A COMMON FIXED POINT THEOREM FOR SIX WEAKLY COMPATIBLE MAPPINGS IN $\mathcal{M}$-FUZZY METRIC SPACES

S. SEDGHI, K. P. R. RAO AND N. SHOBE

Abstract. In this paper, we give some new definitions of $\mathcal{M}$-fuzzy metric spaces and we prove a common fixed point theorem for six mappings under the condition of weakly compatible mappings in complete $\mathcal{M}$-fuzzy metric spaces.

1. Introduction and Preliminaries

The concept of fuzzy sets was introduced initially by Zadeh [39] in 1965. Since then, to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. Especially, Deng [2], Erceg [10], Kaleva and Seikkala [19] and Kramosil and Michalek [20] have introduced the concepts of fuzzy metric spaces in different ways. George and Veeramani [14] and Kramosil and Michalek [20] have introduced the concept of fuzzy topological spaces induced by fuzzy metric which have very important applications in quantum particle physics particularly in connections with both string and $\epsilon^{(\infty)}$ theory which were given and studied by El Naschie [6-9] and Tanaka et.al [36]. Recently Gregori et.al [15,16] and Rafi et.al [28] studied some properties in fuzzy and intuitionistic fuzzy metric spaces. Many authors [1,10-14,17,18,21,22,25-27,29-33,35,37,38] have studied the fixed point theory in fuzzy (probabilistic) metric spaces. On the other hand, there have been a number of generalizations of metric spaces.Recently Dhage[5] introduced the concept of D-metric and has studied some fixed point theorems in [5,3,4]. Unfortunately,almost all theorems of Dhage are not valid(see [23,24]).Sedgi and Shobe [34] introduced D^*-metric space by altering the tetrahedran inequality in D-metric and using D^*-metric analogy,they defined $\mathcal{M}$-fuzzy metric space and studied some fixed point theorems. In this paper we define $\mathcal{M}$-fuzzy metric space using triangular norm and prove some results in it. We also prove a common fixed point theorem for six self maps in a $\mathcal{M}$-fuzzy metric space.

Definition 1.1. A triangular norm (shortly t-norm) is a binary operation T : $[0,1] \times [0,1] \longrightarrow [0,1] = I$ which is a continuous t-norm if it satisfies the following conditions

(1) T is associative and commutative,
(2) T is continuous,

Received: April 2007; Revised: August 2007; Accepted: November 2007
Key words and phrases: $\mathcal{M}$-fuzzy metric spaces, Weakly compatible maps, Hadzic-type t-norm, Common fixed point theorem.

(3) $T(a,1) = a$ for all $a \in [0,1]$,
(4) $T(a,b) \leq T(c,d)$ whenever $a \leq c$ and $b \leq d$, for each $a,b,c,d \in [0,1]$.

Some examples of continuous t-norm are the Lukasiewicz t-norm $T_L : I \times I \longrightarrow I, T(a,b) = \max(a+b-1,0)$,$T_P(a,b) = ab$, and $T_M(a,b) = min\{a,b\}$.
t–norms are recursively defined by $T^1(x_1,x_2) = T(x_1,x_2)$ and

$$T^n(x_1,\cdots,x_{n+1}) = T(T^{n-1}(x_1,\cdots,x_n),x_{n+1})$$

for $n \geq 2$ and $x_i \in [0,1]$, for all $i \in \{1,2,\ldots,n+1\}$.

Now, we define the concept of $\mathcal{M}$-fuzzy metric spaces with the help of continuous t-norms as a generalization of fuzzy metric space due to George and Veeramani [14].

Definition 1.2. A 3-tuple $(X,\mathcal{M},T)$ is called a $\mathcal{M}$-fuzzy metric space if X is an arbitrary (non-empty) set, T is a continuous t-norm, and $\mathcal{M}$ is a fuzzy set on $X^3 \times (0,\infty)$ satisfying the following conditions: for all $x,y,z,a \in X$ and $t,s > 0$,
(FM-1) $\mathcal{M}(x,y,z,t) > 0$,
(FM-2) $\mathcal{M}(x,y,z,t) = 1$ if and only if $x = y = z$,
(FM-3) $\mathcal{M}(x,y,z,t) = \mathcal{M}(p\{x,y,z\},t)$ (symmetry), where p is a permutation function,
(FM-4) $T(\mathcal{M}(x,y,a,t),\mathcal{M}(a,z,z,s)) \leq \mathcal{M}(x,y,z,t+s)$,
(FM-5) $\mathcal{M}(x,y,z,\cdot) : (0,\infty) \to [0,1]$ is continuous.

Lemma 1.3. *Let $(X,\mathcal{M},T)$ be a $\mathcal{M}$-fuzzy metric space. For any $x,y \in X$ and $t > 0$, we have*
(1) $\mathcal{M}(x,x,y,t) = \mathcal{M}(x,y,y,t)$.
(2) $\mathcal{M}(x,y,z,\cdot)$ *is nondecreasing.*

Proof. (1) Let $\epsilon > 0$. Then by (FM-4) we have

$$\mathcal{M}(x,x,y,\epsilon+t) \geq T(\mathcal{M}(x,x,x,\epsilon),\mathcal{M}(x,y,y,t)) = \mathcal{M}(x,y,y,t), \tag{1.1}$$

$$\mathcal{M}(y,y,x,\epsilon+t) \geq T(\mathcal{M}(y,y,y,\epsilon),\mathcal{M}(y,x,x,t)) = \mathcal{M}(y,x,x,t). \tag{1.2}$$

By taking limit $\epsilon \to 0$ in (1.1) and (1.2), we get $\mathcal{M}(x,x,y,t) = \mathcal{M}(x,y,y,t)$.
(2) By (FM-4) we have $T(\mathcal{M}(x,y,a,t),\mathcal{M}(a,z,z,s)) \leq \mathcal{M}(x,y,z,t+s)$ for any $z,a \in X$ and $t,s > 0$. Let $a = z$, then we have $T(\mathcal{M}(x,y,z,t),\mathcal{M}(z,z,z,s)) \leq \mathcal{M}(x,y,z,t+s)$ so that $\mathcal{M}(x,y,z,t+s) \geq \mathcal{M}(x,y,z,t)$. □

In the following examples, we know that both d-metric and fuzzy metric induce a $\mathcal{M}$-fuzzy metric.

Example 1.4. Let (X,d) be a metric space. Denote $T(a,b) = a.b$ for all $a,b \in [0,1]$. For each $t \in]0,\infty[$, let

$$\mathcal{M}(x,y,z,t) = \frac{t}{t+D(x,y,z)}$$

where $D(x,y,z) = \max\{d(x,y),d(y,z),d(x,z)\}$ for all $x,y,z \in X$. Then $(X,\mathcal{M},T)$ is a $\mathcal{M}$-fuzzy metric space. We call the $\mathcal{M}$-fuzzy metric $\mathcal{M}$, induced by the metric d, as the standard $\mathcal{M}$-fuzzy metric.

Example 1.5. Let $X = [0, 1]$. Let $T(a, b) = \min\{a, b\}$ for all $a, b \in [0, 1]$ and let $\mathcal{M}$ be the fuzzy set on $X \times X \times X \times (0, +\infty)$ defined as follows:

$$\mathcal{M}(x, y, z, t) = \frac{t}{t + |x - y| + |y - z| + |z - x|},$$

for all $t > 0$. Then $(X, \mathcal{M}, T)$ is a fuzzy metric space.

Example 1.6. Let (X, M, T) be a fuzzy metric space. If we define $\mathcal{M} : X^3 \times (0, \infty) \longrightarrow [0, 1]$ by

$$\mathcal{M}(x, y, z, t) = T(T(M(x, y, t), M(y, z, t)), M(z, x, t))$$

for every x, y, z in X, then $(X, \mathcal{M}, T)$ is a $\mathcal{M}$-fuzzy metric space.

Proof. Let $x, y, z \in X$ and $t > 0$.

(FM-1) It is easy to see that $\mathcal{M}(x, y, z, t) > 0$.

(FM-2)$\mathcal{M}(x, y, z, t) = 1 \Leftrightarrow M(x, y, t) = M(y, z, t) = M(z, x, t) = 1 \Leftrightarrow x = y = z$.

(FM-3) It is easy to see that $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$, where p is a permutation function.

(FM-4) Since $M(x, y, \cdot)$ is nondecreasing, we have

$$\begin{aligned} \mathcal{M}(x, y, z, t + s) &= T(T(M(x, y, t + s), M(y, z, t + s)), M(z, x, t + s)) \\ &\geq T^4(M(x, y, t), M(y, a, t), M(a, z, s), M(z, a, s), M(a, x, t)) \\ &= T^4(\mathcal{M}(x, y, a, t), M(a, z, s), M(z, a, s), M(z, z, s)) \\ &= T(\mathcal{M}(x, y, a, t), \mathcal{M}(a, z, z, s)) \end{aligned}$$

for any $s > 0$.

(FM-5) $\mathcal{M}(x, y, z, \cdot) : (0, \infty) \to [0, 1]$ is continuous.
Hence $(X, \mathcal{M}, T)$ is a $\mathcal{M}$-fuzzy metric space. □

Let $(X, \mathcal{M}, T)$ be a $\mathcal{M}$-fuzzy metric space. For $t > 0$, the open ball $B_{\mathcal{M}}(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by

$$B_{\mathcal{M}}(x, r, t) = \{y \in X : \mathcal{M}(x, y, y, t) > 1 - r\}.$$

A subset A of X is called open set if for each $x \in A$ there exist $t > 0$ and $0 < r < 1$ such that $B_{\mathcal{M}}(x, r, t) \subseteq A$.

Proposition 1.7. *In a $\mathcal{M}$-fuzzy metric space, every open ball is an open set.*

Proof. Let $B_{\mathcal{M}}(x, r, t)$ be an open ball and $y \in B_{\mathcal{M}}(x, r, t)$. Then $\mathcal{M}(x, y, y, t) > 1 - r$ and there exists $0 < t_0 < t$ such that $\mathcal{M}(x, y, y, t_0) > 1 - r$. Put $r_0 = \mathcal{M}(x, y, y, t_0)$. Since $r_0 > 1 - r$, there exists $0 < s < 1$ such that $r_0 > 1 - s > 1 - r$. Now, for a given r_0 and s with $r_0 > 1 - s$, we can find $0 < r_1 < 1$ such that $T(r_0, r_1) \geq 1 - s$. Now consider the ball $B_{\mathcal{M}}(y, 1 - r_1, t - t_0)$. We claim that $B_{\mathcal{M}}(y, 1 - r_1, t - t_0) \subset B_{\mathcal{M}}(x, r, t)$. Let $z \in B_{\mathcal{M}}(y, 1 - r_1, t - t_0)$. Then

$\mathcal{M}(y,z,z,t-t_0) > r_1$ and hence by Lemma 1.3,

$$\begin{aligned}\mathcal{M}(x,z,z,t) &= \mathcal{M}(z,z,x,t) \geq T(\mathcal{M}(y,x,x,t_0),\mathcal{M}(z,z,y,t-t_0))\\ &= T(\mathcal{M}(x,y,y,t_0),\mathcal{M}(y,z,z,t-t_0)) \geq T(r_0,r_1)\\ &\geq 1-s\\ &> 1-r.\end{aligned}$$

Thus $z \in B_{\mathcal{M}}(x,r,t)$ and hence $B_{\mathcal{M}}(y,1-r_1,t-t_0) \subset B_{\mathcal{M}}(x,r,t)$.
Thus $B_{\mathcal{M}}(x,r,t)$ is an open set. □

Remark 1.8. Let $(X,\mathcal{M},T)$ be a $\mathcal{M}$-fuzzy metric space. Define

$$\tau_{\mathcal{M}} = \{A \subset X : \forall x \in A, \exists\ t > 0 \text{ and } 0 < r < 1 \text{ such that } B_{\mathcal{M}}(x,r,t) \subset A\}.$$

Then $\tau_{\mathcal{M}}$ is a topology on X.

Theorem 1.9. *Every $\mathcal{M}$-fuzzy metric space is Hausdorff.*

Proof. Let $(X,\mathcal{M},T)$ be the given $\mathcal{M}$-fuzzy metric space. Let x,y be two distinct points of X. Then $0 < \mathcal{M}(x,y,y,t) < 1$. Put $\mathcal{M}(x,y,y,t) = r$ for some $r \in (0,1)$. For each r with $r < r_0 < 1$, there exists r_1 such that $T(r_1,r_1) \geq r_0$. Now consider the open balls $B_{\mathcal{M}}(x,1-r_2,\frac{1}{2}t)$ and $B_{\mathcal{M}}(y,1-r_2,\frac{1}{2}t)$. Clearly, $B_{\mathcal{M}}(x,1-r_2,\frac{1}{2}t) \cap B_{\mathcal{M}}(y,1-r_2,\frac{1}{2}t) = \emptyset$. For if there exists $z \in B_{\mathcal{M}}(x,1-r_2,\frac{1}{2}t) \cap B_{\mathcal{M}}(y,1-r_2,\frac{1}{2}t)$, then

$$\begin{aligned}r &= \mathcal{M}(x,y,y,t) = \mathcal{M}(x,x,y,t) \geq T(\mathcal{M}(x,x,z,\tfrac{1}{2}t),\mathcal{M}(z,y,y,\tfrac{1}{2}t))\\ &= T(\mathcal{M}(x,z,z,\tfrac{1}{2}t),\mathcal{M}(y,z,z,\tfrac{1}{2}t))\\ &\geq T(r_1,r_1) \geq r_0\\ &> r,\end{aligned}$$

which is a contradiction. Hence $(X,\mathcal{M},T)$ is Hausdorff. □

Definition 1.10. Let $(X,\mathcal{M},T)$ be a $\mathcal{M}$-fuzzy metric space and $\{x_n\}$ be a sequence in X.

(1) $\{x_n\}$ is said to be convergent to a point $x \in X$ (denoted by $\lim_{n\to\infty} x_n = x$) if $\lim_{n\to\infty} \mathcal{M}(x,x,x_n,t) = 1$ for all $t > 0$.

(2) $\{x_n\}$ is called a Cauchy sequence if if for each $0 < \epsilon < 1$ and $t > 0$, there exist $n_0 \in \mathbb{N}$ such that $\mathcal{M}(x_n,x_n,x_m,t) > 1-\epsilon$ for all $n,m \geq n_0$.

(3) A $\mathcal{M}$-fuzzy metric in which every Cauchy sequence is convergent is said to be complete.

2. The Main Results

Definition 2.1. Let $(X,\mathcal{M},T)$ be a $\mathcal{M}$-fuzzy metric space. $\mathcal{M}$ is said to be continuous function on $X^3 \times (0,\infty)$ if

$$\lim_{n\to\infty} \mathcal{M}(x_n,y_n,z_n,t_n) = \mathcal{M}(x,y,z,t)$$

whenever a sequence $\{(x_n, y_n, z_n, t_n)\}$ in $X^3 \times (0, \infty)$ converges to a point $(x, y, z, t) \in X^3 \times (0, \infty)$, i.e.

$$\lim_{n\to\infty} x_n = x, \lim_{n\to\infty} y_n = y, \lim_{n\to\infty} z_n = z \text{ and } \lim_{n\to\infty} \mathcal{M}(x, y, z, t_n) = \mathcal{M}(x, y, z, t).$$

Lemma 2.2. *Let $(X, \mathcal{M}, T)$ be a $\mathcal{M}$-fuzzy metric space. Then $\mathcal{M}$ is continuous function on $X^3 \times (0, \infty)$.*

Proof. Let $x, y, z \in X$ and $t > 0$, and let $\{(x'_n, y'_n, z'_n, t'_n)\}$ be a sequence in $X^3 \times (0, \infty)$ that converges to (x, y, z, t). Since $\{\mathcal{M}(x'_n, y'_n, z'_n, t'_n)\}$ is a sequence in $(0, 1]$, there is a subsequence $\{(x_n, y_n, z_n, t_n)\}$ of sequence $\{(x'_n, y'_n, z'_n, t'_n)\}$ such that sequence $\{\mathcal{M}(x_n, y_n, z_n, t_n)\}$ converges to some point of $[0, 1]$. Fix $\delta > 0$ such that $\delta < \frac{t}{2}$. Then there is $n_0 \in \mathbb{N}$ such that $|t - t_n| < \delta$ for all $n \geq n_0$. Hence we have

$$\begin{aligned}
&\mathcal{M}(x_n, y_n, z_n, t_n)\\
&\quad \geq \mathcal{M}(x_n, y_n, z_n, t - \delta) \geq T(\mathcal{M}(x_n, y_n, z, t - \frac{4\delta}{3}), \mathcal{M}(z, z_n, z_n, \frac{\delta}{3}))\\
&\quad \geq T^2(\mathcal{M}(x_n, z, y, t - \frac{5\delta}{3}), \mathcal{M}(y, y_n, y_n, \frac{\delta}{3}), \mathcal{M}(z, z_n, z_n, \frac{\delta}{3}))\\
&\quad \geq T^3(\mathcal{M}(z, y, x, t - 2\delta), \mathcal{M}(x, x_n, x_n, \frac{\delta}{3}), \mathcal{M}(y, y_n, y_n, \frac{\delta}{3}), \mathcal{M}(z, z_n, z_n, \frac{\delta}{3}))
\end{aligned}$$

and

$$\begin{aligned}
&\mathcal{M}(x, y, z, t + 2\delta)\\
&\quad \geq \mathcal{M}(x, y, z, t_n + \delta) \geq T(\mathcal{M}(x, y, z_n, t_n + \frac{2\delta}{3}), \mathcal{M}(z_n, z, z, \frac{\delta}{3}))\\
&\quad \geq T^2(\mathcal{M}(x, z_n, y_n, t_n + \frac{\delta}{3}), \mathcal{M}(y_n, y, y, \frac{\delta}{3}), \mathcal{M}(z_n, z, z, \frac{\delta}{3}))\\
&\quad \geq T^3(\mathcal{M}(z_n, y_n, x_n, t_n), \mathcal{M}(x_n, x, x, \frac{\delta}{3}), \mathcal{M}(y_n, y, y, \frac{\delta}{3}), \mathcal{M}(z_n, z, z, \frac{\delta}{3}))
\end{aligned}$$

for all $n \geq n_0$. By taking limit $n \to \infty$, we obtain

$$\lim_{n\to\infty} \mathcal{M}(x_n, y_n, z_n, t_n) \geq T^3(\mathcal{M}(x, y, z, t - 2\delta), 1, 1, 1) = \mathcal{M}(x, y, z, t - 2\delta)$$

and

$$\mathcal{M}(x, y, z, t + 2\delta) \quad \geq \quad \lim_{n\to\infty} T^3(\mathcal{M}(x_n, y_n, z_n, t_n), 1, 1, 1) = \lim_{n\to\infty} \mathcal{M}(x_n, y_n, z_n, t_n),$$

respectively. So, by continuity of the function $t \mapsto \mathcal{M}(x, y, z, t)$, we immediately deduce that

$$\lim_{n\to\infty} \mathcal{M}(x_n, y_n, z_n, t_n) = \mathcal{M}(x, y, z, t).$$

Therefore $\mathcal{M}$ is continuous on $X^3 \times (0, \infty)$. □

Definition 2.3. Let A and S be mappings from a $\mathcal{M}$-fuzzy metric space $(X, \mathcal{M}, *)$ into itself. Then the mappings A and S are said to be

(1) weakly compatible if they commute at a coincidence point, that is, $Ax = Sx$ implies $ASx = SAx$.

(2) compatible if for all $t > 0$,

$$\lim_{n\to\infty} \mathcal{M}(ASx_n, SAx_n, SAx_n, t) = 1$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some $x \in X$.

We also mention the following families of t-norms:

Definition 2.4. It is said that the t-norm T is of Hadzic-type (H-type for short) and $T \in \mathcal{H}$ if the family $\{T^n\}_{n\in\mathbf{N}}$ of its iterates defined, for each x in [0,1], by

$$T^0(x) = 1,\ T^{n+1}(x) = T(T^n(x), x),\ \forall n \geq 0,$$

is equicontinuous at $x = 1$, that is,

$$\forall\epsilon \in (0,1)\ \exists\delta \in (0,1)\ \textit{such that}\ x > 1-\delta \Longrightarrow T^n(x) > 1-\epsilon,\ \forall n \geq 1,$$

There is a nice characterization of continuous t-norm T of the class $\mathcal{H}$ [27].

(i) If there exists a strictly increasing sequence $\{b_n\}_{n\in\mathbf{N}}$ in [0,1] such that $\lim_{n\to\infty} b_n = 1$ and $T(b_n, b_n) = b_n$ $\forall n \in \mathbf{N}$, then T is of Hadzic-type.

(ii) If T is continuous and $T \in \mathcal{H}$, then there exists a sequence $\{b_n\}_{n\in\mathbf{N}}$ as in (i).

The t-norm T_M is an trivial example of a t-norm of H-type, but there are t-norms T of Hadzic-type with $T \neq T_M$ (see, e.g.,[17]).

Definition 2.5. [17]. If T is a t-norm and $(x_1, x_2, \cdots, x_n) \in [0,1]^n (n \in \mathbf{N})$, then $T_{i=1}^n x_i$ is defined recurrently by 1, if $n = 0$ and $T_{i=1}^n x_i = T(T_{i=1}^{n-1} x_i, x_n)$ for all $n \geq 1$. If $\{x_i\}_{i\in\mathbf{N}}$ is a sequence of numbers from [0,1], then $T_{i=1}^\infty x_i$ is defined as $\lim_{n\to\infty} T_{i=1}^n x_i$ (this limit always exists) and $T_{i=n}^\infty x_i$ as $T_{i=1}^\infty x_{n+i}$. In fixed point theory in probablistic metric spaces there are of particular interest t-norms T and sequences $\{x_n\} \subset [0,1]$ such that $\lim_{n\to\infty} x_n = 1$ and $\lim_{n\to\infty} T_{i=1}^\infty x_{n+i} = 1$.

Throughout this section, a binary operation $T : [0,1] \times [0,1] \longrightarrow [0,1]$ is a continuous t-norm of Hadzic-type with $\lim_{t\to\infty} \mathcal{M}(x,y,z,t) = 1$, for every $x, y, z \in X$.

Lemma 2.6. *Let $(X, \mathcal{M}, T)$ be a $\mathcal{M}$-fuzzy metric space. If sequence $\{x_n\}$ in X exists such that for every $n \in \mathbf{N}$,$0 < k < 1$ and $t > 0$,*

$$\mathcal{M}(x_n, x_n, x_{n+1}, k^n t) \geq \mathcal{M}(x_0, x_0, x_1, t)$$

then sequence $\{x_n\}$ is a Cauchy sequence.

Proof. Since t-norm T of Hadzic-type, hence we have

$$\forall\epsilon \in (0,1)\ \exists\delta \in (0,1)\ \textit{such that}\ x > 1-\delta \Longrightarrow T^n(x) > 1-\epsilon,\ \forall n \geq 1.$$

Since, $\lim_{t\to\infty} \mathcal{M}(x_0, x_0, x_1, t) = 1$, there exists $t_0 > 0$ such that $\mathcal{M}(x_0, x_0, x_1, t_0) > 1-\delta$, for some $\delta \in (0,1)$. Therefore,

$$T^n(\mathcal{M}(x_0, x_0, x_1, t_0)) > 1-\epsilon,\ \forall n \geq 1.$$

Since $\sum_{n=0}^{\infty} k^n t_0 < \infty$, we have for every $t > 0$ there exists $n_0 \in \mathbf{N}$ such that $\forall n \geq n_0$ we have,

$$\sum_{i=n}^{\infty} k^i t_0 < t.$$

Thus for every $n \geq n_0$ and $\forall m \in \mathbf{N}$,

$$\begin{aligned}
\mathcal{M}(x_n, x_n, x_{n+m+1}, t) &\geq \mathcal{M}(x_n, x_n, x_{n+m+1}, \sum_{i=n}^{\infty} k^i t_0) \\
&\geq \mathcal{M}(x_n, x_n, x_{n+m+1}, \sum_{i=n}^{n+m} k^i t_0) \\
&\geq T_{i=n}^{n+m} \mathcal{M}(x_i, x_i, x_{i+1}, k^i t_0) \\
&= T_{i=0}^{m} \mathcal{M}(x_{i+n}, x_{i+n}, x_{i+n+1}, k^{i+n} t_0) \\
&\geq T^m \mathcal{M}(x_0, x_0, x_1, t_0) \\
&> 1 - \epsilon,
\end{aligned}$$

for each $0 < \epsilon < 1$ and $t > 0$. Hence sequence $\{x_n\}$ is Cauchy . □

Now we prove a common fixed point theorem for six self maps.

Theorem 2.7. *Let A, B, R, S, C and Q be self-mappings of a fuzzy metric space $(X, \mathcal{M}, T)$ satisfying:*

(i)*$Q(X) \subseteq CS(X)$, $R(X) \subseteq AB(X)$ and $CS(X)$ or $AB(X)$ is a closed subset of X,*

(ii) *The pair (R, CS) and (Q, AB) are weakly compatible and $CS = SC$, $BQ = QB$,$RS = SR$ and $AB = BA$,*

(iii) $\mathcal{M}(Qx, Ry, Ry, kt)\times$
$T(\mathcal{M}(Qx, Ry, Ry, kt), \mathcal{M}(ABx, Qx, Qx, kt))\, \mathcal{M}(CSy, Ry, Ry, kt)$
$\geq [p(t)\mathcal{M}(ABx, Qx, Qx, t) + q(t)\mathcal{M}(ABx, CSy, CSy, t)]\mathcal{M}(ABx, Ry, Ry, 2kt)$

for every $x, y \in X$, all $t > 0$ and some $k \in (0, 1)$, where $p, q : \mathbb{R}^+ \longrightarrow (0, 1]$ be two functions such that $p(t) + q(t) = 1$.
Then, A, B, C, S, Q and R have a unique common fixed point in X.

Proof. Let $x_0 \in X$ be an arbitrary point . By (i), there exist $x_1, x_2 \in X$ such that

$$Qx_0 = CSx_1 = y_0 \text{ and } Rx_1 = ABx_2 = y_1.$$

Inductively, construct sequence $\{y_n\}$ in X such that

$$y_{2n} = Qx_{2n} = CSx_{2n+1} \text{ and } y_{2n+1} = ABx_{2n+2} = Rx_{2n+1},$$

for $n = 0, 1, 2, \cdots$.

Now, we prove $\{y_n\}$ is a Cauchy sequence. Let $d_m(t) = \mathcal{M}(y_m, y_{m+1}, y_{m+1}, t)$. Then, by (iii) we have

$$\mathcal{M}(Qx_{2n}, Rx_{2n+1}, Rx_{2n+1}, kt)\times \begin{pmatrix} T(\mathcal{M}(Qx_{2n}, Rx_{2n+1}, Rx_{2n+1}, kt), \mathcal{M}(ABx_{2n}, Qx_{2n}, Qx_{2n}, kt)) \\ \times\mathcal{M}(CSx_{2n+1}, Rx_{2n+1}, Rx_{2n+1}, kt) \end{pmatrix}$$
$$\geq \begin{pmatrix} p(t)\mathcal{M}(ABx_{2n}, Qx_{2n}, Qx_{2n}, t) \\ +q(t)\mathcal{M}(ABx_{2n}, CSx_{2n+1}, CSx_{2n+1}, t) \end{pmatrix} \mathcal{M}(ABx_{2n}, Rx_{2n+1}, Rx_{2n+1}, 2kt)$$

Thus

$$\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, kt) \begin{pmatrix} T(\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, kt), \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, kt)) \\ \times\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, kt) \end{pmatrix}$$
$$\geq \begin{pmatrix} p(t)\mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) \\ +q(t)\mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) \end{pmatrix} \mathcal{M}(y_{2n-1}, y_{2n+1}, y_{2n+1}, 2kt).$$

Hence $d_{2n}(kt)\mathcal{M}(y_{2n-1}, y_{2n+1}, y_{2n+1}, 2kt)$

$$\geq [p(t)d_{2n-1}(t) + q(t)d_{2n-1}(t)]\mathcal{M}(y_{2n-1}, y_{2n+1}, y_{2n+1}, 2kt).$$

Thus

$$d_{2n}(kt) \geq d_{2n-1}(t)$$

Putting $x = x_{2n+2}, y = x_{2n+1}$ in (iii) we have

$$\mathcal{M}(Qx_{2n+2}, Rx_{2n+1}, Rx_{2n+1}, kt)\times \begin{pmatrix} T(\mathcal{M}(Qx_{2n+2}, Rx_{2n+1}, Rx_{2n+1}, kt), \mathcal{M}(ABx_{2n+2}, Qx_{2n+2}, Qx_{2n+2}, kt)) \\ \mathcal{M}(CSx_{2n+1}, Rx_{2n+1}, Rx_{2n+1}, kt) \end{pmatrix}$$
$$\geq \begin{pmatrix} p(t)\mathcal{M}(ABx_{2n+2}, Qx_{2n+2}, Qx_{2n+2}, t) \\ +q(t)\mathcal{M}(ABx_{2n+2}, CSx_{2n+1}, CSx_{2n+1}, t) \end{pmatrix} \mathcal{M}(ABx_{2n+2}, Rx_{2n+1}, Rx_{2n+1}, 2kt).$$

Thus

$$\mathcal{M}(y_{2n+2}, y_{2n+1}, y_{2n+1}, kt)\times \begin{pmatrix} T(\mathcal{M}(y_{2n+2}, y_{2n+1}, y_{2n+1}, kt), \mathcal{M}(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt)) \\ \times\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, kt) \end{pmatrix}$$
$$\geq \begin{pmatrix} p(t)\mathcal{M}(y_{2n+1}, y_{2n+2}, y_{2n+2}, t) \\ +q(t)\mathcal{M}(y_{2n+1}, y_{2n}, y_{2n}, t) \end{pmatrix} \mathcal{M}(y_{2n+1}, y_{2n+1}, y_{2n+1}, 2kt).$$

Therefore

$$\begin{aligned} d_{2n+1}(kt) &\geq d_{2n+1}(kt)[T(\mathcal{M}(y_{2n+2}, y_{2n+1}, y_{2n+1}, kt), \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, kt))] \\ &\geq p(t)d_{2n+1}(t) + q(t)d_{2n}(t) \\ &\geq p(t)d_{2n+1}(kt) + q(t)d_{2n}(t). \end{aligned}$$

Thus

$$(1 - p(t))d_{2n+1}(kt) \geq q(t)d_{2n}(t).$$

It follows that

$$d_{2n+1}(kt) \geq \frac{q(t)}{1 - p(t)} d_{2n}(t) = d_{2n}(t)$$

Hence for every $n \in \mathbb{N}$ we have $d_n(kt) \geq d_{n-1}(t)$. Now , we have

$$\mathcal{M}(y_n, y_{n+1}, y_{n+1}, t) \geq \mathcal{M}(y_{n-1}, y_n, y_n, \frac{t}{k}) \geq \cdots \geq \mathcal{M}(y_0, y_1, y_1, \frac{t}{k^n})$$

So,by Lemma 2.6, sequence $\{y_n\}$ is Cauchy and the completeness of X, $\{y_n\}$ converges to y in X. Hence

$$\lim_{n\to\infty} Qx_{2n} \quad = \quad \lim_{n\to\infty} CSx_{2n+1} = \lim_{n\to\infty} Rx_{2n+1} = \lim_{n\to\infty} ABx_{2n+2} = y.$$

Let $AB(X)$ be a closed subset of X , then there exists $v \in X$ such that $ABv = y$. Putting $x = v, y = x_{2n+1}$ in (iii) we get

$$\mathcal{M}(Qv, Rx_{2n+1}, Rx_{2n+1}, kt)\begin{pmatrix} T(\mathcal{M}(Qv, Rx_{2n+1}, Rx_{2n+1}, kt), \mathcal{M}(ABv, Qv, Qv, kt)) \\ \times\mathcal{M}(CSx_{2n+1}, Rx_{2n+1}, Rx_{2n+1}, kt) \end{pmatrix}$$
$$\geq \begin{pmatrix} p(t)\mathcal{M}(ABv, Qv, Qv, t) \\ +q(t)\mathcal{M}(ABv, CSx_{2n+1}, CSx_{2n+1}, t) \end{pmatrix}\mathcal{M}(ABv, Rx_{2n+1}, Rx_{2n+1}, 2kt).$$

Letting $n \to \infty$, we get

$$\mathcal{M}(Qv, y, y, kt)\begin{pmatrix} T(\mathcal{M}(Qv, y, y, kt), \mathcal{M}(y, Qv, Qv, kt)) \\ \times\mathcal{M}(y, y, y, kt) \end{pmatrix}$$
$$\geq \begin{pmatrix} p(t)\mathcal{M}(y, Qv, Qv, t) \\ +q(t)\mathcal{M}(y, y, y, t) \end{pmatrix}\mathcal{M}(y, y, y, 2kt).$$

Thus

$$\begin{aligned} \mathcal{M}(Qv, y, y, kt) &\geq \mathcal{M}(Qv, y, y, kt)[T(\mathcal{M}(Qv, y, y, kt), \mathcal{M}(Qv, y, y, kt))] \\ &\geq p(t)\mathcal{M}(y, Qv, Qv, t) + q(t) \\ &\geq p(t)\mathcal{M}(y, y, Qv, kt) + q(t) \end{aligned}$$

So,

$$\mathcal{M}(Qv, y, y, kt) \geq \frac{q(t)}{1 - p(t)} = 1.$$

Hence $Qv = y$. Since the pair (Q, AB) is weakly compatible we have $ABQv = QABv$, hence $ABy = Qy$. Now from (iii), we have

$$\mathcal{M}(Qy, Rx_{2n+1}, Rx_{2n+1}, kt)\begin{pmatrix} T(\mathcal{M}(Qy, Rx_{2n+1}, Rx_{2n+1}, kt), \mathcal{M}(ABy, Qy, Qy, kt)) \\ \times\mathcal{M}(CSx_{2n+1}, Rx_{2n+1}, Rx_{2n+1}, kt) \end{pmatrix}$$
$$\geq \begin{pmatrix} p(t)\mathcal{M}(ABy, Qy, Qy, t) \\ +q(t)\mathcal{M}(ABy, CSx_{2n+1}, CSx_{2n+1}, t) \end{pmatrix}\mathcal{M}(ABy, Rx_{2n+1}, Rx_{2n+1}, 2kt).$$

Letting $n \to \infty$, we get

$$\mathcal{M}(Qy, y, y, kt)\begin{pmatrix} T(\mathcal{M}(Qy, y, y, kt), \mathcal{M}(Qy, Qy, Qy, kt)) \\ \times\mathcal{M}(y, y, y, kt) \end{pmatrix}$$
$$\geq \begin{pmatrix} p(t)\mathcal{M}(y, Qy, Qy, t) \\ +q(t)\mathcal{M}(y, y, y, t) \end{pmatrix}\mathcal{M}(ABy, y, y, 2kt).$$

Thus

$$\mathcal{M}(Qy, y, y, kt)\mathcal{M}(Qy, y, y, 2kt) \quad \geq \quad [p(t)\mathcal{M}(y, Qy, Qy, t) + q(t)]\mathcal{M}(Qy, y, y, 2kt)$$

It follows that

$$\mathcal{M}(Qy, y, y, kt) \geq p(t)\mathcal{M}(y, y, Qy, kt) + q(t),$$

so that ,

$$\mathcal{M}(Qy, y, y, kt) \geq \frac{q(t)}{1-p(t)} = 1.$$

Thus $Qy = y$.Hence $ABy = Qy = y$.Since $y = Qy \in Q(X) \subseteq CS(X)$, there exists $w \in X$ such that $CSw = y$. From (iii), we have

$$\mathcal{M}(Qy, Rw, Rw, kt)\begin{pmatrix} T(\mathcal{M}(Qy, Rw, Rw, kt), \mathcal{M}(ABy, Qy, Qy, kt)) \\ \times\mathcal{M}(CSw, Rw, Rw, kt) \end{pmatrix}$$

$$\geq \begin{pmatrix} p(t)\mathcal{M}(ABy, Qy, Qy, t) \\ +q(t)\mathcal{M}(ABy, CSw, CSw, t) \end{pmatrix} \mathcal{M}(ABy, Rw, Rw, 2kt).$$

$$\mathcal{M}(y, Rw, Rw, , kt)\begin{pmatrix} T(\mathcal{M}(y, Rw, Rw, kt), \mathcal{M}(y, y, y, kt)) \\ \times\mathcal{M}(y, Rw, Rw, kt) \end{pmatrix}$$

$$\geq \begin{pmatrix} p(t)\mathcal{M}(y, y, y, t) \\ +q(t)\mathcal{M}(y, y, y, t) \end{pmatrix} \mathcal{M}(y, Rw, Rw, 2kt).$$

Thus $\mathcal{M}(y, Rw, Rw, kt)\mathcal{M}(y, Rw, Rw, 2kt)$

$$\geq (p(t) + q(t))\mathcal{M}(y, Rw, Rw, 2kt) = \mathcal{M}(y, Rw, Rw, 2kt).$$

Hence $\mathcal{M}(y, Rw, Rw, kt) = 1$ so that $Rw = y$.
Since the pair (R, CS) is weakly compatible , we have $CSRw = RCSw$ and hence $CSy = Ry$. By (iii), we get

$$\mathcal{M}(Qy, Ry, Ry, kt)\begin{pmatrix} T(\mathcal{M}(Qy, Ry, Ry, kt), \mathcal{M}(ABy, Qy, Qy, kt)) \\ \times\mathcal{M}(CSy, Ry, Ry, kt) \end{pmatrix}$$

$$\geq \begin{pmatrix} p(t)\mathcal{M}(ABy, Qy, Qy, t) \\ +q(t)\mathcal{M}(ABy, CSy, y, t) \end{pmatrix} \mathcal{M}(ABy, Ry, Ry, 2kt).$$

Thus

$$\begin{aligned} \mathcal{M}^2(y, Ry, Ry, kt) &\geq \mathcal{M}(y, Ry, Ry, kt)\begin{pmatrix} T(\mathcal{M}(y, Ry, Ry, kt), \mathcal{M}(y, y, y, kt)) \\ \times\mathcal{M}(Ry, Ry, Ry, kt) \end{pmatrix} \\ &\geq \begin{pmatrix} p(t)\mathcal{M}(y, y, y, t) \\ +q(t)\mathcal{M}(y, Ry, y, kt) \end{pmatrix} \mathcal{M}(y, Ry, y, 2kt) \\ &\geq [p(t) + q(t)\mathcal{M}(y, Ry, y, kt)]\mathcal{M}(y, Ry, y, kt) \end{aligned}$$

This implies that

$$\mathcal{M}(y, Ry, y, kt) \geq \frac{p(t)}{1-q(t)} = 1.$$

Hence $Ry = y$. Since $AB = BA$ and $QB = BQ$,we have $AB(By) = B(ABy) = By$, and $QBy = BQy = By$. Similarly, since $CS = SC$ and $RS = SR$ we have $CS(Sy) = S(CSy) = Sy$ and $RSy = SRy = Sy$. By (iii), we have

$$\mathcal{M}(QBy, Ry, Ry, , kt)\begin{pmatrix} T(\mathcal{M}(QBy, Ry, Ry, kt), \mathcal{M}(AB(By), QBy, QBy, kt)) \\ \times\mathcal{M}(CSy, Ry, Ry, kt) \end{pmatrix}$$

$$\geq \begin{pmatrix} p(t)\mathcal{M}(AB(By), QBy, QBy, t) \\ +q(t)\mathcal{M}(AB(By), CSy, CSy, t) \end{pmatrix} \mathcal{M}(AB(By), Ry, Ry, 2kt).$$

Thus

$$\begin{aligned} \mathcal{M}(By, y, y, kt) & \begin{pmatrix} T(\mathcal{M}(By, y, y, kt), \mathcal{M}(By, By, By, kt)) \\ \times\mathcal{M}(y, y, y, kt) \end{pmatrix} \\ & \geq \begin{pmatrix} p(t)\mathcal{M}(By, By, By, t) \\ +q(t)\mathcal{M}(By, y, y, t) \end{pmatrix} \mathcal{M}(By, y, y, 2kt) \end{aligned}$$

Hence

$$\mathcal{M}^2(By, y, y, kt) \geq [p(t) + q(t)M(By, y, y, kt)]\mathcal{M}(By, y, y, kt).$$

$$\mathcal{M}(By, y, y, kt) \geq p(t) + q(t)M(By, y, y, kt).$$

$$\mathcal{M}(By, y, y, kt) \geq \frac{p(t)}{1 - q(t)} = 1.$$

It follows that $By = y$. From (iii), we have

$$\begin{aligned} \mathcal{M}(Qy, RSy, RSy, , kt) & \begin{pmatrix} T(\mathcal{M}(Qy, RSy, RSy, kt), \mathcal{M}(ABy, Qy, Qy, kt)) \\ \times\mathcal{M}(CSy, RSy, RSy, kt) \end{pmatrix} \\ & \geq \begin{pmatrix} p(t)\mathcal{M}(ABy, Qy, Qy, t) \\ +q(t)\mathcal{M}(ABy, CSy, CSy, t) \end{pmatrix} \mathcal{M}(ABy, RSy, RSy, 2kt). \end{aligned}$$

Thus

$$\begin{aligned} \mathcal{M}^2(y, Sy, Sy, kt) & \geq \mathcal{M}(y, Sy, Sy, kt) \begin{pmatrix} T(\mathcal{M}(y, Sy, Sy, kt), \mathcal{M}(y, y, y, kt)) \\ \times\mathcal{M}(Sy, Sy, Sy, kt) \end{pmatrix} \\ & \geq \begin{pmatrix} p(t)\mathcal{M}(y, y, y, t) \\ +q(t)\mathcal{M}(y, Sy, Sy, t) \end{pmatrix} \mathcal{M}(y, Sy, Sy, 2kt) \\ & \geq [p(t) + q(t)\mathcal{M}(y, Sy, Sy, kt)]\mathcal{M}(y, Sy, Sy, kt) \end{aligned}$$

Hence

$$\mathcal{M}(y, Sy, Sy, kt) \geq \frac{p(t)}{1 - q(t)} = 1$$

so that $Sy = y$. Therefore,

$$Sy = By = Qy = Ry = ABy = CSy = Ay = Cy = y.$$

To prove uniqueness, let x be another common fixed point of Q, A, B, C, R, S.Then

$$\begin{aligned} \mathcal{M}(Qx, Ry, Ry, kt) & \begin{pmatrix} T(\mathcal{M}(Qx, Ry, Ry, kt), \mathcal{M}(ABx, Qx, Qx, kt)) \\ \times\mathcal{M}(CSy, Ry, Ry, kt) \end{pmatrix} \\ & \geq \begin{pmatrix} p(t)\mathcal{M}(ABx, Qx, Qx, t) \\ +q(t)\mathcal{M}(ABx, CSy, y, t) \end{pmatrix} \mathcal{M}(ABx, Ry, Ry, 2kt). \end{aligned}$$

Thus

$$\begin{aligned} \mathcal{M}(x, y, y, kt)\mathcal{M}(x, y, y, kt) & \geq [p(t)) + q(t)\mathcal{M}(x, y, y, t)]\mathcal{M}(x, y, y, 2kt) \\ & \geq [p(t) + q(t)\mathcal{M}(x, y, y, kt)]\mathcal{M}(x, y, y, kt) \end{aligned}$$

Therefore,

$$\mathcal{M}(x, y, y, kt) \geq p(t) + q(t)\mathcal{M}(x, y, y, kt).$$

Hence

$$\mathcal{M}(x, y, y, kt) \geq \frac{p(t)}{1 - q(t)} = 1.$$

So $x = y$.

□

Now we give an Example to illustrate our Theorem.

Example 2.8. Let $X = [0, 1]$,$T(a, b) = \min\{a, b\}$ and define $A, B, C, Q, R, S : X \longrightarrow X$ as

$$Qx = Rx = Bx = Sx = 1, Ax = Cx = \begin{cases} 1 & \text{if } x \textit{ is rational}, \\ 0 & \text{if } x \textit{ is irrational}. \end{cases}$$

for all $x \in X$.
Let $p(t)$ and $q(t)$ be any arbitrary functions mapping from $\mathbb{R}^+ \longrightarrow (0, 1]$ such that $p(t) + q(t) = 1$ and

$$\mathcal{M}(x, y, z, t) = \frac{t}{t + |x - y| + |y - z| + |z - x|}.$$

Then all conditions of Theorem 2.7 are satisfied and 1 is the unique common fixed point of A, B, C, Q, R and S.

Acknowledgement: The authors are thankful to the referees for their valuable suggestions to improve this paper.

References

[1] J. C. Chang, H. Chen, S. M. Shyu and W. C. Lian, *Fixed point theorems in fuzzy real line*, Computers and Mathematics with Applications, **47**(2004), 845-851.

[2] Z. K. Deng, *Fuzzy pseudo-metric spaces*, J. Math. Anal. Appl., **86** (1982), 74-95.

[3] B. C. Dhage, *A common fixed point principe in D-metric spaces*, Bull. Calcutta Math. Soc., **91** (1999), 475-480.

[4] B. C. Dhage, A. M. Pathan and B. E. Rhoades, *A general existence priciple for fixed point theorem in D-metric spaces*, Int. J. Math. Math. Sci., **23** (2000), 441-448.

[5] B. C. Dhage, *Generalised metric spaces and mappings with fixed point*, Bull. Calcutta Math. Soc., **84(4)** (1992), 329-336.

[6] M. S. El Naschie, *A review of E-infinity theory and the mass spectrum of high energy particle physics*, Chaos, Solitons and Fractals, **19** (2004), 209-236.

[7] M. S. El Naschie, *On a fuzzy Kahler-like Manifold which is consistent with two-slit experiment*, Int. J. of Nonlinear Science and Numerical Simulation, **6** (2005), 95-98.

[8] M. S. El Naschie, *On the uncertainty of Cantorian geometry and two-slit experiment*, Chaos, Solitons and Fractals, **9** (1998), 517-529.

[9] M. S. El Naschie, *The idealized quantum two-slit gedanken experiment revisited-Criticism and reinterpretation*, Chaos, Solitons and Fractals, **27** (2006), 9-13.

[10] M. A. Erceg, *Metric spaces in fuzzy set theory*, J. Math. Anal. Appl., **69** (1979), 205-230.

[11] J. X. Fang, *On fixed point theorems in fuzzy metric spaces*, Fuzzy Sets and Systems, **46** (1992), 107-113.

[12] M. Grabiec, *Fixed point in fuzzy metric spaces*, Fuzzy Sets and Systems, **27** (1988), 385-389.

[13] V. Gregori and A. Sapena, *On fixed-point theorem in fuzzy metric spaces*, Fuzzy Sets and Systems, **125** (2002), 245-252.

[14] A. George and P. Veeramani, *On some result in fuzzy metric space*, Fuzzy Sets and Systems, **64** (1994), 395-399.

[15] V. Gregori, S. Romaguera and P. Veeramani, *A note on intuitionistic fuzzy metric spaces*, Chaos , Solitons and Fractals, **28**(2006), 902-905.

[16] V. Gregori and S. Romaguera, *Some proerties of fuzzy metric spaces*, Fuzzy Sets and Systems, **115** (2000), 485-489.

[17] O. Hadzic and E. Pap, *Fixed Point Theory in Probabilistic Metric Spaces*, Mathematics and Its Applications, Vol. 536, Kluwer Academic Publishers, Dordrecht, 2001.

[18] O. Hadzic, *Fixed point theorems in probabilistic metric spaces*, Serbian Academy of Sciences and Arts, Institute of Mathematics, University of Novi Sad, Yugoslavia, 1995.

[19] O. Kaleva and S. Seikkala, *On fuzzy metric spaces*, Fuzzy Sets and Systems, **12** (1984), 215-229.

[20] I. Kramosil and J. Michalek, *Fuzzy metric and statistical metric spaces*, Kybernetica, **11** (1975), 326-334.

[21] V. Lakshmikantham and A. S. Vatsala, *Existence offixed points of fuzzy mappings via theory of fuzzy differential equations*, Journal of Computational and Applied Mathematics, **113** (2000), 195-200.

[22] D. Miheţ, *A Banach contraction theorem in fuzzy metric spaces*, Fuzzy Sets and Systems, **144** (2004), 431-439.

[23] S. V. R. Naidu, K. P. R. Rao and N. Srinivasa Rao, *On convergent sequences and fixed point theorems in D-Metric spaces*, Internat. J. Math. Math. Sci. 2005, **12** (2005), 1969-1988.

[24] S. V. R. Naidu, K. P. R. Rao and N. Srinivasa Rao, *On the topology of D-metric spaces and the generation of D-metric spaces from metric spaces*, Internat. J. Math. Math. Sci.2004, **51** (2004), 2719-2740.

[25] J. J. Nieto and Rodriguez-Lopez, *Existence of extremal solutions for quadratic fuzzy equations*, Fixed Point Theory and Applications, **3**(2005), 321-342.

[26] J. J. Nieto, R. L. Pouso and R. Rodriguez-Lopez, *Fixed point theorems on ordered abstract spaces*, Proceedings of American Mathematical Society(in press).

[27] V. Radu, *Lectures on Probabilistic Analysis*, Surveys, Lecture Notes and Monographs.Series on Probability, Statistics and Applied Mathematics, Vol. 2, Universitatea din Timisoara, Timisoara, 1994.

[28] M. Rafi and M. S. M. Noorani, *Fixed point theorem on Intuitionistic Fuzzy Metric spaces*, Iranian J. Fuzzy System, **3** (2006), 23-29.

[29] J. Rodríguez López and S. Ramaguera, *The Hausdorff fuzzy metric on compact sets*, Fuzzy Sets and Systems, **147** (2004), 273-283.

[30] R. Saadati and J. H. Park, *On the intuitionistic fuzzy topological spaces*, Chaos, Solitons and Fractals, **27** (2006), 331-344.

[31] B. Schweizer and A. Sklar, *Probabilistic Metric Spaces*, Elsevier, North Holand, New York, 1983.

[32] B. Schweizer and A. Sklar, *Statistical metric spaces*, Pacific J. Math., **10** (1960), 313-334.

[33] B. Schweizer, H. Sherwood and R. M. Tardiff, *Contractions on PM-space examples and counterexamples*, Stochastica, **1** (1988), 5-17.

[34] S. Sedghi and N. Shobe, *Fixed point theorem in $\mathcal{M}$-fuzzy metric spaces with property(E)*, Advances in Fuzzy Mathematics, **1(1)** (2006), 55-65.

[35] G. Song, *Comments on "A common fixed point theorem in a fuzzy metric spaces"*, Fuzzy Sets and Systems, **135** (2003), 409-413.

[36] Y. Tanaka, Y. Mizno and T. Kado, *Chaotic dynamics in Friedmann equation*, Chaos, Solitons and Fractals, **24** (2005), 407-422.

[37] R. Vasuki and P. Veeramani, *Fixed point theorems and Cauchy sequences in fuzzy metric spaces*, Fuzzy Sets and Systems, **135** (2003), 415-417.

[38] R. Vasuki, *Common fixed points for R-weakly commuting maps in fuzzy metric spaces*, Indian J. Pure Appl. Math., **30** (1999), 419-423.

[39] L. A. Zadeh, *Fuzzy sets*, Inform and Control, **8** (1965), 338-353.

Shaban Sedghi*, Department of Mathematics, Islamic Azad University-Ghaemshahr Branch, Ghaemshahr P.O.Box 163, Iran
E-mail address: sedghi_gh@yahoo.com

K. P. R. Rao, Department of Applied Mathematics, Acharya Nagarjuna University-Nuzvid Campus, Nuzvid-521201, A.P., India
E-mail address: kprrao2004@yahoo.com

Nabi Shobe, Department of Mathematics, Islamic Azad University-Babol Branch, Iran
E-mail address: nabi_shobe@yahoo.com

*Corresponding author