A Constructive Method to Evaluate the Characteristic Impedance in a Rectangular Transmission Line

Z. Brito-Brito, J. H. Caltenco, R. Linares y M., and J. López-Bonilla

Abstract—**A novel method to determine the characteristic impedance of a rectangular transmission line, through the evaluation of lumped parameters, is presented. Leaving of the current distribution that circulates in the central conductor is also compared with a technique that leaves the determination of the magnetic field inside the structure. The obtained results were satisfactory.**

Index Terms—**Rectangular transmission line, characteristic impedance, septum, per unit length parameters (PUL).**

I. INTRODUCTION

THE CHARACTERIZATION of the rectangular transmission lines with a small incorporated septum (Fig. 1) has recently been the reason of several studies by many authors [1]-[10]. Such devices are useful to determine the dielectric and magnetic properties of ferromagnetic materials and nanometric structures used on the design of electromagnetic shieldings, which satisfies the demands of international standards [11]-[17]. On the reviewed literature, information of these kind of devices has not been reported yet, in which the dimensions and the effects caused by the presence of the small septum are taken into account. The effects of the septum are considered in this work and take advantage of the geometric properties of a cavity with these characteristics, which contain ferromagnetic material weakly magnetized of nanometric dimensions.

Once the material is placed inside the line the dielectric properties of the medium surrounding the septum change, which can be calculated from the per unit length parameter. As the impedance has changed, it is necessary to determine those parameters.

The determination of the per unit length parameters has been treated by different authors, who usually neglect the thickness of the septum because the structures are wide with respect to the thickness, we consider $t/w \ll 1$.

The characteristic impedance can be calculated from the vector magnetic potential [18] inside the cell, using Green's function to calculate the fields inside the line, but this method is only valid when the maximum septum thickness is limited to $t/w = 0.02$.

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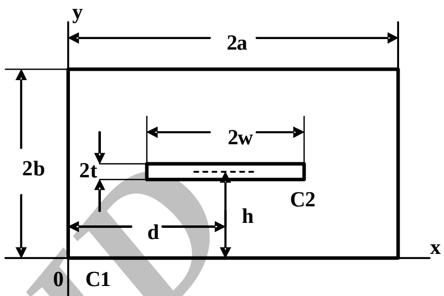

Fig. 1. Cross section of rectangular transmission line.

If we use a variational method, the impedance from the capacitance can be calculated as a function of the electrostatic energy stored in the surrounding field to the two conductors [19] but neglecting the thickness of the septum.

Another way to find the impedance is through conformal transformation, which calculates the capacitance between the walls and at the corners of the structure from two transformations [20], but it is only valid when the sides of the conductors are large with respect to the separation of the walls. The Schwarz-Christoffel transformation [21] also uses a transformation that is valid only for symmetric structures; this is possible when the thickness of the septum is neglected.

II. PROBLEM FORMULATION

One of the most important parameters in the study of transmission lines is the characteristic impedance, which is calculated from the lumped line parameters, in terms of the geometry and the electrical properties of the material that are constructed. In the case of rectangular transmission lines the methodologies that are used to find those parameters normally neglect the thickness of the septum or they approximate it to a cylindrical structure.

A different method to calculate the characteristic impedance defined by the per unit length parameters PUL of a rectangular transmission line without neglecting the thickness of the septum is presented as a contribution in this paper.

In the Electromagnetic Compatibility Laboratory at the Electronic Program of the SEPI ESIME IPN, it is necessary to have a small rectangular transmission line with a narrow septum that allows the use of the line's inner space, to be employed in the material's characterization in small quantities.

The analytical and experimental results agreed, so the method that is presented is an effective tool to calculate the characteristic impedance of the rectangular transmission line.

III. PROBLEM SOLUTION

The characteristic impedance expressed from the unit length parameters is given by

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (1)$$

where the resistance R, inductance L, conductance G and capacitance C are calculated through the selection of a current distribution flowing in the conductors for a specific range of frequency.

At low frequencies (<100 kHz), the current distribution is uniform at the centre of the septum and also at the external conductor, but as the frequency increases the tangential current distribution becomes infinite at the edge of the septum as $r^{-1/2}$ [22], [23], so S. K. Das and B. N. Das in [22] propose that the current distribution at the septum for the x walls (I_a) neglecting its thickness, but only for high frequencies, as

$$Js = \frac{I_a}{\sqrt{1 - (\frac{x - d}{w})^2}} \,. \quad (2)$$

However in this paper we take into account the effects of the thickness of septum, the current distribution at the y walls (I_b) is necessary, with the same characteristics proposed at [22], expressed by the y axis as

$$Js = \frac{I_b}{\sqrt{1 - (\frac{y - h}{t})^2}} \,. \quad (3)$$

The first parameter to be calculated is the inductance, which reflects the total magnetic flux penetrating in the circuit with respect to the total current lines. This is expressed in terms of the magnetic energy stored in the inductor

$$W_m = \frac{1}{4} L I_0^{\,2} \,, \quad (4)$$

where I_0 is the total current flowing on the conductor C_2 and it is given by the line integral of the current density around C_2 [23]

$$I_0 = \oint_{C2} \left|\mathbf{J}_s\right| dl \,. \quad (5)$$

besides, the time-average magnetic energy for TEM waves is [23]

$$W_m = \tfrac{1}{4}\mu \iint \left|\mathbf{J}_s\right|^2 dx\, dy \,. \quad (6)$$

thus, the inductance in terms of the current distribution from (4), (5) and (6) is given by

$$L = \frac{\mu \iint \left|\mathbf{J}_s\right|^2 dx dy}{(\oint_{C2} \left|\mathbf{J}_s\right| dl)^2} \quad (7)$$

The capacitance per unit length is

$$C = \frac{\mu\varepsilon}{L} \quad (8)$$

The shunt conductance G has the expression

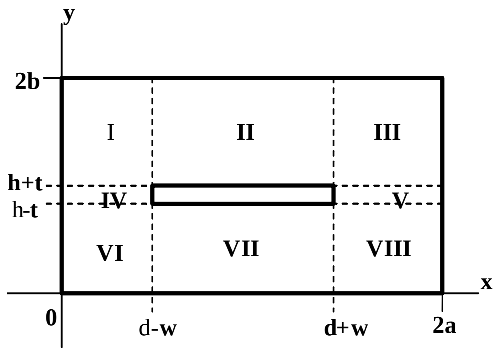

Fig. 2. Regions for evaluating the numerator of inductance.

$$G = \frac{I_S}{V_0} = \frac{\sigma Q}{\varepsilon V_0} = \frac{\sigma C}{\varepsilon} = \omega\varepsilon \tan\delta_1 \quad (9)$$

Although G is given by an expression which is similar to that of a static field problem, it is a function of frequency because of the factor ω arising in the expression for σ, and also because $\tan\delta_1$ is a function of frequency [23].

The resistance R per unit length may be defined by the power loss in the conductors

$$P_L = \frac{1}{2} I_0^{\,2} R \,, \quad (10)$$

where the power loss can be written in terms of the current distribution

$$P_L = \tfrac{1}{2} R_m \oint_{C1+C2} \left|\mathbf{J}_S\right|^2 dl \,. \quad (11)$$

So the resistance from (10), (5) and (11) is

$$R = (\frac{\omega\mu}{2\sigma})^{1/2} \frac{\oint_{C1+C2} \left|\mathbf{J}_S\right|^2 dl}{(\oint_{C2} \left|\mathbf{J}_S\right| dl)^2} \,. \quad (12)$$

The integral of the numerator in (7) is evaluated by dividing the cross section into eight regions as shown in Fig. 2, so that

$$\begin{aligned} \iint \left|\mathbf{J}_S\right|^2 dxdy = & \int_0^{d-w}\int_{h+t}^{2b} \left|\mathbf{J}_S\right|^2 dydx + \int_{d-w}^{d+w}\int_{h+t}^{2b} \left|\mathbf{J}_S\right|^2 dydx \\ & + \int_{d+w}^{2a}\int_{h+t}^{2b} \left|\mathbf{J}_S\right|^2 dydx + \int_0^{d-w}\int_{h-t}^{h+t} \left|\mathbf{J}_S\right|^2 dydx \\ & + \int_{d+w}^{2a}\int_{h-t}^{h+t} \left|\mathbf{J}_S\right|^2 dydx + \int_0^{d-w}\int_0^{h-t} \left|\mathbf{J}_S\right|^2 dydx \\ & + \int_{d-w}^{d+w}\int_0^{h-t} \left|\mathbf{J}_S\right|^2 dydx + \int_{d+w}^{2a}\int_0^{h-t} \left|\mathbf{J}_S\right|^2 dydx \end{aligned} \quad (13)$$

and the denominator of the inductance can be written as:

$$\oint_{C2} \left|\mathbf{J}_S\right| dl = 2\int_{d-w}^{d+w} \left|\mathbf{J}s\right| dx + 2\int_{h-t}^{h+t} \left|\mathbf{J}s\right| dy \,. \quad (14)$$

While the numerator of the resistance can be evaluated by

$$\begin{aligned} \oint_{C1+C2} \left|\mathbf{J}_S\right|^2 dl = & 2\int_0^{2a} \left|\mathbf{J}s\right|^2 dx + 2\int_0^{2b} \left|\mathbf{J}s\right|^2 dy \\ & + 2\int_{d-w}^{d+w} \left|\mathbf{J}s\right|^2 dx + 2\int_{h-t}^{h+t} \left|\mathbf{J}s\right|^2 dy \end{aligned} \quad (15)$$

In solving the integrals, the component of the current distribution when it is integrated the domain x may be (2), while for the domain y may be (3).

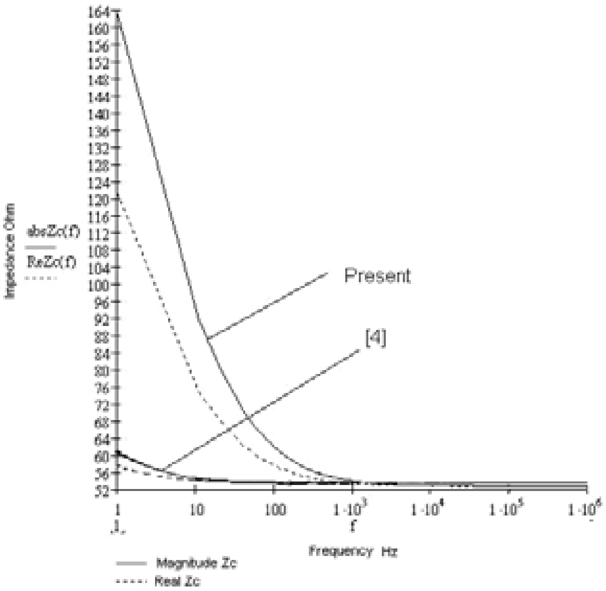

Fig. 3. Characteristic impedance for a given dimensions.

IV. Results

The characteristic impedance of a rectangular transmission line for given dimensions in [22] is plotted in Fig. 3, and this method is compared versus the reported in [22], so the dimensions are:

$$a = 1\,\text{m} \quad b = 0.5\,\text{m} \quad w = 0.6\,\text{m} \quad t = 3\times10^{-3}\,\text{m}\,.$$

The characteristic impedance for a rectangular transmission line constructed of brass at the external conductor and copper at the septum, having the next dimensions, is plotted in Fig. 4, where it is compared with the result of measuring the line.

$$a = 7.64\times10^{-3}\,\text{m} \quad b = 3.87\times10^{-3}\,\text{m}$$

$$w = 685.8\times10^{-6}\,\text{m} \quad t = 152.4\times10^{-6}\,\text{m}$$

V. Conclusions

The characteristic impedances for two rectangular lines are computed. The first compares the results with a method reported previously, and the second compares the results between the measurements and the analytical methods.

In Fig. 3 it is seen that current results well agree with the method reported by Das [22], with differences between them less than 5 % for a frequency range 1 kHz to 1 MHz, because the current distribution proposed in this paper is just valid for high frequencies and can not describe the line for a low band.

In Fig 4 the characteristic impedance is plotted by calculating using the method proposed by this paper and measured using two network analyzer, the HP4195A for the lower band and the HP8510A for the higher band, for the rectangular transmission line of small dimensions that may be used for property characterization. It is seen that the values agree with a margin of 5% for a frequency range of 10 MHz to 800 MHz. The differences at lower frequencies may be due to the current distribution which is valid only for high values, not for low frequencies. The differences at higher values may be due to the methodology used, which is based on TEM waves inside the line. In the case of small dimension lines for frequencies near 1 GHz

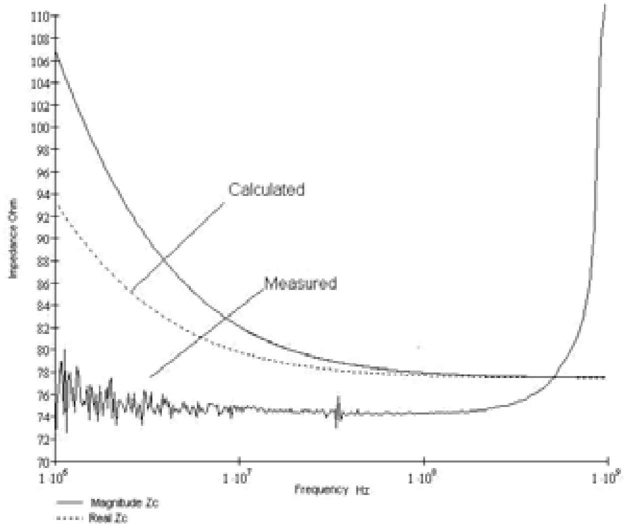

Fig. 4. Characteristic impedance for a given dimensions.

the higher-order waveguide-type modes begin to propagate [24].

However it can be concluded that for a frequency range of 10 MHz to 800 MHz, the line design has a good accuracy to predict the characteristic impedance of the line. Then, if the medium inside the line is changed, the changes in its impedance may be known.

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