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ON ANTI FUZZY IDEALS IN NEAR-RINGS

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ABSTRACT. In this paper, we apply the Biswas' idea of anti fuzzy subgroups to ideals of near-rings. We introduce the notion of anti fuzzy ideals of near-rings, and investigate some related properties.

1. Introduction

W. Liu [14] has studied fuzzy ideals of a ring, and many researchers [5, 10, 12, 17] are engaged in extending the concepts. S. Abou-Zaid [1] introduced the notion of a fuzzy subnear-ring, and studied fuzzy ideals of a near-ring, and many followers [6, 7, 8, 10, 11] discussed further properties of fuzzy ideals in near-rings. In [2], R. Biswas introduced the concept of anti fuzzy subgroups of groups, and K. H. Kim and Y. B. Jun studied the notion of anti fuzzy *R*-subgroups of near-ring in [9]. In this paper, we introduce the notion of anti fuzzy ideals of near-rings, and investigate some related properties.

2. Preliminaries

A *near-ring* ([16]) is a non-empty set R with two binary operations "+" and "." satisfying the following axioms:

- (i) (R, +) is a group,
- (ii) (R, \cdot) is a semigroup,
- (iii) $x \cdot (y+z) = x \cdot y + x \cdot z$, for all $x, y, z \in R$.

Precisely speaking, it is a left near-ring because it satisfies the left distributive law. We will use the word "near-ring" instead of "left near-ring". We denote $x \cdot y$ by xy. If $(R, +, \cdot)$ is a near-ring, then an *ideal* ([1]) of R is a subset I of R such that

- (i) (I, +) is a normal subgroup of (R, +),
- (ii) $RI \subset I$,
- (iii) $(r+i)s rs \in I$, for all $i \in I$ and $r, s \in R$.

Let R and S be two near-rings. A map $f : R \to S$ is a homomorphism of near-rings if f(x+y) = f(x) + f(y) and f(xy) = f(x)f(y) for all $x, y \in R$.

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A fuzzy set μ in a set R is a function $\mu : R \to [0, 1]$. Denote by $\text{Im}(\mu)$ the image set of μ . For $t \in [0, 1]$, the set

$$\mu_t^{\geq} = \{ x \in R | \mu(x) \geq t \} \ (\text{resp. } \mu_t^{\leq} = \{ x \in R | \mu(x) \leq t \})$$

is called a *upper* (resp. *lower*) *t*-*level cut* of μ . Clearly, $\mu_t^{\geq} \cup \mu_t^{\leq} = R$ for $t \in [0, 1]$, and if $t_1 < t_2$, then $\mu_{t_1}^{\leq} \subseteq \mu_{t_2}^{\leq}$ and $\mu_{t_2}^{\geq} \subseteq \mu_{t_1}^{\geq}$. If μ is a fuzzy set in R, then the *complement* of μ , denoted by μ^c , is the fuzzy set in R given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in R$ ([3], [17], [18]).

Let R be a near-ring. A fuzzy subnear-ring of R is a fuzzy set μ of R such that

- (F1) $\mu(x-y) \ge \min\{\mu(x), \mu(y)\},\$
- (F2) $\mu(xy) \ge \min\{\mu(x), \mu(y)\},\$

for all $x, y \in R$. And a *fuzzy ideal* of R is a fuzzy subnear-ring μ of R such that

(F3)
$$\mu(y + x - y) \ge \mu(x)$$
,

- (F4) $\mu(xy) \ge \mu(y)$,
- (F5) $\mu((x+z)y xy) \ge \mu(z),$

for all $x, y, z \in R$ ([10]). Note that μ is a fuzzy left ideal of R if it satisfies (F1), (F2), (F3) and (F4), and μ is a fuzzy right ideal of R if it satisfies (F1), (F2), (F3) and (F5). Let R be a near-ring and μ a fuzzy subset of R. Then the upper *t*-level cut μ_t^{\geq} of μ is a subnear-ring (resp. ideal) of R for all $t \in [0, \mu(0)]$ if and only if μ is a fuzzy subnear-ring (resp. ideal) of R ([1, p145, Theorem 4.2]).

3. Anti Fuzzy Ideals

Definition 3.1. Let R be a near-ring. A fuzzy set μ of R is called an *anti fuzzy* subnear-ring of R if for all $x, y \in R$,

(AF1) $\mu(x-y) \le \max\{\mu(x), \mu(y)\},\$

(AF2) $\mu(xy) \le \max\{\mu(x), \mu(y)\}.$

Definition 3.2. Let R be a near-ring. An anti fuzzy subnear-ring μ of R is called an *anti fuzzy ideal* of R if for all $x, y, z \in R$,

(AF3) $\mu(y+x-y) \le \mu(x),$

(AF4)
$$\mu(xy) \le \mu(y)$$
,

(AF5) $\mu((x+z)y - xy) \le \mu(z).$

Note that μ is an anti fuzzy left ideal of R if it satisfies (AF1), (AF2), (AF3) and (AF4), and μ is an anti fuzzy right ideal of R if it satisfies (AF1), (AF2), (AF3) and (AF5).

Example 3.3. Let $R = \{a, b, c, d\}$ be a set with two binary operations as follows:

+	$a \ b \ c \ d$	$\cdot \mid a \mid b \mid c \mid d$	
a	a b c d		
b	$b \ a \ d \ c$	$b \mid a \mid $	
c	$c \hspace{0.1in} d \hspace{0.1in} b \hspace{0.1in} a$	$c \mid a \mid $	
d	$d \ c \ a \ b$	$d \mid a \mid a \mid b \mid b$	

Then $(R, +, \cdot)$ is a near-ring. We define a fuzzy subset $\mu : R \longrightarrow [0, 1]$ by $\mu(c) = \mu(d) > \mu(b) > \mu(a)$. Then μ is an anti fuzzy right (resp. left) ideal of R.

Every anti fuzzy right (resp. left) ideal of a near-ring R is an anti fuzzy subnear-ring of R, but the converse is not true as shown in the following example.

Example 3.4. Let $R = \{a, b, c, d\}$ be a set with two binary operations as follows:

+	a b c d	•	a	b	c	d
a	a b c d	a	a	a	a	a
b	$b \ a \ d \ c$	b	a	b	c	d
c	c d b a	c	a	a	a	a
d	$d \ c \ a \ b$	d	a	a	a	a

Then $(R, +, \cdot)$ is a near-ring. We define a fuzzy subset $\mu : R \longrightarrow [0, 1]$ by $\mu(c) = \mu(d) > \mu(b) > \mu(a)$. Then μ is an anti fuzzy subnear-ring of R. But μ is not an anti fuzzy right ideal of R, since $\mu((a + b)c - ac) = \mu(c) > \mu(b)$.

Proposition 3.5. If μ is an anti fuzzy subnear-ring of a near-ring R, then $\mu(0) \leq \mu(x)$ for all $x \in R$.

Proof. It follows immediately from [AF1].

Proposition 3.6. Let R be a near-ring. Then a fuzzy set μ is an anti fuzzy subnear-ring in R if and only if μ^c is a fuzzy subnear-ring in R.

Proof. Let μ be an anti fuzzy subnear-ring in R. Then we have that for each $x, y \in R$,

$$\begin{aligned} \mu^{c}(x-y) &= 1 - \mu(x-y) \\ &\geq 1 - \max\{\mu(x), \mu(y)\} \\ &= \min\{1 - \mu(x), 1 - \mu(y)\} \\ &= \min\{\mu^{c}(x), \mu^{c}(y)\}, \end{aligned}$$

and

$$\begin{aligned} \mu^{c}(xy) &= 1 - \mu(xy) \\ &\geq 1 - \max\{\mu(x), \mu(y)\} \\ &= \min\{1 - \mu(x), 1 - \mu(y)\} \\ &= \min\{\mu^{c}(x), \mu^{c}(y)\}. \end{aligned}$$

Hence μ^c is a fuzzy subnear-ring in *R*. The converse is proved similarly.

Proposition 3.7. Let R be a near-ring and μ a fuzzy set in R. Then μ is an anti fuzzy ideal in R if and only if μ^c is a fuzzy ideal in R.

Proof. Let μ be an anti fuzzy ideal in R. Then μ^c is a fuzzy subnear-ring in R, and we have that for all $x, y, z \in R$,

$$\mu^{c}(y+x-y) = 1 - \mu(y+x-y) \ge 1 - \mu(x) = \mu^{c}(x),$$

$$\mu^{c}(xy) = 1 - \mu(xy) \ge 1 - \mu(y) = \mu^{c}(y),$$

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 $\mu^{c}((x+z)y - xy) = 1 - \mu((x+z)y - xy) \ge 1 - \mu(z) = \mu^{c}(z).$

Hence μ^c is a fuzzy ideal in R. The converse is proved similarly.

Let μ be a fuzzy set of a set R. Then $\mu_t^{\leq} = (\mu^c)_{1-t}^{\geq}$ for all $t \in [0, 1]$.

Theorem 3.8. Let μ be a fuzzy set in a near-ring R. Then μ is an anti fuzzy ideal of R if and only if the lower t-level cut μ_t^{\leq} is an ideal of R for each $t \in [\mu(0), 1]$.

Proof. (\Rightarrow) Let μ be an anti fuzzy ideal of R and $t \in [\mu(0), 1]$. Then μ^c is a fuzzy ideal of R, hence $\mu_t^{\leq} = (\mu^c)_{1-t}^{\geq}$ is an ideal of R from [1, Theorem 4.2].

(⇐) Let μ_t^{\leq} be an ideal of R for all $t \in [\mu(0), 1]$ and $s \in [0, 1 - \mu(0)] = [0, \mu^c(0)]$. Then $1 - s \in [\mu(0), 1]$ and $(\mu^c)_s^{\geq} = \mu_{1-s}^{\leq}$ is an ideal of R. Hence $(\mu^c)_s^{\geq}$ is an ideal of R for all $s \in [0, \mu^c(0)]$, and μ^c is a fuzzy ideal of R, whence μ is an anti fuzzy ideal of R.

Proposition 3.9. Let μ be an anti fuzzy subnear-ring R and $t_1, t_2 \in [0, 1]$ with $t_1 < t_2$. Then two lower level cuts $\mu_{t_1}^{\leq}$ and $\mu_{t_2}^{\leq}$ are equal if and only if there is no $x \in R$ such that $t_1 < \mu(x) \leq t_2$.

Proof. From the definition of lower level cuts, it follows that $\mu_t^{\leq} = \mu^{-1}([\mu(0), t])$ for $t \in [0, 1]$. Let $t_1, t_2 \in [0, 1]$ be such that $t_1 < t_2$. Then

$$\mu_{t_1}^{\leq} = \mu_{t_2}^{\leq} \iff \mu^{-1}([\mu(0), t_1]) = \mu^{-1}([\mu(0), t_2]) \iff \mu^{-1}((t_1, t_2]) = \emptyset.$$

Proposition 3.10. If I is an ideal of a near-ring R, then for each $t \in [0, 1]$, there exists an anti fuzzy ideal μ of R such that $\mu_t^{\leq} = I$.

Proof. Let $t \in [0, 1]$ and define a fuzzy set $\mu : R \to [0, 1]$ by

$$\mu(x) = \begin{cases} t & \text{if } x \in I, \\ 1 & \text{if } x \notin I, \end{cases}$$

for each $x \in R$. Then $\mu_s^{\leq} = I$ for any $s \in [t, 1) = [\mu(0), 1)$, and $\mu_1^{\leq} = R$, whence μ_s^{\leq} is an ideal of R for all $s \in [\mu(0), 1]$. Hence μ is an anti fuzzy ideal of R from Theorem 3.8, and $\mu_t^{\leq} = I$.

For a family of fuzzy sets $\{\mu_i \mid i \in \Lambda\}$ in a near-ring R, the union $\bigvee_{i \in \Lambda} \mu_i$ of $\{\mu_i \mid i \in \Lambda\}$ is defined by

$$(\bigvee_{i\in\Lambda}\mu_i)(x) = \sup\{\mu_i(x)|i\in\Lambda\},\$$

for each $x \in R$.

Proposition 3.11. If $\{\mu_i | i \in \Lambda\}$ is a family of anti fuzzy ideals of a near-ring R, then so is $\bigvee_{i \in \Lambda} \mu_i$.

Proof. Let $\{\mu_i \mid i \in \Lambda\}$ be a family of anti fuzzy ideals of R and $x, y \in R$. Then we have that

$$\begin{aligned} (\bigvee_{i \in \Lambda} \mu_i)(x - y) &= \sup\{\mu_i(x - y) | i \in \Lambda\} \\ &\leq \sup\{\max\{\mu_i(x), \mu_i(y)\} | i \in \Lambda\} \\ &= \max\{\sup\{\mu_i(x) | i \in \Lambda\}, \sup\{\mu_i(y) | i \in \Lambda\}\} \\ &= \max\{(\bigvee_{i \in \Lambda} \mu_i)(x), (\bigvee_{i \in \Lambda} \mu_i)(y)\}, \\ (\bigvee \mu_i)(xy) &= \sup\{\mu_i(xy) | i \in \Lambda\} \end{aligned}$$

$$\begin{aligned} \sup_{i \in \Lambda} & \leq \sup\{\max\{\mu_i(x), \mu_i(y)\} | i \in \Lambda\} \\ & \leq \sup\{\max\{\sup\{\mu_i(x) | i \in \Lambda\}, \sup\{\mu_i(y) | i \in \Lambda\}\} \\ & = \max\{\sup\{\mu_i(x) | i \in \Lambda\}, \sup\{\mu_i(y) | i \in \Lambda\}\} \\ & = \max\{(\bigvee_{i \in \Lambda} \mu_i)(x), (\bigvee_{i \in \Lambda} \mu_i)(y)\}. \end{aligned}$$

Hence $\bigvee_{i \in \Lambda} \mu_i$ is an anti fuzzy subnear-ring of R. For any $x, y, z \in R$, we have that

$$(\bigvee_{i \in \Lambda} \mu_i)(y + x - y) = \sup\{\mu_i(y + x - y) | i \in \Lambda\}$$

$$\leq \sup\{\mu_i(x) | i \in \Lambda\}$$

$$= (\bigvee_{i \in \Lambda} \mu_i)(x),$$

$$(\bigvee_{i \in \Lambda} \mu_i)(xy) = \sup\{\mu_i(xy) | i \in \Lambda\}$$

$$\leq \sup\{\mu_i(y) | i \in \Lambda\}$$

$$= (\bigvee_{i \in \Lambda} \mu_i)(y),$$

and

$$(\bigvee_{i \in \Lambda} \mu_i)((x+z)y - xy)) = \sup\{\mu_i((x+z)y - xy) | i \in \Lambda\}$$

$$\leq \sup\{\mu_i(z) | i \in \Lambda\}$$

$$= (\bigvee_{i \in \Lambda} \mu_i)(z).$$

Hence $\bigvee_{i \in \Lambda} \mu_i$ is an anti fuzzy ideal of R.

Theorem 3.12. If μ is an anti fuzzy ideal of a near-ring R, then $\mu(x) = \inf\{t \in I\}$ $[0,1] \mid x \in \mu_t^{\leq} \}$ for each $x \in R$.

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Proof. For each $x \in R$, let $T_x = \{t \in [0,1] \mid x \in \mu_t^{\leq}\}$ and $\alpha = \inf T_x$. Then for any $t \in T_x$, $\mu(x) \leq t$, whence $\mu(x)$ is a lower bound of T_x , hence $\mu(x) \leq \inf T_x = \alpha$. And let $\beta = \mu(x)$. Then $x \in \mu_{\beta}^{\leq}$ and $\beta \in T_x$, hence $\alpha = \inf T_x \leq \beta = \mu(x)$. \Box

Definition 3.13. Let R and S be two near-rings and f a function of R into S.

(1) If ν is a fuzzy set in S, then the *preimage* of ν under f is the fuzzy set in R defined by

$$f^{-1}(\nu)(x) = \nu(f(x)),$$

for each $x \in R$.

(2) If μ is a fuzzy set of R, then the *image* of μ under f is the fuzzy set in S defined by

$$f(\mu)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu(x), & \text{if } f^{-1}(y) \neq \emptyset\\ 0, & \text{otherwise,} \end{cases}$$

for each $y \in S$.

Theorem 3.14. Let $f : R \to S$ be an onto homomorphism of near-rings.

- (1) If ν is a fuzzy subnear-ring of S, then $f^{-1}(\nu)$ is a fuzzy subnear-ring of R.
- (2) If μ is a fuzzy subnear-ring of R, then $f(\mu)$ is a fuzzy subnear-ring of S.

Proof. (1) Let $x_1, x_2 \in R$. Then we have that

$$f^{-1}(\nu)(x_1 - x_2) = \nu(f(x_1) - f(x_2))$$

$$\geq \min\{\nu(f(x_1)), \nu(f(x_2))\}$$

$$= \min\{f^{-1}(\nu)(x_1), f^{-1}(\nu)(x_2)\},\$$

and

$$f^{-1}(\nu)(x_1x_2) = \nu(f(x_1)f(x_2))$$

$$\geq \min\{\nu(f(x_1)), \nu(f(x_2))\}$$

$$= \min\{f^{-1}(\nu)(x_1), f^{-1}(\nu)(x_2)\}.$$

Hence $f^{-1}(\nu)$ is a fuzzy subnear-ring of R.

(2) Let $y_1, y_2 \in S$. Then we have

$$\{x \mid x \in f^{-1}(y_1 - y_2)\} \supseteq \{x_1 - x_2 \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\},\$$

and hence

$$\begin{aligned} f(\mu)(y_1 - y_2) &= \sup\{\mu(x) \mid x \in f^{-1}(y_1 - y_2)\} \\ &\geq \sup\{\mu(x_1 - x_2) \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\} \\ &\geq \sup\{\min\{\mu(x_1), \mu(x_2)\} \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\} \\ &= \min\{\sup\{\mu(x_1) \mid x_1 \in f^{-1}(y_1)\}, \sup\{\mu(x_2) \mid x_2 \in f^{-1}(y_2)\}\} \\ &= \min\{f(\mu)(y_1), f(\mu)(y_2)\}, \end{aligned}$$

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and since
$$\{x \mid x \in f^{-1}(y_1y_2)\} \supseteq \{x_1x_2 \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\},\$$

 $f(\mu)(y_1y_2) = \sup\{\mu(x) \mid x \in f^{-1}(y_1y_2)\}$
 $\ge \sup\{\mu(x_1x_2) \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\}$
 $\ge \sup\{\min\{\mu(x_1), \mu(x_2)\} \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\}$
 $= \min\{\sup\{\mu(x_1) \mid x_1 \in f^{-1}(y_1)\}, \sup\{\mu(x_2) \mid x_2 \in f^{-1}(y_2)\}\}$
 $= \min\{f(\mu)(y_1), f(\mu)(y_2)\}.$

Hence $f(\mu)$ is a fuzzy subnear-ring of S.

Theorem 3.15. Let $f : R \to S$ be an onto homomorphism of near-rings.

- If ν is a fuzzy ideal in S, then f⁻¹(ν) is a fuzzy ideal in R.
 If μ is a fuzzy ideal in R, then f(μ) is a fuzzy ideal in S.

Proof. (1) Let ν be a fuzzy ideal in S. Then $f^{-1}(\nu)$ is a fuzzy subnear-ring of R from Theorem 3.14, and we have that for any $x_1, x_2, x_3 \in R$,

$$f^{-1}(\nu)(x_1 + x_2 - x_1) = \nu(f(x_1) + f(x_2) - f(x_1))$$

$$\geq \nu(f(x_2))$$

$$= f^{-1}(\nu)(x_2),$$

$$f^{-1}(\nu)(x_1x_2) = \nu(f(x_1)f(x_2))$$

$$\geq \nu(f(x_2))$$

$$= f^{-1}(\nu)(x_2),$$

and

$$f^{-1}(\nu)((x_1 + x_2)x_3 - x_1x_3) = \nu((f(x_1) + f(x_2))f(x_3) - f(x_1)f(x_3))$$

$$\geq \nu(f(x_2))$$

$$= f^{-1}(\nu)(x_2).$$

Hence $f^{-1}(\nu)$ is a fuzzy ideal in R.

(2) Let μ be a fuzzy ideal in R. Then $f(\mu)$ is a fuzzy subnear-ring of S from Theorem 3.14, and we have that for any $y_1, y_2, y_3 \in S$,

$$\begin{split} f(\mu)(y_1 + y_2 - y_1) &= \sup\{\mu(x) | x \in f^{-1}(y_1 + y_2 - y_1)\} \\ &\geq \sup\{\mu(x_1 + x_2 - x_1) | x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\} \\ &\geq \sup\{\mu(x_2) | x_2 \in f^{-1}(y_2)\} \\ &= f(\mu)(y_2), \\ f(\mu)(y_1y_2) &= \sup\{\mu(x) | x \in f^{-1}(y_1y_2)\} \\ &\geq \sup\{\mu(x_1x_2) | x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\} \\ &\geq \sup\{\mu(x_2) | x_2 \in f^{-1}(y_2)\} \\ &= f(\mu)(y_2), \end{split}$$

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$$\begin{aligned} f(\mu)((y_1 + y_2)y_3 - y_1y_3) \\ &= \sup\{\mu(x)|x \in f^{-1}((y_1 + y_2)y_3 - y_1y_3)\} \\ &\geq \sup\{\mu((x_1 + x_2)x_3 - x_1x_3)|x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2), x_3 \in f^{-1}(y_3)\} \\ &\geq \sup\{\mu(x_2)|x_2 \in f^{-1}(y_2)\} \\ &= f(\mu)(y_2). \end{aligned}$$

Hence $f(\mu)$ is a fuzzy ideal in S.

Definition 3.16. Let R and S be two near-rings and f a function of R into S. If μ is a fuzzy set in R, then the *anti image* of μ under f is the fuzzy set $f_{-}(\mu)$ in S defined by

$$f_{-}(\mu)(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \mu(x), & \text{if } f^{-1}(y) \neq \emptyset\\ 1, & \text{otherwise,} \end{cases}$$

for each $y \in S$.

Theorem 3.17. Let $f : R \to S$ be an onto homomorphism of near-rings. Then we have that

- (1) if ν is a fuzzy set in S, then $f^{-1}(\nu^c) = (f^{-1}(\nu))^c$,
- (2) if μ is a fuzzy set in R, then $f(\mu^c) = (f_{-}(\mu))^c$ and $f_{-}(\mu^c) = (f(\mu))^c$.

Proof. (1) Let ν is a fuzzy set in S. Then for each $x \in R$,

$$f^{-1}(\nu^{c})(x) = \nu^{c}(f(x))$$

= $1 - \nu(f(x))$
= $1 - f^{-1}(\nu)(x)$
= $(f^{-1}(\nu))^{c}(x).$

Hence $f^{-1}(\nu^c) = (f^{-1}(\nu))^c$.

(2) Let μ is a fuzzy set in R. Then for each $x \in R$,

$$f(\mu^{c})(y) = \sup_{x \in f^{-1}(y)} \mu^{c}(x)$$

=
$$\sup_{x \in f^{-1}(y)} (1 - \mu(x))$$

=
$$1 - \inf_{x \in f^{-1}(y)} \mu(x)$$

=
$$1 - f_{-}(\mu)(y)$$

=
$$(f_{-}(\mu))^{c}(y),$$

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and

$$f_{-}(\mu^{c})(y) = \inf_{\substack{x \in f^{-1}(y)}} \mu^{c}(x)$$

= $\inf_{\substack{x \in f^{-1}(y)}} (1 - \mu(x))$
= $1 - \sup_{\substack{x \in f^{-1}(y)}} \mu(x)$
= $1 - f(\mu)(y)$
= $(f(\mu))^{c}(y).$

Hence $f(\mu^c) = (f_{-}(\mu))^c$ and $f_{-}(\mu^c) = (f(\mu))^c$.

Theorem 3.18. Let $f : R \to S$ be an onto homomorphism of near-rings. Then we have that

- (1) if ν is an anti fuzzy subnear-ring of S, then $f^{-1}(\nu)$ is an anti fuzzy subnear-ring in R,
- (2) if μ is an anti fuzzy subnear-ring of R, then $f_{-}(\mu)$ is an anti fuzzy subnear-ring of S.

Proof. (1) Let ν is an anti fuzzy subnear-ring in S. Then ν^c is a fuzzy subnear-ring in S from Proposition 3.6, and $f^{-1}(\nu^c)$ is a fuzzy subnear-ring in R from Theorem 3.14. Hence $(f^{-1}(\nu))^c$ is a fuzzy subnear-ring in R, and $f^{-1}(\nu)$ is an anti fuzzy subnear-ring in R.

(2) Let μ be an anti fuzzy subnear-ring in R. Then μ^c is a fuzzy subnear-ring in R, and $f(\mu^c)$ is a fuzzy subnear-ring in S by Theorem 3.14. Since $f(\mu^c) = (f_-(\mu))^c$, $(f_-(\mu))^c$ is also a fuzzy subnear-ring in S, and $f_-(\mu)$ is an anti fuzzy subnear-ring in S.

Theorem 3.19. Let $f : R \to S$ be an onto homomorphism of near-rings. Then we have that

- (1) if ν is an anti fuzzy ideal of S, then $f^{-1}(\nu)$ is an anti fuzzy ideal in R,
- (2) if μ is an anti fuzzy ideal of R, then $f_{-}(\mu)$ is an anti fuzzy ideal of S.

Proof. The proof of theorem is straightforward, and so is omitted.

References

- [1] S. Abou-Zaid, On fuzzy subnear-rings and ideals, Fuzzy Sets and Sys 44, (1991), 139-146.
- [2] R. Biswas, Fuzzy subgroups and anti fuzzy subgroups, Fuzzy Sets and Sys 35, (1990), 121-124.
- [3] P. S. Das, Fuzzy groups and level subgroups, J. Math. Anal. and Appl. 84 , (1981), 264-269.
- [4] V. N. Dixit, R. Kumar and N. Ajmal, Fuzzy ideals and fuzzy prime ideals of a ring, Fuzzy Sets and Sys 44, (1991), 127-138.
- [5] V. N. Dixit, R. Kumar and N. Ajmal, On fuzzy rings, Fuzzy Sets and Systems 49, (1992), 205-213.
- [6] S. M. Hong, Y. B. Jun and H. S. Kim, *Fuzzy ideals in near-rings*, Bull. Korean Math. Soc. 35 (No. 3), (1998), 455-464.
- [7] C. K. Hur and H. S. Kim, On fuzzy relations of near-rings, Far East J. Math. Sci. (to appear).
- [8] Y. B. Jun and H. S. Kim, On fuzzy prime ideals under near-ring homomorphisms, (submitted).

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- K. H. Kim and Y. B. Jun, Anti fuzzy R-subgroups of near-rings, Scientiae Mathematicae 2 (No.2), (1999), 147-153.
- [10] S. D. Kim and H. S. Kim, On fuzzy ideals of near-rings, Bull. Korean Math. Soc. 33, (1996), 593-601.
- [11] C. K. Kim and H. S. Kim, On normalized fuzzy ideals of near-rings, Far East J. Math. Sci. , (), .
- [12] R. Kumar, Fuzzy irreducible ideals in rings, Fuzzy Sets and Systems 42, (1991), 369-379.
- [13] R. Kumar, Certain fuzzy ideals of rings redefined, Fuzzy Sets and Systems 46, (1992), 251-260.
- [14] W. Liu, Fuzzy invariant subgroups and fuzzy ideals, Fuzzy Sets and Systems 8, (1982), 133-139.
- [15] D. S. Mailk, Fuzzy ideals of artinian rings, Fuzzy Sets and Systems 37, (1990), 111-115.
- [16] J. D. P. Meldrum, Near-rings and their links with groups, Pitman, Boston, (1985).
- [17] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35, (1971), 512-517.
- [18] L. A. Zadeh, Fuzzy sets, Inform. and Control. 8, (1965), 338-353.

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