AN AGGREGATED FUZZY RELIABILITY INDEX FOR SLOPE STABILITY ANALYSIS

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ABSTRACT. While sophisticated analytical methods like Morgenstern-Price or finite element methods are available for more realistic analysis of stability of slopes, assessment of the exact values of soil parameters is practically impossible. Uncertainty in the soil parameters arises from two different sources: scatter in data and systematic error inherent in the estimate of soil properties. Hence, stability of a slope should be expressed using a factor of safety accompanied by a reliability index.

In this paper, the theory of fuzzy sets is used to deal simultaneously with the uncertain nature of soil parameters and the inaccuracy involved in the analysis. Soil parameters are defined using suitable fuzzy sets and the uncertainty inherent in the value of factor of safety is assessed accordingly. It is believed that this approach accounts for the uncertainty in soil parameters more realistically compared to the conventional probabilistic approaches reported in the literature. A computer program is developed that carries out the large amount of calculations required for evaluating the fuzzy factor of safety based on the concept of domain interval analysis. An aggregated fuzzy reliability index (A*FRI*) is defined and assigned to the calculated factor of safety. The proposed method is applied to a case study and the results are discussed in details. Results from sensitivity analysis describe where the exploration effort or quality control should be concentrated. The advantage of the proposed method lies in its fast calculation speed as well as its ease of data acquisition from experts' opinion through fuzzy sets.

1. Introduction

Earth slopes, whether natural or man-made are susceptible to slide and the failures may cause tragic disasters. As a case history, the slide in the reservoir of the Vaiont Dam in Italy in 1963 resulted in more than 2000 victims and demolished downstream villages [1]. Other examples of slope instability include those related to embankments such as earth and rockfill dams. For this group the Carsington Dam failure is a typical case [2].

Many researchers have tried to improve the methods of slope stability analysis in order to obtain more accurate results. A complete history of these studies has been gathered by Duncan [3]. However, the main source of error lies in the ambiguity and uncertainty inherent in the soil parameters. Therefore, application of sophisticated and advanced methods to real problems does not necessarily give a more realistic answer. This is due to the fact that the natural and disperse characteristic this is due to of the soil will not yield exact and crisp soil parameters. In analyzing a slope stability problem, vagueness is involved in the shear strength parameters c, ϕ, the specific weight of the soil γ, the location of water table, and the boundary of soil layers.

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Briefly, four problems arise in measurement of soil strength parameters [4]: scatter in data originated from natural variations in soil; scatter in data because of in-situ testing errors; systematic error from averaging on a limited number of data and errors like sample disturbance or errors in Vane shear tests. Hence, the real factor of safety varies about the calculated answer in a range that depends on the imprecision of the input data. This fact has encouraged many researchers to employ the probabilistic methods for the slope stability analysis of earth slopes ([4], [5], [6]). In addition, numerous researches have also been pursued in the field of reliability analysis of earth slopes ([7], [8], [9], [10], [11], [12], [13], [14]).

The imprecise characteristic of soil properties has encouraged application of fuzzy sets in geotechnical engineering instead of using probabilistic methods that have still remain a mystery to engineers. The theory of fuzzy sets is generally regarded as the most effective tool for processing qualitative information and inexact data. If the form of uncertainty happens to arise because of imprecision, ambiguity, or vagueness, then the variable is probably fuzzy and can be represented by a membership function [15]. So, all soil parameters having uncertainty may be defined as fuzzy numbers.

One of the most important aims of the reliability analysis is that the engineer, instead of assuming conservative values for soil parameters, can use the best estimates and deal with the products of the uncertainties in the analysis. In other words, "*the power of reliability analysis is not that one can get a better estimate of uncertainties inherent in the problem but that one can deal with them more explicitly and coherently*" [4]. When uncertainties in soil parameters are taken into account, they give a lower bound estimate of the actual possibility of failure because many other unknown parameters are ignored in the reliability analysis. An important use of reliability index is to achieve consistent safety factors for the slopes with different heights and lengths and even different geological conditions, in a single project [16]. In this paper, the theory of fuzzy sets is combined with Bishop's simplified method of slope stability analysis and an aggregated fuzzy reliability index is proposed to determine the reliability of the calculated fuzzy factor of safety. The influence of approximations involved in different soil parameters on the factor of safety and its reliability is also discussed.

2. Fuzzy Sets

Zadeh [17] entered inexactness in mathematical border with the so-called theory of fuzzy sets. Unlike a conventional (crisp) set, a fuzzy set allows its members to have incomplete membership values. In crisp sets, a partial membership to a set has no meaning and all the membership values are either 1 or 0. A common notation to represent a fuzzy set is [15]:

$$\underset{\sim}{A} = \left\{ \frac{\mu(x_1)}{x_1} + \frac{\mu(x_2)}{x_2} + \frac{\mu(x_3)}{x_3} + \cdots \right\} \tag{1}$$

where $\mu(x_i)$ is the membership of element x_i in fuzzy set A. A membership function can have any arbitrary shape. This shape depends on the nature of the parameter under consideration. Usually for parameters with natural origin, like soil properties, a bell-shaped membership function is considered. However, there is no major difference in

the results if another type such as a triangular membership function is taken into account [18]. Application of fuzzy sets in geotechnical engineering and modeling the uncertainties involved, leads to extended algebraic operations on fuzzy numbers. A large number of operations are needed in practical applications. Fuzzy calculations can be performed either with the aid of analytical extension theory [19] or a numerical method. Because of extensive complexity and non-linearity in the Bishop's simplified method for calculating the factor of safety, it would be time consuming to use the extension theory even on a fast computer [20]. Approximate numerical techniques such as L-R representation [19] have been introduced, but their accuracy is not assured when the calculations are highly nonlinear. Other numerical methods require smoothing in answer to eliminate irregularities which, in turn, increases the calculation time. In short, for a highly non-linear formula such as Bishop's simplified method formula, most of the approximate discretization techniques that employ the min-max operation on the fuzzy sets can produce unusual results. For this investigation, an advanced numerical method introduced by Dong, Shah, and Wang known as DSW algorithm [20] is used. The method employs α-cut intervals to carry out calculations. In this algorithm, algebraic operations are done on α-cut intervals of input data and the results are placed at the same α-cut level of the answer. The output is calculated by varying α values from 0 to 1. A typical α-cut representation for an arbitrary fuzzy set is shown in Figure 1.

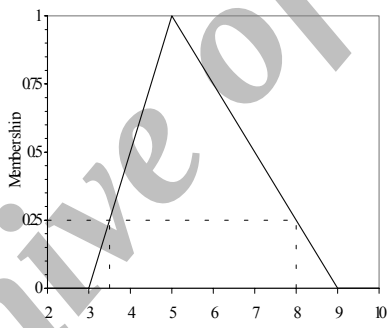


Figure 1- α-cut representation for a fuzzy set

The fuzzy calculations carried out on two arbitrary fuzzy sets are as follow. Consider fuzzy sets X and Y with their boundaries at each cut level as [a,b] and [c,d], respectively. For each α-cut interval [20]:

(2) $$\beta[a,b]=\{[\ \beta a,\ \beta b]\ \textit{if}\ \beta>0;\ [\beta b,\ \beta a]\ \textit{if}\ \beta<0\}$$

(3) $$[a,b]+[c,d] = [a+c,b+d]$$

(4) $$[a,b]-[c,d] = [a-d,b-c]$$

(5) $$[a,b]\times[c,d]=[\textit{min}(ac,ad,bc,bd)\ ,\ \textit{max}(ac,ad,bc,bd)]$$

(6) $$[a,b] \div [c,d] = [a,b] \times [1/d, 1/c],\ 0 \notin [c,d]$$

and while the calculations are treated as binary algebraic operations, following conclusion is obtained:

(7) $$tan([a,b]) = [\tan(a), tan(b)],\ 0 < [a,b] < \pi/2$$

The last equation is based on the concepts of min-max operations utilized to extend DSW algorithm to all algebraic operations [20]. This algorithm is based on two simple concepts: (1) the optimal pair of operands as given by nonlinear programming, and (b) the intervals of the operands corresponding to the same membership. Simplicity of form and efficiency in computation are perhaps the two most important ingredients for successful large scale application of fuzzy sets in engineering [20].

3. π-curve Membership Functions

Another membership function that is used in fuzzy calculations is the π-curve [21]. A typical π-curve is shown in Figure 2. the π-curve is a continuous function and symmetric in shape. The area under the curve is one-half the range over which the function is defined. The range reflects the degree of fuzziness about the mean value and as the degree of fuzziness increases, so will the width of the range.

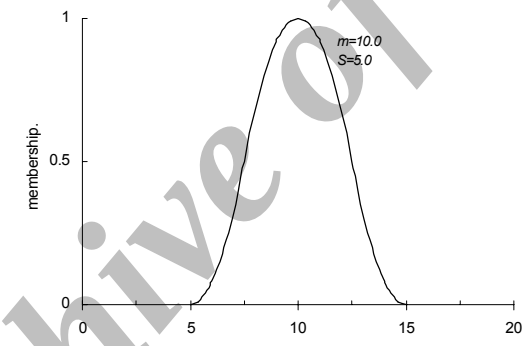


Figure 2– Typical π-curve membership function

If m and S represent the center of the curve and half of its support, respectively, then the π-curve is defined as:

(8) $$f(x) = \begin{cases} 2\{[x-(m-S]/S\}^2, m-S \le x \le (m-S/2) \\ 1-2[(x-m)/S]^2, (m-S/2) \le x \le (m+S/2) \\ 2\{[(m+S)-x]/S\}^2, (m+s/2) \le x \le m+S \end{cases}$$

The advantage of using this type of membership function is in its finite lower and upper value at zero membership value while maintaining the general characteristics of

a bell-shaped membership function. Half widths of π-curves for different degrees of fuzziness for the case studied are listed in Table 1. Values of input parameters corresponding to 50% membership are depicted in Table 1 as well.

Table 1– Width of π-curve membership functions at different system fuzziness

			Overall System Fuzziness					
Layer	**Parameter**	**Var.**	**100%**	**80%**	**60%**	**40%**	**20%**	**10%**
Fill	γ=20kN/m^3	5%	S=2.0	S=1.60	S=1.20	S=0.80	S=0.40	S=0.20
	C=0kPa	-	-	-	-	-	-	-
	ϕ=30°	10%	S=6.0	S=4.80	S=3.60	S=2.40	S=1.20	S=0.60
Crust	γ=18.81kN/m^3	5%	S=1.9	S=1.52	S=1.14	S=0.76	S=0.38	S=0.19
	C=40kPa	25%	S=20.0	S=16.0	S=12.0	S=8.0	S=4.0	S=2.0
	ϕ=0	-	-	-	-	-	-	-
Marine Clay	γ=18.81kN/m^3	5%	S=1.9	S=1.52	S=1.14	S=0.76	S=0.38	S=0.19
	C=34.5kPa	20.2%	S=14.0	S=11.2	S=8.4	S=5.6	S=2.8	S=1.4
	ϕ=0	-	-	-	-	-	-	-
Lacustrine Clay	γ=20.31kN/m^3	5%	S=2.0	S=1.60	S=1.20	S=0.80	S=0.40	S=0.20
	C=31.2kPa	32%	S=20.0	S=16.0	S=12.0	S=8.0	S=4.0	S=2.0
	ϕ=0	-	-	-	-	-	-	-
Layer Boundary		±1.0m	S=2.0	S=1.60	S=1.20	S=0.8	S=0.4	S=0.2

Note: S is the half-width of π-curve membership function.

4. Calculation of the Fuzzy Factor of Safety

Once solving the problem with average soil properties and locating the critical slip surface, stability analysis was performed for all α-cut levels using fuzzified soil parameters and assuming a fixed position for the critical slip surface. Note that no uncertainty was assumed for the location of the slip circle. Two values for the factor of safety were determined at each α-cut level, indicating a lower and an upper bound. By aggregating these results, a shape function for the factor of safety was obtained. The level of output accuracy depends on the distance between the α-cuts. An increment of 0.01 was selected for the α-cut intervals. More precision was reached near α-cut=1.0 by reducing the increment to 0.001. A computer program was written to perform all necessary calculations. Dividing the sliding wedge into 30 equally spaced slices, their fuzzy weights were determined. For calculation of slice weights when the soil profile consists of more than one layer and there is uncertainty for the layer boundary locations, calculation of the slice weight was carried out for all possible combinations of boundary locations. The minimum and maximum weights of each slice were determined and used subsequently in the factor of safety formula for each α-cut level. For instance, if the nth slice had three soil layers, there were two boundaries inside that slice and the combinations shown in Figure 3 were possible. At each α-cut level, weights and pore pressures of all slices were determined. Then, assuming an arbitrary initial fuzzy factor of safety, the final fuzzy factor of safety was obtained using the Bishop's simplified equation given by:

$$\widetilde{F} = \frac{\sum_{i=1}^{i=n}[\widetilde{c}b_i + (\widetilde{w}_i - \widetilde{u}_i b_i)\tan\widetilde{\phi}][\sec\alpha_i /(1 + \tan\alpha_i \tan\widetilde{\phi}/\widetilde{F})]}{\sum_{i=1}^{i=n}\widetilde{W}_i \sin\alpha_i} \quad (9)$$

In Equation 9, the fuzzy parameters are distinguished with a hat symbol. It is obvious that by dividing two fuzzy numbers, a fuzzy factor of safety is obtained for the left side of the equation. The fuzzy factor of safety was obtained from Equation 9 using an iterative procedure since *FOS* (factor of safety) appears on both sides. At each α-cut, an interval for *FOS* was determined. Next, if the absolute difference between these *FOS* values and the corresponding values from the previous iteration for the same α-cut interval is less than a tolerance, convergence is achieved for that level. Else, the factor of safety is updated and the iteration is repeated until the required precision is reached for that α-cut level.

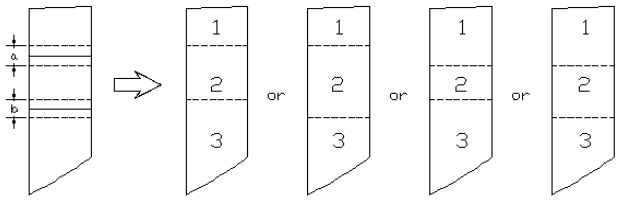


Figure 3- Four different possible cases for a slice weight with three

5. Reliability Index

In probabilistic methods, the reliability index is defined as the number of standard deviations separating the best estimate of factor of safety (its mean value) from its defined failure value, i.e. 1.0. Therefore, the higher the reliability index, the lower is the probability of failure. Figure 4 shows how a system with a higher factor of safety can have a lower reliability index. To evaluate the probability distribution for safety factor, first order second moment approach is used because direct differentiation of the factor of safety formula is not feasible [4].

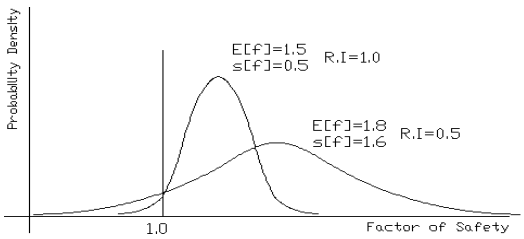


Figure 4- Comparison of two situations with different means and standard deviations

6. Procedure for Reliability Analysis

A factor of safety alone is not a good representative of the risk of failure for a slope. Hence, an accompanying reliability index is essential. Conventionally, the reliability index is obtained by carrying out stability calculations for a slip circle that gives the

minimum factor of safety. The method is better described by solving a case study, i.e. the case investigated by Christian et al. [4]. The case involves two 6m and 12m high dykes. The reliability index was calculated for the slip surface having minimum factor of safety. Among the available limit equilibrium methods for calculating the safety factor of an earth slope, Bishop's simplified method of slices is often the chosen algorithm for stability calculations. This method is known to give acceptable results comparable to the more accurate but sophisticated methods such as Morgenstern-Price's method ([3], [22]). Despite a restriction in applying only to circular shape failures, it has been shown that this limitation does not have a major influence on the evaluated minimum factor of safety ([3], [23]). Moreover, it has been shown that when the shape of the slip surface is set free to find its critical form, the result is very close to a circular arc ([24], [25], [26]). The difference between results of various limit equilibrium methods is within a range of 6%. This difference is insignificant where the error in the input data is seldom smaller [3]. Based on Bishop's simplified method of slices, critical slip circles were found and details of the dyke profiles as well as the critical slip surfaces are shown in Figures 5 and 6. For each of these slip surfaces, fuzzy calculations were performed considering the maximum amount of uncertainty in the input parameters. These results were chosen as a base (maximum fuzziness of the

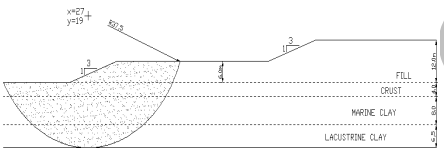


Figure 5- Soil profile and slip circle for 6m high dyke

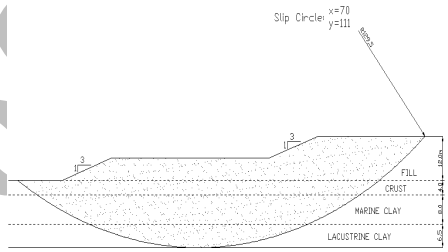


Figure 6- Soil profile and slip circle for 12m high dyke

system) to be used further as a reference in the sensitivity analysis comparisons. Where available, the maximum amount of uncertainty in each input parameter was derived indirectly from the probabilistic parameters given by Christian et al. [4]. This uncertainty was calculated from covariance of each parameter and assigned to the 50% membership degree of the pertinent fuzzy variables. In other words, the possibility (*and not the probability*) of existence of a parameter by one covariance difference from its mean value was defined to be 50% when the probability distribution of that parameter was known. When there is not enough information about the probability distribution of a parameter, choosing a suitable spread for the membership function is based upon experts' opinion.

7. Proposed Reliability Index

After the fuzzy factor of safety is determined, its degree of fuzziness can be calculated. Degree of fuzziness (*DOF*) indicates how far a fuzzy set is from a crisp number. For

slope stability problems, the uncertainty in input parameters causes uncertainty in the factor of safety. The influence of uncertainty involved in each input parameter on the resulting uncertainty varies from one parameter to another and will be discussed subsequently. The degree of fuzziness of a fuzzy set is determined from the following formula [19]:

$$DOF = -\int \{\mu(x)\ln(\mu(x)) + (1-\mu(x))(\ln(1-\mu(x))\} \cdot dx \tag{10}$$

Ayyub [27] defined a fuzzy reliability index (FR_e) for a fuzzy factor of safety as:

$$FR_e = \frac{\int_{x>1} \mu(x) \cdot dx}{\int_x \mu(x) \cdot dx} \tag{11}$$

where x and $\mu(x)$ are the factor of safety and the corresponding membership value, respectively. The Fuzzy Reliability Index (FR_e) denotes the fraction of area under the factor of safety membership function with FOS>1. This index has the following properties: 1- In case all values of the factor of safety are greater than or equal to 1.0, FR_e=1 and it indicates absolute safety. 2- FR_e=0 when all values of factor of safety are less than one, i.e. absolute failure. 3- Systematically FR_e goes toward 1 as the system reliability increases.

For this research, reliability indices are calculated for all FOSs, so the above formula is expressed in a more general form:

$$FR_e = \frac{\int_{x>FOS} \mu(x) \cdot dx}{\int_x \mu(x) \cdot dx} \tag{12}$$

Nevertheless, this reliability index can hardly grant the changes in reliability when the overall fuzziness of the system changes. Disadvantages of the Ayyub's Reliability Index for application in slope stability risk analysis are: 1- While the absolute safety has no actual meaning for a slope stability problem, the reliability index should be capable of showing the difference between the two cases shown in Figure 7a. 2- It cannot show the difference of reliability between two systems that are both completely in safe region but their degree of fuzziness differ considerably (Figure 7b). 3- The importance of the membership value for the factor of safety under consideration is not combined. Therefore, Ayyub's FR_e cannot indicate which case is safer for conditions shown in Figure 7.

Based on the above discussion, a new reliability index was defined to take the influence of the shape of the factor of safety function into account as well as its fuzziness and the possibility of all FOS considered. Combining Ayyub's generalized reliability index and the membership value for a particular factor of safety together with DOF, an *Aggregated Fuzzy Reliability Index* (*AFRI*) is proposed as:

$$AFRI(x) = \frac{\mu(x) \cdot FR_e(x)}{DOF} \tag{13}$$

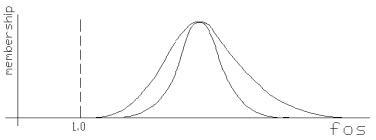
where x is the factor of safety, $\mu(x)$ is the membership value, $FR_e(x)$ is the generalized Ayyub's reliability index given by Equation 12, and DOF is the degree of fuzziness of the factor of safety given by Equation 10. This equation indicates that the reliability index is directly proportional to the membership value of the factor of safety and its fuzzy reliability index FR_e, and inversely proportional to the degree of fuzziness of the fuzzy factor of safety. The proposed *AFRI* has the following properties: 1- In the factor of safety curve, there is a point with maximum *AFRI* value. It corresponds to the most possible and reliable factor of safety (Figure 8). 2- The absolute safety is obtained when there is no fuzziness in the factor of safety, i.e. DOF=0, this leads to an infinite *AFRI*. This means there is no real absolute safety. 3- As the uncertainty increases for a certain problem, *AFRI* should decrease at any given factor of safety. 4- *AFRI* is defined for all possible factors of safety, however, there is no safety assured for the factors of safety below 1.0.

8. Results and Discussion

As described before, two different situations for the failure of the dyke may occur. One is the failure of the first 6m high bench of the 12m high dyke with a small slip circle, and the other is the failure of the 12m high dyke. Both of these cases were analyzed by Christian et al. [4] and the corresponding reliability indices were determined. The results from their work showed that the 6m slip circle had a FOS=1.500 with a reliability index of 1.84, while the FOS for the higher dyke was 1.453 and the corresponding reliability index was 2.66. So it was concluded that the reliability for the stability of the higher dyke is more than the 6m high dyke while the conventional factor of safety analysis showed the opposite. The stability analyses were performed via Bishop's simplified method of slices.

For the approach proposed in this paper, there are two ways to represent the reliability of the system; the first approach is to consider the conventional minimum FOS and its corresponding *AFRI*. In this case, the *AFRI* does not indicate the maximum reliability of the

(a) Two curves are completely outside failure zone



(b) Same mean values but different fuzziness

Figure 7- Cases in which Ayyub's *RI* gives equal values.

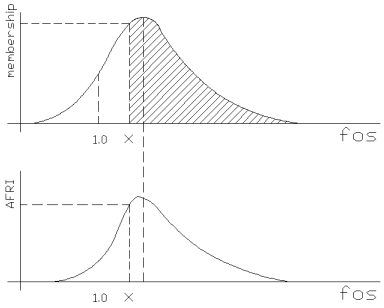


Figure 8- Graphical representation of AFRI

system. Another way of representing reliability of a system is to determine the maximum *AFRI* and its corresponding *FOS* value. Because a reliability analysis is presented here, the second method of representing the reliability was adopted.

Therefore, when speaking of reliability and its corresponding *FOS*, the maximum *AFRI* is assumed.

Searching for a slip circle with minimum FOS using Bishop's simplified method of slices, resulted in *FOS*=1.242 for the 6m high dyke and *FOS*=1.1814 for the 12m high dyke. The reliability analyses using the proposed method, showed *AFRI*=0.554 for the lower dyke and *AFRI*=0.7137 for the higher dyke. However, the *FOS* with the maximum membership value is somewhat bigger than the latter values. In this case, for the 6m high dyke *FOS*=1.517 with *AFRI*=0.447 and for the 12m high dyke *FOS*=1.44 with *AFRI*=0.597 were obtained. Therefore, it maybe concluded again that the higher dyke despite its lower *FOS*, has a higher reliability index (Figures 9 and 10).

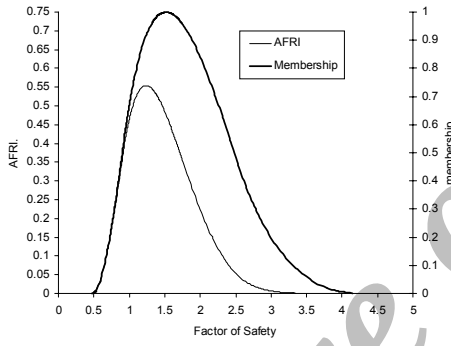


Figure 9- Variation of AFRI with FOS for 6m high dyke

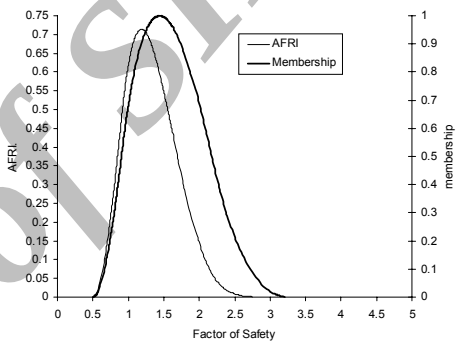


Figure 10- Variation of AFRI with FOS for 12m high dyke

There is always a factor of safety, which has the highest *AFRI* among all *possible* factors of safety for a given slip surface. The point with the highest *AFRI* always lies at the left side of the factor of safety with the highest membership function (i.e. μ=1.0). The latter value is the factor of safety that is obtained by conventional slope stability analyses, because the soil parameters that are used to determine this value are all the average values. Hence, it maybe stated that *the most reliable factor of safety is less than what is obtained by conventional analyses*. One of the main sources that may affect the amount of uncertainty inherent in a slope stability problem is the exploration effort. Besides, the engineer judgment in assuming a reasonable value for each unknown parameter depends on his experience and information about that specific site. The accuracy of data used in the analysis is related to these two matters. It is assumed here that as the exploration effort and the engineer's experience increase, the overall uncertainty in soil parameters decreases. So, to simulate this scenario, all the parameter uncertainties were reduced simultaneously by multiplying them by

reduction factors equal to 0.8, 0.6, 0.4, 0.2 and 0.1. The minimum uncertainty that could be reached was assumed to be 10% of the maximum uncertainty values. The percentage of the maximum uncertainty applied to all parameters is called *overall system fuzziness*.

The results from the above analyses are shown in Figure 11. It is seen that as the overall system uncertainty decreases, the *FOS* membership function becomes slimmer and degree of fuzziness decreases. In addition, the *FOS* with maximum *AFRI* also moves toward the conventional *FOS* as its reliability increases (Figures12 and 13).

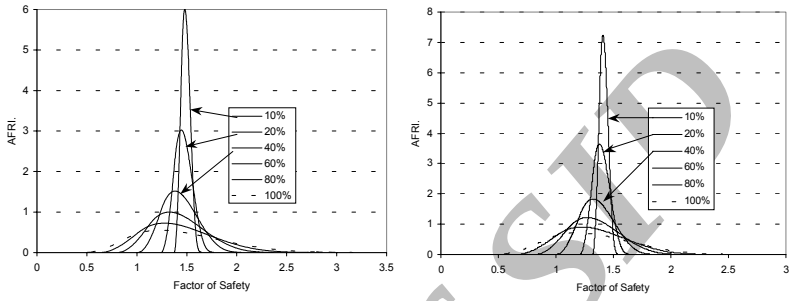


Figure 11- Variation of AFRI with overall system fuzziness (a) 6m high dyke, (b) 12m high dyke

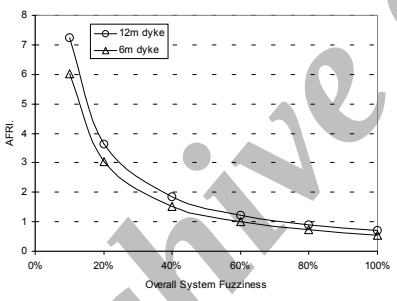


Figure 12- Variation of AFRI for the FOS with maximum reliability versus overall system fuzziness

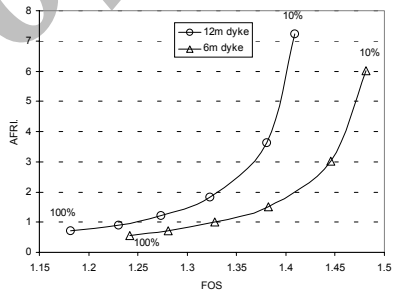


Figure 13- Variation of AFRI vs. FOS with maximum reliability for different overall system fuzziness

To illustrate the influence of uncertainty inherent in each parameter on the overall reliability of the slope, sensitivity ananlyses were performed on all system parameters. Fixing all uncertainties to their maximum values, the effect of each parameter uncertainty on the reliability of the system was studied by changing spread of its membership function. Maximum uncertainty value for each parameter is listed in Table 1. Figures 14 and 15 show variation of *FOS* for the 6m and 12m high dykes with corresponding membership values and *AFRI* when the maximum uncertainty in input parameters was assumed. From these figures, it is clear that the smaller dyke has a higher *FOS*, whereas the 12m high dyke has a higher *AFRI* as indicated in Figure 15.

Therefore, it may be concluded that a single *FOS* is not an adequate measure to describe the stability condition of a given slope in order to compare it with other

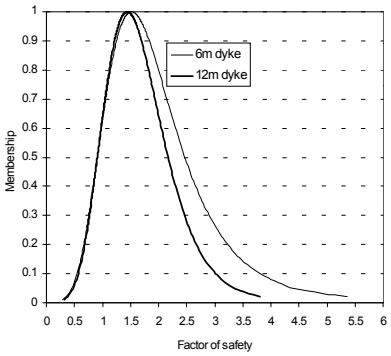


Figure 14- FOS membership functions for 6m and 12m high dykes with 100% fuzziness

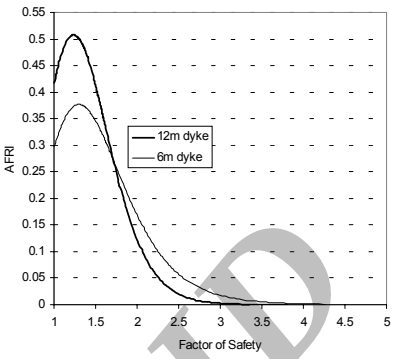


Figure 15- AFRI curves for 6m and 12m high dykes with 100% fuzziness

slopes. Furthermore, summary of the results from sensitivity analyses carried out for these two dykes are shown in Figures 16 (a) and (b).

Since we are dealing with reliability, it is better to discuss first the effects of uncertainty inherent in each parameter on the variation of *AFRI*. As indicated in Figure 16, the maximum variation of *AFRI* is caused by uncertainty of the lacustrine clay cohesion for both the 6m and 12m high dykes. Referring to Figures 5 and 6, it is seen that as far as a major fragment of the slip circle passes through this layer in both cases, uncertainty in the cohesion of this layer has a strong influence on the overall reliability of the problem. The next important factor is the unit weight of the fill layer. As may be suggested from Figures 5 and 6, this uncertainty has more influence on the reliability of the 12m high dyke since a larger part of the slip is located within the fill layer. This parameter affects the slice weights and therewith not only decreases the uncertainty in the slice weights but also the shear strength of the frictional layer due to its overburdening effect. On the third level of importance, is the cohesion of the Marine clay layer. Again, referring to Figures 5 and 6, it can be seen that this layer contains the second longest portion of the slip surface. The uncertainty in the cohesion of the crust layer has a stronger influence on the reliability of the 12m high dyke, due to the longer length of the slip circle within this layer. The uncertainty in friction angle of the fill layer also has a pronounced influence on the higher dyke reliability. The bandwidth of layer boundary location shows a conflicting behavior on the resulting reliability of the two cases. While it changes *AFRI* by 7.96% for the 6m high dyke, it only affects *AFRI* by 2.96% for the 12m high dyke. The uncertainty in the other individual parameters does not have any major effect on the *FOS*. It is believed that the value of uncertainty in the parameters of the three clay layers varies simultaneously because of the joined subsurface explorations. Therefore, the effects were considered and

consequently more effort is needed to evaluate more precise data for their cohesion since it has a significant influence on the results.

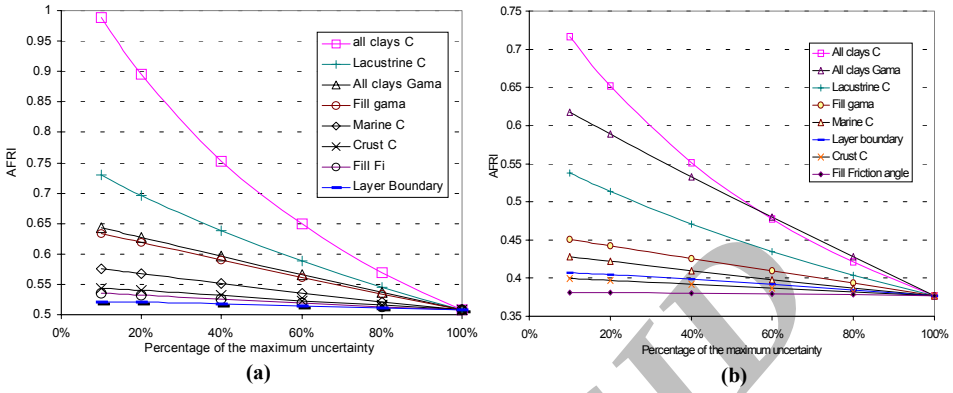


Figure 16- Effect of each parameter uncertainty on the maximum AFRI (a) 6m high dyke, (b) 12m high dyke

9. Conclusion

Possibility is the upper bound of the probability measure [19]. When dealing with fuzzy sets as a tool for measuring possibilities, these sets must reflect properly the uncertainties in the soil parameters. It may seem somehow overestimating to put uncertainty values that correspond to one covariance at 50% memberships but this degree of membership is a clear bound between unknown and known information and it is easier for engineers to deal with it. Perhaps the fuzzy regression is a good way to determine a more exact distribution for the input parameters.

The proposed method evaluates the reliability index for the slope stability analysis considering uncertainties inherent in the soil parameters. It is clear that accuracy of the results, like any other analytical approach, depends on the accuracy of the input parameters. Although the factor of safety determined from the proposed procedure is lower than that given by conventional methods of slope stability but the reliability of this factor of safety is the most among all possible factors of safety. The results of slope stability analyses are represented by curves plotting reliability index against its corresponding factor of safety at different degrees of fuzziness. According to these curves, the most reliable factor of safety can be determined. By comparing the reliability indices of different slopes, the most reliable slope is also established.
Recommendations for the further works may consist of:

(1) Incorporating a fuzzy number for the *failure* safety factor (a fuzzy number of "about one").
(2) Adjusting the location of the critical slip surface each time the parameters change at different α-cuts.
(3) Employing noncircular slip surfaces.

(4) Locating the slip surface with minimum *AFRI* instead of calculating *AFRI* for the slip surface with the minimum factor of safety.

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Abbreviations:

AFRI: Aggregated Fuzzy Reliability Index
DOF: Degree of Fuzziness
FOS: Factor of Safety
MF: Membership Function

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