COMBINING FUZZY QUANTIFIERS AND NEAT OPERATORS FOR SOFT COMPUTING

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Abstract. This paper will introduce a new method to obtain the order weights of the Ordered Weighted Averaging (OWA) operator. We will first show the relation between fuzzy quantifiers and neat OWA operators and then offer a new combination of them. Fuzzy quantifiers are applied for soft computing in modeling the optimism degree of the decision maker. In using neat operators, the ordering of the inputs is not needed resulting in better computation efficiency. The theoretical results will be illustrated in a water resources management problem. This case study shows that more sensitive decisions are obtained by using the new method.

1. Introduction

Yager [12] initiated Ordered Weighted Averaging (OWA) as an aggregation operator and then it has been applied in many fields including multi criteria decision-making. An n-dimensional OWA operator is a mapping $F : I^n \mapsto I$, where $I = [0,1]$, that has an associated n-dimensional vector $w_j = (w_1, w_2, ..., w_n)$ of order weights with $w_j \geq 0$ for all j, and $\sum_{j=1}^{n} w_j = 1$. It is defined as follows:

$$F(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j b_j = w_1 b_1 + w_2 b_2 + ... + w_n b_n \tag{1}$$

where b_j is the jth largest element of the set of the aggregated objects $\{a_1, a_2, ..., a_n\}$ and n is the number of the inputs. The elements a_j of the input vector are usually the evaluations of a project with respect to n criteria. As an important characteristic of the OWA, it has a large variety by the different selections of the (order) weights. These weights depend on the optimism degree (well known as Orness degree) of the decision maker (DM) [16]. The greater the weights at the beginning of the vector are, the higher is the optimism degree (risk acceptance). Yager [12] has defined the optimism degree, θ, as

$$\theta = \frac{1}{n-1} \sum_{j=1}^{n} (n-j) w_j \tag{2}$$

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The most frequently used methods to obtain the OWA weights are listed in Table 1.

Method	Approach	Reference	Year
Fuzzy quantifiers	Using the fuzzy linguistic quantifiers to characterize the aggregation inputs.	Yager [12]	1988
Maximum entropy	Maximizing the entropy measure of Shannon [9] for the order weights in a defined value of optimism degree.	O'Hagan [6]	1990
S-OWA	Defining two specific Equations for Or-like and And-like OWA operators.	Yager [13]	1993
Neat OWA	Using the BADD (BAsic Defuzzification Distribution transformation) OWA operator in which the weights depend on the inputs and the results are neat OWA.	Yager [13]; Yager and Filev [14]	1993; 1994
Learning method	Obtaining the weights by minimizing the distance of outputs of OWA operator from the real data.	Filev and Yager [1]	1998
Exponential OWA	Defining two specific graphs to obtain the weights for optimistic and pessimistic OWA operators.	Filev and Yager [1]	1998
Minimal variability	Minimizing the variance of the weights in a defined value of optimism degree.	Fullr and Majlender [2]	2003
Gaussian method	Obtaining the weights by the Normal distribution.	Xu [11]; Yager [17]	2005; 2007

Table 1. Major methods to obtain the OWA weights [19]

Introducing new methods and interpreting their relation to previously introduced methods is in the focus of the research in this field. In this paper we will concentrate on two methods. They are the fuzzy quantifiers and the neat OWA operators, which will be presented in Sections 2 and 3, respectively. In Section 4 we will also introduce a new method, which relates and combines these two methods. Section 5 presents a real life application in a watershed management problem.

2. **Fuzzy Quantifiers**

Classical logic uses only two kind of quantifiers; the existential quantifier, *there exist*, and the universal quantifier, *all*, in forming logical propositions [15]. We use however many linguistic terms such as *at least*, *few*, *many*, and *about half*, which Zadeh [18] called them linguistic quantifiers. He proposed the modeling of these linguistic quantifiers by using fuzzy sets. In this paper these types of linguistic inputs are modeled by Regular Increasing Monotonic (RIM) quantifiers. An RIM quantifier, Q, characterizes aggregation imperatives, in which the more objects are included, the higher is the approval. This quantifier has the following properties:

$$R(Q) = [0,1], Q(0) = 0, Q(1) = 1 \text{ and } Q(r_1) \geq Q(r_2) \text{ if } r_1 \geq r_2. \tag{3}$$

Yager [12] proposed a vector, $\{b_j\}$, $j = 1, 2, ..., n$, to evaluate an alternative with respect to n criteria, such that $b_j = 1$ for $j \leq k$ and $b_j = 0$ for $j > k$. This indicates that k of the criteria are completely satisfied and the remaining are completely unsatisfied. In this situation we have:

$$F(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j b_j = \sum_{j=1}^{k} w_j = S_k.$$

Thus S_k is the approval degree of the DM for the k/n portion of the criteria. Its margin is defined as the weight of satisfying only one criterion in that order, or $w_j = S_j - S_{j-1}$. Then Yager [12] used fuzzy quantifiers to build better model in describing this meaning and suggested obtaining the weights of OWA operator as

$$w_j = Q(\frac{j}{n}) - Q(\frac{j-1}{n}), j = 1, 2, ..., n. \tag{4}$$

Then from Equations (1) and (4), we have the combined goodness measure of each decision alternative as

$$F = \sum_{j=1}^{n} [Q(\frac{j}{n}) - Q(\frac{j-1}{n})] b_j. \tag{5}$$

The other methods shown in Table 1 do not take advantage of the usage of fuzzy quantifiers to model the optimism/pessimism character of the DM.

The semantic of Equation (4) is however questionable since it assumes full satisfaction by only k criteria, and it relates the overall satisfaction to the orders of the criteria and not to their values. We will return to this point later in Section 4.

3. Neat OWA Operators

Yager [13], and Yager and Filev [14], introduced BADD-OWA operators the weights of which can be determined as either

$$w_j = \frac{b_j^\beta}{\sum_{l=1}^{n} b_l^\beta}. \tag{6}$$

or

$$w_j = \frac{(1-b_j)^\beta}{\sum_{l=1}^{n} (1-b_l)^\beta}. \tag{7}$$

The general form is as follow:

$$w_j = \frac{f(b_j)}{\sum_{l=1}^{n} f(b_l)}. \tag{8}$$

where $\{b_j\}$ is the ordered set of the inputs and $f(x)$ is the generating function. The aggregated result by OWA will then be:

$$F(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j b_j = \sum_{j=1}^{n} \frac{f(b_j)}{\sum_{l=1}^{n} f(b_l)} b_j = \sum_{j=1}^{n} \frac{f(a_j)}{\sum_{l=1}^{n} f(a_l)} a_j. \tag{9}$$

It is clear from Equation (9) that these types of operators are independent of the ordering, so they are called neat operators. Marimin et al. [5] used neat OWA operators to aggregate the linguistic labels for expressing fuzzy preferences in a group decision making problem. Peláez and Doña [7] introduced MA-OWA operator with weights being calculated from the cardinality of the elements to aggregate, which are also independent of the order. They [8] also introduced QMA-OWA to model the majority idea used in group decision making by the linguistic quantifiers. Liu and Lou [4] discussed the important characteristic of the generating function of the neat operators. They also introduced some specific generating functions by maximizing the entropy of the weights and also minimizing their variance. Wu et al. [10] introduced an argument dependent approach based on maximizing the entropy. They showed that the obtained weights follow the normal distribution and the resulting OWA operator is neat.

An additional advantage of using neat OWA operators in comparison to the other methods (listed in Table 1) is due to the fact that in this case more attention is given to the context of the problem (e.g. to the values b_j). It is however a disadvantage that the weights should be calculated separately for each alternative. These operators cannot model the optimistic or pessimistic character of the DM.

4. **New Method: Revised OWA**

Notice first that the derivative of the fuzzy quantifier Q is the following:

$$\frac{dQ}{dr} = \lim_{\Delta r \to 0} \frac{Q(r) - Q(r - \Delta r)}{\Delta r} \tag{10}$$

In the special case when n is large we may select $\Delta r = 1/n$, and so

$$\frac{dQ}{dr} = \lim_{n \to \infty} \frac{Q(r) - Q(r - 1/n)}{1/n}$$

Yager [13] evaluated the value of dQ/dr at $r = j/n$ by using Equation (4) as

$$\left.\frac{dQ}{dr}\right|_{r=j/n} = \lim_{n \to \infty} \frac{Q(j/n) - Q((j-1)/n)}{1/n} = \frac{w_j}{1/n}$$

so

$$w_j^* = \frac{1}{n} \left.\frac{dQ}{dr}\right|_{r=j/n} \tag{11}$$

In general, these values however do not satisfy the necessary condition of OWA weights since their sum differs from unity. For example, select an RIM quantifier as $Q(r) = r^2$. The resulting weights by Equation (11) are as follows:

$$w_j^* = \frac{1}{n}\frac{dQ}{dr}\bigg|_{r=j/n} = \frac{1}{n}(2r)|_{r=j/n} = \frac{2j}{n^2}$$

where

$$\sum_{j=1}^{n} w_j^* = \frac{2}{n^2}(1+2+...+n) = \frac{2}{n^2}\frac{n(n+1)}{2} = 1 + \frac{1}{n} > 1.0$$

The weights then can be normalized as

$$w_j = \frac{w_j^*}{\sum_{l=1}^{n} w_l^*} \tag{12}$$

Zarghami and Szidarovszky [20] introduced a new method to obtain the weights by applying Equation (11) with the slight difference that j/n is replaced by $1 - b_j$. The main reason of this choice is to make the DM satisfied by the evaluations of the criteria and not only by the orders of the criteria. Thus

$$w_j^* = \frac{1}{n}\frac{dQ}{dr}\bigg|_{r=1-b_j} \tag{13}$$

where $b_1 \geq b_2 \geq ... \geq b_n$ or $(1-b_1) \leq (1-b_2) \leq ... \leq (1-b_n)$. The reason of using the term $(1-b_j)$ is due to the opposite ordering of the criteria in Equation (11) in comparison to the ordering of the b_j values in the case of RIM quantifiers. After normalizing the w_j^* values, the weights are obtained as follow:

$$w_j = \frac{Q'(1-b_j)}{\sum_{l=1}^{n} Q'(1-b_l)}. \tag{14}$$

The generating function f can be defined as the derivative of Q, so

$$w_j = \frac{f(r_j)}{\sum_{l=1}^{n} f(r_l)} = \frac{f(1-b_j)}{\sum_{l=1}^{n} f(1-b_l)} \tag{15}$$

This method of weights selection can be called Revised OWA. The weights obtained by Equation (15) satisfy all conditions of the OWA weights. The construction of the weights clearly implies the following facts.

Theorem 4.1. *The Revised OWA operator with weights (14) and with any fuzzy quantifier is a neat operator.*

Theorem 4.2. *The Revised OWA satisfies the following properties:*
1. Commutativity: the result does not depend on the ordering of the inputs.
2. Idempotency: $F(a, ..., a) = a$.
The new operator does not have the monotonicity property. An example of a non-monotone neat OWA operator is presented in the literature [13].

There is a clear difference between fuzzy quantifiers and neat OWA operators. Fuzzy quantifiers model the DM's characteristics by using linguistic quantifiers, while neat OWA operators model the features and the context of the decision making problem. Their combination will give more sense to the DM and provides more sensitive final decisions.

Example 4.3. Consider the commonly used RIM quantifiers $Q(r) = r^\alpha$ with a positive parameter α, where $f(r) = \alpha r^{\alpha-1}$. If this RIM quantifier is used in Equation (14) then we have

$$w_j = \frac{\alpha(1-b_j)^{\alpha-1}}{\sum_{l=1}^{n} \alpha(1-b_l)^{\alpha-1}} = \frac{(1-b_j)^{\alpha-1}}{\sum_{l=1}^{n}(1-b_l)^{\alpha-1}}$$

By introducing the new variable $\beta = \alpha - 1$, we have the simplified form

$$w_j = \frac{(1-b_j)^{\beta}}{\sum_{l=1}^{n}(1-b_l)^{\beta}}. \tag{16}$$

Notice that Equation (16) represents a well known family of neat OWA operators which are introduced previously by Equation (7).

Example 4.4. Assume now that the weights are obtained as

$$w_j = \frac{b_j^{\beta}}{\sum_{l=1}^{n} b_l^{\beta}}. \tag{17}$$

Then the generating function is $f(r_j) = (\beta+1)b_j^{\beta} = (\beta+1)(1-r_j)^{\beta}$, and the corresponding quantifier is

$$Q(r) = 1 - (1-r)^{\beta+1} = 1 - (1-r)^{\alpha}.$$

4.1. **Relation Between the Optimism Degree and the Parameters of the Neat Operators.** The optimism degree of the DM, θ, can be calculated by combining Equations (2) and (4) and letting n tend to infinity [15]:

$$\theta = \int_0^1 Q(r)\, dr. \tag{18}$$

By using the generating function,

$$\theta = \int_0^1 \int_0^r f(t)\, dt\, dr = \int_0^1 \int_t^1 f(t)\, dr\, dt = \int_0^1 (1-t) f(t)\, dt. \tag{19}$$

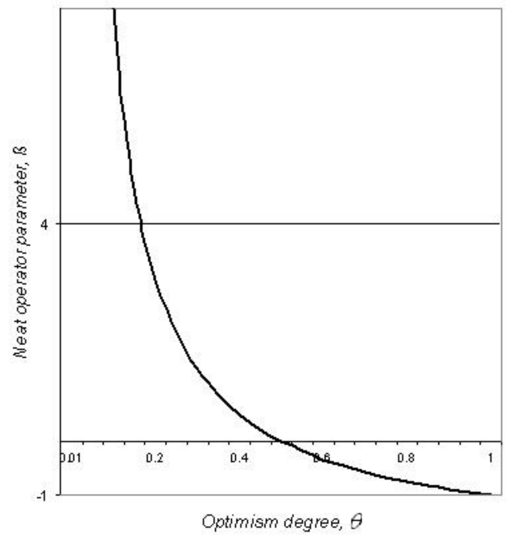

FIGURE 1. The relation between the optimism degree and the neat operator parameter

In the case of Example 4.3 we have $f(t) = (\beta + 1)t^{\beta}$, and therefore the optimism degree, θ, is as follows:

$$\theta = \int_0^1 (1-t)f(t)\,dt = \int_0^1 (1-t)(\beta+1)t^{\beta}\,dt = \frac{1}{2+\beta}. \tag{20}$$

Liu [3] has already suggested (without any reasoning) the similar relation of $\theta = 1/(1+\beta)$ for neat operators by using the weights (16), which is slightly different than the result obtained in Equation (20).

Figure 1 shows the relation between θ and β. If the DM is neutral about the decision making problem then his/her optimism degree in OWA is $\theta = 0.5$. By using Equation (20) we have $\beta = 0.0$, then Equation (17) reduces to $w_j = 1/n$ which represents the simple arithmetic average operator. If β is greater than zero, then the DM is pessimistic since $\theta < 0.5$. If β tends to infinity, then the neat OWA reduces to the min operator. If β is between -1.0 and zero, then the DM is optimistic and the neat OWA will reduce to the max operator if $\beta = -1.0$.

5. **Case Study**

Twelve water resources projects under construction are selected in the Sefidrud watershed, Northwestern region of Iran [19]. These projects are reservoirs and their water distribution networks. The DM in the watershed governing board required the most preferred alternative among these projects with respect to seven criteria. The evaluations of these projects with respect to the criteria were done by a group of experts. These numbers are multiplied by the weights of the criteria, normalized and then ordered. The resulted evaluation matrix is shown in Table 2.

	Ordered inputs						
Alternatives	b_1	b_2	b_3	b_4	b_5	b_6	b_7
$A1$	0.85	0.78	0.78	0.64	0.59	0.49	0.10
$A2$	0.85	0.78	0.70	0.65	0.59	0.59	0.48
$A3$	0.65	0.55	0.48	0.47	0.44	0.42	0.20
$A4$	0.95	0.71	0.67	0.52	0.47	0.42	0.40
$A5$	0.85	0.78	0.67	0.49	0.48	0.12	0.10
$A6$	0.82	0.80	0.70	0.56	0.46	0.39	0.28
$A7$	0.85	0.80	0.70	0.56	0.55	0.13	0.12
$A8$	0.85	0.80	0.78	0.65	0.40	0.13	0.12
$A9$	0.85	0.78	0.59	0.48	0.46	0.20	0.13
$A10$	0.85	0.78	0.48	0.47	0.46	0.20	0.13
$A11$	0.85	0.78	0.71	0.70	0.48	0.46	0.13
$A12$	0.95	0.82	0.78	0.70	0.42	0.37	0.13

Table 2. Weighted, normalized and ordered inputs of twelve alternatives

In the application of Revised OWA operator, the optimism degree of the DM has to be first determined. In this study seven RIM linguistic quantifiers of the form $Q(r) = r^\alpha$ are defined in questioning the DM how many criteria he/she wants to consider. They are shown in Table 3. The DM is considered to be more pessimistic by evaluating the projects with respect to more criteria. The quantifier of *many* of the criteria was chosen from Table 3.

Quantifier	Index of quantifier, α	Optimism degree, θ
At least one of them	$\alpha \to 0.0, (\alpha = 0.001)$	0.999
Few of them	0.1	0.909
Some of them	0.5	0.667
Half of them	1.0	0.500
Many of them	2.0	0.333
Most of them	10.0	0.091
All of them	$\alpha \to \infty, (\alpha = 1000)$	0.001

Table 3. Particular RIM quantifiers, $Q(r) = r^\alpha$ [19]

Due to the uncertainty in this selection process, we will repeat the procedure for all quantifiers. The combined goodness measures for the twelve projects are calculated by using the Revised OWA (Equation (14)). The results are shown in Figure 2.

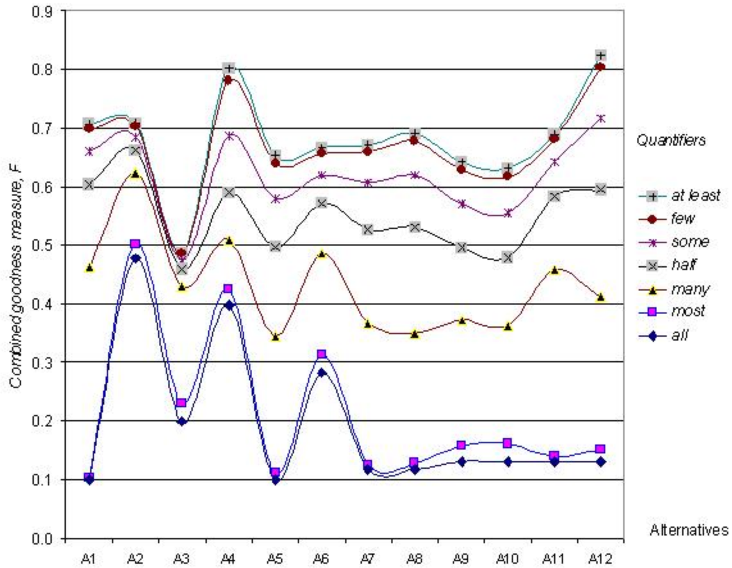

Figure 2. The combined goodness measure of the alternatives by the Revised OWA

$A2$ is the most preferred project by using the quantifiers of *all*, *most*, *many* and *half* of the criteria. For the remaining three quantifiers in the optimistic Section (*some*, *few* and *at least* of the criteria) the $A12$ is the most preferred project as it is shown in Table 4.

6. Conclusions

The original method to obtain the order weights of OWA operators with fuzzy quantifiers method is based on the comparison of the satisfaction values from two adjacent orders of the criteria. The real satisfaction of the DM however depends on the evaluations of the alternatives with respect to the criteria and not only on the orders of the criteria. The new method, Revised OWA, uses the criteria values to achieve a better characterization of the DM's satisfaction. Therefore it is a context based model in which the ordering of the initial inputs is not required since it is a neat operator. Therefore this method is more efficient in the computation of the OWA weights. Application in a real case study illustrates the advantages of the new method.

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	Fuzzy quantifiers						
Alternatives	*All*	*Most*	*Many*	*Half*	*Some*	*Few*	*At least*
A1	12	12	4	2	4	4	4
A2	1	1	1	1	3	3	3
A3	4	4	6	12	12	12	12
A4	2	2	2	4	2	2	2
A5	11	11	12	9	9	9	9
A6	3	3	3	6	7	8	8
A7	9	10	9	8	8	7	7
A8	10	9	11	7	6	6	5
A9	5	6	8	10	10	10	10
A10	6	5	10	11	11	11	11
A11	7	8	5	5	5	5	6
A12	8	7	7	3	1	1	1

Table 4. Ranks of the projects with respect to the seven different quantifiers

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