DATA ENVELOPMENT ANALYSIS WITH FUZZY RANDOM INPUTS AND OUTPUTS: A CHANCE-CONSTRAINED PROGRAMMING APPROACH

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ABSTRACT. In this paper, we deal with fuzzy random variables for inputs and outputs in Data Envelopment Analysis (DEA). These variables are considered as fuzzy random flat LR numbers with known distribution. The problem is to find a method for converting the imprecise chance-constrained DEA model into a crisp one. This can be done by first, defuzzification of imprecise probability by constructing a suitable membership function, second, defuzzification of the parameters using an α-cut and finally, converting the chance-constrained DEA into a crisp model using the method of Cooper [4].

1. Introduction

Most research work in DEA deals with precise and deterministic information. The papers related to uncertainty are either deal with random information see [1, 4, 5, 6 &18] or fuzzy information for inputs and outputs see [9, 10, 12, 20, 21 & 22]. However, we have not noticed any contribution that may incorporate the hybrid uncertainty (i.e. fuzziness and randomness) in DEA. When the parameters (inputs and outputs) of a DEA model are fuzzy random variables we have a fuzzy chance-constrained DEA problem. Therefore, we consider a DEA model to evaluate the efficiency of DMUs when data are fuzzy random variables. In addition, the probability of the constraints may also be described as fuzzy relation. Kawakernaak [13] and Puri & Ralescu [19], introduced fuzzy random variables and Guangyuan & Yue [7&8], Liu [14&15], Chakraborti [2&3] and Luhandjula [16&17] presented some important methods for solving mathematical programming with fuzzy random variable coefficients.

In this paper, we consider the CCR model of DEA with chance-constrained programming approach, in which inputs and outputs are fuzzy random variables; we assume that these fuzzy random variables are flat LR fuzzy numbers. Our objective is to convert the fuzzy chance-constraint DEA into crisp DEA. For this purpose, first of all, the model is defuzzified by using a suitable membership function for the fuzzy relation of the probability. In the second stage the fuzziness of the parameters is removed by an α-cut approach and finally the randomness is rectified by classical mean-variance method of Cooper [4]. The structure of this paper is as follows: Section 2 presents the fuzzy

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chance-constrained DEA and the procedure of its conversion into a crisp DEA model. To demonstrate the process, a numerical example is given in section 3. Section 4 consists of a conclusion.

2. Fuzzy Chance Constrained DEA

We formulate the fuzzy chance-constrained DEA as follows:

Let $\tilde{x}_j(\omega) = (\tilde{x}_{1j}(\omega),\ldots,\tilde{x}_{mj}(\omega))^T$ and $\tilde{y}_j(\omega) = (\tilde{y}_{1j}(\omega),\ldots,\tilde{y}_{sj}(\omega))^T$ represent $(m \times 1)$ a $(s \times 1)$ fuzzy random input and output vectors, for each DMU$_j$ (j=1,...,n), where

$$\tilde{x}_{ij}(\omega) = (x^l{}_{ij}(\omega), x^u{}_{ij}(\omega), p^x_{ij}(\omega), q^x_{ij}(\omega))_{LR}{}^T \quad ; i=1,\ldots,m \ \&$$
$$\tilde{y}_{rj}(\omega) = (y^l{}_{rj}(\omega), y^u{}_{rj}(\omega), p^y_{rj}(\omega), q^y_{rj}(\omega))_{LR}{}^T \quad ; r=1,\ldots,s$$

are LR flat fuzzy random variables related to a random variable ω (p and q are the left and right spreads). We present the fuzzy chance-constrained CCR model as follows:

$$\begin{aligned}
&\text{Min} \quad \theta \\
&\text{s.t}: \\
&\qquad \text{Prob}\left(\theta\, \tilde{x}_{i0}(\omega) \geq \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij}(\omega)\right) \underset{\sim}{\geq} p \qquad ,i = 1,\ldots, m \\
&\qquad \text{Prob}\left(\tilde{y}_{r0}(\omega) \leq \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj}(\omega)\right) \underset{\sim}{\geq} p \qquad ,r = 1,\ldots, s \\
&\qquad\qquad \lambda_j \geq 0 \qquad\qquad , j = 1,\ldots, n
\end{aligned} \tag{1}$$

"$\underset{\sim}{\geq}$" signifies that the constraints are fuzzily satisfied with probability p. Next we demonstrate a view of the efficient frontier of such a model for single input and single output. For simplicity of presentation, we consider y_0 to be a random variable and $\tilde{x}_0(\omega)$ to be a triangular fuzzy random variable. In Figure 1, the lines 1, 2 and 3 represent the inner part, mean and the outer part of the efficiency frontier. Each α-cut intersects this frontier in random variables. For instance, the abscissa of the point R for is a random variable x with distribution $N(x_0^m(\omega),\sigma)$ and its ordinate is a random variable y with distribution $N(y_0,\sigma)$.

In what follows, the process of conversion has been developed in three phases. In phase I the imprecise probability is defuzzified. In phase II defuzzification of the parameters is carried out, thereby converting the problem into a chance-constrained

DEA. Finally the conversion of this chance-constrained DEA into a crisp model is performed in phase III.

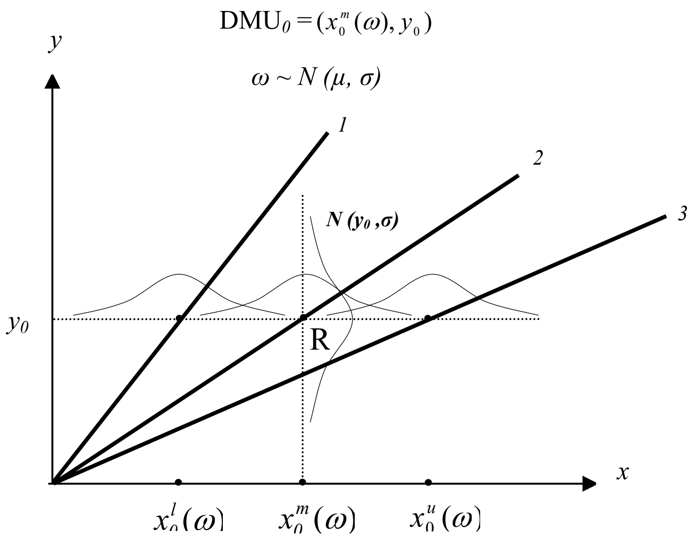

FIGURE 1. Efficiency frontier of a fuzzy *chance-constrained CCR* model.

Phase I: It is clear that $\theta\,\tilde{x}_{i0}(\omega) \ge \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij}(\omega)$ and $\tilde{y}_{r0}(\omega) \le \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj}(\omega)$ are fuzzy events. For simplicity, let C_i denote the event $\theta\,\tilde{x}_{i0}(\omega) \ge \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij}(\omega)$. The membership function $\mu_{prob}(p)$ explains the grade to which $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, ..., \lambda_n)$ fits the constraints. The probability of this fuzzy event is the measure of the degree to which $\boldsymbol{\lambda}$ is reliable. According to [2], we define the membership function for the $\mathrm{Prob}(C_l) \underset{\sim}{\ge} p$ as follows:

$$\mu_{\mathrm{Prob}}(p) = \begin{cases} 1 & \text{if } \mathrm{Prob}(C_l) \ge p \\ \dfrac{\mathrm{Prob}(C_l) - (p - \Delta p)}{\Delta p} & \text{if } p - \Delta p \le \mathrm{Prob}(C_l) \le p \\ 0 & \text{otherwise} \end{cases}$$

where Δp is the tolerance of the probability.

It is evident that:

$$\frac{\mathrm{Prob}(C_l) - (p - \Delta p)}{\Delta p} \ge 0$$

Therefore,

$$\text{Prob}\left(\theta\,\tilde{x}_{i0}(\omega) \geq \sum_{j=1}^{n}\lambda_j \tilde{x}_{ij}(\omega)\right) \geq p - \Delta p \qquad , i = 1,\dots,\text{m}$$

and similarly,

$$\text{Prob}\left(\tilde{y}_{r0}(\omega) \leq \sum_{j=1}^{n}\lambda_j \tilde{y}_{rj}(\omega)\right) \geq p - \Delta p \qquad , r = 1,\dots,\text{s}$$

So, (1) can be converted into:

$$\begin{aligned}
&\text{Min} \quad \theta \\
&\text{s.t}: \quad \text{Prob}\left(\theta\,\tilde{x}_{i0}(\omega) \geq \sum_{j=1}^{n}\lambda_j \tilde{x}_{ij}(\omega)\right) \geq p - \Delta p \qquad , i = 1,\dots,\text{m} \\
&\qquad\quad \text{Prob}\left(\tilde{y}_{r0}(\omega) \leq \sum_{j=1}^{n}\lambda_j \tilde{y}_{rj}(\omega)\right) \geq p - \Delta p \qquad , r = 1,\dots,\text{s} \\
&\qquad\quad \lambda_j \geq 0 \qquad , \text{j} = 1,\dots,\text{n}
\end{aligned} \tag{2}$$

Phase II: In this phase the fuzziness of the coefficients are dealt with. For this, we apply the concept of α-cut as in [20]. By introducing the α-cut of constraints and summation of LR flat fuzzy numbers, we will have the following problem:

$$\begin{aligned}
&\text{Min} \quad \theta \\
&\text{s.t}: \\
&\text{Prob}\left(\begin{matrix}\left[\theta\left(x_{i0}^{l}(\omega) - p_{i0}^{x}(\omega)L_{xi0}^{-1}(\alpha)\right), \theta\left(x_{i0}^{u}(\omega) + q_{i0}^{x}(\omega)R_{xi0}^{-1}(\alpha)\right)\right] \geq \\ \left[\sum_{j=1}^{n}\lambda_j\left(x_{ij}^{l}(\omega) - p_{ij}^{x}(\omega)L_{xij}^{-1}(\alpha)\right), \sum_{j=1}^{n}\lambda_j\left(x_{ij}^{u}(\omega) + q_{ij}^{x}(\omega)R_{xij}^{-1}(\alpha)\right)\right]\end{matrix}\right) \geq p \quad , i = 1,\dots,m \\
&\text{Prob}\left(\begin{matrix}\left[y_{r0}^{l}(\omega) - p_{r0}^{y}(\omega)L_{yr0}^{-1}(\alpha), y_{r0}^{u}(\omega) + q_{r0}^{y}(\omega)R_{yr0}^{-1}(\alpha)\right] \geq \\ \left[\sum_{j=1}^{n}\lambda_j\left(y_{rj}^{l}(\omega) - p_{rj}^{y}(\omega)L_{yrj}^{-1}(\alpha)\right), \sum_{j=1}^{n}\lambda_j\left(y_{rj}^{u}(\omega) + q_{rj}^{y}(\omega)R_{yrj}^{-1}(\alpha)\right)\right]\end{matrix}\right) \geq p \quad , r = 1,\dots,s \\
&\qquad\qquad \lambda_j \geq 0 \qquad , \text{j} = 1,\dots,\text{n}
\end{aligned} \tag{3}$$

According to Saati et al. [20], to evaluate the efficiency of DMUs, the lower level of inputs and upper level of outputs (that is the best part of DMUs) for each DMU are compared with the inner part of efficiency frontier.

The best part of a DMU$_0$ is $\left(x_{i0}^{l}(\omega) - p_{i0}^{x}(\omega) L_{xi0}^{-1}(\alpha),\ y_{r0}^{u}(\omega) + q_{r0}^{y}(\omega) R_{yr0}^{-1}(\alpha) \right)$ and the inner part of the frontier is

$$\left(\sum_{j=1}^{n} \lambda_j (x_{ij}^{u}(\omega) + q_{ij}^{x}(\omega) R_{xij}^{-1}(\alpha)), \sum_{j=1}^{n} \lambda_j (y_{rj}^{l}(\omega) - p_{rj}^{y}(\omega) L_{yrj}^{-1}(\alpha)) \right).$$

Therefore, model (3) can be written as follows:

$$\begin{aligned}
&\text{Min} \quad \theta \\
&\text{s.t}: \\
&\text{Prob} \begin{pmatrix} \theta \left(x_{i0}^{l}(\omega) - p_{i0}^{x}(\omega) L_{xi0}^{-1}(\alpha) \right) \geq \\ \sum_{j=1}^{n} \lambda_j \left(x_{ij}^{u}(\omega) + q_{ij}^{x}(\omega) R_{ij}^{-1}(\alpha) \right) \end{pmatrix} \geq p \qquad i = 1,\ldots, m \\
&\text{Prob} \begin{pmatrix} y_{r0}^{u}(\omega) + q_{r0}^{y}(\omega) R_{yr0}^{-1}(\alpha) \leq \\ \sum_{j=1}^{n} \lambda_j \left(y_{rj}^{l}(\omega) - p_{rj}^{y}(\omega) L_{yrj}^{-1}(\alpha) \right) \end{pmatrix} \geq p \qquad r = 1,\ldots, s \\
&\lambda_j \geq 0 \qquad j = 1,\ldots, n.
\end{aligned} \tag{4}$$

This problem is a parametric chance-constrained DEA, while $\alpha \in (0,1]$ is a parameter.

Phase III: Now we can convert the chance-constrained DEA (4) into the following crisp nonlinear programming by Cooper [4]:

$$\begin{aligned}
&\text{Min} \quad \theta \\
&\text{s.t}: \\
&\theta \left(x_{i0}^{l}(\varpi) - p_{i0}^{x}(\varpi) L_{xi0}^{-1}(\alpha) \right) - \sum_{j=1}^{n} \lambda_j \left(x_{ij}^{u}(\varpi) + q_{ij}^{x}(\varpi) R_{xij}^{-1}(\alpha) \right) + \varphi^{-1}(1 - p + \Delta p) \sigma_x^{i} \geq 0 \\
&\qquad ,i = 1,\ldots,m \\
&y_{r0}^{u}(\varpi) + q_{r0}^{y}(\varpi) R_{yr0}^{-1}(\alpha) - \sum_{j=1}^{n} \lambda_j \left(y_{rj}^{l}(\varpi) - p_{rj}^{y}(\varpi) L_{yrj}^{-1}(\alpha) \right) - \varphi^{-1}(1 - p + \Delta p) \sigma_y^{r} \leq 0 \\
&\qquad ,r = 1,\ldots,s \\
&\lambda_j \geq 0 \qquad ,j = 1,\ldots,n
\end{aligned} \tag{5}$$

where :

$\overline{\omega}$ is the mean of the random variable ω ,
φ is the standard normal distribution function ,
φ^{-1} is inverse of φ ,

$$(\sigma_x^i)^2 = \sum_{j=1}\sum_{k=1}\lambda_j\lambda_k Cov(x_{ij}^u(\omega)+q_{ij}^x(\omega)R_{xij}^{-1}(\alpha),\ x_{ik}^u(\omega)+q_{ik}^x(\omega)R_{xik}^{-1}(\alpha))$$
$$-2\theta\sum_{j\neq 0}\lambda_j Cov(x_{ij}^u(\omega)+q_{ij}^x(\omega)R_{ij}^{-1}(\alpha),\ \ x_{i0}^l(\omega)-p_{i0}^x(\omega)L_{xi0}^{-1}(\alpha))$$
$$+\theta^2 Var(x_{i0}^l(\omega)-p_{i0}^x(\omega)L_{xi0}^{-1}(\alpha)),$$

$$(\sigma_y^r)^2 = \sum_{i=1}\sum_{j=1}\lambda_i\lambda_j Cov(y_{ri}^l(\omega)-p_{ri}^y(\omega)L_{yri}^{-1}(\alpha)\,,y_{rj}^l(\omega)-p_{rj}^y(\omega)L_{yrj}^{-1}(\alpha))$$
$$-2\sum_{i=1}\lambda_i Cov(y_{ri}^l(\omega)-p_{ri}^y(\omega)L_{yri}^{-1}(\alpha),y_{j0}^u(\omega)+q_{r0}^y(\omega)R_{yr0}^{-1}(\alpha))$$
$$+Var(y_{r0}^u(\omega)+q_{r0}^y(\omega)R_{yr0}^{-1}(\alpha)).$$

We have an optimal solution for each α . Thus, for different $\alpha\in(0,1]$, the optimal solutions can be obtained based on the choice of decision maker with regard to α , and the appropriate solution may be selected.

The non-linearity in (5) is due to σ_x^i and σ_y^r . The $(\sigma_y^r)^2$ and $(\sigma_x^i)^2$ are as follows:

$$(\sigma_x^i)^2 = [\theta,-\lambda_1,-\lambda_2,\ldots,-\lambda_n]\,\mathbf{V}_{(n+1)(n+1)}[\theta,-\lambda_1,-\lambda_2,\ldots,-\lambda_n]^t$$
$$(\sigma_y^r)^2 = [1,-\lambda_1,-\lambda_2,\ldots,-\lambda_n]\,\mathbf{V}'_{(n+1)(n+1)}[1,-\lambda_1,-\lambda_2,\ldots,-\lambda_n]^t$$

where $\mathbf{V}$ and $\mathbf{V}'$ are variance-covariance matrices for each set of the constraints respectively. Since these matrices are positive definite, so σ_x^i and σ_y^r are convex [11]. Therefore we can claim that the problem (5) is convex programming problem.

As a special case when the data are triangular fuzzy random numbers, i.e. $\widetilde{x}_{ij}(\omega)=(x^m{}_{ij}(\omega),x^l{}_{ij}(\omega),x^u{}_{ij}(\omega))^T$ and $\widetilde{y}_{rj}(\omega)=(y^m{}_{rj}(\omega),y^l{}_{rj}(\omega),y^u{}_{rj}(\omega))^T$,*i=1,…,m ; r=1, … ,s* the model (5) is as follows:

$$\begin{aligned}
&\text{Min} \quad \theta \\
&\text{s.t:} \\
&\theta\left(\alpha\, x_{i0}^{m}(\overline{\omega})+(1-\alpha)x_{i0}^{l}(\overline{\omega})\right)-\sum_{j=1}^{n}\lambda_j\left(\alpha\, x_{ij}^{m}(\overline{\omega})+(1-\alpha)x_{ij}^{u}(\overline{\omega})\right)+\varphi^{-1}(1-p+\Delta p)\sigma_x^{i}\geq 0 \\
&\qquad\qquad ,i=1,\ldots,m \\
&\alpha\, y_{r0}^{m}(\overline{\omega})+(1-\alpha)y_{r0}^{u}(\overline{\omega})-\sum_{j=1}^{n}\lambda_j\left(\alpha\, y_{rj}^{m}(\overline{\omega})+(1-\alpha)y_{rj}^{l}(\overline{\omega})\right)-\varphi^{-1}(1-p+\Delta p)\sigma_y^{r}\leq 0 \\
&\qquad\qquad ,r=1,\ldots,s \\
&\lambda_j\geq 0 \qquad\qquad ,j=1,\ldots,n
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
(\sigma_x^{i})^2 &= \sum_{j=1}\sum_{k=1}\lambda_j\lambda_k Cov(\alpha\, x_{ij}^{m}(\omega)+(1-\alpha)x_{ij}^{u}(\omega),(\alpha\, x_{ik}^{m}(\omega)+(1-\alpha)x_{ik}^{u}(\omega)) \\
&-2\theta\sum_{j\neq 0}\lambda_j Cov(\alpha\, x_{ij}^{m}(\omega)+(1-\alpha)x_{ij}^{u}(\omega),\alpha\, x_{i0}^{m}(\omega)+(1-\alpha)x_{i0}^{l}(\omega)) \\
&+\theta^2 Var(\alpha\, x_{i0}^{m}(\omega)+(1-\alpha)x_{i0}^{l}(\omega)),
\end{aligned}$$

$$\begin{aligned}
(\sigma_y^{r})^2 &= \sum_{i=1}\sum_{j=1}\lambda_i\lambda_j Cov(\alpha\, y_{ri}^{m}(\omega)+(1-\alpha)y_{ri}^{l}(\omega),(\alpha\, y_{rj}^{m}(\omega)+(1-\alpha)y_{rj}^{l}(\omega)) \\
&-2\sum_{i=1}\lambda_i Cov(\alpha\, y_{ri}^{m}(\omega)+(1-\alpha)y_{ri}^{l}(\omega),\alpha\, y_{r0}^{m}(\omega)+(1-\alpha)y_{r0}^{u}(\omega)) \\
&+Var(\alpha\, y_{r0}^{m}(\omega)+(1-\alpha)y_{r0}^{u}(\omega)).
\end{aligned}$$

3. **Illustration Example**

The efficiency of 4 farms D_1, D_2, D_3 and D_4 , with areas of 5, 5, 4 and 7 acres respectively, are to be evaluated. In all the farms, the crop cultivated is wheat. The amount of the yield is a random variable normally distributed with mean 2.5, 3, 5 and 3.5. The variance is 1 for all. The amount of rainfall is estimated as a fuzzy random variable with parameters $\widetilde{\omega}=(\omega,\omega-1,\omega+1)$, $\widetilde{\rho}=(\rho,\rho-1,\rho+1)$, $\widetilde{\gamma}=(\lambda,\lambda-1,\lambda+1)$ and $\widetilde{\eta}=(\eta,\eta-1,\eta+1)$ where $\omega\sim N(6,1)$, $\rho\sim N(2,1)$, $\gamma\sim N(4,1)$ *and* $\eta\sim N(1.5,1)$. The yield is the output of the model and the area and rainfall are inputs. The data are listed in table 1 and the efficiencies of DMUs with the proposed method for different α values are listed in Table 2.

	D_1	D_2	D_3	D_4
I_1	$\tilde{\omega}$	$\tilde{\rho}$	$\tilde{\gamma}$	$\tilde{\eta}$
I_2	5	5	4	7
O_1	$N(2.5,1)$	$N(3,1)$	$N(5,1)$	$N(3.5,1)$

TABLE 1. Data for Numerical Example

α	D_1	D_2	D_3	D_4
0.00	*1*	*1*	*1*	*1*
0.25	*0.94*	*0.99*	*1*	*0.90*
0.50	*0.86*	*0.95*	*1*	*0.81*
0.75	*0.81*	*0.91*	*1*	*0.74*
1.00	*0.75*	*0.87*	*1*	*0.63*

TABLE 2. The Efficiencies by Proposed Method

As seen, the efficiencies are decreased by increasing α, but D_3 is efficient for all α. In case of $\alpha=1$, (6) is equvalent to the chance-constrained CCR model. Furthermore, DMU*s* are ranked as D_3, D_2, D_1 and D_4.

4. **Conclusion**

In this paper a CCR model is suggested for chance-constrained DEA with fuzzy random data. We assume that the fuzzy random variables are flat LR fuzzy numbers. For non-linear cases after α-cut, the relation (3) and (4) must be modified according to the distribution of the fuzzy number. We propose a method for converting this problem into a crisp chance-constrained DEA model based on α-cut and fuzzy probability measure. In this method the lower level of inputs and upper level of outputs are compared with the inner part of efficiency frontier. The illustrative example shows the applicability of the model. It is suggested that the efficiency of this algorithm be studied for larger problems.

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