ON ANTI FUZZY IDEALS IN NEAR-RINGS

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Abstract. In this paper, we apply the Biswas' idea of anti fuzzy subgroups to ideals of near-rings. We introduce the notion of anti fuzzy ideals of near-rings, and investigate some related properties.

1. Introduction

W. Liu [14] has studied fuzzy ideals of a ring, and many researchers [5, 10, 12, 17] are engaged in extending the concepts. S. Abou-Zaid [1] introduced the notion of a fuzzy subnear-ring, and studied fuzzy ideals of a near-ring, and many followers [6, 7, 8, 10, 11] discussed further properties of fuzzy ideals in near-rings. In [2], R. Biswas introduced the concept of anti fuzzy subgroups of groups, and K. H. Kim and Y. B. Jun studied the notion of anti fuzzy R-subgroups of near-ring in [9]. In this paper, we introduce the notion of anti fuzzy ideals of near-rings, and investigate some related properties.

2. Preliminaries

A *near-ring* ([16]) is a non-empty set R with two binary operations "+" and "·" satisfying the following axioms:

(i) $(R, +)$ is a group,
(ii) $(R, \cdot)$ is a semigroup,
(iii) $x \cdot (y + z) = x \cdot y + x \cdot z$, for all $x, y, z \in R$.

Precisely speaking, it is a left near-ring because it satisfies the left distributive law. We will use the word "near-ring" instead of "left near-ring". We denote $x \cdot y$ by xy. If $(R, +, \cdot)$ is a near-ring, then an *ideal* ([1]) of R is a subset I of R such that

(i) $(I, +)$ is a normal subgroup of $(R, +)$,
(ii) $RI \subset I$,
(iii) $(r + i)s - rs \in I$, for all $i \in I$ and $r, s \in R$.

Let R and S be two near-rings. A map $f : R \to S$ is a *homomorphism* of near-rings if $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ for all $x, y \in R$.

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A *fuzzy set* μ in a set R is a function $\mu : R \to [0,1]$. Denote by $\mathrm{Im}(\mu)$ the image set of μ. For $t \in [0,1]$, the set

$$\mu_t^{\geq} = \{x \in R | \mu(x) \geq t\} \quad (\text{resp. } \mu_t^{\leq} = \{x \in R | \mu(x) \leq t\})$$

is called a *upper* (resp. *lower*) *t-level cut* of μ. Clearly, $\mu_t^{\geq} \cup \mu_t^{\leq} = R$ for $t \in [0,1]$, and if $t_1 < t_2$, then $\mu_{t_1}^{\leq} \subseteq \mu_{t_2}^{\leq}$ and $\mu_{t_2}^{\geq} \subseteq \mu_{t_1}^{\geq}$. If μ is a fuzzy set in R, then the *complement* of μ, denoted by μ^c, is the fuzzy set in R given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in R$ ([3], [17], [18]).

Let R be a near-ring. A *fuzzy subnear-ring* of R is a fuzzy set μ of R such that

(F1) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$,
(F2) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$,

for all $x, y \in R$. And a *fuzzy ideal* of R is a fuzzy subnear-ring μ of R such that

(F3) $\mu(y + x - y) \geq \mu(x)$,
(F4) $\mu(xy) \geq \mu(y)$,
(F5) $\mu((x + z)y - xy) \geq \mu(z)$,

for all $x, y, z \in R$ ([10]). Note that μ is a fuzzy left ideal of R if it satisfies (F1), (F2), (F3) and (F4), and μ is a fuzzy right ideal of R if it satisfies (F1), (F2), (F3) and (F5). Let R be a near-ring and μ a fuzzy subset of R. Then the upper t-level cut $\mu_t^{\geq}$ of μ is a subnear-ring (resp. ideal) of R for all $t \in [0, \mu(0)]$ if and only if μ is a fuzzy subnear-ring (resp. ideal) of R ([1, p145, Theorem 4.2]).

3. Anti Fuzzy Ideals

Definition 3.1. Let R be a near-ring. A fuzzy set μ of R is called an *anti fuzzy subnear-ring* of R if for all $x, y \in R$,

(AF1) $\mu(x - y) \leq \max\{\mu(x), \mu(y)\}$,
(AF2) $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$.

Definition 3.2. Let R be a near-ring. An anti fuzzy subnear-ring μ of R is called an *anti fuzzy ideal* of R if for all $x, y, z \in R$,

(AF3) $\mu(y + x - y) \leq \mu(x)$,
(AF4) $\mu(xy) \leq \mu(y)$,
(AF5) $\mu((x + z)y - xy) \leq \mu(z)$.

Note that μ is an anti fuzzy left ideal of R if it satisfies (AF1), (AF2), (AF3) and (AF4), and μ is an anti fuzzy right ideal of R if it satisfies (AF1), (AF2), (AF3) and (AF5).

Example 3.3. Let $R = \{a, b, c, d\}$ be a set with two binary operations as follows:

$+$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

Then $(R, +, \cdot)$ is a near-ring. We define a fuzzy subset $\mu : R \longrightarrow [0, 1]$ by $\mu(c) = \mu(d) > \mu(b) > \mu(a)$. Then μ is an anti fuzzy right (resp. left) ideal of R.

Every anti fuzzy right (resp. left) ideal of a near-ring R is an anti fuzzy subnear-ring of R, but the converse is not true as shown in the following example.

Example 3.4. Let $R = \{a, b, c, d\}$ be a set with two binary operations as follows:

$+$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	b	c	d
c	a	a	a	a
d	a	a	a	a

Then $(R, +, \cdot)$ is a near-ring. We define a fuzzy subset $\mu : R \longrightarrow [0, 1]$ by $\mu(c) = \mu(d) > \mu(b) > \mu(a)$. Then μ is an anti fuzzy subnear-ring of R. But μ is not an anti fuzzy right ideal of R, since $\mu((a + b)c - ac) = \mu(c) > \mu(b)$.

Proposition 3.5. *If μ is an anti fuzzy subnear-ring of a near-ring R, then $\mu(0) \leq \mu(x)$ for all $x \in R$.*

Proof. It follows immediately from [AF1]. □

Proposition 3.6. *Let R be a near-ring. Then a fuzzy set μ is an anti fuzzy subnear-ring in R if and only if μ^c is a fuzzy subnear-ring in R.*

Proof. Let μ be an anti fuzzy subnear-ring in R. Then we have that for each $x, y \in R$,

$$\begin{aligned} \mu^c(x - y) &= 1 - \mu(x - y) \\ &\geq 1 - \max\{\mu(x), \mu(y)\} \\ &= \min\{1 - \mu(x), 1 - \mu(y)\} \\ &= \min\{\mu^c(x), \mu^c(y)\}, \end{aligned}$$

and

$$\begin{aligned} \mu^c(xy) &= 1 - \mu(xy) \\ &\geq 1 - \max\{\mu(x), \mu(y)\} \\ &= \min\{1 - \mu(x), 1 - \mu(y)\} \\ &= \min\{\mu^c(x), \mu^c(y)\}. \end{aligned}$$

Hence μ^c is a fuzzy subnear-ring in R. The converse is proved similarly. □

Proposition 3.7. *Let R be a near-ring and μ a fuzzy set in R. Then μ is an anti fuzzy ideal in R if and only if μ^c is a fuzzy ideal in R.*

Proof. Let μ be an anti fuzzy ideal in R. Then μ^c is a fuzzy subnear-ring in R, and we have that for all $x, y, z \in R$,

$$\mu^c(y + x - y) = 1 - \mu(y + x - y) \geq 1 - \mu(x) = \mu^c(x),$$
$$\mu^c(xy) = 1 - \mu(xy) \geq 1 - \mu(y) = \mu^c(y),$$

and

$$\mu^c((x+z)y-xy)=1-\mu((x+z)y-xy)\geq 1-\mu(z)=\mu^c(z).$$

Hence μ^c is a fuzzy ideal in R. The converse is proved similarly. □

Let μ be a fuzzy set of a set R. Then $\mu_t^{\leq}=(\mu^c)_{1-t}^{\geq}$ for all $t\in[0,1]$.

Theorem 3.8. *Let μ be a fuzzy set in a near-ring R. Then μ is an anti fuzzy ideal of R if and only if the lower t-level cut $\mu_t^{\leq}$ is an ideal of R for each $t\in[\mu(0),1]$.*

Proof. ($\Rightarrow$) Let μ be an anti fuzzy ideal of R and $t\in[\mu(0),1]$. Then μ^c is a fuzzy ideal of R, hence $\mu_t^{\leq}=(\mu^c)_{1-t}^{\geq}$ is an ideal of R from [1, Theorem 4.2].

($\Leftarrow$) Let $\mu_t^{\leq}$ be an ideal of R for all $t\in[\mu(0),1]$ and $s\in[0,1-\mu(0)]=[0,\mu^c(0)]$. Then $1-s\in[\mu(0),1]$ and $(\mu^c)_s^{\geq}=\mu_{1-s}^{\leq}$ is an ideal of R. Hence $(\mu^c)_s^{\geq}$ is an ideal of R for all $s\in[0,\mu^c(0)]$, and μ^c is a fuzzy ideal of R, whence μ is an anti fuzzy ideal of R. □

Proposition 3.9. *Let μ be an anti fuzzy subnear-ring R and $t_1,t_2\in[0,1]$ with $t_1<t_2$. Then two lower level cuts $\mu_{t_1}^{\leq}$ and $\mu_{t_2}^{\leq}$ are equal if and only if there is no $x\in R$ such that $t_1<\mu(x)\leq t_2$.*

Proof. From the definition of lower level cuts, it follows that $\mu_t^{\leq}=\mu^{-1}([\mu(0),t])$ for $t\in[0,1]$. Let $t_1,t_2\in[0,1]$ be such that $t_1<t_2$. Then

$$\mu_{t_1}^{\leq}=\mu_{t_2}^{\leq}\ \Leftrightarrow\ \mu^{-1}([\mu(0),t_1])=\mu^{-1}([\mu(0),t_2])\ \Leftrightarrow\ \mu^{-1}((t_1,t_2])=\emptyset.$$

□

Proposition 3.10. *If I is an ideal of a near-ring R, then for each $t\in[0,1]$, there exists an anti fuzzy ideal μ of R such that $\mu_t^{\leq}=I$.*

Proof. Let $t\in[0,1]$ and define a fuzzy set $\mu:R\to[0,1]$ by

$$\mu(x)=\begin{cases} t & \text{if } x\in I,\\ 1 & \text{if } x\notin I,\end{cases}$$

for each $x\in R$. Then $\mu_s^{\leq}=I$ for any $s\in[t,1)=[\mu(0),1)$, and $\mu_1^{\leq}=R$, whence $\mu_s^{\leq}$ is an ideal of R for all $s\in[\mu(0),1]$. Hence μ is an anti fuzzy ideal of R from Theorem 3.8, and $\mu_t^{\leq}=I$. □

For a family of fuzzy sets $\{\mu_i\mid i\in\Lambda\}$ in a near-ring R, the union $\bigvee_{i\in\Lambda}\mu_i$ of $\{\mu_i\mid i\in\Lambda\}$ is defined by

$$(\bigvee_{i\in\Lambda}\mu_i)(x)=\sup\{\mu_i(x)|i\in\Lambda\},$$

for each $x\in R$.

Proposition 3.11. *If $\{\mu_i|i\in\Lambda\}$ is a family of anti fuzzy ideals of a near-ring R, then so is $\bigvee_{i\in\Lambda}\mu_i$.*

Proof. Let $\{\mu_i \mid i \in \Lambda\}$ be a family of anti fuzzy ideals of R and $x, y \in R$. Then we have that

$$\begin{aligned}(\bigvee_{i\in\Lambda} \mu_i)(x-y) &= \sup\{\mu_i(x-y)|i \in \Lambda\} \\ &\leq \sup\{\max\{\mu_i(x), \mu_i(y)\}|i \in \Lambda\} \\ &= \max\{\sup\{\mu_i(x)|i \in \Lambda\}, \sup\{\mu_i(y)|i \in \Lambda\}\} \\ &= \max\{(\bigvee_{i\in\Lambda} \mu_i)(x), (\bigvee_{i\in\Lambda} \mu_i)(y)\},\end{aligned}$$

$$\begin{aligned}(\bigvee_{i\in\Lambda} \mu_i)(xy) &= \sup\{\mu_i(xy)|i \in \Lambda\} \\ &\leq \sup\{\max\{\mu_i(x), \mu_i(y)\}|i \in \Lambda\} \\ &= \max\{\sup\{\mu_i(x)|i \in \Lambda\}, \sup\{\mu_i(y)|i \in \Lambda\}\} \\ &= \max\{(\bigvee_{i\in\Lambda} \mu_i)(x), (\bigvee_{i\in\Lambda} \mu_i)(y)\}.\end{aligned}$$

Hence $\bigvee_{i\in\Lambda} \mu_i$ is an anti fuzzy subnear-ring of R.

For any $x, y, z \in R$, we have that

$$\begin{aligned}(\bigvee_{i\in\Lambda} \mu_i)(y+x-y) &= \sup\{\mu_i(y+x-y)|i \in \Lambda\} \\ &\leq \sup\{\mu_i(x)|i \in \Lambda\} \\ &= (\bigvee_{i\in\Lambda} \mu_i)(x),\end{aligned}$$

$$\begin{aligned}(\bigvee_{i\in\Lambda} \mu_i)(xy) &= \sup\{\mu_i(xy)|i \in \Lambda\} \\ &\leq \sup\{\mu_i(y)|i \in \Lambda\} \\ &= (\bigvee_{i\in\Lambda} \mu_i)(y),\end{aligned}$$

and

$$\begin{aligned}(\bigvee_{i\in\Lambda} \mu_i)((x+z)y-xy)) &= \sup\{\mu_i((x+z)y-xy)|i \in \Lambda\} \\ &\leq \sup\{\mu_i(z)|i \in \Lambda\} \\ &= (\bigvee_{i\in\Lambda} \mu_i)(z).\end{aligned}$$

Hence $\bigvee_{i\in\Lambda} \mu_i$ is an anti fuzzy ideal of R. □

Theorem 3.12. *If μ is an anti fuzzy ideal of a near-ring R, then $\mu(x) = \inf\{t \in [0,1] \mid x \in \mu_t^{\leq}\}$ for each $x \in R$.*

Proof. For each $x \in R$, let $T_x = \{t \in [0,1] \mid x \in \mu_t^{\leq}\}$ and $\alpha = \inf T_x$. Then for any $t \in T_x$, $\mu(x) \leq t$, whence $\mu(x)$ is a lower bound of T_x, hence $\mu(x) \leq \inf T_x = \alpha$. And let $\beta = \mu(x)$. Then $x \in \mu_\beta^{\leq}$ and $\beta \in T_x$, hence $\alpha = \inf T_x \leq \beta = \mu(x)$. □

Definition 3.13. Let R and S be two near-rings and f a function of R into S.

(1) If ν is a fuzzy set in S, then the *preimage* of ν under f is the fuzzy set in R defined by

$$f^{-1}(\nu)(x) = \nu(f(x)),$$

for each $x \in R$.

(2) If μ is a fuzzy set of R, then the *image* of μ under f is the fuzzy set in S defined by

$$f(\mu)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x), & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise,} \end{cases}$$

for each $y \in S$.

Theorem 3.14. *Let $f : R \to S$ be an onto homomorphism of near-rings.*

(1) *If ν is a fuzzy subnear-ring of S, then $f^{-1}(\nu)$ is a fuzzy subnear-ring of R.*

(2) *If μ is a fuzzy subnear-ring of R, then $f(\mu)$ is a fuzzy subnear-ring of S.*

Proof. (1) Let $x_1, x_2 \in R$. Then we have that

$$\begin{aligned} f^{-1}(\nu)(x_1 - x_2) &= \nu(f(x_1) - f(x_2)) \\ &\geq \min\{\nu(f(x_1)), \nu(f(x_2))\} \\ &= \min\{f^{-1}(\nu)(x_1), f^{-1}(\nu)(x_2)\}, \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\nu)(x_1 x_2) &= \nu(f(x_1) f(x_2)) \\ &\geq \min\{\nu(f(x_1)), \nu(f(x_2))\} \\ &= \min\{f^{-1}(\nu)(x_1), f^{-1}(\nu)(x_2)\}. \end{aligned}$$

Hence $f^{-1}(\nu)$ is a fuzzy subnear-ring of R.

(2) Let $y_1, y_2 \in S$. Then we have

$$\{x \mid x \in f^{-1}(y_1 - y_2)\} \supseteq \{x_1 - x_2 \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\},$$

and hence

$$\begin{aligned} f(\mu)(y_1 - y_2) &= \sup\{\mu(x) \mid x \in f^{-1}(y_1 - y_2)\} \\ &\geq \sup\{\mu(x_1 - x_2) \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\} \\ &\geq \sup\{\min\{\mu(x_1), \mu(x_2)\} \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\} \\ &= \min\{\sup\{\mu(x_1) \mid x_1 \in f^{-1}(y_1)\}, \sup\{\mu(x_2) \mid x_2 \in f^{-1}(y_2)\}\} \\ &= \min\{f(\mu)(y_1), f(\mu)(y_2)\}, \end{aligned}$$

and since $\{x \mid x \in f^{-1}(y_1y_2)\} \supseteq \{x_1x_2 \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\}$,

$$\begin{aligned} f(\mu)(y_1y_2) &= \sup\{\mu(x) \mid x \in f^{-1}(y_1y_2)\} \\ &\geq \sup\{\mu(x_1x_2) \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\} \\ &\geq \sup\{\min\{\mu(x_1), \mu(x_2)\} \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\} \\ &= \min\{\sup\{\mu(x_1) \mid x_1 \in f^{-1}(y_1)\}, \sup\{\mu(x_2) \mid x_2 \in f^{-1}(y_2)\}\} \\ &= \min\{f(\mu)(y_1), f(\mu)(y_2)\}. \end{aligned}$$

Hence $f(\mu)$ is a fuzzy subnear-ring of S. □

Theorem 3.15. *Let $f : R \to S$ be an onto homomorphism of near-rings.*
(1) *If ν is a fuzzy ideal in S, then $f^{-1}(\nu)$ is a fuzzy ideal in R.*
(2) *If μ is a fuzzy ideal in R, then $f(\mu)$ is a fuzzy ideal in S.*

Proof. (1) Let ν be a fuzzy ideal in S. Then $f^{-1}(\nu)$ is a fuzzy subnear-ring of R from Theorem 3.14, and we have that for any $x_1, x_2, x_3 \in R$,

$$\begin{aligned} f^{-1}(\nu)(x_1 + x_2 - x_1) &= \nu(f(x_1) + f(x_2) - f(x_1)) \\ &\geq \nu(f(x_2)) \\ &= f^{-1}(\nu)(x_2), \end{aligned}$$

$$\begin{aligned} f^{-1}(\nu)(x_1x_2) &= \nu(f(x_1)f(x_2)) \\ &\geq \nu(f(x_2)) \\ &= f^{-1}(\nu)(x_2), \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\nu)((x_1 + x_2)x_3 - x_1x_3) &= \nu((f(x_1) + f(x_2))f(x_3) - f(x_1)f(x_3)) \\ &\geq \nu(f(x_2)) \\ &= f^{-1}(\nu)(x_2). \end{aligned}$$

Hence $f^{-1}(\nu)$ is a fuzzy ideal in R.

(2) Let μ be a fuzzy ideal in R. Then $f(\mu)$ is a fuzzy subnear-ring of S from Theorem 3.14, and we have that for any $y_1, y_2, y_3 \in S$,

$$\begin{aligned} f(\mu)(y_1 + y_2 - y_1) &= \sup\{\mu(x) | x \in f^{-1}(y_1 + y_2 - y_1)\} \\ &\geq \sup\{\mu(x_1 + x_2 - x_1) | x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\} \\ &\geq \sup\{\mu(x_2) | x_2 \in f^{-1}(y_2)\} \\ &= f(\mu)(y_2), \end{aligned}$$

$$\begin{aligned} f(\mu)(y_1y_2) &= \sup\{\mu(x) | x \in f^{-1}(y_1y_2)\} \\ &\geq \sup\{\mu(x_1x_2) | x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\} \\ &\geq \sup\{\mu(x_2) | x_2 \in f^{-1}(y_2)\} \\ &= f(\mu)(y_2), \end{aligned}$$

and

$$\begin{aligned}
&f(\mu)((y_1+y_2)y_3-y_1y_3)\\
&\qquad = \sup\{\mu(x)|x\in f^{-1}((y_1+y_2)y_3-y_1y_3)\}\\
&\qquad \geq \sup\{\mu((x_1+x_2)x_3-x_1x_3)|x_1\in f^{-1}(y_1),x_2\in f^{-1}(y_2),x_3\in f^{-1}(y_3)\}\\
&\qquad \geq \sup\{\mu(x_2)|x_2\in f^{-1}(y_2)\}\\
&\qquad = f(\mu)(y_2).
\end{aligned}$$

Hence $f(\mu)$ is a fuzzy ideal in S. □

Definition 3.16. Let R and S be two near-rings and f a function of R into S. If μ is a fuzzy set in R, then the *anti image* of μ under f is the fuzzy set $f_{-}(\mu)$ in S defined by

$$f_{-}(\mu)(y)=\begin{cases}\inf\limits_{x\in f^{-1}(y)}\mu(x), & \text{if } f^{-1}(y)\neq\emptyset\\ 1, & \text{otherwise,}\end{cases}$$

for each $y\in S$.

Theorem 3.17. *Let $f:R\to S$ be an onto homomorphism of near-rings. Then we have that*

(1) *if ν is a fuzzy set in S, then $f^{-1}(\nu^c)=(f^{-1}(\nu))^c$,*
(2) *if μ is a fuzzy set in R, then $f(\mu^c)=(f_{-}(\mu))^c$ and $f_{-}(\mu^c)=(f(\mu))^c$.*

Proof. (1) Let ν is a fuzzy set in S. Then for each $x\in R$,

$$\begin{aligned}
f^{-1}(\nu^c)(x) &= \nu^c(f(x))\\
&= 1-\nu(f(x))\\
&= 1-f^{-1}(\nu)(x)\\
&= (f^{-1}(\nu))^c(x).
\end{aligned}$$

Hence $f^{-1}(\nu^c)=(f^{-1}(\nu))^c$.

(2) Let μ is a fuzzy set in R. Then for each $x\in R$,

$$\begin{aligned}
f(\mu^c)(y) &= \sup_{x\in f^{-1}(y)}\mu^c(x)\\
&= \sup_{x\in f^{-1}(y)}(1-\mu(x))\\
&= 1-\inf_{x\in f^{-1}(y)}\mu(x)\\
&= 1-f_{-}(\mu)(y)\\
&= (f_{-}(\mu))^c(y),
\end{aligned}$$

and

$$\begin{aligned} f_{-}(\mu^c)(y) &= \inf_{x\in f^{-1}(y)} \mu^c(x) \\ &= \inf_{x\in f^{-1}(y)} (1-\mu(x)) \\ &= 1-\sup_{x\in f^{-1}(y)} \mu(x) \\ &= 1-f(\mu)(y) \\ &= (f(\mu))^c(y). \end{aligned}$$

Hence $f(\mu^c) = (f_{-}(\mu))^c$ and $f_{-}(\mu^c) = (f(\mu))^c$). □

Theorem 3.18. *Let $f : R \to S$ be an onto homomorphism of near-rings. Then we have that*

(1) *if ν is an anti fuzzy subnear-ring of S, then $f^{-1}(\nu)$ is an anti fuzzy subnear-ring in R,*
(2) *if μ is an anti fuzzy subnear-ring of R, then $f_{-}(\mu)$ is an anti fuzzy subnear-ring of S.*

Proof. (1) Let ν is an anti fuzzy subnear-ring in S. Then ν^c is a fuzzy subnear-ring in S from Proposition 3.6, and $f^{-1}(\nu^c)$ is a fuzzy subnear-ring in R from Theorem 3.14. Hence $(f^{-1}(\nu))^c$ is a fuzzy subnear-ring in R, and $f^{-1}(\nu)$ is an anti fuzzy subnear-ring in R.

(2) Let μ be an anti fuzzy subnear-ring in R. Then μ^c is a fuzzy subnear-ring in R, and $f(\mu^c)$is a fuzzy subnear-ring in S by Theorem 3.14. Since $f(\mu^c) = (f_{-}(\mu))^c$, $(f_{-}(\mu))^c$ is also a fuzzy subnear-ring in S, and $f_{-}(\mu)$ is an anti fuzzy subnear-ring in S. □

Theorem 3.19. *Let $f : R \to S$ be an onto homomorphism of near-rings. Then we have that*

(1) *if ν is an anti fuzzy ideal of S, then $f^{-1}(\nu)$ is an anti fuzzy ideal in R,*
(2) *if μ is an anti fuzzy ideal of R, then $f_{-}(\mu)$ is an anti fuzzy ideal of S.*

Proof. The proof of theorem is straightforward, and so is omitted. □

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