METACOMPACTNESS IN L-TOPOLOGICAL SPACES

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Abstract. In this paper the concept of metacompactness in L-topological spaces is introduced by means of point finite families of L-fuzzy sets. This fuzzy metacompactness is a natural generalization of Lowen fuzzy compactness. Further a characterization of fuzzy metacompactness in the weakly induced L-topological spaces is also obtained.

1. **Introduction**

In [7] Fu-Gui Shi and Cheng-You Zheng introduced the concept of α-locally finite family to characterize fuzzy compactness and using this they have defined paracompactness in L-topological spaces in [8], which is a natural generalization of the Lowen fuzzy compactness. In this paper we define α-point finite families and metacompactness in L-topological spaces. Besides getting a characterization for metacompactness in the weakly induced L-topological spaces that involve the concept of well monotone and directed α-Q-covers, it is also seen that the metacompactness obtained is closed hereditary.

2. **Preliminaries and Basic Definitions**

Let L be a complete lattice. Its universal bounds are denoted by $\perp$ and $\top$. We presume that L is consistent. ie., $\perp$ is distinct from $\top$. Thus $\perp \leq \alpha \leq \top$ for all $\alpha \in L$. We note $\bigvee \phi = \perp$ and $\bigwedge \phi = \top$. The two point lattice $\{\perp, \top\}$ is denoted by **2**. A unary operation $'$ on L is a quasi-complementation. It is an involution (ie., $\alpha'' = \alpha$ for all $\alpha \in L$) that inverts the ordering. (ie., $\alpha \leq \beta$ implies $\beta' \leq \alpha'$). In $(L,')$ the DeMorgan laws hold: $(\bigvee A)' = \bigwedge\{\alpha'; \alpha \in A\}$ and $(\bigwedge A)' = \bigvee\{\alpha'; \alpha \in A\}$ for every $A \subset L$. Moreover, in particular, $\perp' = \top$ and $\top' = \perp$.

A molecule or co-prime element in a lattice L is a join irreducible element in L and the set of all non zero co-prime elements of L is denoted by $M(L)$. A complete lattice L is completely distributive if it satisfies either of the logically equivalent CD1 or CD2 below:

CD1: $\bigwedge_{i\in I}(\bigvee_{j\in J_i} a_{i,j}) = \bigvee_{\phi\in\Pi_{i\in I}J_i}(\bigwedge_{i\in I} a_{i,\phi(i)})$

CD2: $\bigvee_{i\in I}(\bigwedge_{j\in J_i} a_{i,j}) = \bigwedge_{\phi\in\Pi_{i\in I}J_i}(\bigvee_{i\in I} a_{i,\phi(i)})$

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for all $\{\{a_{ij}; j \in J_i\}; i \in I\} \subset P(L) \setminus \{\phi\}, I \neq \phi$

If L is a complete lattice, then for a set X, L^X is the complete lattice of all maps from X into L, called L-sets or L-subsets of X. Under point-wise ordering, $a \leq b$ in L^X if and only if $a(x) \leq b(x)$ in L for all $x \in X$. If $A \subset X$, $1_A \in \mathbf{2}^X \subset L^X$ is the characteristic function of A. The constant member of L^X with value α is denoted by α itself. We use the same notation to represent crisp set as well as its characteristic function. Wang [9] proved that a complete lattice is completely distributive if and only if for each $a \in L$, there exists $B \subseteq L$ such that (i) $a = \bigvee A$ and (ii) if $A \subseteq L$ and $a \leq \bigvee B$, then for each $b \in B$, there exists $c \in A$ such that $b \leq c$. B is called the minimal set of a and $\beta(a)$ denote the union of all minimal sets of a. Again $\beta^*(a) = \beta(a) \cap M(L)$. Clearly $\beta(a)$ and $\beta^*(a)$ are minimal sets of a.

For $\alpha \in L$ and $A \in L^X$, we use the following notations.

$$\begin{aligned} A_{[\alpha]} &= \{x \in X : A(x) \geq \alpha\} \\ A^{[\alpha]} &= \{x \in X : A(x) \leq \alpha\} \\ A^{(\alpha)} &= \{x \in X : A(x) \not\leq \alpha\} \\ A_{(\alpha)} &= \{x \in X : \alpha \in \beta(A(x))\} \end{aligned}$$

Clearly L^X has a quasi complementation $'$ defined point-wisely $\alpha'(x) = \alpha(x)'$ for all $\alpha \in L$ and $x \in X$. Thus the DeMorgan laws are inherited by $(L^X, ')$.

Let $(L, ')$ be a complete lattice equipped with an order reversing involution and X be any non empty set. A subfamily $\tau \subset L^X$ which is closed under the formation of sups and finite infs (both formed in L^X) is called an L-topology on X and its members are called open L-sets. The pair (X, τ) is called an L-topological space (L-ts). The category of all L-topological spaces, together with L-continuous mappings and the composition and identities of Set is denoted by L-**Top**. Quasi complements of open L-sets are called closed L-sets.

We know that the set of all non zero co-prime elements in a completely distributive lattice is $\bigvee$-generating. Moreover for a continuous lattice L and a topological space $(X, T), T = \imath_L \omega_L(T)$ is not true in general. By proposition 3.5 in Kubiak [4] we know that one sufficient condition for $T = \imath_L \omega_L(T)$ is that L is completely distributive.

In [10] Wang extended the Lowen functor ω for completely distributive lattices as follows: For a topological space (X, T), $(X, \omega(T))$ is called the induced space of (X, T) where $\omega(T) = \{A \in L^X; \forall \alpha \in M(L), A^{(\alpha')} \in T\}$. In 1992 Kubiak also extended the Lowen functor ω_L for a complete lattice L. In fact when L is completely distributive, $\omega_L = \omega$.

An L-topological space (X,τ) is called weakly induced space if $\forall\alpha \in M(L), \forall A \in \tau$ it is true that $A^{(\alpha')} \in [\tau]$ where $[\tau]$ is the set of all crisp open sets in τ.

Based on these facts, in this paper we use a complete, completely distributive lattice L in L^X. For a standardized basic fixed-basis terminology, we follow Hoehle and Rodabaugh [3]. Also L-$Pnt(X)$ denote the collection of all L-fuzzy points in the L-ts (X,τ).

A closed remote neighbourhood (R-nbd) of x_λ is a closed L-set P such that $x_\lambda \not\leq P$. An open L-set Q is called an open Q-neighbourhood (Q-nbd) of x_λ if Q' is a closed R-nbd of x_λ. Set of all Q-nbds of x_λ is denoted by $\mathbf{Q}(x_\lambda)$ and the set of all closed R-nbds of x_λ is denoted by $\eta(x_\lambda)$.

A is called α-nonempty if $A_{[\alpha]} \neq \phi$. Moreover if there exists $\gamma \in \beta^*(\alpha)$ such that A is γ-nonempty, then A is called α^--nonempty. If $A \wedge B$ is α-nonempty (α^--nonempty), we say that A is α-nonempty (α^--nonempty) in B.

Definition 2.1. [9, 10] Let (X,τ) be an L-ts, $D \in L^X$ and $\alpha \in M(L)$. $\mathbf{A} \subseteq \tau'$ is called an α-R neighborhood family of D, briefly α-RF of D , if for each $x_\alpha \leq D$, there exists $A \in \mathbf{A}$ such that $x_\alpha \not\leq A$. A is called an $\alpha^- - R$ neighborhood family of D, briefly $\alpha^- - RF$ of D , if there exists $\gamma \in \beta^*(\alpha)$ such that $\mathbf{A}$ is a γ-RF of D.

If $\{A\}$ is an α-$RF(\alpha^- RF)$ of D, then we call A an α-R-neighborhood (α^--R-neighborhood) of D.

Definition 2.2. [8] Let (X,τ) be an L-ts, $D \in L^X$ and $\alpha \in M(L)$. $\mathbf{A} \subseteq \tau$ is called an α-Q-cover of D, if for each $x_\alpha \leq D$, there exists $A \in \mathbf{A}$ such that $x_\alpha \not\leq A'$. $\mathbf{A}$ is called an an α^--Q cover of D, if there exists $\gamma \in \beta^*(\alpha)$ such that $\mathbf{A}$ is a γ-Q-cover of D.

If $\{A\}$ is an α-Q-cover (α^--Q-cover) of D, then we call A an α-Q-neighborhood (α^--Q-neighborhood) of D.

Clearly $\mathbf{A}$ is an α-RF of D if and only if $\mathbf{A}' = \{A' : A \in \mathbf{A}\}$ is an α-Q-cover of D.

Definition 2.3. [7] Let $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$, $D \in L^X$, $\alpha \in M(L)$. If $\forall\, x_\alpha \leq D$, $\exists\, P \in \eta(x_\alpha)$ and a finite subset T_0 of T such that $\forall t \in T - T_0$, $A_t \leq P$, then $\mathbf{A}$ is called α-locally finite in D. If there exists $\gamma \in \beta^*(\alpha)$ such that $\mathbf{A}$ is γ-locally finite in D, then $\mathbf{A}$ is called α^--locally finite in D.

Definition 2.4. Let $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$, $D \in L^X$, $\alpha \in M(L)$. Then $\mathbf{A}$ is called α-point finite in D if $\forall\, x_\alpha \leq D$, there exists at most finitely many $t \in T$ such that $x_\alpha \leq A_t$. If there exists $\gamma \in \beta^*(\alpha)$ such that A is γ-point finite in D, then $\mathbf{A}$ is called α^--point finite in D.

Obviously α^--locally finite implies α-locally finite and α^--point finite implies α-point finite.

More over we say that $\mathbf{A}$ is locally finite (point finite) in D if $\mathbf{A}$ is α-locally finite (α-point finite) in D for every co-prime element $\alpha \in L$.

Proposition 2.5. *Every α-locally finite (α^--locally finite) family is α-point finite (α^--point finite).*

Proof. Proof of Proposition 2.5 follows immediately from the definitions. □

Definition 2.6. A collection $\boldsymbol{A}$ refines a collection $\boldsymbol{B}$ ($\boldsymbol{A}< \boldsymbol{B}$) if for every $A \in \boldsymbol{A}$, there exists $B \in \boldsymbol{B}$ such that $A \leq B$.

Definition 2.7. [7] Let (X, τ) be an L-ts, $D \in L^X$. D is called fuzzy compact if $\forall \alpha \in M(L)$ and $\forall \gamma \in \beta^*(\alpha)$, every constant α-net in D has a cluster point $x_\gamma \leq D$

Definition 2.8. [7] Let (X, τ) be an L-ts, $D \in L^X$. D is called fuzzy countably compact if $\forall \alpha \in M(L)$ and $\forall \gamma \in \beta^*(\alpha)$, each α-sequence in D has a cluster point $x_\gamma \leq D$.

3. Fuzzy Metacompactness

Definition 3.1. [8] Let (X, τ) be an L-ts, $D \in L^X$. D is called fuzzy paracompact if for each $\alpha \in M(L)$ and for each α^--Q-cover $\mathbf{A}$ of D, there exists an α-Q-cover $\mathbf{B}$ of D such that $\mathbf{B}$ is a refinement of $\mathbf{A}$ and $\mathbf{B}_{(0)} \wedge D$ is α-locally finite in D,where $\mathbf{B}_{(0)} = \{B_{(0)} : B \in \mathbf{B}\}$. When $D = X$, (X, τ) is called fuzzy paracompact.

Definition 3.2. Let (X, τ) be an L-ts, $D \in L^X$. D is called fuzzy metacompact if for each $\alpha \in M(L)$ and for each α^--Q-cover $\mathbf{A}$ of D, there exists an α-Q-cover $\mathbf{B}$ of D such that $\mathbf{B}$ is a refinement of $\mathbf{A}$ and $\mathbf{B}_{(0)} \wedge D$ is α-point finite in D, where $\mathbf{B}_{(0)} = \{B_{(0)} : B \in \mathbf{B}\}$. When $D = X$, (X, τ) is called fuzzy metacompact.

Clearly we have the following implications

$$\text{fuzzy compact} \Rightarrow \text{fuzzy paracompact} \Rightarrow \text{fuzzy metacompat.}$$

From the above implication and the fact that the fuzzy unit interval $I(L)$ is fuzzy compact, we have the following corollary.

Corollary 3.3. *The fuzzy unit interval $I(L)$ is fuzzy metacompact.*

Now we give an example of a fuzzy metacompact space which is not fuzzy paracompact.

Example 3.4. Let X be the deleted Tychnoff plank $T_\infty = T - (\omega_1, \omega)$ where T is the Tychnoff's plank given by $[0, \omega_1] \times [0, \omega]$ where ω_1 is the first uncountable ordinal and ω is the first infinite ordinal. Let $\alpha \in [0, 1)$,. Define for each $\varsigma \in [0, \omega)$ and $\beta \in [0, \omega_1), U^\beta_\varsigma = \{(\beta, \gamma) : \varsigma < \gamma \leq \omega\}$ and for each $\lambda \in [0, \omega_1)$ and $\delta \in [0, \omega), V^\delta_\lambda = \{(\gamma, \delta) : \lambda < \gamma \leq \omega_1\}$. Let T be the I-topology generated by taking each point p of $[0, \omega_1] \times [0, \omega]$ as fuzzy points with value η with $\alpha < \eta \leq 1$ and U^β_ς and V^δ_λ as the open sets. Now (X, T) is fuzzy metacompact. For, any α^--Q-cover of X by open I-sets has an α-Q-cover refinement consisting of one basic neighbourhood for each fuzzy point of X. Any such α-Q-cover refinement $\mathbf{U}$ is point finite, since an arbitrary fuzzy point x_α can have at most three members of $\mathbf{U}$ such that $x_\alpha \leq U$, where $U \in \mathbf{U}$.

Now the space (X, T) is not fuzzy paracompact. For, consider the α^--Q-cover of X by sets $U_0 = X - B$ and $U_n = V^{n-1}_0$ for $n = 1, 2, 3$, where $A = \{(\omega, n) : 0 \leq n < \omega\}$, has no locally finite refinement. For, if possible let $\{W_\mu\}$ be a locally finite refinement. Now for each $n \in N$, we may define an ordinal α_n to be the least ordinal such that characterestic function of $V^n_{\alpha_n}$ is contained in just one W_μ. If $\alpha = Sup\{\alpha_n\} < \omega_1$, every R-neighbourhood of (α, ω) will contain infinitely many members of $\{W_\mu\}$.

Theorem 3.5. *Let (X, τ) be an L-ts and $D \in L^X$. Then D is fuzzy metacompact if and only if for each $\alpha \in M(L)$ and for each α^--Q-cover $\boldsymbol{A}$ of D, there exists an α^--Q-cover $\boldsymbol{B}$ of D such that $\boldsymbol{B}$ is a refinement of $\boldsymbol{A}$ and $\boldsymbol{B}_{(0)} \wedge D$ is α^--point finite in D.*

Proof. Sufficiency part: Since every α^--Q-cover of D, is an α-Q-cover of D, and every α^--point finite family is α-point finite, sufficiency part follows clearly.
Necessary part: Assume that D is metacompact. Let $\alpha \in M(L)$ and $\mathbf{A}$ be an α^--Q-cover of D. Then by definition $\exists \gamma \in \beta^*(\alpha)$ such that $\mathbf{A}$ is a γ-Q-cover of D. Now we have $\gamma \in \beta^*(\alpha) = \beta^*(\bigvee\{\lambda : \lambda \in \beta^*(\alpha)\}) = \bigcup\{\beta^*(\lambda) : \lambda \in \beta^*(\alpha)\}$. Therefore it follows that there is a $\lambda \in \beta^*(\alpha)$ such that $\gamma \in \beta^*(\lambda)$. Thus $\mathbf{A}$ is a λ^--Q-cover of D. Since D is fuzzy metacompact , $\exists$ a λ-Q-cover $\mathbf{B}$ of D which refines $\mathbf{A}$ and $\mathbf{B}_{(0)} \wedge D$ is α^--point finite in D. □

Theorem 3.6. *Let (X, τ) be an L-ts and $D \in L^X$. Then if D is fuzzy metacompact, then $\forall B \in \tau'$, $D \wedge B$ is fuzzy metacompact.*

Proof. Let $\mathbf{A}$ be an α^--Q-cover of $D \wedge B$, where $\alpha \in M(L)$. By the definition of α^--Q-cover, $\exists \gamma \in \beta^*(\alpha)$ such that $\mathbf{A}$ is a γ-Q-cover of $D \wedge B$. Take $\mathbf{B} = \mathbf{A} \bigcup \{B'\}$. Then clearly $\mathbf{B}$ is a γ-Q-cover of D. Since D is fuzzy metacompact, it follows that $\mathbf{B}$ has a refinement $\mathbf{C}$ which is an α-Q-cover of D and $\mathbf{C}_{(0)} \wedge D$ is point finite in D. Let $\mathbf{F} = \{C \in \mathbf{C} : B' \not\geq C\}$. Now $\mathbf{F}$ is an α-Q-cover of $D \wedge B$ and $\mathbf{F}$ is a refinement of $\mathbf{A}$. Obviously $\mathbf{F}_{(0)} \wedge D$ is α-point finite in D and hence in $D \wedge B$. Hence $\mathbf{F}_{(0)} \wedge D \wedge B$ is α-point finite in $D \wedge B$. Thus $D \wedge B$ is fuzzy metacompact. □

Definition 3.7. Let (X, τ) be an L-ts. $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$ is a closure preserving collection if for every subfamily $\mathbf{A}_0$ of $\mathbf{A}$, $cl[\vee \mathbf{A}_0] = \vee[cl\mathbf{A}_0]$.

Proposition 3.8. *A point finite closure preserving closed collection is always locally finite.*

Proof. Proof follows clearly from Definitions 2.3, 2.4 and 3.7. □

Remark 3.9. A collection **U** is locally finite implies that so is $\{clU : U \in \mathbf{U}\}$. But this does not hold for point finite families.

Definition 3.10. [5] Let (X, τ) be an L-ts. Then by $[\tau]$ we denote the family of support sets of all crisp subsets in τ . (X, $[\tau]$) is a topology and it is the background space. (X, τ) is weakly induced if each $U \in \tau$ is a lower semi continuous function from the background space $(X, [\tau])$ to L.

Theorem 3.11. *If (X, τ) is a weakly induced L-ts, then (X, τ) is fuzzy metacompact if and only if $(X, [\tau])$ is metacompact.*

Proof. Let(X, τ) be weakly induced and fuzzy metacompact. Let **A** be an open cover of $(X, [\tau])$. Take $\alpha \in M(L)$ and $\gamma \in \beta^*(\alpha)$. Then clearly **A** is a γ-Q-cover of (X, τ). Since (X, τ) is fuzzy metacompact, $\exists$ an α-Q-cover **B** which refines **A** and $\mathbf{B}_{(0)}$ is α-point finite. Now we can easily show that $\mathbf{U} = \{B^{(\alpha')} : B \in \mathbf{B}\}$ is a point finite open refinement of **A**, proving that $(X, [\tau])$ is metacompact.

Conversely assume that $(X, [\tau])$ is metacompact. Let **A** be an $\alpha^- Q$-cover of (X, τ), where $\alpha \in M(L)$. Then $\exists \gamma \in \beta^*(\alpha)$ such that**A** is a γ-Q-cover of (X, τ). Hence $\mathbf{A}^{(\gamma')} = \{A^{(\gamma')} : A \in \mathbf{A}\}$ is an open cover of $(X, [\tau])$ by the weakly induced property. Since $(X, [\tau])$ is metacompact, this cover has a point finite open refinement say $\mathbf{B} = \{B_t : t \in T\}$. Let $\Omega = \{B_t \wedge A : B_t \leq A^{(\gamma')}, B_t \in \mathbf{B}, A \in \mathbf{A}\}$. Now clearly Ω is a refinement of **A** which is a γ-Q-cover also. Hence it is an α^--Q-cover of (X, τ). Now we will show that $\Omega_{(0)}$ is γ-point finite in (X, τ). For, any $x_\gamma \in M(L^X)$, since **B** is point finite in $(X, [\tau])$, $\exists$ at the most finitely many $t \in T$ such that $x \in B_t$. Now $\Omega_{(0)} = \{(B_t \wedge A)_{(0)} : B_t \leq A^{(\gamma')}, B_t \in \mathbf{B}, A \in \mathbf{A}\}$ and it follows that $x_\gamma \leq (B_t \wedge A)_{(0)}$ for at the most finitely many $t \in T$. Hence $\Omega_{(0)}$ is γ^--point finite and hence (X, τ) is fuzzy metacompact. □

Definition 3.12. A collection **U** of fuzzy subsets of an L-topological space (X, τ) is said to be well monotone if the subset relation $' <'$ is a well order on **U**.

Definition 3.13. A collection **U** of fuzzy subsets of an L-topological space (X, τ) is said to be directed if $U, V \in \mathbf{U}$ implies there exists $W \in \mathbf{U}$ such that $U \vee V < W$.

Theorem 3.14. *If (X, τ) is a weakly induced L-ts, then the following are equivalent.*
(i) (X, τ) is fuzzy metacompact .
(ii) For every $\alpha \in M(L)$, every well monotone open α^--Q-cover of X has an α^--point finite open refinement which is also an α^--Q-cover of X.

Proof. $(i) \Rightarrow (ii)$ Obvious
$(ii) \Rightarrow (i)$ By Theorem 3.11 it is enough to prove that $(X, [\tau])$ is metacompact. But by a characterization of metacompactness (Burke Dennis [1]), it is enough to prove

that every well monotone open cover of $(X, [\tau])$ has a point finite refinement.
Let $\mathbf{U} = \{U_t : t \in T\}$ be a well monotone open cover of $(X, [\tau])$. Then clearly $\mathbf{U}$ is an open well monotone α^--Q-cover of X for every $\alpha \in M(L)$. So it has an α-point finite refinement say $\mathbf{A} = \{A_t : t \in T\}$. Take $\mathbf{B} = \{A_t^{(\alpha')} : t \in T\}$. Since (X, τ) is weakly induced, $\mathbf{B} \subset [\tau]$. Now if possible let there be some $x \in X$ such that $x \in B$ for infinitely many $B \in \mathbf{B}$. ie., $x_\alpha \leq A_t$ for infinitely many $t \in T$. This is a contradiction to that $\mathbf{A}$ is point finite. Again let $x \in A_t^{(\alpha')}$ for some $t \in T$. Since $\{A_t : t \in T\}$ refines $\{U_t : t \in T\}$, it follows that $\alpha \leq A_t(x) \leq U_t$. This implies that $U_t \neq 0$. Thus $x \in U_t$ and hence $\mathbf{B}$ is a refinement of $\{U_t : t \in T\}$. This completes the proof.

□

Lemma 3.15. *Let (X, τ) be a weakly induced L-ts, and $\alpha \in M(L)$ .Then if every directed open α^--Q-cover of X has a closure preserving closed refinement which is also an α^--Q-cover of X, then (X, τ) is metacompact.*

Proof. Let $\mathbf{U} = \{U_t : t \in T\}$ be a directed open cover of X. Then clearly $\mathbf{U}$ is a directed α^--Q-cover of X for every $\alpha \in M(L)$ and hence it is having a closure preserving closed refinement say $\mathbf{A} = \{A_t : t \in T\}$, which is also an α^--Q-cover of X. Now consider the collection $\mathbf{B} = \{A_t^{[\alpha']} : t \in T\}$. Since X is weakly induced, clearly $\mathbf{B} \subset \tau'$ and we will show that $\mathbf{B}$ is the required closure preserving closed refinement. For, let $x \in A_t^{[\alpha']}$. Since $\mathbf{A}$ refines $\mathbf{U}$, it follows that $U_t(x) \neq 0$ for any $t \in T$. And hence $x \in U_t$ and $A_t^{[\alpha']} \subset U_t$. Hence $\mathbf{B}$ refines $\mathbf{U}$. Moreover it easily follows that $\mathbf{B}$ is closure preserving from the fact that $\mathbf{A}$ is closure preserving. This completes the proof. □

Lemma 3.16. *Let (X, τ) be a weakly induced metacompact L-ts and $\alpha \in M(L)$. Then every directed open α^--Q-cover of X has a closure preserving closed refinement which is also an α^--Q-cover of X.*

Proof. Let $\mathbf{U}$ be a directed α^--Q-cover of X. Now $\mathbf{V}= \{U^{(\alpha')} : U \in \mathbf{U}\}$ is a directed open cover of $(X, [\tau])$ and since $(X, [\tau])$ is metacompact, it follows that $\mathbf{V}$ has a closure preserving closed refinement say $\mathbf{W}$. This $\mathbf{W}$ is the required closure preserving closed refinement of $\mathbf{U}$, which is also an α^--Q-cover of X. □

Theorem 3.17. *Let (X, τ) be an L-ts and $\alpha \in M(L)$. Then the following are equivalent.*
(i)Every directed α^--Q-cover of X has a closure preserving closed refinement which is also an α^--Q-cover of X.
(ii)For every α^--Q-cover $\mathbf{U}$ of X, $\mathbf{U}^F$ has a closure preserving closed refinement which is also an α^--Q-cover of X. (Where $\mathbf{U}^F$ is the collection of all unions of finite sub collections from $\mathbf{U}$).

Proof. (i) $\Rightarrow$ (ii) Clearly $\mathbf{U}^F$ is directed and hence has a closure preserving refinement.
(ii) $\Rightarrow$ (i) Let $\mathbf{U}$ be a directed α^--Q-cover of X. Since $\mathbf{U}$ is directed, $\mathbf{U}^F$ is a refinement of $\mathbf{U}$. Then by (ii), $\mathbf{U}^F$ has a closure preserving closed refinement say $\mathbf{V}$

which is also an α^--Q-cover of X. Now $\mathbf{V}$ refines $\mathbf{U}^F$ and $\mathbf{U}^F$ refines $\mathbf{U}$. Hence it follows that $\mathbf{V}$ is the required closure preserving closed refinement of $\mathbf{U}$. $\square$

Combining the results in 3.14, 3.15, 3.16 and 3.17 , we have the following characterization of metacompactness in L-topological spaces.

Theorem 3.18. *If (X, τ) is a weakly induced L-ts, then the following are equivalent.*
(i) (X, τ) is fuzzy metacompact .
(ii) $(X, [\tau])$ is metacompact.
(iii) For every $\alpha \in M(L)$, every well monotone open α^--Q-cover of X has an α^--point finite open refinement which is also an α^--Q-cover of X.
(iv)For every $\alpha \in M(L)$, every directed open α^--Q-cover of X has a closure preserving closed refinement which is also an α^--Q-cover of X.
(v) For every $\alpha \in M(L)$ and every α^--Q-cover $\boldsymbol{U}$ of X, $\boldsymbol{U}^F$ has a closure preserving closed refinement which is also an α^--Q-cover of X.

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