Achieving a Better Performance Using FHT Instead of FFT in ADSL Systems

S. Ghazi Maghrebi, M. Lotfizad, and M. Ghanbari

***Abstract*—In this paper, we introduce a new constellation scheme to have better performance (in terms of BER) with respect to the conventional rectangular QAM constellation. We compared the new one and two new kinds of circular constellations with QAM. They were chosen so that they have the same average power. Also we compared the effects of using FHT and FFT for all constellations. Based on the results, the new constellations to provide slightly better performance in channels which have maximum SNR=20 dB. Also it is shown that the new constellation has better performance for the noisy channels (AWGN and burst) with ISI and low SNR. Also we have shown the effect of channel length with respect to cyclic prefix in this system.**

***Index Terms*—ADSL, FFT, FHT, multitone.**

I. Introduction

ADSL provides a high bit-rate downstream channel as well as lower bit-rate upstream one. The increasing demand for high-bandwidth services such as corporate communications, fast Internet access, and digital audio and video delivery presents both a great challenge and a great opportunity [1]. To respond to this challenge, and to take advantage of the opportunity, the telecommunications industry is developing communications equipment based on clever implementations of highly efficient modulation techniques. Most of these efforts are focused on the last mile as the bottleneck for providing high-performance bandwidth to consumers. Additionally, since the telephone has the highest penetration rate of all media to consumers solving the last-mile problem implicitly requires that the bandwidth be delivered over telephone lines [2].

The multi-carrier modulation (MCM) has features that make it ideal for communications over dispersive channels in which both the insertion loss of the channel and the injected noise are frequency dependent. Practical wired-line and wireless channels present such dispersion and frequency dependency, making DMT and OFDM ideal modulation for such environments [3].

ADSL provides a high bit-rate downstream channel and a lower bit-rate upstream channel over twisted pair copper wire. The transmission method selected for ADSL is based on discrete multi-tone modulation (DMT). Various aspects of a high performance ADSL transceiver system is accomplished by the powerful DMT modulation technology. DMT divides the available bandwidth into parallel sub-channels or tones. MCM has features that make it ideal for communications over band limited, highly-dispersive channels in which both the insertion loss of the channel and the injected noise are frequency dependent [4].

The incoming serial bit-stream is divided into parallel streams, which are then used to QAM- modulate the different tones. Very efficient modulation and equalization techniques are necessary in the ADSL transmission environments because of severe channel attenuation, inter-symbol interference(ISI), and a host of other line impairments, including crosstalk, additive white Gaussian noise(AWGN), and impulse noise[5]. As a result, the powerful DMT modulation technique was chosen by ANSI as the standard line code for ADSL. Due to an imperfect balance between the twisted pair channels, different noises occur on the telephone lines. The most common noises include crosstalk, radio-frequency interference (RFI) and impulse noise. The impact of impulse noise on practical systems depends on the impulse power, duration, inter-arrival and spectral characteristics [5]. In this paper we have applied the AWGN and impulse noise to the channel separately for each of the constellations.

The rectangular QAM is employed in the conventional DMT. In the US, 64-QAM and 256-QAM are the mandated modulation schemes for digital cable and in UK, 16-QAM and 64-QAM are currently used for digital terrestrial television. The rectangular QAM constellation is, in general, sub-optimal in the sense that it does not maximally space the constellation points for a given energy.

After modulation with an IFFT, a cyclic prefix (CP) is added to each symbol. If the cyclic prefix length is equal or longer than the channel impulse response duration minus one, then the demodulation can be implemented by means of an FFT, followed by a (complex) 1-tap frequency domain equalizer (FEQ) per tone to compensate for the channel amplitude and phase effects. A long prefix however results in a large overhead with respect to the data rate [6].

On the other hand, the partitioning of a channel bandwidth into subchannels can be achieved using many normal bases, but the one particular basis which has found favor for DSL applications is the Fourier transform, and particularly the IDFT. One key advantage of the DFT and IDFT is the availability of efficient computational methods, such as the FFT [7]. Another normal basis that can be used in ADSL is the Fast Hadamard Transform (FHT), and because of its simple structure, it uses less computational operations in comparison to the FFT, so we used it instead of the FFT, and consequently the results show a better performance of the system. In this paper, we have shown

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S. Ghazi Maghrebi is with Islamic Azad University Science and Research Branch, Tehran, I. R. Iran. (e-mail:s_ghazi2002@yahoo.com).

M. Lotfizad is with the Department of Electronics, Tarbiat Modares University, Tehran, I. R. Iran (lotfizad@modares.ac.ir).

M. Ghanbari is with the School of Computer Science and Electronic Engineering, University of Essex, United Kingdom. (e-mail: ghan@essex.ac.uk).

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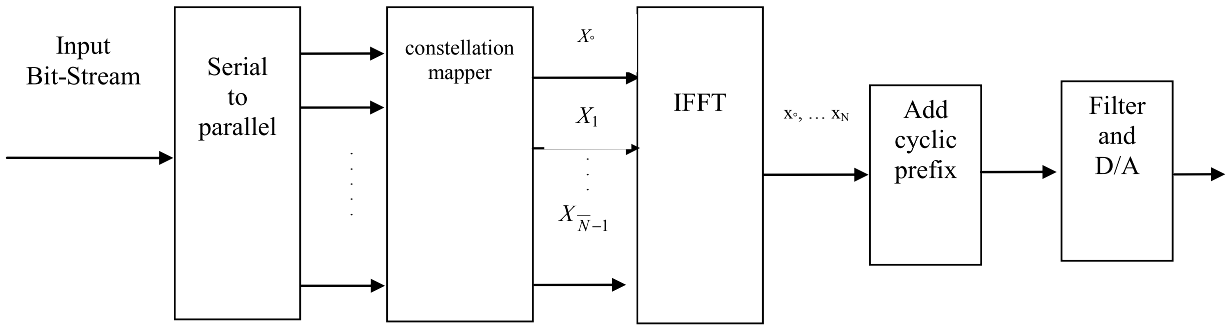


Fig. 1. DMT transmitter block diagram.

that by using the FHT, which has less complexity, instead of the FFT we will obtain a lower BER in the same situation for some constellations. Also we have introduced a new QAM constellation with better performance, especially when we use the FHT instead of FFT.

In this paper, we first analyze DMT system and then in section III we describe QAM modulation and its specifications and its major parameters. In section IV we explained FHT and finally in section V the results of simulations are shown.

II. Analysis of DMT

MCM is a technique of breaking up the frequency spectrum into multiple sub-carriers, each of which is orthogonal to the other sub-carriers. Each of the sub-carriers implements the QAM which enables easier transmission and reception of each sub-carrier [8].

DMT over single carrier modulation (SCM) has major advantages such as higher performance, immunity to noise and line conditions, simplicity in design and implementation, flexibility and rate adaptation capabilities [3].

The most popular manifestation of MCM is the DMT that is used in ADSL. In the block diagram of a DMT transmitter in Fig. 1 a sampled and digitized analog signal is divided into parallel blocks with a serial to parallel converter**.** These blocks are the input to the constellation mapping**,** which is basically representing segments of bits as spectral coefficients**.** The power necessary to transmit the actual signal is proportional to the area, so the points must not be too far apart**.** The FFT of a *N*-Point sequence causes greatly reducing the computational complexity from N^2 operations that would be required for most matrix multiplications to $NLog_2^{(N)}$ operations**.** An additional advantage of using the IDFT for the multi-carrier modulation is that the basis functions are fixed and independent of the channel**.** The DFT of an *N*-dimensional sequence $\mathbf{x} = [x_\circ \;\; x_2 .... \;\; x_{N\text{-}1}]^T$ is given by $X = [X_\circ \;\; X_2 .... \;\; X_{N\text{-}1}]^T$ where

$$X_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n e^{-j(\frac{2\pi}{N})kn} \quad , \quad \forall k \in [0, N-1] \qquad (1)$$

The IDFT of **X** is then given by

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j(\frac{2\pi}{N})kn} \quad , \quad \forall k \in [0, N-1] \qquad (2)$$

The DFT can be written in matrix form as $\mathbf{X}=\mathbf{Q}\mathrm{x}$, and therefore the IDFT matrix can be written as $\mathrm{x}=\mathbf{Q}^*\mathbf{X}$.

We need to eventually transmit a real-valued signal, but after the constellation mapping, the blocks are all complex-valued**.**

$$Q = \frac{1}{\sqrt{N}} \begin{bmatrix} e^{-j(\frac{2\pi}{N})(N-1)(N-1)} & e^{-j(\frac{2\pi}{N})(N-2)(N-1)} & \cdots & 1 \\ e^{-j(\frac{2\pi}{N})(N-1)(N-2)} & e^{-j(\frac{2\pi}{N})(N-2)(N-2)} & \cdots & 1 \\ \vdots & \vdots & & \\ e^{-j(\frac{2\pi}{N})(N-1)} & e^{-j(\frac{2\pi}{N})(N-2)} & \cdots & 1 \\ 1 & 1 & \cdots & \end{bmatrix} \qquad (3)$$

Mirroring ensures that the transmitted signal is real-valued by using the fact that the IFFT of a conjugate-symmetric vector is real; **i.e.,** mirroring gives each block a conjugate-symmetry**.**

In DMT, the subchannels overlap in such a way that they remain orthogonal at the sub-carrier frequencies. Each symbol of the DMT transmitter is applied to the channel, where each symbol is the result of an IDFT operation, can be considered to be windowed in the time domain by a rectangular pulse, which is caused by the finite duration of each symbol. Denoting the sub-carrier spacing as Δf , the n th DMT symbol is the sum of components that can be written in the time domain as

$$x_{n,k}(t) = [X_k e^{j2\pi\Delta f kt} + X_k^* e^{-j2\pi\Delta f kt}].w(t) \qquad (4)$$

where $x_{n,k}(t)$ represents the components of the n th symbol due to the k th sub-channel. The rectangular window is defined as

$$w(t) = \begin{cases} 1 & t \in \left(0, \dfrac{1}{\Delta f}\right) \\ 0 & t \notin \left(0, \dfrac{1}{\Delta f}\right) \end{cases} \qquad (5)$$

The Fourier transform of $w(t)$ is a Sinc function with zeros at multiples of Δf . Therefore, at any sub-carrier frequency, the value of aggregate signal (the sum of all the Sinc functions corresponding to the sub-channels) is due only to the signal on that sub channel. Thus, the signal at any multiple of Δf is independent of all other sub-channels.

The result of the IDFT yields a composite signal whose frequency spectrum is a sum of Sinc functions that are centered at integer multiples of the sub-channel spacing (sub carrier frequencies), with zeros at all other sub-carrier frequencies.

In practice, however, the frequency response across any selected sub-channel is not likely to be perfectly flat, so ISI is not completely eliminated by the partitioning process.

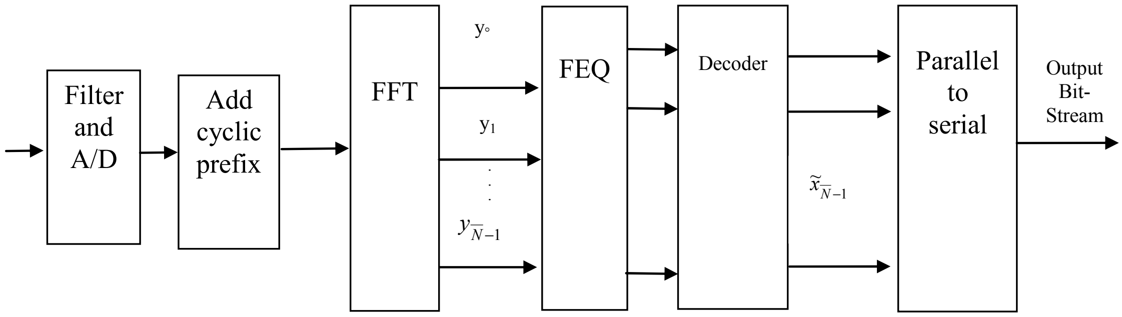


Fig. 2. DMT receiver block diagram.

Instead, if the noise on each sub-channel is assumed to be white and Gaussian, inter-symbol and inter-sub-channel interference caused by the channel's impulse response length being grater than one can be eliminated by use of a CP, otherwise the ISI caused by each symbol is confined to the CP of the following symbol. Consequently, by discarding the CP samples in the receiver prior to demodulation using the DFT, ISI can be eliminated completely [6].

However, the duration of the CP required to eliminate ISI is a function of the channel impulse response duration, which is a function of the loop length, and the required length can not be known precisely a priori. To compensate for residual ISI that occurs if the channel impulse response duration exceeds the chosen CP duration, to shorten the channel impulse response length to within the CP duration, a time-domain equalizer (TEQ) may be used in the receiver [9].

We implemented a channel with a frequency response, which added two forms of interference: the inter-symbol-interference (ISI) and the inter-channel-interference (ICI).

The ISI arises from the fact that the channel performs a linear convolution of its impulse response with the time-domain waveform. The ICI comes from the fact that the carrier frequencies for DMT lose their orthogonality due to the frequency response of the channel. Without orthogonal carriers, the FFT can not exactly recover the correct spectral coefficients. Cyclic padding solves this problem by turning the linear convolution of the channel impulse response with the signal into a cyclic convolution. The addition of a CP to each symbol solves both the ISI and, ICI problems [6]. A CP (of length L) is inserted at the beginning of each block to combat problems introduced by the channel. In wired mediums in high frequency electro-magnetic waves are quickly attenuated, while low frequency waves retain much of their power, even over long distances. On this basis, we model the frequency response of our channel as a low-pass filter.

On the other side of the channel shown in Fig. 2, the received signal is again broken up into parallel blocks. The CPs are removed, the FFT of each block is taken and each one is de-mirrored. A de-constellation mapping then occurs, which converts the complex values back to bits and increases the block length back to N .

After equalization, the effect of the channel's low-pass filter is removed, but the additive noise is still there. To enable the bit-stream to be recovered, a nearest-neighbor approximation is performed on each point. The blocks of bits are concatenated back into a single bit-stream, which then undergoes a D/A conversion back to a sampled analog signal.

In ADSL, the value of N in the downstream direction is 512 which would result in theory $\mathrm{PAR} \simeq 55$, the CP duration is 32 samples, and the sampling rate is 2208 kHz. The sub-carrier spacing is 4.3125 kHz. To aid ADSL receiver synchronization, a special symbol known as the synchronization symbol is transmitted after every 68 data symbols [6].

As a result, the synchronization symbol adds 544/68=8 samples of additional overhead to each data symbol. Therefore, the data symbol rate is calculated as $1/T = 2208/(512+32+8) = 4$ kHz. Consequently, the CP results in an overhead penalty of approximately 8 percent [6].

III. QAM MODULATION

There are two main scenarios usually considered in DSL deployments. In the first scenario, a DSL system is deployed over unused copper pairs (dark copper) or replace a digital loop carrier (DLC) system (T1 or E1). Because only DSL uses the wire, a base-band transmission can be used to utilize frequencies almost down to zero, benefiting from low attenuation of the twisted pair in low frequencies. In the second scenario, a DSL system has to share a wire with POTS or basic rate ISDN. To ensure minimal impact on their operation, DSL systems use only frequencies above the spectrum of these base-band services. Accordingly, pass-band transmission is preferable for this scenario, and the most popular modulation technique is the well-known QAM.

Different systems use a wide variety of different constellations, compromising between the complexity of the implementation and the efficiency of the constellation for the given transmit power. The QAM which is used in conventional DMT is a class of non-constant envelope schemes that can achieve higher bandwidth efficiency than MPSK with the same average signal power. One important point is that the shape of the power spectral density (PSD) of a QAM scheme is determined by the base-band pulse shape, and the magnitude of the PSD is determined by the average power (or average amplitude) of the QAM signal set. It is also worthwhile to point out that the shape of the PSD of a QAM scheme is independent of the constellation. In other words, no matter what the constellation is, i.e. rectangular, circular or others, the PSD shape is the same as long as the $p(t)$ is the same; the PSD magnitude is also the same, as long as the average signal power is also the same [10].

For most practical cases, the values of K_I and K_Q are even (to get a symmetric signal space) and can be calculated as: $K_I = K_Q = 2\times\lceil 2M/2-1 \rceil$, where the ceiling function $\lceil . \rceil$ rounds up to the nearest integer and K_I and K_Q are the number of possible setting of in-phase component and quadrature component respectively. The ceiling function is necessary for non-integer values of $M/2$. For instance, in the case of a 32-QAM, $M=5$, $K_I = K_Q = 6$. The output signal of a QAM modulator can be written as

$$S_{QAM} = \sum_n I_n g(t-nT)Cos(2\pi f_c t) - \sum_n Q_n g(t-nT)Sin(2\pi f_c t) \quad (6)$$

The fact that after demodulation the signal has a base-band format is sometimes an advantage, because it simplified the implementation of QAM transceivers operating with high carrier frequencies [6].

In DSL, additional data encoding schemes, such as forward error-correction (FEC) coding or trellis coding, are usually used prior to the constellation encoding. The spectral magnitude function of the QAM signal may be expressed as

$$\left|S_{QAM}(f)\right| \approx \left|\frac{Sin(\pi T(f-f_c))}{\pi T(f-f_c)}\right| \quad (7)$$

The main parameters of modulated signals are the average power per symbol, peak-to-average ratio (*PAR*), and the minimum Euclidean distance. The relative average power per symbol is

$$P = \frac{1}{2^M}\sum_{k=1}^{2^M}(I_k^2 + Q_k^2) \quad (8)$$

where I_k and Q_k are the in-phase and quadrature components respectively. The *PAR*, in dB, is calculated as a ratio between the peak and the average power:

$$PAR = 10\log_{10}\left(\frac{\max_k(P_k)}{p}\right)+3 \quad (dB) \quad (9)$$

The Euclidian distance $d(k,i)$ is the geometric distance between points k and i of the constellation diagram. The minimum $d(k,i)$ normalized to the averaged symbol power p indicates the relative noise immunity of the particular constellation: $d_N = \min_{k,i}[d(k,i)]/\sqrt{P}$. That value, in turn, expresses the difference in SNR necessary to ensure the same average probability of symbol error when different constellations are used. Taking 2-QAM as a convenient reference, the noise immunity of other constellations can be estimated, in dB, as

$$\eta = 20\log_{10}\frac{d_N[2^M - QAM]}{d_N[2 - QAM]} \quad (dB) \quad (10)$$

In the QAM, the constellation points are usually arranged in a square grid with equal vertical and horizontal spacing, although other configurations are possible. By moving to a higher order constellation, it is possible to transmit more bits per symbol. However, if the mean energy of the constellation is to remain the same, the points must be closer together and are thus more susceptible to noise and other disturbances; this results in a higher bit error rate and so higher-order QAM can deliver more data less reliable than the lower-order QAM[6]. The rectangular QAM constellations are, in general, sub-optimal in the sense that they do not space maximally the constellation points for a given energy. The non-rectangular constellation, dealt with below, achieves marginally better bit-error rate (BER) but is harder to modulate and demodulate. For evaluating the performance of an even number of bits per symbol, k, the exact expressions are as

$$P_{sc} = 2\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3}{M-1}\frac{E_s}{N_0}}\right) \quad (11)$$

So

$$P_s = 1-(1-P_{sc})^2 \quad (12)$$

where P_s is the probability of symbol-error and P_{sc} is the probability of symbol-error per carrier, E_s is the energy per symbol and N_o is the noise power spectral density (W/Hz). Where $Q(.)$ is the error function. The bit-error rate will depend on the exact assignment of bits to symbols, but for Gray-coded assignment with equal bits per carrier we have

$$P_{bc} = \frac{4}{k}\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3k}{M-1}\frac{E_b}{N_0}}\right) \quad (13)$$

So

$$P_b = 1-(1-P_{bc})^2 \quad (14)$$

where P_b is the probability of bit-error and P_{bc} is the probability of bit-error per carrier. It is hard to establish expressions for the error-rates of non-rectangular QAM since it necessarily depends on the constellation. Nevertheless, an obvious upper bound to the rate is related to the minimum Euclidean distance of the constellation

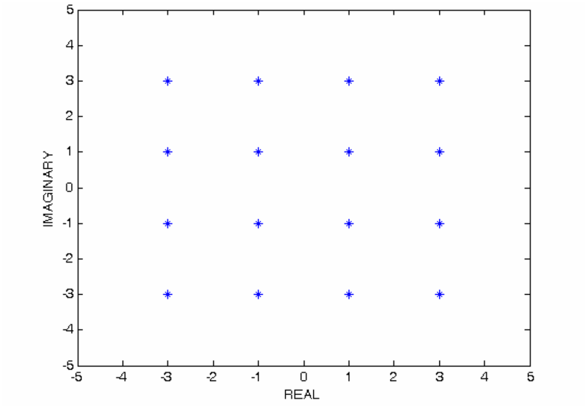
$$P_s \le 4Q\left(\sqrt{\frac{3kE_b}{(M-1)N_0}}\right) \quad (15)$$

Again, the bit-error rate will depend on the assignment of bits to symbols [9].

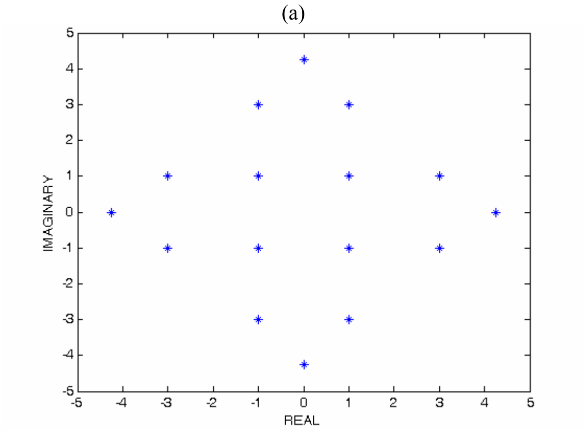
IV. The Fast Hadamard Transform (FHT)

The Hadamard transform in symmetric form lends itself to applications ranging across many technical fields. The Hadamard transform of an $N=2^n$ dimensional vector $\mathbf{x}$ is simply $y = \mathbf{H}_n\mathbf{x}$. Because the Hadamard matrix consists of ±1 s, the computation consists of adding and subtracting the components of $\mathbf{x}$. It involves no multiplies, and if the components of **x** are all integers, the computation does not require any floating-point operations at all. However, by implementing the Hadamard transform with a straightforward matrix multiplication as written, it requires a hefty $N^2 = 2^{2n}$ operations. Instead, we should implement one of the fast Hadamard transform (FHT) algorithms that exploit many of the symmetries discussed above. Many of these fast algorithms require only $N\log_2^N = n2^n$ additions.

Because the rows in a Hadamard matrix are orthogonal, we can use the FHT to decompose any signal into its constituent Hadamard components. It functions just like an



(a)



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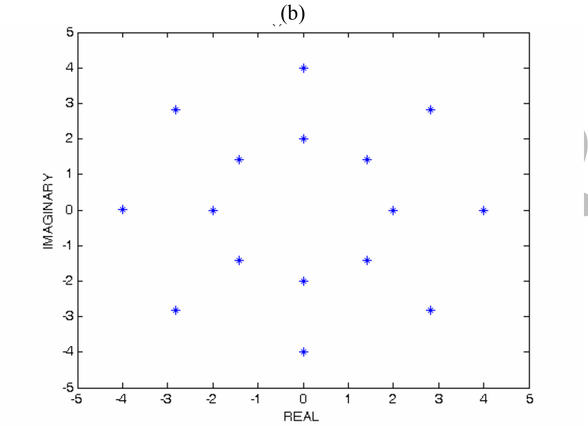

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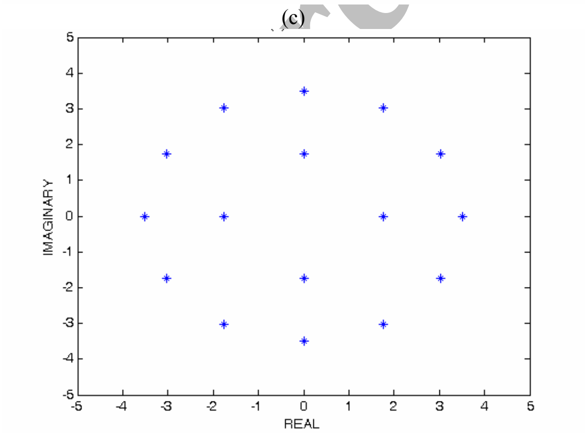
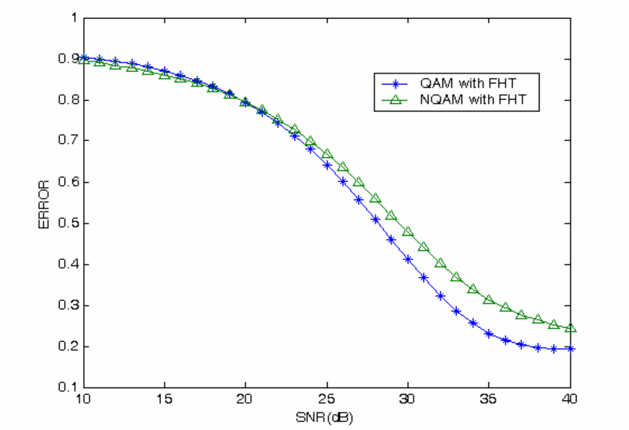


(d)

Fig. 3. Different constellations, (a) Rectangular 16-QAM consallation, (b) Non-Rectangular 16-QAM consallation, (c) Circular (I) constellation, and (d) Circular (II) constellation.

FFT, except that the Hadamard components are based on sequency rather than frequency**.** But because an FHT requires fewer computations (it omits the FFT's twiddle factors), we can implement the FHT on smaller and cheaper hardware**.** One key difference between the FHT and the FFT is that the magnitude of an FFT is invariant to phase shifts in the signal. This fact is not true for the FHT because a circular shift in one row of the Hadamard matrix does not leave it orthogonal to other rows. In fact, the shifted row might be closer (in Hamming distance) to other rows than to the original. Thus data alignment is critically important to FHT applications. For this reason, the FHT is less useful than the FFT in most spectral-analysis applications [11].



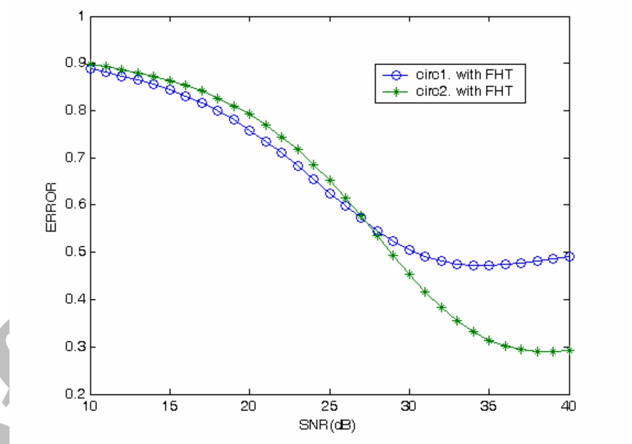


Fig. 4. The effects of FHT on different constellations.

TABLE I
MAIN PARAMETERS OF CONSTELLATIONS

Parameter	Rectangular QAM	Non-rectangular QAM	Circular (I)	Circular (II)
P	10	10	10	10
PAR[dB]	5.55	5.55	5.04	3.90
d_N	0.632	0.50	0.39	0.41
η [dB]	-10	-11.96	-14.17	-13.38

V. SIMULATION RESULTS

In this paper, we applied four constellations to DMT system as shown in Fig. 3 with their specifications given in Table I. As shown their average powers are the same and in this situation we compared their performances in terms of BER.

For the first experiment, we applied a file as the input of the DMT system with 129 taps impulse response length and AWGN noise (with mean=0) and compared using FFT and FHT in DMT system separately **(**Fig. 4).

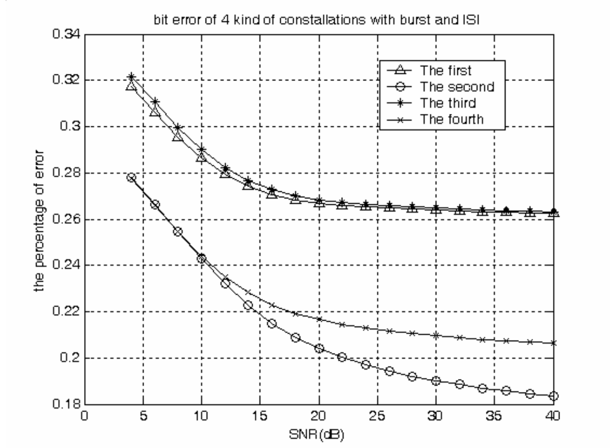


Fig. 5. Comparison of 4 constellations with AWGN channel len. $L/2$.

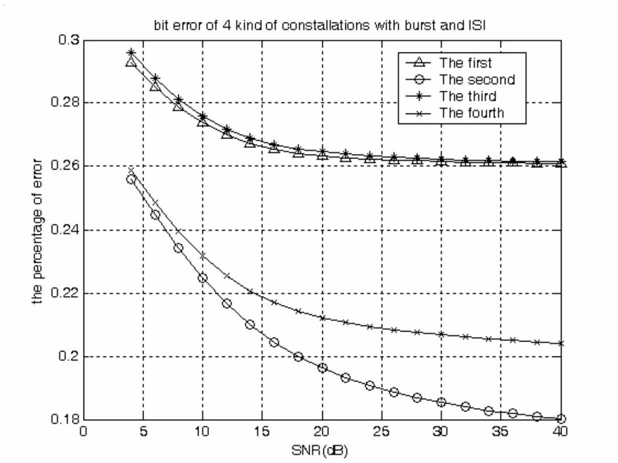


Fig. 6. Comparison of 4 constellations with AWGN channel len. $L-1$.

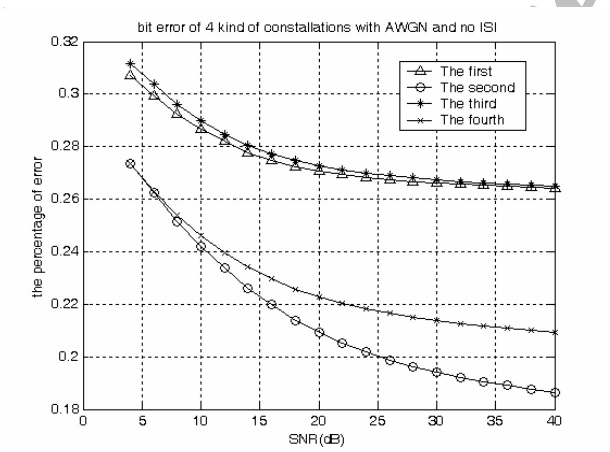


Fig. 7. Comparison of 4 constellations with AWGN channel len. L.

As can be seen, two kinds of QAM and NQAM and two kinds of circular constellations have the same BER at SNR=20 dB and SNR=27 dB respectively**.** Also the new constellation performs slightly better than the conventional QAM in the channels with low BER. Also it is very interesting that in the channel with SNR greater than 27 dB the circular constellation (form II) has a better performance with respect to form I, especially if we use the FHT.

Also, we applied these four constellations to different channels. In Fig. 5 through 7, the channel impulse response duration is shorter than CP length ($L=16$). As it is shown for the shorter channel and lower SNR, the new constellation is the same as the conventional constellation.

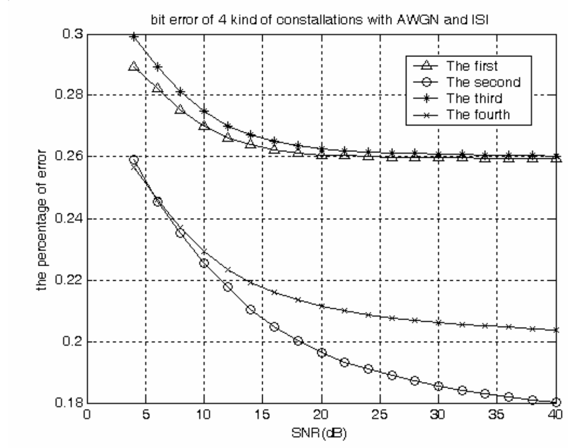


Fig. 8. Comparison of 4 constellations with AWGN channel len. $L+1$.

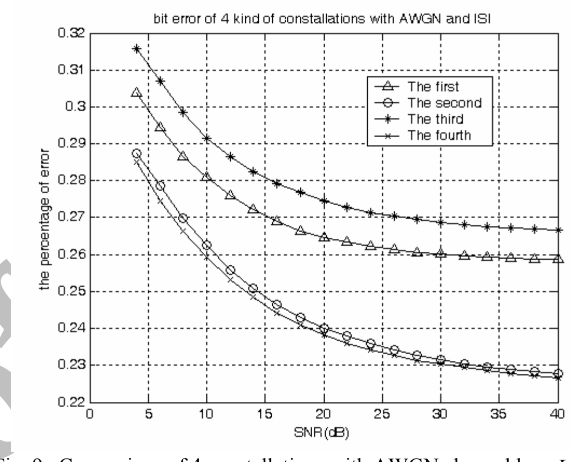


Fig. 9. Comparison of 4 constellations with AWGN channel len. $L+10$.

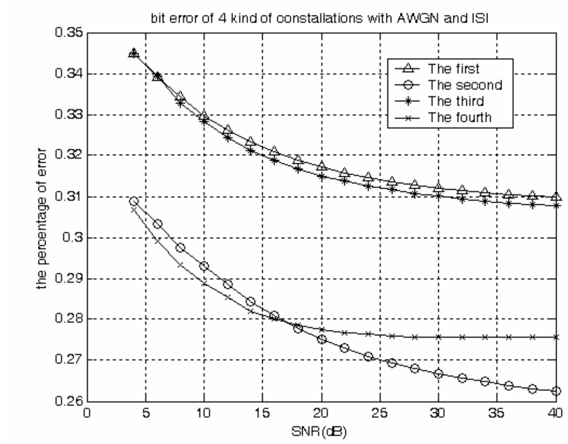


Fig. 10. Comparison of 4 constellations with AWGN channel len. $L+50$.

As shown in Fig. 8 through 11, by increasing the length of the channel impulse response (increasing ISI) with the new constellation the system performs better than others. In these Figures we compared the effect of varying the channel length with respect to the CP length and the BER results. Also it is impressive to note that the Fig. 9 shows that for too long channels (the longer channel, the more ISI) the new constellation is much better for SNR<25 dB.

Also in Fig. 12 to 13, the effects of burst noise are shown and it is obvious that the new constellation has a better performance with respect to others especially for long channels. Finally, we can conclude that new constellation can replace the conventional rectangular QAM in DMT.

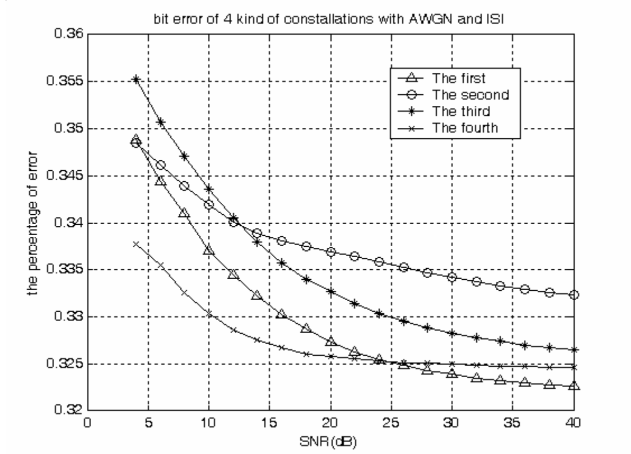


Fig. 11. Comparison of 4 constellations with AWGN channel length $L+100$.

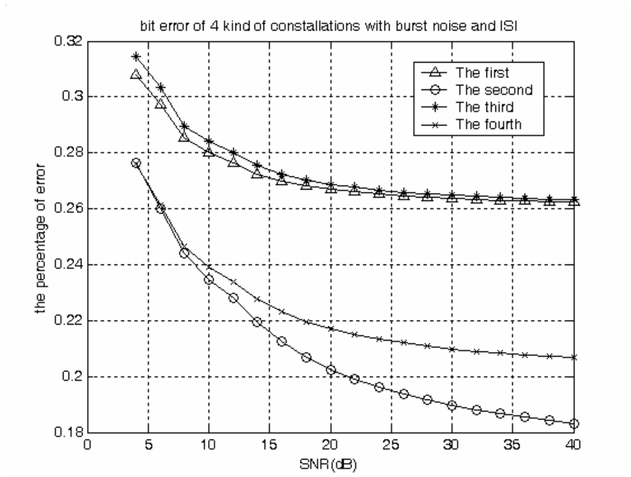


Fig. 12. Comparison of 4 constellations with burst channel length L.

VI. SUMMARY AND CONCLUSION

In this work, three new constellations, the non rectangular QAM and two circular forms, are introduced. Two kinds of noise, AWGN and burst, are applied to the channels. Also we compared the effects of two kinds of orthogonal bases, FFT and FHT, on DMT modulation. Based on the results using these new constellations, the DMT has a lower BER with respect to the conventional rectangular QAM. Also it is shown that by using the FHT instead of the FFT the system probability of error in terms of BER will be improved.

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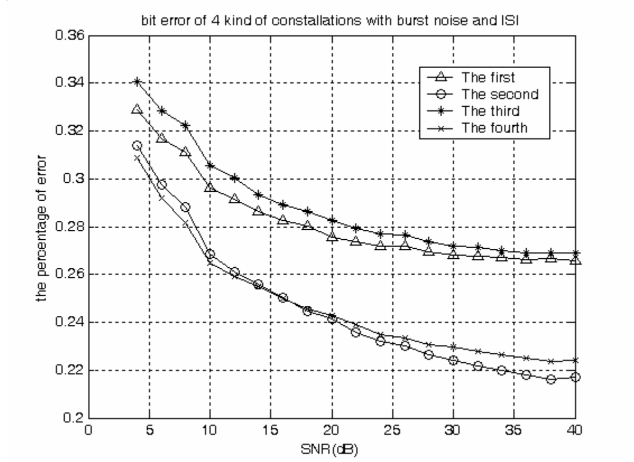

Fig. 13. Comparison of 4 constellations with burst channel $L+10$.

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S. Ghazi Maghrebi was born in Iran in 1963. He received the B.S. degree in electrical engineering from Kerman University, Iran, in 1988, and the M.S. degree from the Khajeh Nasir-edin-Toosi University, Iran, in 1995. Currently, he is pursuing his Ph.D. in Islamic Azad University, Iran. He is a Lecturer at the Communication Department of Islamic Azad University, Iran. His current research interests are digital communication, signal processing, adaptive filtering.

M. Lotfizad was born in Tehran, Iran, in 1955. He received the B.S. degree in electrical engineering from Amir Kabir University, Iran, in 1980, and the M.S. and Ph.D. degrees from the University of Wales, UK, in1985 and 1988, respectively. He then joined the Engineering Faculty Tarbiat Modarres University, Iran. He has also been a Consultant to several industrial and government organizations. His current research interests are signal processing, adaptive filtering, speech processing, and specialized.

Mohammad Ghanbari is a professor of video networking and the School of Computer Science and Electronic Engineering, University of Essex, United Kingdom. He is a world-renowned expert in particular for video coding and networking. Professor Ghanbari is a Fellow of IEEE, Fellow of IET, CEng and has contributed to 11 patents, produced 9 books/ book chapters and published more than 150 Journal papers and presented about 300 conference papers.