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# SOME RESULTS OF INTUITIONISTIC FUZZY WEAK DUAL HYPER K-IDEALS

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Abstract. In this note we consider the notion of intuitionistic fuzzy (weak) dual hyper $K$-ideals and obtain related results. Then we classify this notion according to level sets. After that we determine the relationships between intuitionistic fuzzy (weak) dual hyper $K$-ideals and intuitionistic fuzzy (weak) hyper $K$-ideals. Finally, we define the notion of the product of two intuitionistic fuzzy (weak) dual hyper $K$-ideals and prove several Decomposition Theorems.

## 1. Introduction

Hyperalgebraic structure theory was introduced by F. Marty [7] in 1934. Imai and Iseki [5] introduced the notion of a BCK-algebra in 1966. Borzooei, Jun, Hasankhani and Zahedi et.al. [3] applied hyperstructures to BCK-algebras and introduced the concept of hyper $K$-algebras which are a generalization of BCK-algebras. The idea of "intuitionistic fuzzy set" was first introduced by Atanassov [1] as a generalization of fuzzy sets. In this note, we consider intuitionistic fuzzification of the notion of (weak) dual hyper $K$-ideals obtain related results.

## 2. Preliminaries

**Definition 2.1.** [3] Let $H$ be a nonempty set and "$\circ$" be a *hyperoperation* on $H$, i.e. "$\circ$" is a function from $H \times H$ to $P^*(H) = P(H) - \{\emptyset\}$. Then $(H, \circ, 0)$ is called a *hyper $K$-algebra* if it contains a constant "0" and satisfies the following axioms:
(HK1) $(x \circ z) \circ (y \circ z) < x \circ y$,
(HK2) $(x \circ y) \circ z = (x \circ z) \circ y$,
(HK3) $x < x$,
(HK4) $x < y,\ y < x \Rightarrow x = y$,
(HK5) $0 < x$.
for all $x, y, z \in H$, where the relation $x < y$ is defined by $0 \in x \circ y$. For every $A, B \subseteq H$, $A < B$ is defined by $\exists a \in A$, $\exists b \in B$ such that $a < b$.

Note that if $A, B \subseteq H$, then by $A \circ B$ we mean the subset $\bigcup_{a \in A, b \in B} a \circ b$ of $H$.

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**Theorem 2.2.** [3] *Let $(H, \circ, 0)$ be a hyper $K$-algebra. Then for all $x, y, z \in H$ and for all non-empty subsets $A$, $B$ and $C$ of $H$ the following statements hold:*
(i) $x \circ y < z \Leftrightarrow x \circ z < y$, (ii) $x \circ (x \circ y) < y$,
(iii) $x \circ y < x$, (iv) $x \in x \circ 0$,
(v) $A \circ B < A$

**Definition 2.3.** [3] Let $(H, \circ, 0)$ be a hyper $K$-algebra. If there exists an element $1 \in H$ such that $1 < x$ for all $x \in H$, then $H$ is called a *bounded hyper $K$-algebra* and 1 is said to be the unit of $H$. In a bounded hyper $K$-algebra, we denote $1 \circ x$ by $Nx$.

**Definition 2.4.** [10] Let $D$ be a nonempty subset of a hyper $K$-algebra $(H, \circ, 0)$ and $1 \in D$. Then,
(i) $D$ is called a *weak dual hyper $K$-ideal* of $H$, if $N(Nx \circ Ny) \subseteq D$ and $y \in D$ imply that $x \in D$.
(ii) $D$ is called a *dual hyper $K$-ideal* of $H$, if $N(Nx \circ Ny) \bigcap D \neq \emptyset$ and $y \in D$ imply that $x \in D$.

**Definition 2.5.** [3] Let $(H, \circ, 0)$ be a hyper $K$-algebra. An element $a \in H$ is called a *left(resp. right) scalar* if $|a \circ x| = 1$(resp. $|x \circ a| = 1$) for all $x \in H$.

**Theorem 2.6.** [11] *Let $(H, \circ, 0)$ be a bounded hyper $K$-algebra and $NNx = x$, for all $x \in H$. Then:*
(i) *1 is a left scalar,*
(ii) *0 is a right scalar,*
(iii) *If $x < y$, then $Ny < Nx$.*

**Theorem 2.7.** [10] *Let $(H, \circ, 0)$ be a bounded hyper $K$-algebra and let $NNx = x$, for all $x$ in $H$ and $\emptyset \neq D \subseteq H$. Then $D$ is a (weak) dual hyper $K$-ideal if and only if $ND$ is a (weak) hyper $K$-ideal.*

**Definition 2.8.** [3] Let $(H_1, \circ_1, 0_1)$ and $(H_2, \circ_2, 0_2)$ be two hyper $K$-algebras. Then a function $f : H_1 \longrightarrow H_2$ is called a *homomorphism* if $\forall x, y \in H$, $f(x \circ_1 y) = f(x) \circ_2 f(y)$ and $f(0_1) = 0_2$.

**Definition 2.9.** [13] Let $\mu$ be a fuzzy set of a nonempty set $H$ and $t \in [0, 1]$. Then the set

$$U(\mu; t) = \{x \in H | \ \ \mu(x) \geq t\}$$

$$(resp. \ \ L(\mu; t) = \{x \in H | \ \ \mu(x) \leq t\})$$

is called an *upper (resp. lower) level set* of $\mu$.

**Definition 2.10.** Let $\mu$ and $\nu$ be fuzzy sets of $X$ and $Y$, respectively. Then the fuzzy sets $\mu \times \nu$ and $\mu \otimes \nu$ of $X \times Y$, which are called *the product* and *anti-product* of $\mu$ and $\nu$, respectively, are defined by

$$(\mu \times \nu)(x, y) = \min\{\mu(x), \nu(y)\}$$

$$(\mu \otimes \nu)(x, y) = \max\{\mu(x), \nu(y)\}$$

**Definition 2.11.** [1] *An intuitionistic fuzzy set* (briefly, IFS) $A$ on a nonempty set $X$ is an object having the form

$$A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$$

where the function $\mu_A : X \to [0, 1]$ and $\gamma_A : X \to [0, 1]$ denote the degree of membership and the degree of nonmembership, respectively, and

$$0 \leq \mu_A(x) + \gamma_A(x) \leq 1$$

for all $x \in X$. An intuitionistic fuzzy set $A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$ on $X$ can be identified with an ordered pair $(\mu_A, \gamma_A)$ in $I^X \times I^X$. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \gamma_A)$ for the $IFS$ $A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$.

**Definition 2.12.** [12] Let $f : X \longrightarrow Y$ be a function. An intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ of $X$ is said to be *$f$-invariant*, if $f(x) = f(y)$ implies that $\mu_A(x) = \mu_A(y)$ and $\gamma_A(x) = \gamma_A(y)$ for all $x, y \in H$.

**Definition 2.13.** [12] Let $f : X \longrightarrow Y$ be a function and $A$ be an intuitionistic fuzzy set of $X$. Then the intuitionistic fuzzy set $f(A) = (f(\mu_A), f(\gamma_A))$ of $Y$ is defined by:

$$f(\mu_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$f(\gamma_A)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \gamma_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}.$$

**Definition 2.14.** Let $f : X \longrightarrow Y$ be a function and $B$ be a fuzzy set of $Y$. Then the fuzzy set $f^{-1}(B)$ of $X$ is defined by:

$$f^{-1}(B)(x) = B(f(x)).$$

**Definition 2.15.** An $IFS$ $A = (\mu_A, \gamma_A)$ of $H$ is said to satisfy the *sup-inf property* if for any subset $T$ of $H$ there exist $x_0, y_0 \in T$ such that $\mu_A(x_0) = \sup\limits_{x \in T} \mu_A(x)$ and $\gamma_A(y_0) = \inf\limits_{y \in T} \gamma_A(y)$.

**Definition 2.16.** [12] Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of $(H, \circ, 0)$. Then,
(i) $A$ is called an *intuitionistic fuzzy week hyper $K$-ideal* of $H$ if

$$\mu_A(0) \geq \mu_A(x) \geq \min(\inf_{a \in x \circ y} \mu_A(a), \mu_A(y))$$

and

$$\gamma_A(0) \leq \gamma_A(x) \leq \max(\sup_{b \in x \circ y} \gamma_A(b), \gamma_A(y))$$

for all $x, y \in H$.
(ii) $A$ is called an *intuitionistic fuzzy hyper $K$-ideal* of $H$ if

$$\mu_A(0) \geq \mu_A(x) \geq \min(\sup_{a \in x \circ y} \mu_A(a), \mu_A(y))$$

and

$$\gamma_A(0) \leq \gamma_A(x) \leq \max(\inf_{b \in x \circ y} \gamma_A(b), \gamma_A(y))$$

for all $x, y \in H$.

**Theorem 2.17.** [12] *Let $A = (\mu_A, \gamma_A)$ be an $IFS$ of $(H, \circ, 0)$ which satisfy the sup-inf property. Then $A$ is an intuitionistic fuzzy hyper $K$-ideal if and only if for all $s, t \in [0, 1]$ the nonempty level sets $U(\mu_A; t)$ and $L(\gamma_A; s)$ are hyper $K$-ideals of $H$.*

**Theorem 2.18.** [12] *Let $A = (\mu_A, \gamma_A)$ be an $IFS$ of $(H, \circ, 0)$. Then $A$ is an intuitionistic fuzzy weak hyper $K$-ideal if and only if for all $s, t \in [0, 1]$ the nonempty level sets $U(\mu_A; t)$ and $L(\gamma_A; s)$ are weak hyper $K$-ideals of $H$.*

**Definition 2.19.** [12] Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of $(H, \circ, 0)$. Then,
(i) $A$ satisfies the *additive condition*, whenever for all $x, y \in H$, $x < y$ implies that $\mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$.
(ii) $A$ satisfies the *anti-additive condition* if $x < y$ implies that $\mu_A(x) \leq \mu_A(y)$ and $\gamma_A(x) \geq \gamma_A(y)$.

## 3. Intuitionistic Fuzzy (Weak) Dual Hyper $K$-ideals

In what follows let $H$ denote a bounded hyper $K$-algebra.

**Definition 3.1.** An $IFS$ $A = (\mu_A, \gamma_A)$ of $H$ is called an *intuitionistic fuzzy weak dual hyper $K$-ideal* of $H$ if it satisfies the following conditions:

$$\mu_A(1) \geq \mu_A(x) \geq \min(\inf_{a \in N(Nx \circ Ny)} \mu_A(a), \mu_A(y))$$

and

$$\gamma_A(1) \leq \gamma_A(x) \leq \max(\sup_{b \in N(Nx \circ Ny)} \gamma_A(b), \gamma_A(y))$$

for all $x, y \in H$.

**Example 3.2.** Let $H = \{0, 1, 2\}$ be a hyper $K$-algebra with the following table.

| $\circ$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | $\{0\}$ | $\{0,1,2\}$ | $\{0,1,2\}$ |
| 1 | $\{1\}$ | $\{0\}$ | $\{2\}$ |
| 2 | $\{2\}$ | $\{0,1,2\}$ | $\{0,1,2\}$ |

Define the $IFS$ $A = (\mu_A, \gamma_A)$ on $H$ as follows:

$$\mu_A(2) = 0.2,\ \mu_A(0) = \mu_A(1) = 0.5,\ \gamma_A(2) = 0.6,\ \gamma_A(0) = \gamma_A(1) = 0.3$$

Then $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy weak dual hyper $K$-ideal.

**Definition 3.3.** An $IFS$ $A = (\mu_A, \gamma_A)$ of $H$ is called an *intuitionistic fuzzy dual hyper $K$-ideal* of $H$ if it satisfies the following conditions:

$$\mu_A(1) \geq \mu_A(x) \geq \min(\sup_{a \in N(Nx \circ Ny)} \mu_A(a), \mu_A(y))$$

and

$$\gamma_A(1) \leq \gamma_A(x) \leq \max(\inf_{b\in N(Nx\circ Ny)} \gamma_A(b), \gamma_A(y))$$

for all $x, y \in H$.

**Example 3.4.** The following table shows a hyper $K$-algebra structure on $H = \{0, 1, 2, 3\}$.

| $\circ$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | $\{0\}$ | $\{0\}$ | $\{0\}$ | $\{0\}$ |
| 1 | $\{1\}$ | $\{0\}$ | $\{3\}$ | $\{1, 2\}$ |
| 2 | $\{2\}$ | $\{0\}$ | $\{0\}$ | $\{2\}$ |
| 3 | $\{3\}$ | $\{0\}$ | $\{1, 2, 3\}$ | $\{0, 3\}$ |

Define the $IFS$ $A = (\mu_A, \gamma_A)$ on $H$ as follows:

$\mu_A(0) = \mu_A(3) = 0.2,\ \mu_A(1) = \mu_A(2) = 0.5,\ \gamma_A(0) = \gamma_A(3) = 0.6,\ \gamma_A(1) = \gamma_A(2) = 0.4$

Then $A$ is an intuitionistic fuzzy dual hyper $K$-ideal.

**Theorem 3.5.** *Every intuitionistic fuzzy dual hyper $K$-ideal is an intuitionistic fuzzy weak dual hyper $K$-ideal.*

*Proof.* Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy dual hyper $K$-ideal. Then for all $x, y \in H$

$$\mu_A(1) \geq \mu_A(x) \geq \min(\sup_{a\in N(Nx\circ Ny)} \mu_A(a), \mu_A(y))$$

and

$$\gamma_A(1) \leq \gamma_A(x) \leq \max(\inf_{b\in N(Nx\circ Ny)} \gamma_A(b), \gamma_A(y)).$$

Since $\sup_{a\in N(Nx\circ Ny)} \mu_A(a) \geq \inf_{a\in N(Nx\circ Ny)} \mu_A(a)$ and $\inf_{b\in N(Nx\circ Ny)} \gamma_A(b) \leq \sup_{b\in N(Nx\circ Ny)} \gamma_A(b)$,

hence

$$\mu_A(1) \geq \mu_A(x) \geq \min(\inf_{a\in N(Nx\circ Ny)} \mu_A(a), \mu_A(y))$$

and

$$\gamma_A(1) \leq \gamma_A(x) \leq \max(\sup_{b\in N(Nx\circ Ny)} \gamma_A(b), \gamma_A(y))$$

for all $x, y \in H$. It follows that $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy weak dual hyper $K$-ideal. □

Example 3.2 shows that the converse of the above theorem is not true in general.

**Theorem 3.6.** *Let $A = (\mu_A, \gamma_A)$ be an $IFS$ of $H$ which satisfies the sup-inf property. Then $A$ is an intuitionistic fuzzy dual hyper $K$-ideal if and only if for all $s, t \in [0, 1]$ the nonempty level sets $U(\mu_A; t)$ and $L(\gamma_A; s)$ are dual hyper $K$-ideals of $H$.*

*Proof.* Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy dual hyper $K$-ideal and $U(\mu_A; t) \neq \emptyset \neq L(\gamma_A; s)$. By Definition 3.3, it is clear that $1 \in U(\mu_A; t) \bigcap L(\gamma_A; s)$. Let $N(Nx \circ Ny) \bigcap U(\mu_A; t) \neq \emptyset$ and $y \in U(\mu_A; t)$. Then $\mu_A(y) \geq t$ and there exists $r \in N(Nx \circ Ny)$ such that $\mu_A(r) \geq t$. Thus

$$\mu_A(x) \geq \min( \sup_{a \in N(Nx \circ Ny)} \mu_A(a), \mu_A(y)) \geq \min(\mu_A(r), \mu_A(y)) \geq t$$

and so $x \in U(\mu_A; t)$. Now let $N(Nx \circ Ny) \bigcap L(\gamma_A; s) \neq \emptyset$ and $y \in L(\gamma_A; s)$. Then by a similar argument we can get that $x \in L(\gamma_A; s)$.
Conversely, let for all $s, t \in [0, 1]$ the nonempty level sets $U(\mu_A; t)$ and $L(\gamma_A; s)$ be dual hyper $K$-ideals. Let $x, y \in H$, $\mu_A(x) = t$ and $\gamma_A(y) = s$. Since $x \in U(\mu_A; t)$ and $y \in L(\gamma_A; s)$, hence by hypothesis we get that $1 \in U(\mu_A; t) \bigcap L(\gamma_A; s)$. It follows that $\mu_A(1) \geq \mu_A(x)$ and $\gamma_A(1) \leq \gamma_A(y)$. Now let $k = \min( \sup_{a \in N(Nx \circ Ny)} \mu_A(a), \mu_A(y))$ and $h = \max( \inf_{b \in N(Nx \circ Ny)} \gamma_A(b), \gamma_A(y))$. Since $A = (\mu_A, \gamma_A)$ satisfies the sup-inf property, there exist $x_0, y_0 \in N(Nx \circ Ny)$ such that

$$\mu_A(x_0) = \sup_{a \in N(Nx \circ Ny)} \mu_A(a) \geq \min( \sup_{a \in N(Nx \circ Ny)} \mu_A(a), \mu_A(y)) = k$$

and

$$\gamma_A(y_0) = \inf_{b \in N(Nx \circ Ny)} \gamma_A(b) \leq \max( \inf_{b \in N(Nx \circ Ny)} \gamma_A(b), \gamma_A(y)) = h$$

, we have $x_0 \in U(\mu_A; k)$ and $y_0 \in L(\gamma_A; h)$. Since $x_0 \in U(\mu_A; k) \bigcap N(Nx \circ Ny)$ and $y_0 \in L(\gamma_A; h) \bigcap N(Nx \circ Ny)$, then $N(Nx \circ Ny) \bigcap U(\mu_A; k) \neq \emptyset$ and $N(Nx \circ Ny) \bigcap L(\gamma_A; h) \neq \emptyset$. Also $\mu_A(y) \geq k$ and $\gamma_A(y) \leq h$ imply that $y \in U(\mu_A; k) \bigcap L(\gamma_A; h)$. Thus, by hypothesis, we get that $x \in U(\mu_A; k) \bigcap L(\gamma_A; h)$. So for all $x, y \in H$

$$\mu_A(x) \geq k = \min( \sup_{a \in N(Nx \circ Ny)} \mu_A(a), \mu_A(y))$$

and

$$\gamma_A(x) \leq h = \max( \inf_{b \in N(Nx \circ Ny)} \gamma(b), \gamma(y))$$

for all $x, y \in H$. Therefore $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy dual hyper $K$-ideal. □

**Theorem 3.7.** *Let $A = (\mu_A, \gamma_A)$ be an $IFS$ of $H$. Then $A$ is an intuitionistic fuzzy weak dual hyper $K$-ideal if and only if for all $s, t \in [0, 1]$ the nonempty level sets $U(\mu_A; t)$ and $L(\gamma_A; s)$ are weak dual hyper $K$-ideals of $H$.*

*Proof.* The proof is similar to the proof of Theorem 3.6. □

**Theorem 3.8.** *Let $NNx = x$ for all $x \in H$ and $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy dual hyper $K$-ideal. Then $A$ satisfies the anti-additive condition.*

*Proof.* Let $x < y$. Then $Ny < Nx$, by Theorem 2.6 and so $0 \in Ny \circ Nx$. Thus $\sup_{a \in N(Ny \circ Nx)} \mu_A(a) \geq \mu_A(1)$ and $\inf_{b \in N(Ny \circ Nx)} \gamma_A(b) \leq \gamma_A(1)$. Hence, by hypothesis, we get that

$$\mu_A(y) \geq \min(\sup_{a \in N(Ny \circ Nx)} \mu_A(a), \mu_A(x)) \geq \min(\mu_A(1), \mu_A(x)) = \mu_A(x)$$

and

$$\gamma_A(y) \leq \max(\inf_{b \in N(Ny \circ Nx)} \gamma_A(b), \gamma_A(x)) \leq \max(\gamma_A(1), \gamma_A(x)) = \gamma_A(x).$$ □

The following example shows that the above theorem is not true for intuitionistic fuzzy weak dual hyper $K$-ideals.

**Example 3.9.** In Example 3.2, $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy weak dual hyper $K$-ideal, $NNx = x$ for all $x \in H$ and $0 < 2$ while $0.5 = \mu_A(0) \not\leq \mu_A(2) = 0.2$ and $0.3 = \gamma_A(0) \not\geq \gamma_A(2) = 0.6$.

**Theorem 3.10.** *If $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy (weak) dual hyper $K$-ideal, then the set*

$$H_A = \{x \in H | \mu_A(x) = \mu_A(1) \quad , \quad \gamma_A(x) = \gamma_A(1)\}$$

*is a (weak) dual hyper $K$-ideal.*

*Proof.* Obviously $1 \in H_A$. Let $N(Nx \circ Ny) \bigcap H_A \neq \emptyset$ and $y \in H_A$. Then $\mu_A(y) = \mu_A(1)$, $\gamma_A(y) = \gamma_A(1)$ and there exists $s \in N(Nx \circ Ny)$ such that $\mu_A(s) = \mu_A(1)$ and $\gamma_A(s) = \gamma_A(1)$. So, by hypothesis, we have

$$\mu_A(1) \geq \mu_A(x) \geq \min(\sup_{a \in N(Nx \circ Ny)} \mu_A(a), \mu_A(y)) \geq \min(\mu_A(s), \mu_A(y)) = \mu_A(1)$$

and

$$\gamma_A(1) \leq \gamma_A(x) \leq \max(\inf_{b \in N(Nx \circ Ny)} \gamma_A(b), \gamma_A(y)) \leq \max(\gamma_A(s), \gamma_A(y)) = \gamma_A(1).$$

Therefore $\mu_A(x) = \mu_A(1)$ and $\gamma_A(x) = \gamma_A(1)$ i.e., $x \in H_A$. □

**Theorem 3.11.** *Let $1 \in 1 \circ x$ for all $x \in H$ and $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy dual hyper $K$-ideal. Then $H_A = H$.*

*Proof.* By hypothesis we get that

$$\mu_A(1) \geq \mu_A(x) \geq \min(\sup_{a \in N(Nx \circ N1)} \mu_A(a), \mu_A(1)) \geq \min(\mu_A(1), \mu_A(1)) = \mu_A(1)$$

and

$$\gamma_A(1) \leq \gamma_A(x) \leq \max(\inf_{b \in N(Nx \circ N1)} \gamma_A(b), \gamma_A(1)) \leq \max(\gamma_A(1), \gamma_A(1)) = \gamma_A(1)$$

for all $x \in H$. Therefore $\mu_A(x) = \mu_A(1)$ and $\gamma_A(x) = \gamma_A(1)$, for all $x \in H$, i.e. $H_A = H$. □

**Theorem 3.12.** *Let $NNx = x$ for all $x \in H$. Then $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy weak dual hyper $K$-ideal if and only if $A^N = (\mu_A^N, \gamma_A^N)$ is an intuitionistic fuzzy weak hyper $K$-ideal, where for all $x \in H$, $\mu_A^N(x) = \mu_A(Nx)$ and $\gamma_A^N(x) = \gamma_A(Nx)$.*

*Proof.* Since $NNx = x$, for all $x \in H$, hence, by Theorem 2.6 (i), $A^N = (\mu_A^N, \gamma_A^N)$ is an intuitionistic fuzzy set.
Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy weak dual hyper $K$-ideal and $U(\mu_A^N; t) \neq \emptyset \neq L(\gamma_A^N; s)$ for $s, t \in [0, 1]$. Now we prove that $NU(\mu_A; t) = U(\mu_A^N; t)$ and $NL(\mu_A; s) = L(\mu_A^N; s)$. Let $x \in NU(\mu_A; t)$. Then $\exists h \in U(\mu_A; t)$ such that $x \in 1 \circ h$. So $\mu_A(h) \geq t$ and $1 \circ x \subseteq 1 \circ (1 \circ h) = h$. Thus $\mu_A^N(x) = \mu_A(Nx) \geq t$. Therefore $x \in U(\mu_A^N; t)$ i.e., $NU(\mu_A; t) \subseteq U(\mu_A^N; t)$.
Now let $x \in U(\mu_A^N; t)$. Then $\mu_A(Nx) = \mu_A^N(x) \geq t$ . Hence by Theorem 2.6 there exists $h \in H$ such that $Nx = h$. Thus $\mu_A(h) \geq t$ and so $h \in U(\mu_A; t)$. Since $Nh = NNx = x$, hence $x = Nh \subseteq NU(\mu_A; t)$. Therefore $U(\mu_A^N; t) \subseteq NU(\mu_A; t)$. Similarly we prove that $NL(\mu_A; s) = L(\mu_A^N; s)$. Since $U(\mu_A^N; t)$ and $L(\gamma_A^N; s)$ are nonempty, so are $U(\mu_A; t)$ and $L(\gamma_A; s)$ are nonempty too. Thus by Theorem 3.7 $U(\mu_A; t)$ and $L(\gamma_A; s)$ are weak dual hyper $K$-ideals and so, by Theorem 2.7, $NU(\mu_A; t)$ and $NL(\gamma_A; s)$ are weak hyper $K$-ideals. Hence $U(\mu_A^N; t)$ and $L(\gamma_A^N; s)$ are also weak hyper $K$-ideals. Therefore, by Theorem 2.18, $A^N = (\mu_A^N, \gamma_A^N)$ is an intuitionistic fuzzy weak hyper $K$-ideal. The proof of the converse is similar to above if we invoke Theorems 2.7, 3.7 and 2.18. □

**Theorem 3.13.** *Let $NNx = x$ for all $x \in H$ and suppose $A = (\mu_A, \gamma_A)$ satisfies the sup-inf property. Then $A$ is an intuitionistic fuzzy dual hyper $K$-ideal if and only if $A^N = (\mu_A^N, \gamma_A^N)$ is an intuitionistic fuzzy hyper $K$-ideal.*

*Proof.* The proof is similar to the proof of Theorem 3.12. □

**Theorem 3.14.** *Let $NNx = x$ for all $x \in H$. Then $A = (\mu_A, \gamma_A)$ satisfies the additive condition if and only if $A^N = (\mu_A^N, \gamma_A^N)$ satisfies the anti-additive condition.*

*Proof.* Let $A$ satisfy the additive condition and $x < y$. Then by Theorem 2.6 $Ny < Nx$ and so $\mu_A(Nx) \leq \mu_A(Ny)$ and $\gamma_A(Nx) \geq \gamma_A(Ny)$. Thus $\mu_A^N(x) \leq \mu_A^N(y)$ and $\gamma_A^N(x) \geq \gamma_A^N(y)$. Therefore $A^N = (\mu_A^N, \gamma_A^N)$ satisfies the anti-additive condition. The proof of the converse is similar. □

## 4. Decomposition of Intuitionistic Fuzzy (Weak) Dual Hyper $K$-ideals

**Definition 4.1.** Let $A$ be a fuzzy set of $H$ and $A(1) \geq A(x)$ for all $x \in H$. Then
(i) $A$ is called a *fuzzy dual hyper $K$-ideal*, if

$$A(x) \geq \min(\sup_{a \in N(Nx \circ Ny)} A(a), A(y)),$$

(ii) $A$ is called a *fuzzy weak dual hyper $K$-ideal*, if

$$A(x) \geq \min(\inf_{b \in N(Nx \circ Ny)} A(b), A(y)).$$

**Theorem 4.2.** *Let $D$ be a nonempty subset of $H$. Then*
*(i) $D$ is a dual hyper $K$-ideal if and only if $\chi_D$ is a fuzzy dual hyper $K$-ideal.*
*(ii) $D$ is a (weak) dual hyper $K$-ideal if and only if $\chi_D$ is a fuzzy (weak) dual hyper $K$-ideal.*

*Proof.* The proof is easy. □

**Theorem 4.3.** *Every (weak) dual hyper $K$-ideal of a bounded hyper $K$-algebra $H$ is a level set of a fuzzy (weak) dual hyper $K$-ideal.*

*Proof.* Let $D$ be a dual hyper $K$-ideal of $H$ and $A$ be a fuzzy set on $H$ defined by

$$A(x) = \begin{cases} \alpha & \text{if } x \in D \\ 0 & \text{otherwise} \end{cases}$$

where $\alpha \in [0,1]$. It is clear that $U(A,\alpha) = D$. Now we show that $A$ is a fuzzy dual hyper $K$-ideal. If $N(Nx \circ Ny) \bigcap D = \emptyset$ or $y \notin D$, then $A(x) \geq \min(\sup_{a \in N(Nx \circ Ny)} A(a), A(y)) = 0$. If $N(Nx \circ Ny) \bigcap D \neq \emptyset$ and $y \in D$, then by hypothesis we have $x \in D$. Thus $\alpha = A(x) = \min(\sup_{a \in N(Nx \circ Ny)} A(a), A(y)) = \alpha$. Therefore $A$ is a fuzzy dual hyper $K$-ideal. □

**Theorem 4.4.** *An $IFS$ $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy (weak) dual hyper $K$-ideal if and only if the fuzzy sets $\mu_A$ and $\bar{\gamma}_A = 1 - \gamma_A$ are fuzzy (weak) dual hyper $K$-ideals.*

*Proof.* Assume that $A$ is an intuitionistic fuzzy dual hyper $K$-ideal. Obviously $\mu_A$ is a fuzzy dual hyper $K$-ideal. By hypothesis we have $\gamma_A(1) \leq \gamma_A(x) \leq \max(\inf_{b \in N(Nx \circ Ny)} \gamma_A(b), \gamma_A(y))$ for all $x, y \in H$. Thus,

$$1 - \gamma_A(1) \geq 1 - \gamma_A(x) \geq 1 - \max(\inf_{b \in N(Nx \circ Ny)} \gamma_A(b), \gamma_A(y))$$

$$\begin{aligned} \bar{\gamma}_A(1) \geq \bar{\gamma}_A(x) &\geq 1 + \min(\sup_{b \in N(Nx \circ Ny)} -\gamma_A(b), -\gamma_A(y)) \\ &= \min(\sup_{b \in N(Nx \circ Ny)} 1 - \gamma_A(b), 1 - \gamma_A(y)) \\ &= \min(\sup_{b \in N(Nx \circ Ny)} \bar{\gamma}_A(b), \bar{\gamma}_A(y)) \end{aligned}$$

Therefore $\bar{\gamma}_A$ is a fuzzy dual hyper $K$-ideal.

Conversely, let $\mu_A$ and $\bar{\gamma}_A$ be fuzzy dual hyper $K$-ideals. Then for all $x, y \in H$,

$$\mu_A(1) \geq \mu_A(x) \geq \min(\sup_{a \in N(Nx \circ Ny)} \mu_A(a), \mu_A(y))$$

and

$$\bar{\gamma}_A(1) \geq \bar{\gamma}_A(x) \geq \min(\sup_{b \in N(Nx \circ Ny)} \bar{\gamma}_A(b), \bar{\gamma}_A(y)).$$

Thus,

$$\begin{aligned} 1-\gamma_A(1) \geq 1-\gamma_A(x) &\geq \min(\sup_{b\in N(Nx\circ Ny)} 1-\gamma_A(b), 1-\gamma_A(y)) \\ &= 1-\max(\inf_{b\in N(Nx\circ Ny)} \gamma_A(b), \gamma_A(y)) \end{aligned}$$

So $\gamma_A(1) \leq \gamma_A(x) \leq \max(\inf_{b\in N(Nx\circ Ny)} \gamma_A(b), \gamma_A(y))$ for all $x, y \in H$. Therefore $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy dual hyper $K$-ideal. □

**Theorem 4.5.** *Let $A = (\mu_A, \gamma_A)$ be an $IFS$ of $H$. Then $A$ is an intuitionistic fuzzy (weak) dual hyper $K$-ideal if and only if $A_\mu = (\mu_A, \bar{\mu}_A)$ and $A_\gamma = (\bar{\gamma}_A, \gamma_A)$ are intuitionistic fuzzy (weak) dual hyper $K$-ideals, where $\bar{\gamma}_A = 1-\gamma_A$ and $\bar{\mu}_A$ is the same as $\bar{\gamma}_A$.*

*Proof.* The proof follows from Theorem 4.4. □

**Theorem 4.6.** *Let $(H_1, \circ_1, 0_1)$ and $(H_2, \circ_2, 0_2)$ be bounded hyper $K$-algebras, and let $1_{H_1} \circ_1 1_{H_1} = 0_1$, $1_{H_2} \circ_2 1_{H_2} = 0_2$, and $\mu$ and $\nu$ be fuzzy sets of $H_1$ and $H_2$, respectively. If $\mu(1_{H_1}) = \nu(1_{H_2})$, $\mu(1) \geq \mu(x)$ and $\nu(1) \geq \nu(y)$, $\forall (x, y) \in H_1 \times H_2$, then $\mu \times \nu$ is a fuzzy (weak) dual hyper $K$-ideal of $H_1 \times H_2$ if and only if $\mu$ and $\nu$ are fuzzy (weak) dual hyper $K$-ideals of $H_1$ and $H_2$, respectively.*

*Proof.* Let $\mu$ and $\nu$ be fuzzy dual hyper $K$-ideals of $H_1$ and $H_2$, respectively. Then for all $(x, y) \in H_1 \times H_2$:

$$(\mu\times\nu)(1_{H_1}, 1_{H_2}) = \min\{\mu(1_{H_1}), \nu(1_{H_2})\} \geq \min\{\mu(x), \nu(y)\} = (\mu\times\nu)(x, y).$$

Let $(x_1, y_1)$, $(x_2, y_2) \in H_1 \times H_2$. Then

$$\begin{aligned} &\min(\sup_{(a,b)\in N(N(x_1,y_1)\circ N(x_2,y_2))} (\mu\times\nu)(a,b), (\mu\times\nu)(x_2,y_2)) \\ &= \min(\sup_{\substack{a\in N_1(N_1x_1\circ_1 N_1x_2) \\ b\in N_2(N_2y_1\circ_2 N_2y_2)}} \min\{\mu(a), \nu(b)\}, \min\{\mu(x_2), \nu(y_2)\}) \\ &\leq \min(\min\{\sup_{a\in N_1(N_1x_1\circ_1 N_1x_2)} \mu(a), \mu(x_2)\}, \min\{\sup_{b\in N_2(N_2y_1\circ_2 N_2y_2)} \nu(b), \nu(y_2)\}) \\ &\leq \min(\mu(x_1), \nu(y_1)) = (\mu\times\nu)(x_1, y_1). \end{aligned}$$

Therefore $\mu\times\nu$ is a fuzzy dual hyper $K$-ideal.
Conversely, let $\mu\times\nu$ be a fuzzy dual hyper $K$-ideal of $H_1\times H_2$ and $(x, y) \in H_1\times H_2$. Then, by hypothesis, we get that $\mu(1) \geq \mu(x)$ and $\nu(1) \geq \nu(y)$. Let $x, y \in H_1$. By

hypothesis and since $\nu(1) = \mu(1)$, then

$$\begin{aligned}
\mu(x) = \min(\mu(x), \mu(1)) &= \min(\mu(x), \nu(1)) = \mu \times \nu(x, 1) \\
&\geq \min(\sup_{(a,b)\in N(N(x,1)\circ N(y,1))} (\mu \times \nu)(a, b), (\mu \times \nu)(y, 1)) \\
&= \min(\sup_{\substack{a\in N_1(N_1x\circ_1 N_1y) \\ b\in N_2(N_2 1\circ_2 N_2 1)}} \min\{\mu(a), \nu(b)\}, \mu(y)) \\
&= \min(\sup_{a\in N_1(N_1x\circ_1 N_1y)} \min\{\mu(a), \nu(1)\}, \mu(y)) \\
&= \min(\sup_{a\in N_1(N_1x\circ_1 N_1y)} \mu(a), \mu(y))
\end{aligned}$$

Therefore $\mu$ is a fuzzy dual hyper $K$-ideal. Similarly we can prove that $\nu$ is also a fuzzy dual hyper $K$-ideal. □

**Theorem 4.7.** *Let $(H_1, \circ_1, 0_1)$ and $(H_2, \circ_2, 0_2)$ be bounded hyper $K$-algebras, $1_{H_1} \circ_1 1_{H_1} = 0_1$, $1_{H_2} \circ_2 1_{H_2} = 0_2$ and suppose that $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ are two intuitionistic fuzzy sets of $H_1$ and $H_2$, respectively. If $\mu_A(1_{H_1}) = \mu_B(1_{H_2})$, $\gamma_A(1_{H_1}) = \gamma_B(1_{H_2})$, $\mu_A(1_{H_1}) \geq \mu_A(x)$, $\mu_B(1_{H_2}) \geq \mu_B(y)$, $\gamma_A(1_{H_1}) \leq \gamma_A(x)$ and $\gamma_B(1_{H_2}) \leq \gamma_B(y)$ for all $(x, y) \in H_1 \times H_2$, then $A \times B = (\mu_A \times \mu_B, \gamma_A \otimes \gamma_B)$ is an intuitionistic fuzzy (weak) dual hyper $K$-ideal if and only if $A$ and $B$ are intuitionistic fuzzy (weak) dual hyper $K$-ideals, where $(\mu \otimes \nu)(x, y) = \max\{\mu(x), \nu(y)\}$.*

*Proof.* Let $A$ and $B$ be intuitionistic fuzzy dual hyper $K$-ideals. Then, by Theorem4.4, we have $\mu_A$, $\mu_B$, $\bar{\gamma}_A$ and $\bar{\gamma}_B$ are fuzzy dual hyper $K$-ideals . On the other hand, it is easy to check that $\overline{\gamma_A \otimes \gamma_B} = \bar{\gamma}_A \times \bar{\gamma}_B$. Hence, by Theorem 4.6, $\mu_A \times \mu_B$ and $\overline{\gamma_A \otimes \gamma_B}$ are fuzzy dual hyper $K$-ideal. So, by Theorem 4.4, $A \times B = (\mu_A \times \mu_B, \gamma_A \otimes \gamma_B)$ is an intuitionistic fuzzy dual hyper $K$-ideal.
The proof of the converse is similar. □

**Theorem 4.8.** *Let $(H_1, \circ_1, 0_1)$ and $(H_2, \circ_2, 0_2)$ be two bounded hyper $K$-algebras, $N_1N_1x = x$, $N_2N_2y = y$ and $y \circ_2 y = \{0_2\}$, for all $(x, y) \in H_1 \times H_2$. If $\mu$ is a fuzzy dual hyper $K$-ideal of $H_1 \times H_2$, then there are fuzzy dual hyper $K$-ideals $\mu_1$ and $\mu_2$ of $H_1$ and $H_2$, respectively, for which $\mu = \mu_1 \times \mu_2$.*

*Proof.* Define $\mu_1(x) = \mu(x, 1_{H_2})$ and $\mu_2(y) = \mu(1_{H_1}, y)$, $\forall (x, y) \in H_1 \times H_2$. Then, by a proof similar to that of Theorem 4.6, we can see that $\mu_1$ and $\mu_2$ are fuzzy dual hyper $K$-ideal. Now we show that $\mu = \mu_1 \times \mu_2$. By Theorem 3.8, $\mu$ satisfies the fuzzy anti-additive condition. Hence $(x, y) < (x, 1)$ and $(x, y) < (1, y)$ imply that $\mu(x, y) \leq \mu(x, 1) = \mu_1(x)$ and $\mu(x, y) \leq \mu(1, y) = \mu_2(y)$. Thus $\mu(x, y) \leq \min\{\mu_1(x), \mu_2(y)\} = \mu_1 \times \mu_2(x, y)$, for all $(x, y) \in H_1 \times H_2$.
Let $(x, y) \in H_1 \times H_2$. Then:

$$\begin{aligned}
\mu(x, y) &\geq \min(\sup_{(a,b)\in N(N(x,y)\circ N(1,y))} \mu(a, b), \mu(1, y)) \\
&= \min(\sup_{\substack{a\in N_1(N_1x\circ_1 N_1 1)=x \\ b\in N_2(N_2y\circ_2 N_2y)=1}} \mu(a, b), \mu(1, y)) = \min(\mu(x, 1), \mu(1, y)) \\
&= \min\{\mu_1(x), \mu_2(y)\} = \mu_1 \times \mu_2(x, y)
\end{aligned}$$

Therefore $\mu = \mu_1 \times \mu_2, \forall (x,y) \in H_1 \times H_2$. □

**Theorem 4.9.** *Let $(H_1, \circ_1, 0_1)$ and $(H_2, \circ_2, 0_2)$ be two bounded hyper $K$-algebras, $N_1N_1x = x$, $N_2N_2y = y$ and $y \circ_2 y = \{0_2\}$ for all $(x,y) \in H_1 \times H_2$ and suppose that $\mu$ is a fuzzy weak dual hyper $K$-ideal of $H_1 \times H_2$. If $\mu$ satisfies the anti-additive condition, then there exist fuzzy weak dual hyper $K$-ideals $\mu_1$ and $\mu_2$ of $H_1$ and $H_2$, respectively, in which $\mu = \mu_1 \times \mu_2$.*

*Proof.* The proof is similar to the proof of Theorem 4.8. □

**Theorem 4.10.** *(Decomposition Theorem 1) Let $(H_1, \circ_1, 0_1)$ and $(H_2, \circ_2, 0_2)$ be two bounded hyper $K$-algebras and $N_1N_1x = x$, $N_2N_2y = y$ and $y \circ_2 y = \{0_2\}$ for all $(x,y) \in H_1 \times H_2$. If $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy dual hyper $K$-ideal of $H_1 \times H_2$, then there exist intuitionistic fuzzy dual hyper $K$-ideals $A_1 = (\mu_{A_1}, \gamma_{A_1})$ and $A_2 = (\mu_{A_2}, \gamma_{A_2})$ of $H_1$ and $H_2$, respectively, in which $A \times B = (\mu_{A_1} \times \mu_{A_2}, \gamma_{A_1} \otimes \gamma_{A_2})$.*

*Proof.* The proof follows from Theorems 4.8 and 4.4. □

**Theorem 4.11.** *(Decomposition Theorem 2) Let $(H_1, \circ_1, 0_1)$ and $(H_2, \circ_2, 0_2)$ be two bounded hyper $K$-algebras, $N_1N_1x = x$, $N_2N_2y = y$ and $y \circ_2 y = \{0_2\}$ for all $(x,y) \in H_1 \times H_2$. Suppose $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy weak dual hyper $K$-ideal. If $A$ satisfies the anti-additive condition of $H_1 \times H_2$, then there exist intuitionistic fuzzy weak dual hyper $K$-ideals $A_1 = (\mu_{A_1}, \gamma_{A_1})$ and $A_2 = (\mu_{A_2}, \gamma_{A_2})$ of $H_1$ and $H_2$, respectively, for which $A \times B = (\mu_{A_1} \times \mu_{A_2}, \gamma_{A_1} \otimes \gamma_{A_2})$*

*Proof.* The proof follows from Theorems 4.9 and 4.4. □

**Theorem 4.12.** *Let $(H, \circ, 0)$ and $(H', \circ', 0')$ be two bounded hyper $K$-algebras and let $f : H \to H'$ be an onto homomorphism. Then :*
(i) $f(1) = 1$,
(ii) $N(f(x)) = f(Nx)$.

*Proof.* It is clear that $f(1) < 1$. Now since $f$ is onto, then there exists $x \in H$ such that $f(x) = 1$. Thus $0 = f(0) \in f(x \circ 1) = f(x) \circ f(1) = 1 \circ f(1)$, i.e. $1 < f(1)$. Therefore $f(1) = 1$.
(ii) by (i) we have

$$N(f(x)) = 1 \circ f(x) = f(1) \circ f(x) = f(1 \circ x) = f(Nx)$$

□

**Theorem 4.13.** *Let $(H_1, \circ_1, 0_1)$ and $(H_2, \circ_2, 0_2)$ be two bounded hyper $K$-algebras and $f : H_1 \to H_2$ be an onto homomorphism . Then $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy (weak) dual hyper $K$-ideal of $H_2$ if and only if $f^{-1}(A) = (f^{-1}(\mu_A), f^{-1}(\gamma_A))$ is an intuitionistic fuzzy (weak) dual hyper $K$-ideal of $H_1$.*

*Proof.* Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy dual hyper $K$-ideal. Then by hypothesis we have $\mu_A(y) \leq \mu_A(1)$ and $\gamma_A(1) \leq \gamma_A(y)$ for all $y \in H_2$. Hence

$f^{-1}(\mu_A)(x) = \mu_A(f(x)) \leq \mu_A(f(1)) = f^{-1}(\mu_A)(1)$ and $f^{-1}(\gamma_A)(1) = \gamma_A(f(1)) \leq \gamma_A(f(x)) = f^{-1}(\gamma_A)(x)$ for all $x \in H_1$. Let $x, y \in H_1$. Then,

$$\begin{aligned}\min(\sup_{a\in N_1(N_1x\circ_1 N_1y)} f^{-1}(\mu_A)(a) \quad &, \quad f^{-1}(\mu_A)(y)) \\ &= \min(\sup_{a\in N_1(N_1x\circ_1 N_1y)} \mu_A(f(a)), \mu_A(f(y))) \\ &= \min(\sup_{b\in N_2(N_2f(x)\circ_2 N_2f(y))} \mu_A(b), \mu_A(f(y))) \\ &\leq \mu_A(f(x)) = f^{-1}(\mu_A(x)\end{aligned}$$

Similarly we can prove that

$$f^{-1}(\gamma_A)(x) \leq \max(\inf_{b\in N_1(N_1x\circ_1 N_1y)} f^{-1}(\gamma_A)(b), f^{-1}(\gamma_A)(y))$$

for all $x, y \in H_1$. Therefore, $f^{-1}(A) = (f^{-1}(\mu_A), f^{-1}(\gamma_A))$ is an intuitionistic fuzzy dual hyper $K$-ideal.

Conversely, let $f^{-1}(A) = (f^{-1}(\mu_A), f^{-1}(\gamma_A))$ be an intuitionistic fuzzy dual hyper $K$-ideal. Since $f$ is onto, it is easy to check that $\mu_A(x) \leq \mu_A(1)$ and $\gamma_A(x) \geq \gamma_A(1)$ for all $x \in H_2$. Let $x, y \in H_2$. Then there exists $x^{'}, y^{'} \in H_1$ such that $f(x^{'}) = x$ and $f(y^{'}) = y$. Now by hypothesis we have:

$$\begin{aligned}\min(\sup_{b\in N_2(N_2x\circ_2 N_2y)} \mu_A(b), \mu_A(y)) &= \min(\sup_{b\in f(N_1(N_1x'\circ_1 N_1y'))} \mu_A(b), \mu_A(f(y^{'}))) \\ &= \min(\sup_{r\in N_1(N_1x'\circ_1 N_1y')} \mu_A(f(r)), \mu_A(f(y^{'}))) \\ &= \min(\sup_{r\in N_1(N_1x'\circ_1 N_1y')} f^{-1}(\mu_A)(r), f^{-1}(\mu_A)(y^{'})) \\ &\leq f^{-1}(\mu_A)(x^{'}) = \mu_A(f(x^{'})) = \mu_A(x)\end{aligned}$$

Similarly, $\gamma_A(x) \leq \max(\inf_{b\in N_2(N_2x\circ_2 N_2y)} \gamma_A(b), \gamma_A(y))$ for all $x, y \in H_2$. Therefore $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy dual hyper $K$-ideal of $H_2$. □

**Theorem 4.14.** *Let $f : H_1 \to H_2$ be an onto homomorphism of two bounded hyper $K$-algebras. If $A = (\mu_A, \gamma_A)$ is an $f$-invariant and an intuitionistic fuzzy (weak) dual hyper $K$-ideal of $H_1$, then $f(A) = (f(\mu_A), f(\gamma_A))$ is an intuitionistic fuzzy (weak) dual hyper $K$-ideal of $H_2$.*

*Proof.* By hypothesis and Theorem 4.12 we have $f(\mu_A)(1) = \mu_A(1)$ and $f(\gamma_A(1)) = \gamma_A(1)$ and so $f(\mu_A)(y) \leq f(\mu_A)(1)$ and $f(\gamma_A)(1) \leq f(\gamma_A)(y)$ for all $y \in H_2$.
Let $x, y \in H_2$. Then $f(x^{'}) = x$ and $f(y^{'}) = y$. Now by hypothesis we get that; (1)

$$\min(\sup_{a\in N_2(N_2x\circ_2 N_2y)} f(\mu_A)(a), f(\mu_A)(y)) = \min(\sup_{a\in f(N_1(N_1x'\circ_1 N_1y'))} f(\mu_A)(a), f(\mu_A)(y))$$

Consider the following sets

$T_1 = \{f(\mu_A)(a) | \ a \in f(N_1(N_1x^{'} \circ_1 N_1y^{'})\}$ , $T_2 = \{\mu_A(r) | \ r \in N_1(N_1x^{'} \circ_1 N_1y^{'})\}$.

Since $A$ is an $f$-invariant, we can see that $T_1 = T_2$. Also by hypothesis we get that $f(\mu_A)(y) = \sup_{t\in f^{-1}(y)} \mu_A(t) = \sup_{t\in f^{-1}(f(y'))} \mu_A(t) = \mu_A(y^{'})$. Thus (1) is equal to

$$\min(\sup_{r\in N_1(N_1x'\circ_1 N_1y')} \mu_A(r), \mu_A(y^{'})) \leq \mu_A(x^{'}) = f(\mu_A)(x).$$

Similarly

$$f(\gamma_A)(x) \leq \max(\inf_{b\in N_2(N_2x\circ_2 N_2y)} f(\gamma_A)(b), f(\gamma_A)(y))$$

for all $x, y \in H$. Therefore $f(A) = (f(\mu_A), f(\gamma_A))$ is an intuitionistic fuzzy dual hyper $K$-ideal. □

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