Brent Method for Dynamic Economic Dispatch with Transmission Losses

K. Chandram, N. Subrahmanyam, and M. Sydulu

Abstract—**In this paper, Brent method is proposed to solve dynamic economic dispatch (DED) problem with transmission losses. The proposed algorithm involves the selection of the values of incremental fuel costs (lambda) and then the evaluation of optimal lambda is done by Brent method. The constraint of ramp rate limits distinguishes the DED problem from traditional static economic dispatch(ED) problem. The DED problem divides the entire dispatch period into a number of small time intervals and then static economic dispatch problem is solved in each interval by incorporating the ramp rate limits. The proposed method has been tested on 6- and 15- units. The simulation results of the proposed method are compared with conventional lambda iterative method. The simulation results show that the proposed method achieves qualitative solution with less computational time than the conventional lambda iterative method.**

Index Terms—**Brent method, dynamic economic dispatch, ramp rate limits, incremental transmission loss.**

NOMENCLATURE

T	Number of intervals
ng	Number of units
P_i^t	Output power of unit 'i' at hour 't'
a_i, b_i, c_i	Coefficients of fuel cost function
C_T	Total fuel cost
$C_i(P_i^t)$	Fuel cost of unit 'i' at hour 't'
P_D^t	Power demand at hour 't'
P_L^t	Incremental transmission loss
B_{ij}, B_{0i}, B_{00}	B-Loss coefficient
$P_{i,\min}$	Minimum output power of unit 'i'
$P_{i,\max}$	Maximum output power of unit 'i'
λ_i	Incremental fuel cost of unit 'i'
UR_i	Up ramp rate limits of unit 'i'
DR_i	Down ramp rate limits of unit 'i'
P_i^0	Initial output power of unit 'i'

I. INTRODUCTION

DYNAMIC economic dispatch (DED) problem is one of the main functions of power system operation and control. The main objective of DED problem is to determine the optimal schedule of output powers of online generating units over a certain period of time to meet power demands at minimum operating cost. It is a dynamic optimization problem that includes generator constraints and ramp rate limits [1]. The static economic dispatch problem assumes that the amount of power to be supplied by a given set of units is constant for a given interval of time and attempts to minimize the cost of supplying this energy subject to constraints on the static behavior of the generating units. In real time operation of power system, thermal gradients inside the turbine should be kept within safe limits to avoid shortening the life of equipments such as generating units and boiler. This mechanical constraint is translated into a limit on the rate of increase and decrease of the output power during the variation in the power demand and this limit is known as ramp rate limit. The constraint of ramp rate limits distinguishes the DED problem from the static economic dispatch (ED) problem. Due to ramp rate limits the DED problem cannot be solved for a single value of the load. Moreover, it is the most accurate formulation of the economic dispatch problem. Generally, DED problem divides the entire dispatch period into a number of small time intervals and then static economic dispatch problem is solved in each interval by incorporating the ramp rate limits.

Earlier, conventional approaches such as lambda iterative method [2], Gradient projection method [3], linear [4] and dynamic programming [5] methods were used for solving the DED problem. In these methods, computational time increases with the increase of the dimensionality of the problem. In order to get the qualitative solution, Gradient type Hopfield neural network [6] was used to solve DED problem. The major problem associated with the Hopfield neural network is that the unsuitable sigmoid function may increase the computational time to give optimal solution [7]. Stochastic search optimization techniques such as genetic algorithm (GA) [8], evolutionary programming (EP) [9] and particle swarm optimization(PSO) [10] methods have been used to solve DED problem because these algorithms can achieve global optimal solution. Major problem associated with these algorithms is that appropriate control parameters are required. Some times these algorithms take huge computational time due to improper selection of the control parameters. More precisely, hybrid methods combining probabilistic methods and deterministic methods are found to be very effective in solving complex optimization problems [11], [12]. In these methods, initially probabilistic methods are used for search purpose to find near optimal solution and then deterministic methods are used to fine tune that region to get the final solution.

It is observed from the literature survey that most of the conventional and stochastic search methods have some limitations to solve the DED problems within considerable computational time. The conventional lambda iterative method takes more computational time. Some times, it

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exhibits oscillatory behavior towards the end due to the improper selection of initial guess value of lambda (incremental fuel cost) and incremental lambda. Also the heuristic and modern heuristic methods are unable to find the optimal solution within considerable time due to their heuristic nature. Therefore it is necessary to find a suitable method to solve DED problem. In brief, power balance equation in DED problem contains two variables, namely lambda and power demand. At specified power demand, power balance is a highly non linear equation in terms of lambda. Non linear equations with single variables can be solved by root finding methods [13] available in numerical methods. In this paper, Brent method [14] is proposed to solve the DED problem.

The proposed algorithm has been implemented in MATLAB on a Pentium IV, 2.4 GHz personal computer with 512 MB RAM. The paper is organized as follows: Formulation of DED problem is introduced in section II. The description of Brent method is addressed in section III. Implementation of Brent method for solving DED problem is given in Section IV. The simulation results of power system with various generating units are presented in Section V. Conclusions are finally given in the last section.

II. Dynamic Economic Dispatch Problem

The main objective of DED problem is to determine the optimal schedule of output powers of online generating units with predicted power demands over a certain period of time to meet the power demand at minimum operating cost. The mathematical formulation of DED problem is given below.

The fuel cost function of the generating unit is expressed as a quadratic function of real power generation.

The objective function of the DED problem is

$$\operatorname{Min} C_T = \sum_{t=1}^{T}\sum_{i=1}^{ng} C_i(P_i^t) \quad (1)$$

where

$$C_i(P_i^t) = a_i + b_i P_i^t + c_i P_i^{t\,2} \quad (2)$$

The objective function is subjected to various constraints, which are given below.

A. Equality Constraint

$$\sum_{i=1}^{ng} P_i^t = P_D^t + P_L^t \quad (3)$$

The total transmission loss is assumed as a quadratic function of output powers of the generator units [15].

$$P_L^t = \sum_{i=1}^{ng}\sum_{j=1}^{ng} P_i^t B_{ij} P_j^t + B_{i0} P_j^t + B_{00} \quad (4)$$

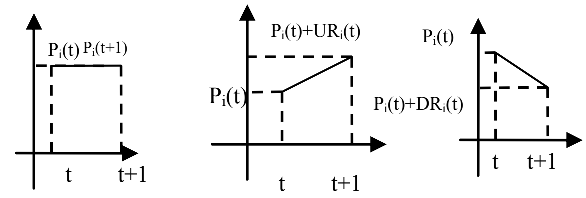
B. Inequality Constraints

1) Generator Limits

$$P_{i,\min} \le P_{i,t} \le P_{i,\max} \quad (5)$$

2) Ramp Rate Limits

The range of actual operation of online generating unit is restricted by its ramp rate limits. These limits can impact the operation of generating unit. The operational decision at the present hour may affect the operational decision at the later hour due to ramp rate limits. In actual operation, three possible situations exist due to variation in power demand from present hour to next hour. First, during the steady state operation, the operation of the online unit is in steady state condition. Second, if the power demand is increased, the power generation of the generating unit also increases. Third, if the power demand is decreased then the power generation of the generating unit also decreases. The ramp rate limits with all possible cases are shown in Fig. 1.



a) Steady state power demand   b) Increased power demand   c) Decreased power demand

Fig. 1. Ramp rate limits of the generating units.

The generator constraints due to ramp rate limits of generating units are given as

A) when generation increases

$$P_{i,t} - P_{i,t-1} \le UR_i \quad (6)$$

B) when generation decreases

$$P_{i,t-1} - P_{i,t} \le DR_i \quad (7)$$

Therefore the generator constraints can be modified as

$$\max(P_{i,\min}, P_{i,t-1} - DR_i) \le P_{i,t} \le \min(P_{i,\max}, P_{i,t-1} + UR_i) \quad (8)$$

From (8), the limits of minimum and maximum output powers are modified as follows

$$P_{i,\min\_ramp} = \max(P_{i,\min}, P_{i,t-1} - DR_i) \quad (9)$$

$$P_{i,\max\_ramp} = \min(P_{i,\max}, P_{i,t-1} + UR_i) \quad (10)$$

Formulation of Lagrange function for the DED problem with ramp rate limits is given by

$$\chi = F_T + \lambda \times (P_D + P_L - \sum_{i=1}^{ng} P_i) \quad (11)$$

The expressions of lambda and output power are

$$\lambda_i = \frac{\beta_i + (2\times\gamma_i\times P_i)}{1-(2\times\sum_{i=1}^{ng} B_{ij}P_j + B_{i0})} \quad (12)$$

$$P_i = \frac{\lambda_i \times (1 - B_{i0} - 2\times \sum_{j=1,i\neq j}^{ng} B_{ij}P_j) - \beta_i}{2\times(\gamma_i + \lambda_i B_{ii})} \quad (13)$$

III. Brent Method

Brent method is a root finding method which combines root bracketing, bisection and inverse quadratic interpolation. It uses a Lagrange interpolation polynomial of degree 2. Brent claims that this method always converges as long as the values of the function are computable within a given region containing a root.

Brent method fits x as a quadratic function of y from the three points x_1, x_2 and x_3 and then the relation between the x and y are obtained as follows from the interpolation formula.

$$x = \frac{(y - f(x_1))(y - f(x_2))x_3}{(f(x_3) - f(x_1))(f(x_3) - f(x_2))} + \frac{(y - f(x_2))(y - f(x_3))x_1}{(f(x_1) - f(x_2))(f(x_1) - f(x_3))} + \frac{(y - f(x_3))(y - f(x_1))x_2}{(f(x_2) - f(x_3))(f(x_2) - f(x_1))} \quad (14)$$

subsequent estimation of root is obtained by setting $y = 0$

$$x = x_2 + \frac{P}{Q} \quad (15)$$

where

$$P = S\left[T\left(R - T\right)\left(x_3 - x_2\right)\right] - \left[\left(1 - R\right)\left(x_2 - x_1\right)\right] \quad (16)$$

$$Q = (T-1)(R-1)(S-1) \quad (17)$$

with

$$R = \frac{f(x_2)}{f(x_3)} \quad (18)$$

$$S = \frac{f(x_2)}{f(x_1)} \quad (19)$$

$$T = \frac{f(x_1)}{f(x_3)} \quad (20)$$

IV. Implementation of Brent Method for Dynamic Economic Dispatch Problem With Transmission Losses

In this section, Brent method has been proposed for solving DED problem with transmission losses.

The power balance equation can be written as

$$f(\lambda, P_D) = \sum_{i=1}^{ng} P_i - (P_D + P_L) \quad (21)$$

It is clear from (21) that $f(\lambda, P_D)$ contains two variables λ and P_D. At specified P_D, $f(\lambda)$ is highly non-linear in terms of λ. Therefore, equation (21) becomes

$$f(\lambda) = 0 \quad (22)$$

where $f(\lambda) = 0$ is a non linear relation in λ. The solution of λ is obtained by Brent method.

Two steps are involved for solving the DED problem

A. Selection of Lambda Value

At required power demand, the best two lambda values are obtained from reduced pre-prepared power demand (RPPD) table. The formulation of pre-prepared power demand table and RPPD table are given below

1) Formation of PPD Table

(i) From (12), lambda values are evaluated at the minimum and maximum output powers of all generators by incorporating ramp rate limits.

(ii) All the lambda values are arranged in ascending order.

(iii) The output powers and power losses are computed for all values of lambda.

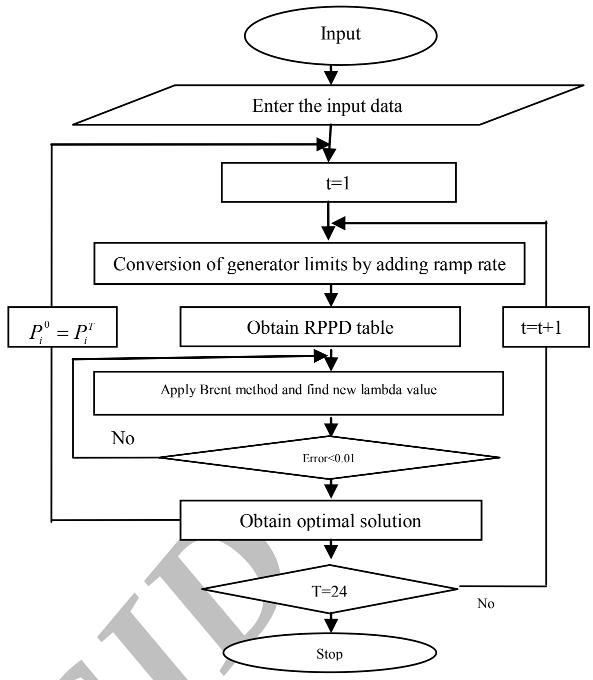


Fig 2. Flow chart of Brent method for solving DED problem.

(iv) All lambda values, output powers, Sum of Output Powers (SOP), power losses and SOP plus power loss are formulated as a table. This table is called PPD table.

2) Formation of RPPD Table

At required power demand, the upper and lower rows of the PPD table are selected such that the power demand lies within the SOP plus loss and these two rows are formulated as a table and it is known as reduced PPD (RPPD) table.

The application of Brent method to find the optimal lambda value from the power balance equation at required power demand in the ED problem is as follows.

At the required power demand

$$x_1 = \lambda_j \quad \text{and } f(x_1) = SOP_j + P_{L\,j} \quad (23)$$

$$x_3 = \lambda_{j+1} \quad \text{and } f(x_3) = SOP_{j+1} + \mathrm{P}_{\mathrm{L\,j+1}} \quad (24)$$

$$x_2 = (\lambda_j + \lambda_{j+1})/2 \quad (25)$$

At x_2, $f(x_2)$ value is evaluated and finally from (15), the optimal lambda value is evaluated by an iterative approach.

Solution of the DED problem by the proposed algorithm is as follows

- Enter the input data
- Lambda values are calculated using (12) for all units at their maximum and minimum output powers by incorporating ramp rate limits and then are arranged in ascending order and finally minimum and maximum lambda values are selected.
- Output powers and power loss are computed for selected lambda values.
- lambda is evaluated by Brent method from the power balance equation. Set the generator constraints by incorporating ramp rate limits.
- Optimal solution is obtained

The complete flow chart of Brent method for solving dynamic economic dispatch is shown in Fig. 2.

TABLE I
FUEL COST DATA OF SIX UNITS SYSTEM

U	a_i (\$)	b_i (\$/MW)	c_i (\$/MW^2)	$P_{i,\min}$ (MW)	$P_{i,\max}$ (MW)
1	240	7	0.007	100	500
2	200	10	0.0095	50	200
3	220	8.5	0.009	80	300
4	200	11	0.009	50	150
5	220	10.5	0.008	50	200
6	190	12	0.0075	50	120

TABLE II
RAMP RATE LIMITS DATA OF SIX UNITS SYSTEM

Unit	P_i^0 (MW)	UR_i (MW/h)	DR_i (MW/h)
1	340	80	120
2	134	50	90
3	240	65	100
4	90	50	90
5	110	50	90
6	52	50	90

TABLE III
DATA OF PRDICTED POWER DEMANDS FOR 6 UNITS SYSTEM

H	1	2	3	4	5	6	7	8
PD (MW)	955	942	935	930	935	963	989	1023
H	9	10	11	12	13	14	15	16
PD (MW)	1126	1150	1201	1235	1190	1251	1263	1250
H	17	18	19	20	21	22	23	24
PD (MW)	1221	1202	1159	1092	1023	984	975	960

V. CASE STUDIES AND SIMULATION RESULTS

This section presents numerical examples and simulation results of two test cases to evaluate the performance of the proposed method. The proposed algorithm has been implemented in MATLAB and executed on Pentium IV, 2.4 GHz personal computer with 512 MB RAM to solve the DED problem of a power system having 6 and 15 generating units with generator constraints and transmission losses. The results obtained from the proposed method were compared in terms of the solution quality and computation efficiency with lambda iterative method.

During the execution of conventional lambda iterative method, the lambda value and incremental lambda values are selected based on the dimensionality of the problem.

Example-1) In this example, 6- units system is considered. The data was extracted from [16]. The fuel cost data of the six thermal units is given in Table I. B-Loss coefficients of 6- units system is given as follows

$$B_{ij} = 10^{-3}\begin{bmatrix} 1.7 & 1.2 & 0.7 & -0.1 & -0.5 & -2.0 \\ 1.2 & 1.4 & 0.9 & 0.1 & -0.6 & -0.1 \\ 0.7 & 0.9 & 3.1 & 0.0 & -1.0 & -0.6 \\ -0.1 & 0.1 & 0.0 & 0.24 & -0.6 & -0.8 \\ -0.5 & -0.6 & -0.1 & -0.6 & 12.9 & -0.2 \\ -2.0 & -1.0 & -0.6 & -0.8 & -0.2 & 15.0 \end{bmatrix}$$

$$B_{oi} = 10^{-3} \bullet [-0.3908 \ \ -1.297 \ \ 7.047 \ \ 0.591 \ \ 2.161 \ \ -6.635]$$

$$B_{00} = 0.056$$

Ramp rate limits data is given in Table II. The data of predicted power demands is given in Table III.

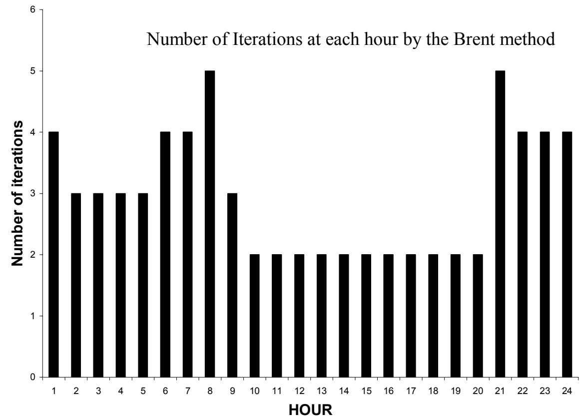


Fig. 3 Number of iterations at each hour for 24 hour by Brent method for 6 units system.

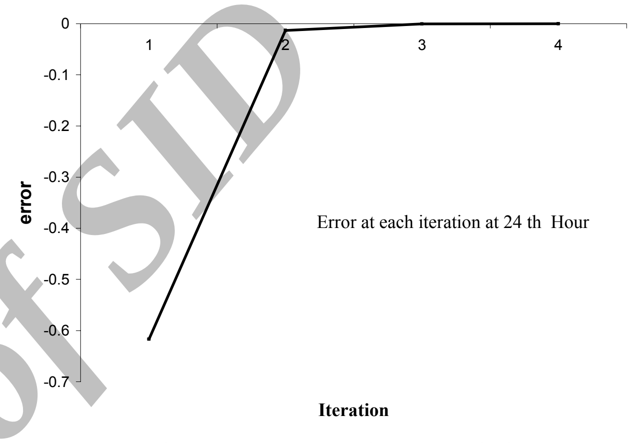


Fig. 4. Error at each iteration at 24*th* Hour by Brent method.

TABLE IV
OUTPUT POWERS AND POWER LOSSES FOR ALL POWER DEMANDS OF 6-UNITS SYSTEM

S.n	OUTPUT POWERS (MW)						Loss (MW)
	P1	P2	P3	P4	P5	P6	
1	380.34	123.69	211.33	84.278	112.78	50	7.4194
2	377.14	121.33	208.84	81.672	110.25	50	7.2344
3	375.42	120.05	207.5	80.271	108.89	50	7.136
4	374.19	119.14	206.55	79.27	107.92	50	7.0662
5	375.42	120.05	207.5	80.271	108.89	50	7.136
6	382.31	125.15	212.86	85.882	114.34	50	7.5348
7	388.72	129.88	217.83	91.098	119.39	50	7.9173
8	397.1	136.07	224.34	97.927	126	50	8.4356
9	419.01	152.26	241.35	115.79	143.21	64.399	10.016
10	423.97	155.93	245.2	119.84	147.09	68.372	10.407
11	434.53	163.73	253.39	128.46	155.35	76.807	11.275
12	441.58	168.94	258.86	134.22	160.85	82.424	11.879
13	432.25	162.05	251.62	126.6	153.57	74.989	11.083
14	444.9	171.39	261.44	136.94	163.44	85.065	12.172
15	447.39	173.23	263.37	138.97	165.39	87.045	12.394
16	444.69	171.24	261.28	136.77	163.28	84.9	12.153
17	438.68	166.79	256.61	131.85	158.59	80.112	11.628
18	434.74	163.88	253.55	128.63	155.51	76.973	11.292
19	425.83	157.31	246.64	121.36	148.55	69.862	10.557
20	411.99	147.07	235.89	110.06	137.7	58.764	9.4798
21	397.1	136.07	224.34	97.927	126	50	8.4356
22	387.48	128.97	216.87	90.094	118.42	50	7.8429
23	385.27	127.33	215.15	88.289	116.67	50	7.7099
24	381.57	124.6	212.28	85.281	113.75	50	7.4914

The number of iterations at each power demand is shown in Fig. 3. It is clear that Brent method provides optimal solution within few iterations.

Output powers and power loss for all power demands are given in Table IV.

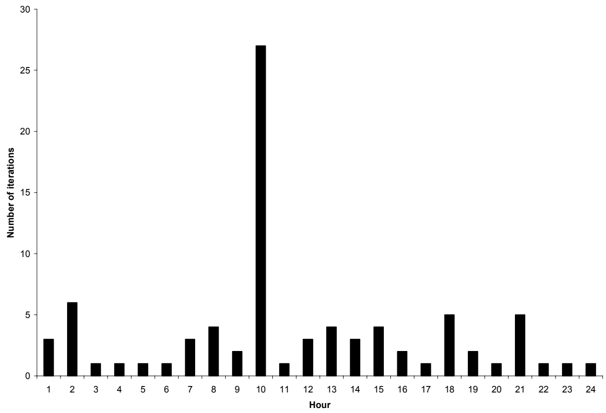


Fig. 5. Number of iterations for 24 hours by the Brent method.

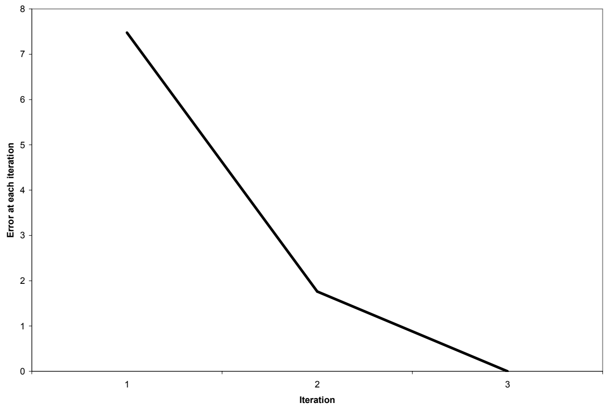


Fig. 6. Error at each iteration at the Power demand of 2236 MW.

The simulation results obtained from the proposed method are compared with lambda iterative method in terms of the solution quality, convergence characteristics and computational time and the statistical data is given in Table V.

Example-2) In this example, the system contains 15 generating units whose characteristics are given in table VI and the data was extracted from [15]. Power demands for 24 hours are given in Table VII.

TABLE V
SIMULATION RESULTS OF LAMBDA ITERATIVE METHOD AND THE PROPOSED METHOD

Methods	Lambda Iterative method	Proposed
Fuel cost	313405.648	313405.403
Iterations	100	2
Time (Sec)	0.125	0.078

TABLE VI
FUEL COST DATA OF 15 GENERATING UNITS

U	a_i ($)	b_i ($/MW)	c_i ($/MW^2)	$P_{i,\min}$ (MW)	$P_{i,\max}$ (MW)
1	671	10.1	0.000299	150	455
2	574	10.2	0.000183	150	455
3	374	8.8	0.001126	20	130
4	374	8.8	0.001126	20	130
5	461	10.4	0.000205	150	470
6	630	10.1	0.000301	135	460
7	548	9.8	0.000364	135	465
8	227	11.2	0.000338	60	300
9	173	11.2	0.000807	25	162
10	175	10.7	0.001203	25	160
11	186	10.2	0.003586	20	80
12	230	9.9	0.005513	20	80
13	225	13.1	0.000371	25	85
14	309	12.1	0.001929	15	55
15	323	12.4	0.004447	15	55

TABLE VII
LOAD DEMAND FOR 24 HOURS

H	1	2	3	4	5	6
PD(MW)	2236	2240	2226	2236	2298	2316
H	7	8	9	10	11	12
PD(MW)	2331	2443	2630	2728	2783	2785
H	13	14	15	16	17	18
PD(MW)	2780	2830	2970	2950	2902	2803
H	19	20	21	22	23	24
PD(MW)	2651	2584	2432	2312	2261	2254

The proposed algorithm has been successfully applied for mixed generating units by considering 15 units system with transmission losses. The exact transmission

$$B_{ij} = 10^{-3}.\begin{bmatrix} 1.4 & 1.2 & 0.7 & -0.1 & -0.3 & -0.1 & -0.1 & -0.1 & -0.3 & -0.5 & -0.3 & -0.2 & 0.4 & 0.3 & -0.1 \\ 1.2 & 1.5 & 1.3 & 0.0 & -0.5 & -0.2 & 0.0 & 0.1 & -0.2 & -0.4 & -0.4 & 0.0 & 0.4 & 1.0 & -0.2 \\ 0.7 & 1.3 & 7.6 & -0.1 & -1.3 & -0.9 & -0.1 & 0.0 & -0.8 & -1.2 & -1.7 & 0.0 & -2.6 & 11.1 & -2.8 \\ -0.1 & 0.0 & -0.1 & 3.4 & -0.7 & -0.4 & 1.1 & 5.0 & 2.9 & 3.2 & -1.1 & 0.0 & 0.1 & 0.1 & -2.6 \\ -0.3 & -0.5 & -1.3 & -0.7 & 9.0 & 1.4 & -0.3 & -1.2 & -1.0 & -1.3 & 0.7 & -0.2 & -0.2 & -2.4 & -0.3 \\ -0.1 & -0.2 & -0.9 & -0.4 & 1.4 & 1.6 & 0.0 & -0.6 & -0.5 & -0.8 & 1.1 & -0.1 & -0.2 & -1.7 & 0.3 \\ -0.1 & 0.0 & -0.1 & 1.1 & -0.3 & 0.0 & 1.5 & 1.7 & 1.5 & 0.9 & -0.5 & 0.7 & 0.0 & -0.2 & -0.8 \\ -0.1 & 0.1 & 0.0 & 5.0 & -1.2 & -0.6 & 1.7 & 16.8 & 8.2 & 7.9 & -2.3 & -3.6 & 0.1 & 0.5 & -7.8 \\ -0.3 & -0.2 & -0.8 & 2.9 & -1.0 & -0.5 & 1.5 & 8.2 & 12.9 & 11.6 & -2.1 & -2.5 & 0.7 & -1.2 & -7.2 \\ -0.5 & -0.4 & -1.2 & 3.2 & -1.3 & -0.8 & 0.9 & 7.9 & 11.6 & 20 & -2.7 & -3.4 & 0.9 & -1.1 & -8.8 \\ -0.3 & -0.4 & -1.7 & -1.1 & 0.7 & 1.1 & -0.5 & -2.3 & -2.1 & -2.7 & 14.0 & 0.1 & 0.4 & -3.8 & 16.8 \\ -0.2 & 0.0 & 0.0 & 0.0 & -0.2 & -0.1 & 0.7 & -3.6 & -2.5 & -3.4 & 0.1 & 5.4 & -0.1 & -0.4 & 2.8 \\ 0.4 & 0.4 & -2.6 & 0.1 & -0.2 & -0.2 & 0.0 & 0.1 & 0.7 & 0.9 & 0.4 & -0.1 & 10.3 & -10.1 & 2.8 \\ 0.3 & 1.0 & 11.1 & 0.1 & -2.4 & -1.7 & -0.2 & 0.5 & -1.2 & -1.1 & -3.8 & -0.4 & -10.1 & 57.8 & -9.4 \\ -0.1 & -0.2 & -2.8 & -2.6 & -0.3 & 0.3 & -0.8 & -7.8 & -7.2 & -8.8 & 16.8 & 2.8 & 2.8 & -9.4 & 128.3 \end{bmatrix}$$

$$B_{i0} = 10^{-3}[-1 \quad -2 \quad 28 \quad -1\ 1 \quad -3 \quad -2 \quad -2\ 6 \quad 39 \quad -17 \quad -00 \quad -32\ 67\ -64]$$

$$B_{00} = 0.0055$$

TABLE VIII
OUTPUT POWERS FOR 24 HOURS BY BRENT METHOD FOR 15-UNITS SYSTEM

H	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	352.59	330.271	130	130	306.35	285.45	366.51	162.88	25	59.416	40.728	20	25	15	15
2	402.4	333.33	130	130	186.35	353.95	446.51	62.88	25	67.916	47.526	20	25	15	15
3	403.93	335.49	130	130	150	356.05	465	60	25	68.177	47.735	20	25	15	15
4	406.43	338.93	130	130	150	359.5	465	60	25	68.604	48.077	20	25	15	15
5	421.95	360.24	130	130	150	380.88	465	60	25	71.254	50.2	20	25	15	15
6	426.46	366.43	130	130	150	387.09	465	60	25	72.024	50.817	20	25	15	15
7	430.22	371.58	130	130	150	392.27	465	60	25	72.666	51.331	20	25	15	15
8	455	410.77	130	130	150	431.68	465	60	25	77.547	55.244	21.758	25	15	15
9	455	455	130	130	172.65	460	465	60	25	119.41	80	50.619	25	15	15
10	455	455	130	130	240.47	460	465	60	25	138.92	80	64.243	25	15	15
11	455	455	130	130	278.67	460	465	60	25	150.01	80	72.033	25	15	15
12	455	455	130	130	280.07	460	465	60	25	150.42	80	72.32	25	15	15
13	455	455	130	130	276.59	460	465	60	25	149.41	80	71.607	25	15	15
14	455	455	130	130	311.38	460	465	60	25	159.57	80	78.772	25	15	15
15	455	455	130	130	391.38	460	465	60	71.9	160	80	80	25	15	15
16	455	455	130	130	437.15	460	465	60	25	160	80	80	25	15	15
17	455	455	130	130	385.78	460	465	60	25	160	80	80	25	15	15
18	455	455	130	130	292.58	460	465	60	25	154.07	80	74.89	25	15	15
19	455	455	130	130	187.16	460	465	60	25	123.56	80	53.511	25	15	15
20	455	455	130	130	150	460	465	60	25	106.04	78.165	41.351	25	15	15
21	455	405.94	130	130	150	426.82	465	60	25	76.945	54.761	21.347	25	15	15
22	425.46	365.05	130	130	150	385.71	465	60	25	71.853	50.68	20	25	15	15
23	412.69	347.52	130	130	150	368.12	465	60	25	69.672	48.933	20	25	15	15
24	410.93	345.11	130	130	150	365.71	465	60	25	69.373	48.693	20	25	15	15

TABLE IX
POWER LOSS AND FUEL COST OF THE BRENT METHOD FOR 15 UNITS SYSTEM

H	Power Loss (MW)	Fuel cost ($)	H	Power Loss (MW)	Fuel cost ($)
1	28.201	28619	13	32.599	34232
2	20.919	28454	14	34.721	34787
3	20.381	28292	15	41.869	36412
4	20.537	28398	16	42.145	36133
5	21.528	29050	17	38.777	35590
6	21.825	29240	18	33.545	34487
7	22.075	29398	19	28.233	32817
8	23.982	30580	20	26.556	32089
9	27.671	32589	21	23.802	30464
10	30.647	33658	22	21.759	29198
11	32.719	34265	23	20.931	28661
12	32.8	34287	24	20.82	28587

TABLE X
TOTAL FUEL COST, AVERAGE ITERATIONS AND TIME OF BRENT METHOD FOR 24 HOURS

	Proposed method
Fuel cost ($)	760287.232
Average iterations	4
Time (Sec)	0.53

loss of the system is represented by B-Loss coefficients [17].

Optimal solution by the Brent method for 24 hours is given in Table VIII. Also the number of iterations for 24 hours is shown in Fig. 5. It is clear from the Fig. 5 that the Brent method provides the optimal solution in few iterations. Error at each iteration at the power demand of 2236 MW is shown in Fig. 6.

VI. CONCLUSIONS

In this paper, Brent method has been proposed for solving the dynamic economic dispatch problem of a power system having 6 and 15 units with the generator constraints, ramp rate limits and transmission losses. A salient feature of the proposed method is that it gives high quality solution with fast convergence characteristics compare to the lambda iterative method. Due to the fast convergence, the computational time is less for getting optimal solution. The proposed algorithm will not depend on any user defined parameters. Furthermore, the computational times of the proposed method are much less than the lambda iterative method and increase linearly with size of the system. The comprehensive numerical results prove the successful implementation and feasibility of the proposed approach for the ED problems.

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