i(k)p^T H}\right|$, then

$$\begin{aligned} s_{k+1} &= b_1 \sum_{i=1}^{d+n} \left(f_i - \bar{l}_i\right) x_i(k) \\ &= b_1 \sum_{i=1}^{d+n} \left[f_i - f_i + D_i \left| \frac{\sqrt{\rho} s_k}{(d+n) x_i(k) p^T H} \right| \right] x_i(k) \\ &= b_1 \sum_{i=1}^{d+n} \left(-D_i \left| \frac{\sqrt{\rho} s_k}{(d+n) x_i(k) p^T H} \right| \right) x_i(k) \end{aligned}$$

$$\begin{aligned} s_{k+1} &= b_1 \sum_{i=1}^{d+n} \left(-D_i \left| \frac{\sqrt{\rho} s_k}{(d+n) x_i(k) b_1} \right| \right) x_i(k) \\ &= \sqrt{\rho} \frac{|s_k|}{d+n} \frac{b_1}{|b_1|} \sum_{i=1}^{d+n} \left(-D_i \frac{x_i(k)}{|x_i(k)|} \right) = d_0 |s_k|, \end{aligned}$$

where

$$d_0 = \frac{\sqrt{\rho}}{d+n} \frac{b_1}{|b_1|} \sum_{i=1}^{d+n} \left(-D_i \frac{x_i(k)}{|x_i(k)|} \right)$$

If $|D_i| \le 1$, $i = 1, 2, ..., d+n$, then

$$\begin{aligned} |d_0| &= \left| \frac{\sqrt{\rho}}{d+n} \frac{b_1}{|b_1|} \sum_{i=1}^{d+n} \left(-D_i \frac{x_i(k)}{|x_i(k)|} \right) \right| = \frac{\sqrt{\rho}}{d+n} \left| \sum_{i=1}^{d+n} \left(-D_i \frac{x_i(k)}{|x_i(k)|} \right) \right| \\ &\le \frac{\sqrt{\rho}}{d+n} \sum_{i=1}^{d+n} \left| -D_i \frac{x_i(k)}{|x_i(k)|} \right| = \frac{\sqrt{\rho}}{d+n} \sum_{i=1}^{d+n} \left(|-D_i| \left| \frac{x_i(k)}{|x_i(k)|} \right| \right) \\ &= \frac{\sqrt{\rho}}{d+n} \sum_{i=1}^{d+n} |D_i| \le \frac{\sqrt{\rho}}{d+n} (d+n) = \sqrt{\rho} < 1, \end{aligned}$$

so that $-\sqrt{\rho}\,|s_k| < s_{k+1} < \sqrt{\rho}\,|s_k|$.

From $\bar{L}^T = L^T - (p^T H)^{-1} w(k) x^{-1}(k) = [\bar{l}_1 \; \bar{l}_2 .. \bar{l}_{d+n}]$, one can obtain $L^T = \bar{L}^T + (p^T H)^{-1} w(k) x^{-1}(k) = [l_1 \; l_2 .. l_{d+n}]$, which implies

$$\begin{aligned} l_i &= \bar{l}_i + (p^T H)^{-1} w(k) \frac{x_i(k)}{x^T(k) x(k)} \\ &= f_i + D_i \left| \frac{\sqrt{\rho} s_k}{(d+n) x_i(k) p^T H} \right| + (p^T H)^{-1} w(k) \frac{x_i(k)}{x^T(k) x(k)} \\ &= f_i + D_i \left| \frac{\sqrt{\rho} s_k}{(d+n) x_i(k) b_1} \right| + \frac{w(k) x_i(k)}{b_1 x^T(k) x(k)}, \end{aligned}$$

or

$$\begin{aligned} L^T &= f^T + \frac{1}{b_1} \mathrm{sgn}(b_1) |s_k| \bar{x}^T + \frac{1}{b_1} w(k) x^{-1}(k) \\ &= \frac{1}{b_1} p^T G + \frac{1}{b_1} \mathrm{sgn}(b_1) |s_k| \bar{x}^T \\ &+ \frac{1}{b_1} \left[p^T Q v(k-1) - k_I \zeta(k+1) \right] x^{-1}(k) \\ &= \frac{1}{b_1} \left\{ p^T G + \mathrm{sgn}(b_1) |s_k| \bar{x}^T + \left[p^T Q v(k-1) - k_I \zeta(k+1) \right] x^{-1}(k) \right\}, \end{aligned}$$

with $\bar{x}^T(k) = \frac{\sqrt{\rho}}{d+n} \left[\frac{D_1}{|x_1(k)|} \quad \frac{D_2}{|x_2(k)|} \quad \cdots \quad \frac{D_{d+n}}{|x_{d+n}(k)|} \right]$ and

$$x^{-1}(k) = \frac{x^T(k)}{x^T(k) x(k)}.$$

Finally

$$\begin{aligned} u(k) &= -L^T(k) x(k) \\ &= -\frac{1}{b_1} \{ p^T G + \mathrm{sgn}(b_1) |s_k| \bar{x}^T + \left[p^T Q v(k-1) - k_I \zeta(k+1) \right] x^{-1}(k) \} x(k) \\ &= -\frac{1}{b_1} \left[p^T G x(k) + \mathrm{sgn}(b_1) |s_k| \bar{x}^T x(k) + p^T Q v(k-1) - k_I \zeta(k+1) \right] \\ &= -\frac{1}{b_1} \left[\begin{array}{l} p^T G x(k) + \frac{\sqrt{\rho}}{d+n} \mathrm{sgn}(b_1) D^T \mathrm{sgn}(x) |s_k| \\ + p^T Q v(k-1) - k_I \zeta(k+1) \end{array} \right] \end{aligned}$$

B. Determination of Integral Control Gain and Coefficients of the Switching Hyperplane

Similar to the methods used in [28] and [29], the sliding motion satisfies $s_k(x, \zeta) = 0$, i.e.

$$\begin{cases} p^T x(k) - k_I \zeta(k) = 0, \\ x_{d+n}(k) + \sum_{i=1}^{d+n-1} p_i x_i(k) - k_I \zeta(k) = 0 \end{cases}$$

the system described by (2) and above equation can be reduced to the following linear equations

$$\begin{cases} x_1(k+1) = x_2(k) = z^{-d-n+2} x_{d+n}(k), \\ x_2(k+1) = x_3(k) = z^{-d-n+3} x_{d+n}(k), \\ \ldots \\ x_i(k+1) = x_{i+1}(k) = z^{-d-n+i+1} x_{d+n}(k), \\ \ldots \\ x_{d+n-1}(k+1) = x_{d+n}(k), \\ x_{d+n}(k) = k_I \zeta(k) - \sum_{i=1}^{d+n-1} p_i x_i(k) \end{cases}$$

i.e., to

$$\begin{cases} x_i(k+1) = x_{i+1}(k) = z^{-d-n+i+1} x_{d+n}(k), \\ i = 1, \cdots, d+n-1, \\ x_{d+n}(k) = k_I \zeta(k) - \sum_{i=1}^{d+n-1} p_i x_i(k) \end{cases}$$

This gives

$$\begin{cases} x_i(k) = z^{-d-n+i} x_{d+n}(k), \; i = 1, \cdots, d+n-1, \\ x_{d+n}(k) = k_I \zeta(k) - \sum_{i=1}^{d+n-1} p_i x_i(k). \end{cases} \quad \text{(B1)}$$

From (5), one can obtain

$$\begin{aligned} \zeta(k) &= e(k-1) + \zeta(k-1) \\ &= r(k-1) - y(k-1) + \zeta(k-1) \\ &= r(k-1) - x_{d+n}(k-1) + \zeta(k-1) \end{aligned}$$

Thus

$$\zeta(k) = \frac{z^{-1}}{1 - z^{-1}} \left[r(k) - x_{d+n}(k) \right] \quad \text{(B2)}$$

By substituting (B2) to the second part of (B1) we get

$$\begin{aligned} x_{d+n}(k) &= \frac{k_I z^{-1}}{1 - z^{-1}} \left[r(k) - x_{d+n}(k) \right] - \sum_{i=1}^{d+n-1} p_i x_i(k) \\ &= \frac{k_I z^{-1}}{1 - z^{-1}} \left[r(k) - x_{d+n}(k) \right] - \sum_{i=1}^{d+n-1} p_i z^{-d-n+i} x_{d+n}(k) \end{aligned}$$

The transfer function of the system described by the above equations is reduced to

$$\begin{aligned} \frac{x_{d+n}(k)}{r(k)} &= \frac{k_I z^{-1}}{k_I z^{-1} + (1 - z^{-1})(1 + \sum_{i=1}^{d+n-1} p_i z^{-d-n+i})} \\ &= k_I z^{-1} [1 + (p_{d+n-1} - 1 + k_I) z^{-1} + \cdots \\ &+ (p_i - p_{i+1}) z^{-d-n+i} + \cdots + (p_d - p_{d+1}) z^{-n} + \cdots \\ &+ (p_1 - p_2) z^{-d-n+1} - p_1 z^{-d-n}]^{-1} \end{aligned}$$

So the characteristic equation of the system is

$$z^{d+n} + (p_{d+n-1} - 1 + k_I) z^{d+n-1} + (p_{d+n-2} - p_{d+n-1}) z^{d+n-2}$$
$$+ \cdots + (p_i - p_{i+1}) z^i + \cdots + (p_d - p_{d+1}) z^d$$
$$+ \cdots + (p_1 - p_2) z - p_1 = 0.$$

Let the desired eigenvalues of the system be $\lambda_1, \lambda_2, \cdots, \lambda_{d+n}$, or, equivalently, let the desired characteristic equation be $(z+\lambda_1)(z+\lambda_2)\cdots(z+\lambda_d)(z+\lambda_{d+1})\cdots(z+\lambda_{d+n}) = 0$ or $z^{d+n} + \alpha_1 z^{d+n-1} + \alpha_2 z^{d+n-2} + \cdots + \alpha_{d+n} = 0$. By comparing the coefficients of the above equation and those of the characteristic equation of the system, the integral control gain and the switching plant coefficients can be chosen to satisfy the following

$$\begin{cases} \alpha_1 = p_{d+n-1} - p_{d+n} + k_I, \\ \ldots \\ \alpha_i = p_{d+n-i} - p_{d+n-i+1}, \\ \ldots \\ \alpha_{d+n-1} = p_1 - p_2, \\ \alpha_{d+n} = -p_1, \end{cases}$$

Solving the above simultaneous equations, gives the desired coefficients in the form

$$\begin{cases} k_I = 1 + \sum_{i=1}^{d+n} \alpha_i, \\ p_{d+n} = 1, \\ p_{d+n-1} = p_{d+n} + \alpha_1 - k_I, \\ p_i = p_{i+1} + \alpha_{d+n-i}, \ i = d+n-2, \cdots, 2, \\ p_1 = -\alpha_{d+n}. \end{cases}$$

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REFERENCES

[1] F. G. Shinskey, *Process Control Systems*, 2nd edition, McGraw-Hill, New York, 1979.

[2] K. J. Astrom and T. Hagglund, *Automatic Tuning of PID Controllers*, Instrument Society of America (ISA), 1988.

[3] C. L. Albert and D. A. Coggan, *Fundamentals of Industrial Control - Practical Guides for Measurement and Control*, Editors, Instrument Society of America, Research Triangle Park, NC, US, 1992.

[4] J. L. Martins de Carvalho, *Dynamical Systems and Automatic Control*, Prentice- Hall, New York, 1993.

[5] O. J. M. Smith, *Feedback Control Systems*, McGraw-Hill, New York, 1958.

[6] V. I. Utkin, "Variable structure systems with sliding modes," *IEEE Trans. Automatic Control*, vol. 22, no. 2, pp. 212-222, Apr. 1977.

[7] S. C. Chung and C.-L. Lin, "A general class of sliding surface for sliding mode control," *IEEE Trans. on Automatic Control*, vol. 43, no. 1, pp. 115-119, Jan. 1998.

[8] J. Y. Hung, W. B. Gao, and J. C. Hung, "Variable structure control: a survey," *IEEE Trans. Industrial Electronics*, vol. 40, no. 1, pp. 2-22, Feb. 1993.

[9] H. N. Iordanou and B. W. Surgenor, "Experimental evaluation of the robustness of discrete sliding mode control versus linear quadratic control," *IEEE Trans. Control Systems Technology*, vol. 5, no. 2, pp. 254-260, Mar. 1997.

[10] T. -L. Chern and Y. -C. Wu, "Design of integral variable structure controller and application to electro-hydraulic velocity servo-systems," *IEE Proceeding-D*, vol. 138, no. 5, pp. 439-444, Sep. 1991.

[11] T. Chern, C. Chuang, and R. Jiang, "Design of discrete integral variable structure control system and application to a brushless DC motor control," *Automatica*, vol. 32, no. 5, pp. 773-779, 1996.

[12] W. Gao, Y. Wang, and A. Homaifa, "Discrete-time variable structure systems," *IEEE Trans. on Industrial Electronics*, vol. 42, no. 2, pp. 117-122, Apr. 1995.

[13] A. Bartoszewicz, "Discrete-Time quasi-sliding-mode control strategies," *IEEE Trans. on Industrial Electronics*, vol. 45, no. 4, pp. 633-637, Aug. 1998.

[14] G. Bartolini, A. Ferrara, and E. Usai, "Chattering avoidance by second-order sliding mode control," *IEEE Trans. on Autom. Control*, vol. 43, no. 2, pp. 241-246, Feb. 1998.

[15] E. A. Misawa, "Discrete-time sliding mode control," *ASME's J. of Dynamic Systems, Measurement and Control*, vol. 119, pp. 503-512, Sep. 1997.

[16] E. A. Misawa, "Discrete-time sliding mode control: the linear case," *ASME's J. of Dynamic Systems, Measurement and Control*, vol. 119, pp. 819-821, Dec. 1997.

[17] C. Edwards and S. K. Spurgeon, "Robust output tracking using a sliding mode controller / observer scheme," *Int. J. Control*, vol. 64, pp. 967-983, 1996.

[18] C. Edwards and S. K. Spurgeon, "Sliding mode output tracking with application to a multivariable high temperature furnace problem," *Int. J. Robust and Nonlinear Control*, vol. 7, pp. 337-351, 1997.

[19] P. Korondi, H. Hashimoto, and V. Utkin, "Direct torsion control of flexible shaft in an observer-based discrete-time sliding mode," *IEEE Trans. on Industrial Electronics*, vol. 45, no. 2, pp. 291-296, Apr. 1998.

[20] K. Furuta, "Sliding mode control of a discrete system," *Systems & Control Letters*, vol. 14, pp. 145-152, 1990.

[21] G. M. Aly and W. G. All, "Digital design of variable structure control systems," Int. J. Systems Sci., vol.21, no.8, pp.1709-1720, 1990.

[22] W. Gao and J. C. Huang, "Variable structure control of nonlinear systems: a new approach," *IEEE Trans. Industrial Electronics*, vol. 40, no. 1, pp. 45-55, Feb. 1993.

[23] L. Hsu, F. Lizarralde, and A. D. de Araújo, "New results on output-feedback variable structure model-reference adaptive control: design and stability analysis," *IEEE Trans. Autom. Control*, vol. 42, no. 3, pp. 386-393, Mar. 1997.

[24] L. Hsu, "Variable structure model reference adaptive control using only I/O measurement: General case," *IEEE Trans. Automat. Contr.*, vol. 35, no. 11, pp. 1238-1243, Nov. 1990.

[25] L. Hsu, A. D. Araújo, and R. R. Costa, "Analysis and design of I/O based variable structure adaptive control," *IEEE Trans. Automat. Contr.*, vol. 39, no. 1, pp. 4-21, Jan. 1994.

[26] L. Hsu and R. R. Costa, "Variable structure model reference adaptive control using only input and output measurement: Part I," *Int. J. Contr.*, vol.49, no.2, pp.399-416, 1989.

[27] B. Bandyopahyyay and S. Janardhanan, "Discrete-time sliding mode control using multirate output feedback," *Lecture Notes in Control and Information Sciences*, vol. 334, pp. 351-371, 2006.

[28] D. H. Sha and V. B. Bajić, "Robust discrete adaptive I/O based sliding mode controller," *Int. J. of Systems Science*, vol. 31, no. 12, pp. 1601-1614, Dec. 2000.

[29] D. H. Sha, V. B. Bajić, and H. Y. Yang, "New model and sliding mode control of hydraulic elevator velocity tracking system," *Simulation Practice and Theory*, vol. 9, no. 6, pp. 365-385, 15 May 2002.

[30] W. S. Su, T.-Y. Lee, and S. Park, "Optimal PID controller tuning method for single-input/single-output processes," *American Institute of Chemical Engineers Journal*, vol. 48, no. 6, pp. 1358-1361, 2002.

[31] B. Kristiansson and B. Lennartson, "Evaluation and simple tuning of PID controllers with high-frequency robustness," *J. of Process Control*, vol. 16, no. 2, pp. 91-102, 2006.

[32] P. Govender and V. B. Bajić, "Nonlinear modifier of PID control for processes with long dead-time," in *Proc. of the International Conference on Communications, Signals and Systems, CSS'96*, vol. 1, pp. 247-250, Brno, Czech Republic, Sep. 1996.

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