A NEW NOTION OF FUZZY PS-COMPACTNESS

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Abstract. In this paper, using pre-semi-open L-sets and their inequality, a new notion of PS-compactness is introduced in L-topological spaces, where L is a complete De Morgan algebra. This notion does not depend on the structure of the basis lattice L and L does not need any distributivity.

1. **Introduction**

It is known that compactness and its stronger and weaker forms play very important roles in topology. Based on fuzzy topological spaces introduced by Chang [4], various kinds of fuzzy compactness [2-4,7,11] have been established. However, these concepts of fuzzy compactness rely on the structure of L and L is required to be completely distributive. In [10], for a complete De Morgan algebra L, Shi introduced a new definition of fuzzy compactness in L-topological spaces using open L-sets and their inequality. This new definition doesn't depend on the structure of L.

In this paper, following the lines of [10], we introduce a new notion of PS-compactness in L-topological spaces by means of pre-semi-open L-sets and their inequality. This notion can also be characterized by pre-semi-closed L-sets and their inequality and is a strong form of semi-compactness[9]. This form of PS-compactness is a good generalization and has many characterizations when L is completely distributive De Morgan algebra.

2. **Preliminaries**

Throughout this paper, $(L, \vee, \wedge, ')$ is a complete De Morgan algebra, X a nonempty set and L^X the set of all L-fuzzy sets (or L-sets for short) on X. The smallest element and the largest element in L^X are denoted by $\underline{0}$ and $\underline{1}$. An element a in L is called a prime element if $b \wedge c \leq a$ implies that $b \leq a$ or $c \leq a$. a in L is called a co-prime element if a' is a prime element [6] The set of nonunit prime elements in L is denoted by $P(L)$ and the set of nonzero co-prime elements in L by $M(L)$.

The binary relation $\prec$ in L is defined as follows: for $a, b \in L, a \prec b$ iff for every subset $D \subseteq L$, the relation $b \leq supD$ always implies the existence of $d \in D$ with $a \leq d$ [5]. In a completely distributive De Morgan algebra L, each element b is a sup of $\{a \in L | a \prec b\}$. The set $\beta(b) = \{a \in L | a \prec b\}$ is called the greatest minimal family of b in the sense of [7,11]. Now, for $b \in L$, we define $\beta^*(b) = \beta(b) \cap M(L)$,

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$\alpha(b) = \{a \in L | a' \prec b'\}$ and $\alpha^*(b) = \alpha(b) \cap P(L)$. For $a \in L$ and $A \in L^X$, we write $A^{(a)} = \{x \in X | A(x) \not\leq a\}$ and $A_{(a)} = \{x \in X | a \in \beta(A(x))\}$ and for a subfamily $\psi \subseteq L^X, 2^{(\psi)}$ will denote the set of all finite subfamilies of ψ.

An L-topological space (or L-ts for short) is a pair (X, δ), where δ is a subfamily of L^X which contains $\underline{0}, \underline{1}$ and is closed for any suprema and finite infima. δ is called an L-topology on X. Each member of δ is called an open L-set and its quasi-complement is called a closed L-set.

Definition 2.1. [7,11] For a topological space (X, τ), let $\omega_L(\tau)$ denote the family of all the lower semi-continuous maps from (X, τ) to L; i.e. $\omega_L(\tau) = \{A \in L^X | A^{(a)} \in \tau, a \in L\}$. Then $\omega_L(\tau)$ is an L-topology on X and $(X, \omega_L(\tau))$ is topologically generated by (X, τ).

Definition 2.2. [7,11] An L-ts (X, δ) is weak induced if, for all $a \in L$ and for all $A \in \delta$, it follows that $A^{(a)} \in [\delta]$, where $[\delta]$ denotes the topology formed by all crisp sets in δ. It is obvious that $(X, \omega_L(\tau))$ is weak induced.

Definition 2.3. [9] Let (X, δ) be an L-ts, $a \in L \setminus \{1\}$, and $A \in L^X$. A family $\mu \subseteq L^X$ is called

(1) an a-shading of A if for any $x \in X$, $(A'(x) \vee \bigvee_{B \in \mu} B(x)) \not\leq a$.

(2) a strong a-shading (briefly S-a-shading) of A if $\bigwedge_{x \in X}(A'(x) \vee \bigvee_{B \in \mu} B(x)) \not\leq a$.

(3) an a-R-neighborhood family (briefly a-R-NF) of A if for any $x \in X$, $(A(x) \wedge \bigwedge_{B \in \mu} B(x)) \not\geq a$.

(4) a strong a-R-neighborhood family (briefly S-a-R-NF)of A if $\bigvee_{x \in X}(A(x) \wedge \bigwedge_{B \in \mu} B(x)) \not\geq a$.

It is obvious that an S-a-shading of A is an a-shading of A, an S-a-R-NF of A is an a-R-NF of A, and μ is an S-a-R-NF of A iff μ' is an S-a-shading of A.

Definition 2.4. [9] Let (X, δ) be an L-ts, $a \in L \setminus \{0\}$ and $A \in L^X$. A family $\mu \subseteq L^X$ is called

(1)a β_a-cover of A if for any $x \in X$, it follows that $a \in \beta(A'(x) \vee \bigvee_{B \in \mu} B(x))$.

(2) a strong β_a-cover (briefly S-β_a-cover) of A if $a \in \beta(\bigwedge_{x \in X}(A'(x) \vee \bigvee_{B \in \mu} B(x)))$.

(3) a Q_a-cover of A if for any $x \in X$, it follows that $A'(x) \vee \bigvee_{B \in \mu} B(x) \geq a$.

It is obvious that an S-β_a-cover of A must be a β_a-cover of A, and a β_a-cover of A must be a Q_a-cover of A.

Definition 2.5. [9] Let $a \in L \setminus \{0\}$ and $A \in L^X$. A family $\mu \subseteq L^X$ is said to have weak a-nonempty intersection in A if $\bigvee_{x \in X}(A(x) \wedge \bigwedge_{B \in \mu} B(x)) \geq a$. μ is said to have the finite weak a-intersection property in A if every finite subfamily ν of μ has weak a-nonempty intersection in A.

Lemma 2.6. [8] *Let L be a complete Heyting algebra, $f : X \to Y$ a map and $f_L^{\to} : L^X \to L^Y$ the extension of f. Then for any family $\psi \subseteq L^Y$,*

$$\bigvee_{y \in Y}(f_L^{\to}(A)(y) \wedge \bigwedge_{B \in \psi} B(y)) = \bigvee_{x \in X}(A(x) \wedge \bigwedge_{B \in \psi} f_L^{\to}(B)(x)).$$

Definition 2.7. [1] Let (X, δ) be an L-ts, $A \in L^X$. Then A is called a pre-semi-open set if $A \leq (A^-)_o$, and A is called a pre-semi-closed set if $A \geq (A^o)_-$, where A^o, A^-, A_o and A_- are the interior, closure, semi-interior and semi-closure of A, respectively.

Definition 2.8. [1,2] Let (X, δ) and (Y, τ) be two L-ts's. A map $f : (X, \delta) \to (Y, \tau)$ is called

(1) pre-semi-continuous if $f_L^{\leftarrow}(B)$ is pre-semi-open in (X, δ) for every $B \in \tau$.

(2) pre-semi-irresolute if $f_L^{\leftarrow}(B)$ is pre-semi-open in (X, δ) for every pre-semi-open L-set B in (Y, τ).

3. Definition and Properties of PS-compactness

Definition 3.1. Let (X, δ) be an L-ts. $A \in L^X$ is called PS-compact if for every family μ of pre-semi-open L-sets, it follows that

$$\bigwedge_{x\in X}(A'(x) \vee \bigvee_{B\in\mu} B(x)) \leq \bigvee_{\nu\in 2^{(\mu)}} \bigwedge_{x\in X}(A'(x) \vee \bigvee_{B\in\nu} B(x)).$$

(X, δ) is called PS-compact if $\underline{1}$ is PS-compact.

Example 3.2. Let $X = \{x\}$ and $L = \{0, 1/3, 2/3, 1\}$. For each $a \in L$ define $a' = 1 - a$. Let $\delta = \{\emptyset, A, X\}$, where $A(x) = 2/3$, then δ is a topology on X. Clearly, any L-set in (X, δ) is PS-compact.

Example 3.3. Let X be an infinite set or a singleton, A and C be two [0, 1]-sets on X defined as $A(x) = 0.5$, for all $x \in X$; $C(x) = 0.6$, for all $x \in X$. Take $\delta = \{\emptyset, A, X\}$, then δ is a topology on X. Obviously, any [0,1]-set in (X, δ) is pre-semi-open, and the set of all semi-open [0,1]-sets in (X, δ) is δ. In this case, we easily obtain that C is not PS-compact, and any [0,1]-set in (X, δ) is semi-compact.

Remark 3.4. Since every semi-open L-set is pre-semi-open[1], every PS-compact L-set is semi-compact. Example 3.3 shows that a semi-compact L-set needn't be PS-compact.

Theorem 3.5. *Let (X, δ) be an L-ts. $A \in L^X$ is PS-compact iff for every family μ of pre-semi-closed L-sets, it follows that*

$$\bigvee_{x\in X}(A(x) \wedge \bigwedge_{B\in\mu} B(x)) \geq \bigwedge_{\nu\in 2^{(\mu)}} \bigvee_{x\in X}(A(x) \wedge \bigwedge_{B\in\nu} B(x)).$$

Proof. This is immediate from Definition 3.1 and quasi-complements. □

Theorem 3.6. *Let (X, δ) be an L-ts and $A \in L^X$. Then the following conditions are equivalent.*

(1) A is PS-compact.

(2) For any $a \in L \setminus \{1\}$, each pre-semi-open S-a-shading μ of A has a finite subfamily which is an S-a-shading of A.

(3) For any $a \in L \setminus \{0\}$, each pre-semi-closed S-a-R-NF ψ of A has a finite subfamily which is an S-a-R-NF of A.

(4) For any $a \in L \setminus \{0\}$, each family of pre-semi-closed L-sets which has the finite weak a-intersection property in A has weak a-nonempty intersection in A.

Proof. The theorem follows immediately from Definition 3.1 and Theorem 3.5. □

Theorem 3.7. *Let L be a complete Heyting algebra. If both C and D are PS-compact, then $C \vee D$ is PS-compact.*

Proof. By Theorem 3.5 for any family μ of pre-semi-closed L-sets, we have

$$\begin{aligned}
&\bigvee_{x\in X} ((C \vee D)(x) \wedge \bigwedge_{B\in\mu} B(x)) \\
&= \{ \bigvee_{x\in X} (C(x) \wedge \bigwedge_{B\in\mu} B(x))\} \vee \{ \bigvee_{x\in X} (D(x) \wedge \bigwedge_{B\in\mu} B(x))\} \\
&\geq \{ \bigwedge_{\nu\in 2^{(\mu)}} \bigvee_{x\in X} (C(x) \wedge \bigwedge_{B\in\nu} B(x))\} \vee \{ \bigwedge_{\nu\in 2^{(\mu)}} \bigvee_{x\in X} (D(x) \wedge \bigwedge_{B\in\nu} B(x))\} \\
&= \bigwedge_{\nu\in 2^{(\mu)}} \bigvee_{x\in X} ((C \vee D)(x) \wedge \bigwedge_{B\in\nu} B(x)).
\end{aligned}$$

This shows that $C \vee D$ is PS-compact. □

Theorem 3.8. *Let (X, δ) be an L-ts and $C, D \in L^X$. If C is PS-compact and D is pre-semi-closed, then $C \wedge D$ is PS-compact.*

Proof. By Theorem 3.5, for any family μ of pre-semi-closed L-sets, we have

$$\begin{aligned}
&\bigvee_{x\in X} ((C \wedge D)(x) \wedge \bigwedge_{B\in\mu} B(x)) \\
&= \bigvee_{x\in X} (C(x) \wedge \bigwedge_{B\in\mu\cup\{D\}} B(x)) \\
&\geq \bigwedge_{\nu\in 2^{(\mu\cup\{D\})}} \bigvee_{x\in X} (C(x) \wedge \bigwedge_{B\in\nu} B(x)) \\
&= \{ \bigwedge_{\nu\in 2^{(\mu)}} \bigvee_{x\in X} (C(x) \wedge \bigwedge_{B\in\nu} B(x))\} \wedge \{ \bigwedge_{\nu\in 2^{(\mu)}} \bigvee_{x\in X} (C(x) \wedge D(x) \wedge \bigwedge_{B\in\nu} B(x))\} \\
&= \{ \bigwedge_{\nu\in 2^{(\mu)}} \bigvee_{x\in X} (C(x) \wedge D(x) \wedge \bigwedge_{B\in\nu} B(x))\} \\
&= \bigwedge_{\nu\in 2^{(\mu)}} \bigvee_{x\in X} ((C \wedge D)(x) \wedge \bigwedge_{B\in\nu} B(x)).
\end{aligned}$$

This shows that $C \wedge D$ is PS-compact. □

Corollary 3.9. *Let (X, δ) be PS-compact and $D \in L^X$ be pre-semi-closed. Then D is PS-compact.*

Definition 3.10. Let (X, δ) and (Y, τ) be two L-ts's. A map $f : (X, \delta) \to (Y, \tau)$ is called

(1) strongly pre-semi-continuous if $f_L^{\leftarrow}(B)$ is pre-semi-open in (X, δ) for every semi-open L-set B in (Y, τ).

(2) strongly pre-semi-irresolute if $f_L^{\leftarrow}(B)$ is semi-open in (X, δ) for every pre-semi-open L-set B in (Y, τ).

It is obvious that a strongly pre-semi-continuous map is pre-semi-continuous, and a strongly pre-semi-irresolute map is pre-semi-irresolute.

From Lemma 2.6 and Definitions 2.7, 2.8, 3.10, we can obtain the following theorems.

Theorem 3.11. *Let L be a complete Heyting algebra and $f : (X, \delta) \to (Y, \tau)$ be an pre-semi-irresolute map. If A is a PS-compact L-set in (X, δ), then $f_L^{\rightarrow}(A)$ is a PS-compact L-set in (Y, τ).*

Theorem 3.12. *Let L be a complete Heyting algebra and $f:(X,\delta)\to(Y,\tau)$ be a pre-semi-continuous map. If A is a PS-compact L-set in (X,δ), then $f_L^{\to}(A)$ is a compact L-set in (Y,τ).*

Theorem 3.13. *Let L be a complete Heyting algebra and $f:(X,\delta)\to(Y,\tau)$ be a strongly pre-semi-continuous map. If A is a PS-compact L-set in (X,δ), then $f_L^{\to}(A)$ is a semi-compact L-set in (Y,τ).*

Theorem 3.14. *Let L be a complete Heyting algebra and $f:(X,\delta)\to(Y,\tau)$ be a strongly pre-semi-irresolute map. If A is a semi-compact L-set in (X,δ), then $f_L^{\to}(A)$ is a PS-compact L-set in (Y,τ).*

4. Further Properties of PS-compactness and Goodness

In this section, we assume that L is a completely distributive de Morgan algebra.

Theorem 4.1. *Let (X,δ) be an L-ts and $A\in L^X$. Then the following conditions are equivalent.*

(1) A is PS-compact.

(2) For any $a\in L\setminus\{0\}$, each pre-semi-closed S-a-R-NF ψ of A has a finite subfamily which is an S-a-R-NF of A.

(3) For any $a\in L\setminus\{0\}$, each pre-semi-closed S-a-R-NF ψ of A has a finite subfamily which is an a-R-NF of A.

(4) For any $a\in L\setminus\{0\}$ and any pre-semi-closed S-a-R-NF ψ of A, there exist a finite subfamily φ of ψ and $b\in\beta(a)$ such that φ is an S-b-R-NF of A.

(5) For any $a\in L\setminus\{0\}$ and any pre-semi-closed S-a-R-NF ψ of A, there exist a finite subfamily φ of ψ and $b\in\beta(a)$ such that φ is a b-R-NF of A.

(6) For any $a\in M(L)$, each pre-semi-closed S-a-R-NF ψ of A has a finite subfamily which is an S-a-R-NF of A.

(7) For any $a\in M(L)$, each pre-semi-closed S-a-R-NF ψ of A has a finite subfamily which is an a-R-NF of A.

(8) For any $a\in M(L)$ and any pre-semi-closed S-a-R-NF ψ of A, there exist a finite subfamily φ of ψ and $b\in\beta^*(a)$ such that φ is an S-b-R-NF of A.

(9) For any $a\in M(L)$ and any pre-semi-closed S-a-R-NF ψ of A, there exist a finite subfamily φ of ψ and $b\in\beta^*(a)$ such that φ is a b-R-NF of A.

(10) For any $a\in L\setminus\{1\}$, each pre-semi-open S-a-shading μ of A has a finite subfamily which is an S-a-shading of A.

(11) For any $a\in L\setminus\{1\}$, each pre-semi-open S-a-shading μ of A has a finite subfamily which is an a-shading of A.

(12) For any $a\in L\setminus\{1\}$ and any pre-semi-open S-a-shading μ of A, there exist a finite subfamily ν of μ and $b\in\alpha(a)$ such that ν is an S-b-shading of A.

(13) For any $a\in L\setminus\{1\}$ and any pre-semi-open S-a-shading μ of A, there exist a finite subfamily ν of μ and $b\in\alpha(a)$ such that ν is a b-shading of A.

(14) For any $a\in P(L)$, each pre-semi-open S-a-shading μ of A has a finite subfamily which is an S-a-shading of A.

(15) For any $a\in P(L)$, each pre-semi-open S-a-shading μ of A has a finite subfamily which is an a-shading of A.

(16) For any $a \in P(L)$ and any pre-semi-open S-a-shading μ of A, there exist a finite subfamily ν of μ and $b \in \alpha^*(a)$ such that ν is an S-b-shading of A.

(17) For any $a \in P(L)$ and any pre-semi-open S-a-shading μ of A, there exist a finite subfamily ν of μ and $b \in \alpha^*(a)$ such that ν is a b-shading of A.

(18) For any $a \in L \setminus \{0\}$, each pre-semi-open S-β_a-cover μ of A has a finite subfamily which is an S-β_a-cover of A.

(19) For any $a \in L \setminus \{0\}$, each pre-semi-open S-β_a-cover μ of A has a finite subfamily which is a β_a-cover of A.

(20) For any $a \in L \setminus \{0\}$ and any pre-semi-open S-β_a-cover μ of A, there exist a finite subfamily ν of μ and $b \in L$ with $a \in \beta(b)$ such that ν is an S-β_b-cover of A.

(21) For any $a \in L \setminus \{0\}$ and any pre-semi-open S-β_a-cover μ of A, there exist a finite subfamily ν of μ and $b \in L$ with $a \in \beta(b)$ such that ν is a β_b-cover of A.

(22) For any $a \in M(L)$, each pre-semi-open S-β_a-cover μ of A has a finite subfamily which is an S-β_a-cover of A.

(23) For any $a \in M(L)$, each pre-semi-open S-β_a-cover μ of A has a finite subfamily which is a β_a-cover of A.

(24) For any $a \in M(L)$ and any pre-semi-open S-β_a-cover μ of A, there exist a finite subfamily ν of μ and $b \in M(L)$ with $a \in \beta^*(b)$ such that ν is an S-β_b-cover of A.

(25) For any $a \in M(L)$ and any pre-semi-open S-β_a-cover μ of A, there exist a finite subfamily ν of μ and $b \in M(L)$ with $a \in \beta^*(b)$ such that ν is a β_b-cover of A.

(26) For any $a \in L \setminus \{0\}$ and any $b \in \beta(a) \setminus \{0\}$, each pre-semi-open Q_a-cover μ of A has a finite subfamily which is a Q_b-cover of A.

(27) For any $a \in L \setminus \{0\}$ and any $b \in \beta(a) \setminus \{0\}$, each pre-semi-open Q_a-cover μ of A has a finite subfamily which is a β_b-cover of A.

(28) For any $a \in L \setminus \{0\}$ and any $b \in \beta(a) \setminus \{0\}$, each pre-semi-open Q_a-cover μ of A has a finite subfamily which is an S-β_b-cover of A.

(29) For any $a \in M(L)$ and any $b \in \beta^*(a)$, each pre-semi-open Q_a-cover μ of A has a finite subfamily which is a Q_b-cover of A.

(30) For any $a \in M(L)$ and any $b \in \beta^*(a)$, each pre-semi-open Q_a-cover μ of A has a finite subfamily which is a β_b-cover of A.

(31) For any $a \in M(L)$ and any $b \in \beta^*(a)$, each pre-semi-open Q_a-cover μ of A has a finite subfamily which is an S-β_b-cover of A.

Proof. (1) $\Leftrightarrow$ (2) : This follows directly from Theorem 3.6.

(2) $\Rightarrow$ (3) : This is easy to prove if one notices that every S-a-R-NF of A is an a-R-NF of A.

(3) $\Rightarrow$ (4) : Let $a \in L \setminus \{0\}$ and ψ be a pre-semi-closed S-a-R-NF of A. Then $\bigvee_{x\in X}(A(x) \wedge \bigwedge_{B\in\psi} B(x)) \not\geq a$. Take $c \in \beta(a)$ such that $\bigvee_{x\in X}(A(x) \wedge \bigwedge_{B\in\psi} B(x)) \not\geq c$. Obviously ψ is a pre-semi-closed S-c-R-NF of A. By (3) we know that ψ has a finite subfamily φ which is a c-R-NF of A. Take $b \in \beta(a)$ such that $c \in \beta(b)$, then φ is an S-b-R-NF of A.

(4) $\Rightarrow$ (5) $\Rightarrow$ (2) : Obvious.

(2) $\Rightarrow$ (6) $\Rightarrow$ (7) $\Rightarrow$ (8) $\Rightarrow$ (9) $\Rightarrow$ (1) : the proof is similar.

(1) $\Leftrightarrow$ (10) : This follows directly from Theorem 3.6.

(10) $\Rightarrow$ (11) : This is easy to prove if one notices that every S-a-shading of A is an a-shading of A.

(11) $\Rightarrow$ (12) : Let $a \in L \setminus \{1\}$ and μ be a pre-semi-open S-a-shading of A. Then $\bigwedge_{x\in X}(A'(x) \vee \bigvee_{B\in\mu} B(x)) \not\leq a$. Take $c \in \alpha(a)$ such that $\bigwedge_{x\in X}(A'(x) \vee \bigvee_{B\in\mu} B(x)) \not\leq c$. Obviously μ is a pre-semi-open S-c-shading of A. By (11) we know that μ has a finite subfamily ν which is a c-shading of A. Take $b \in \alpha(a)$ such that $c \in \alpha(b)$, then ν is an S-b-shading of A.

(12) $\Rightarrow$ (13) $\Rightarrow$ (10) : Obvious.

(10) $\Rightarrow$ (14) $\Rightarrow$ (15) $\Rightarrow$ (16) $\Rightarrow$ (17) $\Rightarrow$ (10) : We can prove these in the similar way.

Similarly we can also prove the other results. □

Lemma 4.2. *Let $(X, \omega_L(\tau))$ be generated topologically by (X, τ). If A is a pre-semi-open L-set in (X, τ), then χ_A is a pre-semi-open set in $(X, \omega_L(\tau))$. If B is a pre-semi-open L-set in $(X, \omega_L(\tau))$, then, $B_{(a)}$ is a pre-semi-open set in (X, τ) for every $a \in L$.*

Proof. This is easy to prove if one notices that $\chi_{D^-} = (\chi_D)^-$ and $\chi_{D_o} = (\chi_D)_o$ and applies Lemma 5.4 in [9]. □

Theorem 4.3. *Let (X, τ) be a topological space and $(X, \omega_L(\tau))$ be generated topologically by (X, τ). Then $(X, \omega_L(\tau))$ is PS-compact iff (X, τ) is PS-compact.*

Proof. Necessity: Let μ be a pre-semi-open cover of (X, τ). Then $\{\chi_A | A \in \mu\}$ is a family of pre-semi-open L-sets in $(X, \omega_L(\tau))$ with $\bigwedge_{x\in X}(\bigvee_{A\in\mu} \chi_A(x)) = 1$. From PS-compactness of $(X, \omega_L(\tau))$, we have that

$$1 = \bigwedge_{x\in X} (\bigvee_{A\in\mu} \chi_A(x)) \leq \bigvee_{\nu\in 2^{(\mu)}} \bigwedge_{x\in X} (\bigvee_{A\in\nu} \chi_A(x)).$$

This implies that there exists $\nu \in 2^{(\mu)}$ such that $\bigwedge_{x\in X}(\bigvee_{A\in\nu} \chi_A(x)) = 1$. Hence, ν is a cover of (X, τ). Therefore (X, τ) is PS-compact.

Sufficiency: Let μ be a family of pre-semi-open L-sets in $(X, \omega_L(\tau))$ and $\bigwedge_{x\in X}(\bigvee_{B\in\mu} B(x)) = a$. If $a = 0$, obviously we have that

$$\bigwedge_{x\in X} (\bigvee_{B\in\mu} B(x)) \leq \bigvee_{\nu\in 2^{(\mu)}} \bigwedge_{x\in X} (\bigvee_{B\in\nu} B(x)).$$

Now we suppose that $a \neq 0$. In this case, for any $b \in \beta(a) \setminus \{0\}$, we have that

$$b \in \beta(\bigwedge_{x\in X} (\bigvee_{B\in\mu} B(x))) \subseteq \bigcap_{x\in X} \beta(\bigvee_{B\in\mu} B(x)) = \bigcap_{x\in X} \bigcup_{B\in\mu} \beta(B(x)).$$

By Lemma 4.2, this implies that $\{B_{(b)} | B \in \mu\}$ is a pre-semi-open cover of (X, τ) and from the PS-compactness of (X, τ), we know that there exists $\nu \in 2^{(\mu)}$ such that $\{B_{(b)} | B \in \nu\}$ is a cover of (X, τ). Hence $b \leq \bigwedge_{x\in X}(\bigvee_{B\in\nu} B(x))$. Moreover, we have that

$$b \leq \bigwedge_{x\in X} (\bigvee_{B\in\nu} B(x)) \leq \bigvee_{\nu\in 2^{(\mu)}} \bigwedge_{x\in X} (\bigvee_{B\in\nu} B(x)).$$

This implies that

$$\bigwedge_{x\in X} (\bigvee_{B\in\mu} B(x)) = a = \bigvee\{b | b \in \beta(a)\} \leq \bigvee_{\nu\in 2^{(\mu)}} \bigwedge_{x\in X} (\bigvee_{B\in\nu} B(x)).$$

Therefore, $(X, \omega_L(\tau))$ is PS-compact. □

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