ROBUST H_∞ CONTROL FOR T–S TIME-VARYING DELAY SYSTEMS WITH NORM BOUNDED UNCERTAINTY BASED ON LMI APPROACH

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Abstract. In this paper we consider the problem of delay-dependent robust H_∞ control for uncertain fuzzy systems with time-varying delay. The Takagi–Sugeno (T–S) fuzzy model is used to describe such systems. Time-delay is assumed to have lower and upper bounds. Based on the Lyapunov-Krasovskii functional method, a sufficient condition for the existence of a robust H_∞ controller is obtained. The fuzzy state feedback gains are derived by solving pertinent LMIs. The proposed method can avoid restrictions on the derivative of the time-varying delay assumed in previous works. The effectiveness of our method is demonstrated by a numerical example.

1. Introduction

Since time delays and perturbations can be often sources of instability for a system, stabilization problems and robust control of nonlinear uncertain systems with time-delay have received considerable attention for decades [3, 5, 8, 9, 10, 13, 16, 17, 20]. Such systems occus very often in real life. Examples are electric power systems, large electric networks, rolling mill systems, economic systems, aerospace systems, several types of social systems and ecological systems. In practice, inevitable uncertainties may enter a nonlinear system in a much more complex way. The uncertainty may include modeling error, parameter perturbations, fuzzy approximation errors, and external disturbances. To the best of the authors knowledge, delay-dependent robust H_∞ control for uncertain fuzzy systems with time-varying delay have not yet been fully investigated and hence we attempt to do so in this paper.

Depending on whether the condition includes the information of delay or not, stability criteria can be classified into two types: delay-independent and delay-dependent. Delay-independent stability conditions are independent of the size of the delays. They can be used to study systems without any information on the time delays [13, 20]. Generally speaking, delay-dependent results are less conservative than those for the delay-independent case, especially for time-delay systems with small delay. But in [1, 6, 9], the stability conditions require that the upper bound of derivative of the time-varying delay be less than 1. Our results do not need this restriction.

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The Takagi–Sugeno (T–S) fuzzy model [12, 15] can represent nonlinear systems using fuzzy rules with consequent part as local linear subsystems. This kind of model provides an effective representation of complex nonlinear systems. When the nonlinear plant is represented by a so-called T–S type fuzzy model, local dynamics in different state-space regions are represented by linear models and the overall model of the system is achieved by fuzzy "blending" of these fuzzy models. The control design is carried out based on the fuzzy model via the so-called parallel distributed compensation (PDC) scheme [15].

Linear matrix inequality (LMI) theory is a useful mathematical tool for optimization problems [2, 4, 7, 11]. Many control problems can be converted to either a feasible problem of an LMI system, or a convex optimization problem which has the LMI restriction.

In this paper, we consider the problem of delay-dependent robust H_∞ control for uncertain fuzzy systems with time-varying delay. First we study the stability and stabilization conditions of the closed-loop fuzzy system with no disturbance. Then we consider the performance index $J(\omega)$ and derive a sufficient condition for the existence of the delay-dependent robust H_∞ controller by the Lyapunov-Krasovskii functional method. Now, introducing free-weighting matrices and solving linear matrix inequalities, state feedback gains can be obtained. Finally, we illustrate the effectiveness of method by a numerical example.

The major contribution of this work is as follows. First, it gives a new method to design the robust H_∞ controller for uncertain fuzzy systems with time-varying delay. Second, it presents time-dependent results which can be used to determine the upper bound of the time-delay by using convex optimization to guarantee robust H_∞ fuzzy stablizable systems for all time-delays. These results are less conservative than those for the delay-independent case. Third, it is able to treat systems with no requirement regarding the information of the derivative of the time-delay; i.e. our method allows for fast time-varying delay.

The paper is organized as follows. In Section 2, we present a T–S fuzzy model for an uncertain system with time-varying delay and state some assumptions. In Section 3, we derive the existence condition of a delay-dependent robust H_∞ controller in LMI form using the Lyapunov-Krasovskii approach. In Section 4, we give a numerical example to demonstrate the results. Section 5 concludes the paper.

Notation. For a symmetric matrix X, $X > 0$ means that X is positive definite. I is an identity matrix of appropriate dimension and X^T denotes the transpose of X. For any nonsingular matrix X, X^{-1} denotes the inverse of X. R^n denotes the n-dimensional Euclidean space and $R^{m\times n}$ is the set of all $m \times n$ matrices. $L_2[0,\infty)$ refers to the space of square summable infinite vector sequences. $\|\cdot\|_2$ stands for the usual $L_2[0,\infty)$ norm. $*$ denotes the transposed element in the symmetric position of a matrix.

2. Problem Formulation and Assumptions

Consider the following parameter uncertain system with time-varying delay described by the Takagi–Sugeno fuzzy model. The ith rule of the model is

Plant Rule i:

If $z_1(t)$ is M_{i1}, $z_2(t)$ is M_{i2},$\cdots$, $z_g(t)$ is M_{ig},

Then

$$\begin{cases} \dot{x}(t) = & (A_{i1} + \Delta A_{i1}(t))x(t) + (A_{i2} + \Delta A_{i2}(t))x(t-h(t)) \\ & +(B_i + \Delta B_i(t))u(t) + B_{\omega i}\omega(t), \\ \tilde{z}(t) = & C_i x(t) + D_i u(t), \\ x(t) = & \varphi(t), \qquad t \in [-h_M, 0], \end{cases} \tag{1}$$

where $i = 1, 2, \cdots, n$. n is the number of rules, $z_1(t), z_2(t), \cdots, z_g(t)$ are the premise variables and $M_{ij}(i = 1, 2, \cdots, n, j = 1, 2, \cdots, g)$ is the fuzzy set, $x(t) \in R^l$ is the state vector, $u(t) \in R^m$ is the input vector, $\omega(t)$ is the disturbance which belongs to $L_2[0, \infty)$, $\tilde{z}(t) \in R^p$ is the controlled output. $A_{i1}, A_{i2}, B_i, B_{\omega i}, C_i$ and D_i $(i = 1, 2, \cdots, n)$ are constant matrices of appropriate dimensions, $\varphi(t)$ is the initial condition of system (1), h_M is the upper bound of time-delay $h(t)$ and $\Delta A_{i1}(t), \Delta A_{i2}(t), \Delta B_i(t)$ $(i = 1, 2, \cdots, n)$ are unknown matrices representing time-varying parameter uncertainties of (1) and satisfying the following assumption.

Assumption 2.1.

$$[\Delta A_{i1}(t), \Delta A_{i2}(t), \Delta B_i(t)] = U_i F_i(t)[E_{i1}, E_{i2}, E_i], \tag{2}$$

where U_i, E_{i1}, E_{i2} and E_i $(i = 1, 2, \cdots, n)$ are known real constant matrices of appropriate dimensions. $F_i(t)$ $(i = 1, 2, \cdots, n)$ is an unknown real time-varying matrix with Lebesgue measurable elements satisfying

$$F_i(t)^T F_i(t) \leq I, \quad i = 1, 2, \cdots, n. \tag{3}$$

Let $\mu_i(z(t))$ be the normalized membership function of the inferred fuzzy set $\rho_i(z(t))$, i.e.

$$\mu_i(z(t)) = \frac{\rho_i(z(t))}{\sum_{i=1}^{n} \rho_i(z(t))},$$

where $z(t) = [z_1(t), z_2(t), \cdots, z_g(t)]$, $\rho_i(z(t)) = \prod_{j=1}^{g} M_{ij}(z_j(t))$. $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. It is assumed that

$$\rho_i(z(t)) \geq 0, \quad i = 1, 2, \cdots, n, \quad \sum_{i=1}^{n} \rho_i(z(t)) > 0, \quad \forall t \geq 0.$$

Then, it can be easily shown that

$$\mu_i(z(t)) \geq 0, \quad i = 1, 2, \cdots, n, \quad \sum_{i=1}^{n} \mu_i(z(t)) = 1, \quad \forall t \geq 0.$$

By using the center-average defuzzifier, product inference and a singleton fuzzifier, the T–S fuzzy model (1) can be expressed by the following global model

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{n} \mu_i(z(t))[\tilde{A}_{i1}x(t) + \tilde{A}_{i2}x(t-h(t)) + \tilde{B}_i u(t) + B_{\omega i}\omega(t)], \\ \tilde{z}(t) = \sum\limits_{i=1}^{n} \mu_i(z(t))[C_i x(t) + D_i u(t)], \\ x(t) = \varphi(t), \qquad t \in [-h_M, 0], \end{cases} \tag{4}$$

where $\tilde{A}_{i1} \triangleq A_{i1}+\Delta A_{i1}(t),\ \tilde{A}_{i2} \triangleq A_{i2}+\Delta A_{i2}(t),\ \tilde{B}_i \triangleq B_i+\Delta B_i(t),\ i = 1, 2, \cdots, n.$

In this paper, a delay-dependent state feedback T–S fuzzy-model-based H_∞ controller will be designed for the robust stabilization of the T–S fuzzy system (4). The ith controller rule is

$$\begin{aligned} R^i: \quad &\text{If} \quad z_1(t) \text{ is } M_{i1},\ z_2(t) \text{ is } M_{i2}, \cdots,\ z_g(t) \text{ is } M_{ig}, \\ &\text{Then} \quad u(t) = K_i x(t), \end{aligned} \tag{5}$$

where K_i $(i = 1, 2, \cdots, n)$ are the controller gains of (5) to be determined. The defuzzified output of the controller rule is given by

$$u(t) = \sum_{i=1}^{n} \mu_i(z(t)) K_i x(t). \tag{6}$$

Combining (4) and (6) , the following closed-loop fuzzy system can be obtained.

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} \mu_i(z(t))\mu_j(z(t))[(\tilde{A}_{i1} + \tilde{B}_i K_j)x(t) + \tilde{A}_{i2}x(t-h(t)) + B_{\omega i}\omega(t)], \\ \tilde{z}(t) = \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} \mu_i(z(t))\mu_j(z(t))[(C_i + D_i K_j)x(t)], \\ x(t) = \varphi(t), \qquad t \in [-h_M, 0]. \end{cases} \tag{7}$$

Assumption 2.2. $h(t)$ is a uniformly continuous time-varying function satisfying

$$0 \le h_m \le h(t) \le h_M. \tag{8}$$

In this paper, we define

$$\alpha = \frac{1}{2}(h_M + h_m), \qquad \beta = \frac{1}{2}(h_M - h_m). \tag{9}$$

Then, $h(t)$ is a function belonging to the interval $[\alpha - \beta,\ \alpha + \beta]$, where β can be taken as the range of variation of the time-varying delay $h(t)$. When $\beta = 0$, $h(t)$ denotes a constant delay.

Definition 2.3. For a prescribed scalar $\gamma > 0$, we define the performance index

$$J(\omega) \triangleq \int_0^\infty [\tilde{z}^T(\theta)\tilde{z}(\theta) - \gamma^2\omega^T(\theta)\omega(\theta)]\mathrm{d}\theta. \tag{10}$$

Remark 2.4. The purpose of this paper is to design a delay-dependent robust H_∞ controller (6) for the T–S global model (4) such that for all admissible uncertainties satisfying (2), (3), and $h(t)$ satisfying (8) for a prescribed scalar $\gamma > 0$,

(I) The closed-loop fuzzy system (7) is asymptotically stable when $\omega(t) = 0$;
(II) The closed-loop fuzzy system (7) satisfies $\|\tilde{z}(t)\|_2 < \gamma\|\omega(t)\|_2$ for all nonzero $\omega(t) \in L_2[0, \infty]$ under the zero initial condition.

3. Main Results

In this section, we present a method to design the robust H_∞ controller for uncertain systems with time-varying delay based on the Lyapunov-Krasovskii approach.

First, we state four lemmas which are the key to proving the main theorem of our paper.

Lemma 3.1. [14] *For any two matrices X and Y, we have*

$$X^TY + Y^TX \leq \varepsilon X^TX + \varepsilon^{-1}Y^TY,$$

where $X \in R^{m\times n}$, $Y \in R^{m\times n}$, and ε is any positive constant.

Lemma 3.2. [18] *If Y, U, and E are matrices of appropriate dimensions, and $Y = Y^T$, then for any matrix F satisfying $F^TF \leq I$,*

$$Y + UFE + E^TF^TU^T < 0$$

if and only if there exists a constant $\varepsilon > 0$ satisfying

$$Y + \varepsilon UU^T + \varepsilon^{-1}E^TE < 0.$$

Lemma 3.3. [19] *For any matrices $R_1 > 0$, $R_2 > 0$, N and T of appropriate dimensions, we have*

$$[a] \quad -2\xi^T(t)N\int_{t-\alpha}^{t}\dot{x}(s)ds \leq \alpha\xi^T(t)NR_1^{-1}N^T\xi(t) + \int_{t-\alpha}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds,$$

$$\begin{aligned}
[b] -2\xi^T(t)T\int_{t-h(t)}^{t-\alpha}\dot{x}(s)ds &= 2\xi^T(t)T\int_{t-\alpha}^{t-h(t)}\dot{x}(s)ds \\
(as\ \ h(t) \leq \alpha) \qquad &\leq \beta\xi^T(t)TR_2^{-1}T^T\xi(t) + \int_{t-\alpha}^{t-h(t)}\dot{x}^T(s)R_2\dot{x}(s)ds \\
&\leq \beta\xi^T(t)TR_2^{-1}T^T\xi(t) + \int_{t-\alpha-\beta}^{t-\alpha+\beta}\dot{x}^T(s)R_2\dot{x}(s)ds, \\
-2\xi^T(t)T\int_{t-h(t)}^{t-\alpha}\dot{x}(s)ds &\leq \beta\xi^T(t)TR_2^{-1}T^T\xi(t) + \int_{t-h(t)}^{t-\alpha}\dot{x}^T(s)R_2\dot{x}ds \\
(as\ \ h(t) \geq \alpha) \qquad &\leq \beta\xi^T(t)TR_2^{-1}T^T\xi(t) + \int_{t-\alpha-\beta}^{t-\alpha+\beta}\dot{x}^T(s)R_2\dot{x}(s)ds.
\end{aligned}$$

Lemma 3.4. *(Schur complements) For a symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, the following conditions are equivalent:*

[a] $S < 0$; [b] $S_{11} < 0$, $S_{11} - S_{12}^TS_{22}^{-1}S_{12} < 0$; [c] $S_{22} < 0$, $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$.

For the purpose given in Remark 2.4, our first result is with respect to the stability of system (7) with no disturbance. For simplicity, let $\mu_i = \mu_i(z(t))$.

Theorem 3.5. *Suppose $\omega(t)=0$, for given scalars $h_m \geq 0$, $h_M \geq 0$ and matrix K_j. If there exist matrices $P>0$, $Q>0$, $R_1>0$, $R_2>0$, N_k, M_k, T_k $(i,j=1,2,\cdots,n,\ k=1,2,3,4)$ of appropriate dimensions such that*

$$\begin{bmatrix} \Omega_{11}^{ii} & * \\ \Omega_{21} & \Omega_{22} \end{bmatrix} < 0, \qquad 1 \leq i \leq n, \tag{11}$$

$$\begin{bmatrix} \Omega_{11}^{ij}+\Omega_{11}^{ji} & * \\ \Omega_{21} & \frac{1}{2}\Omega_{22} \end{bmatrix} < 0, \qquad 1 \leq i < j \leq n, \tag{12}$$

then system (7) is asymptotically stable, where

$$\Omega_{11}^{ij} = \begin{bmatrix} H_{11}^{ij} & * & * & * \\ H_{21}^{ij} & H_{22}^{ij} & * & * \\ H_{31}^{ij} & H_{32}^{ij} & H_{33}^{ij} & * \\ H_{41}^{ij} & H_{42}^{ij} & H_{43}^{ij} & H_{44}^{ij} \end{bmatrix},$$

$$\Omega_{21} = \begin{bmatrix} \alpha N_1^T & \alpha N_2^T & \alpha N_3^T & \alpha N_4^T \\ \beta T_1^T & \beta T_2^T & \beta T_3^T & \beta T_4^T \end{bmatrix}, \; \Omega_{22} = \begin{bmatrix} -\alpha R_1 & * \\ 0 & -\beta R_2 \end{bmatrix},$$

$$\begin{aligned}
H_{11}^{ij} &= Q + N_1 + N_1^T + M_1\tilde{A}_{i1} + M_1\tilde{B}_iK_j + \tilde{A}_{i1}^T M_1^T + K_j^T\tilde{B}_i^T M_1^T, \\
H_{21}^{ij} &= N_2 - T_1^T + M_2\tilde{A}_{i1} + M_2\tilde{B}_iK_j + \tilde{A}_{i2}^T M_1^T, \\
H_{22}^{ij} &= -T_2 - T_2^T + M_2\tilde{A}_{i2} + \tilde{A}_{i2}^T M_2^T, \\
H_{31}^{ij} &= -N_1^T + T_1^T + N_3 + M_3\tilde{A}_{i1} + M_3\tilde{B}_iK_j, \\
H_{32}^{ij} &= -N_2^T + T_2^T - T_3 + M_3\tilde{A}_{i2}, \\
H_{33}^{ij} &= -Q - N_3^T - N_3 + T_3^T + T_3, \\
H_{41}^{ij} &= P + N_4 + M_4\tilde{A}_{i1} + M_4\tilde{B}_iK_j - M_1^T, \\
H_{42}^{ij} &= -T_4 + M_4\tilde{A}_{i2} - M_2^T, \\
H_{43}^{ij} &= -N_4 + T_4 - M_3^T, \\
H_{44}^{ij} &= \alpha R_1 + 2\beta R_2 - M_4 - M_4^T, \qquad 1 \leq i \leq j \leq n.
\end{aligned}$$

Proof. Using the Newton-Leibniz formula, we have

$$x(t) - x(t-\alpha) - \int_{t-\alpha}^{t} \dot{x}(s)\mathrm{d}s = 0,$$

$$x(t-\alpha) - x(t-h(t)) - \int_{t-h(t)}^{t-\alpha} \dot{x}(s)\mathrm{d}s = 0.$$

From (7) we have

$$\sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i\mu_j[(\tilde{A}_{i1} + \tilde{B}_iK_j)x(t) + \tilde{A}_{i2}x(t-h(t)) + B_{\omega i}\omega(t) - \dot{x}(t)] = 0.$$

Based on the equalities mentioned above, for any matrices N_k, M_k and T_k ($i = 1, 2, 3, 4$) of appropriate dimensions, we have

$$\xi^T(t)N\{\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j[x(t) - x(t-\alpha) - \int_{t-\alpha}^{t}\dot{x}(s)\mathrm{d}s]\} = 0, \tag{13}$$

$$\xi^T(t)T\{\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j[x(t-\alpha) - x(t-h(t)) - \int_{t-h(t)}^{t-\alpha}\dot{x}(s)\mathrm{d}s]\} = 0, \tag{14}$$

$$\xi^T(t)M\{\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j[(\tilde{A}_{i1} + \tilde{B}_iK_j)x(t) + \tilde{A}_{i2}x(t-h(t)) + B_{\omega i}\omega(t) - \dot{x}(t)]\} = 0, \tag{15}$$

where $\xi^T(t) = [x^T(t)\ x^T(t-h(t))\ x^T(t-\alpha)\ \dot{x}^T(t)]$, $N^T = [N_1^T\ N_2^T\ N_3^T\ N_4^T]$, $M^T = [M_1^T\ M_2^T\ M_3^T\ M_4^T]$, $T^T = [T_1^T\ T_2^T\ T_3^T\ T_4^T]$.

We first consider the case when $\omega(t) \neq 0$. Choose the Lyapunov-Krasovskii functional

$$\begin{aligned} V(x(t)) =& x^T(t)Px(t) + \int_{t-\alpha}^{t} x^T(s)Qx(s)\mathrm{d}s + \int_{t-\alpha}^{t}\int_{s}^{t}\dot{x}^T(\nu)R_1\dot{x}(\nu)\mathrm{d}\nu\mathrm{d}s \\ &+ \int_{-\alpha-\beta}^{-\alpha+\beta}\int_{t+s}^{t}\dot{x}^T(\nu)R_2\dot{x}(\nu)\mathrm{d}\nu\mathrm{d}s, \end{aligned} \tag{16}$$

where $P > 0$, $Q > 0$, $R_1 > 0$, $R_2 > 0$.

Combining (13)-(15), using Lemma 3.3 and taking the derivative of $V(x(t))$ with respect to t along the trajectory of (7), we have

$$\begin{aligned} \dot{V}(x(t)) =& 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-\alpha)Qx(t-\alpha) \\ &+ \dot{x}^T(t)(\alpha R_1 + 2\beta R_2)\dot{x}(t) - \int_{t-\alpha}^{t}\dot{x}^T(s)R_1\dot{x}(s)\mathrm{d}s - \int_{t-\alpha-\beta}^{t-\alpha+\beta}\dot{x}(s)R_2\dot{x}(s)\mathrm{d}s \\ =& 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-\alpha)Qx(t-\alpha) \\ &+ \dot{x}^T(t)(\alpha R_1 + 2\beta R_2)\dot{x}(t) - \int_{t-\alpha}^{t}\dot{x}^T(s)R_1\dot{x}(s)\mathrm{d}s - \int_{t-\alpha-\beta}^{t-\alpha+\beta}\dot{x}(s)R_2\dot{x}(s)\mathrm{d}s \\ &+ 2\xi^T(t)N\{\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j[x(t) - x(t-\alpha) - \int_{t-\alpha}^{t}\dot{x}(s)\mathrm{d}s]\} \\ &+ 2\xi^T(t)T\{\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j[x(t-\alpha) - x(t-h(t)) - \int_{t-h(t)}^{t-\alpha}\dot{x}(s)\mathrm{d}s]\} \\ &+ 2\xi^T(t)M\{\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j[(\tilde{A}_{i1} + \tilde{B}_iK_j)x(t) + \tilde{A}_{i2}x(t-h(t)) \\ &\ + B_{\omega i}\omega(t) - \dot{x}(t)]\} \\ \leq& 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) + \dot{x}^T(t)(\alpha R_1 + 2\beta R_2)\dot{x}(t) - x^T(t-\alpha)Qx(t-\alpha) \end{aligned}$$

$$
\begin{aligned}
&+2\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j\xi^T(t)N[x(t)-x(t-\alpha)]\\
&+2\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j\xi^T(t)T[x(t-\alpha)-x(t-h(t))]\\
&+2\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j\xi^T(t)M[(\tilde{A}_{i1}+\tilde{B}_iK_j)x(t)+\tilde{A}_{i2}x(t-h(t))+B_{\omega i}\omega(t)\\
&-\dot{x}(t)]+\alpha\xi^T(t)NR_1^{-1}N^T\xi(t)+\beta\xi^T(t)TR_2^{-1}T^T\xi(t)\\
=&\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j\{\zeta^T(t)\Theta_{11}^{ij}\zeta(t)+\zeta^T(t)\begin{bmatrix}\alpha NR_1^{-1}N^T+\beta TR_2^{-1}T^T & *\\ 0 & 0\end{bmatrix}\zeta(t)\}\\
=&\sum_{i=1}^{n}\mu_i^2\{\zeta^T(t)\Theta_{11}^{ii}\zeta(t)+\zeta^T(t)\begin{bmatrix}\alpha NR_1^{-1}N^T+\beta TR_2^{-1}T^T & *\\ 0 & 0\end{bmatrix}\zeta(t)\}\\
&+\sum_{i=1}^{n-1}\sum_{j>i}^{n}\mu_i\mu_j\{\zeta^T(t)[\Theta_{11}^{ij}+\Theta_{11}^{ji}]\zeta(t)\\
&+\zeta^T(t)\begin{bmatrix}2\alpha NR_1^{-1}N^T+2\beta TR_2^{-1}T^T & *\\ 0 & 0\end{bmatrix}\zeta(t)\},
\end{aligned}
$$

where

$$
\zeta^T(t)=[\xi^T(t)\ \ \omega^T(t)],\qquad \Theta_{11}^{ij}=\begin{bmatrix}\Omega_{11}^{ij} & *\\ (2,1) & 0\end{bmatrix},\qquad (2,1)=[H_{51}^{ij}\ \ H_{52}^{ij}\ \ H_{53}^{ij}\ \ H_{54}^{ij}],
$$

$$
H_{51}^{ij}=B_{\omega i}^TM_1^T,\quad H_{52}^{ij}=B_{\omega i}^TM_2^T,\quad H_{53}^{ij}=B_{\omega i}^TM_3^T,\quad H_{54}^{ij}=B_{\omega i}^TM_4^T.
$$

If $\omega(t)=0$ the result is proved using Schur complements. □

Since $J(\omega)$ plays an important role in designing a robust H_∞ controller for uncertain systems with time-varying delay, in the next theorem we consider the performance index (10) of system (7) .

Theorem 3.6. *$J(\omega)<0$, if*

$$
\begin{bmatrix}\Theta_{11}^{ii}+\Gamma^{ii} & *\\ \Theta_{21} & \Theta_{22}\end{bmatrix}<0,\qquad 1\le i\le n, \tag{17}
$$

$$
\begin{bmatrix}\Theta_{11}^{ij}+\Theta_{11}^{ji}+\Gamma^{ij}+\Gamma^{ji} & *\\ \Theta_{21} & \frac{1}{2}\Theta_{22}\end{bmatrix}<0,\qquad 1\le i<j\le n, \tag{18}
$$

where $\Gamma^{ij}=diag\{(C_i+D_iK_j)^T(C_i+D_iK_j)\ \ 0\ \ 0\ \ 0\ \ -\gamma^2I\}$, $\Theta_{21}=[\Omega_{21}\quad 0]$, $\Theta_{22}=\Omega_{22}$.

Proof. Assume that $\varphi(t) = 0, \quad t \in [-h_M, 0]$. From the proof of Theorem 3.5, we have

$$\begin{aligned} J(\omega) &= \int_0^\infty [\tilde{z}^T(\theta)\tilde{z}(\theta) - \gamma^2\omega^T(\theta)\omega(\theta) + \dot{V}(x(\theta))]\mathrm{d}\theta - V(x(\infty)) \\ &\le \int_0^\infty [\tilde{z}^T(\theta)\tilde{z}(\theta) - \gamma^2\omega^T(\theta)\omega(\theta) + \dot{V}(x(\theta))]\mathrm{d}\theta \\ &\le \int_0^\infty \sum_{i=1}^n \sum_{j=1}^n \mu_i\mu_j\{\zeta^T(\theta)[\Theta_{11}^{ij} + \Gamma^{ij}]\zeta(\theta) \\ &\quad + \zeta^T(\theta)\begin{bmatrix} \alpha N R_1^{-1} N^T + \beta T R_2^{-1} T^T & * \\ 0 & 0 \end{bmatrix}\zeta(\theta)\}\mathrm{d}\theta \\ &= \int_0^\infty \sum_{i=1}^n \mu_i^2\{\zeta^T(\theta)[\Theta_{11}^{ii} + \Gamma^{ii}]\zeta(\theta) \\ &\quad + \zeta^T(\theta)\begin{bmatrix} \alpha N R_1^{-1} N^T + \beta T R_2^{-1} T^T & * \\ 0 & 0 \end{bmatrix}\zeta(\theta)\}\mathrm{d}\theta \\ &\quad + \int_0^\infty \sum_{i=1}^{n-1} \sum_{j>i}^n \mu_i\mu_j\{\zeta^T(\theta)[\Theta_{11}^{ij} + \Theta_{11}^{ji} + \Gamma^{ij} + \Gamma^{ji}]\zeta(\theta) \\ &\quad + \zeta^T(\theta)\begin{bmatrix} 2\alpha N R_1^{-1} N^T + 2\beta T R_2^{-1} T^T & * \\ 0 & 0 \end{bmatrix}\zeta(\theta)\}\mathrm{d}\theta. \end{aligned}$$

Using Schur complements, we can prove that $J(\omega) < 0$ when inequalities (17), (18) hold. □

Remark 3.7. It is easy to see that (17) implies (11), and (18) implies (12).

The parameter uncertainties $\Delta A_{i1}(t)$, $\Delta A_{i2}(t)$, $\Delta B_i(t)$ are contained in (17) and (18). So Theorem 3.6 cannot be directly used to determine whether $J(\omega)$ is less than zero. The following theorem is concerned with these uncertainties and provides a sufficient condition for $J(\omega) < 0$.

Theorem 3.8. *$J(\omega) < 0$, if*

$$\begin{bmatrix} \hat{\Theta}_{11}^{ii} + \Gamma^{ii} & * & * \\ \Theta_{21} & \Theta_{22} & * \\ \Theta_{31}^i & 0 & \Theta_{33}^{ii} \end{bmatrix} < 0, \qquad 1 \le i \le n, \tag{19}$$

$$\begin{bmatrix} \hat{\Theta}_{11}^{ij} + \hat{\Theta}_{11}^{ji} + \Gamma^{ij} + \Gamma^{ji} & * & * & * \\ \Theta_{21} & \frac{1}{2}\Theta_{22} & * & * \\ \Theta_{31}^i & 0 & \Theta_{33}^{ij} & * \\ \Theta_{31}^j & 0 & 0 & \Theta_{33}^{ji} \end{bmatrix} < 0, \qquad 1 \le i < j \le n, \tag{20}$$

where

$$\hat{\Theta}_{11}^{ij} = \begin{bmatrix} \hat{\Omega}_{11}^{ij} & * \\ (2,1) & 0 \end{bmatrix}, \quad \Theta_{31}^{i} = \begin{bmatrix} U_i^T M_1^T & U_i^T M_2^T & U_i^T M_3^T & U_i^T M_4^T & 0 \\ U_i^T M_1^T & U_i^T M_2^T & U_i^T M_3^T & U_i^T M_4^T & 0 \\ U_i^T M_1^T & U_i^T M_2^T & U_i^T M_3^T & U_i^T M_4^T & 0 \end{bmatrix},$$
$$\Theta_{33}^{ij} = diag\{-\varepsilon_{ij} I \quad -\varepsilon_{ij} I \quad -\varepsilon_{ij} I\},$$

Θ_{21} *and* Θ_{22} *are as given before and*

$$\hat{\Omega_{11}^{ij}} = \begin{bmatrix} \hat{H}_{11}^{ij} & * & * & * \\ \hat{H}_{21}^{ij} & \hat{H}_{22}^{ij} & * & * \\ \hat{H}_{31}^{ij} & \hat{H}_{32}^{ij} & \hat{H}_{33}^{ij} & * \\ \hat{H}_{41}^{ij} & \hat{H}_{42}^{ij} & \hat{H}_{43}^{ij} & \hat{H}_{44}^{ij} \end{bmatrix},$$
$$\hat{H}_{11}^{ij} = Q + N_1 + N_1^T + M_1 A_{i1} + M_1 B_i K_j + A_{i1}^T M_1^T + K_j^T B_i^T M_1^T + \varepsilon_{ij} E_{i1}^T E_{i1}$$
$$+ \varepsilon_{ij} K_j^T E_i^T E_i K_j,$$
$$\hat{H}_{21}^{ij} = N_2 - T_1^T + M_2 A_{i1} + M_2 B_i K_j + A_{i2}^T M_1^T,$$
$$\hat{H}_{22}^{ij} = -T_2 - T_2^T + M_2 A_{i2} + A_{i2}^T M_2^T + \varepsilon_{ij} E_{i2}^T E_{i2},$$
$$\hat{H}_{31}^{ij} = -N_1^T + T_1^T + N_3 + M_3 A_{i1} + M_3 B_i K_j,$$
$$\hat{H}_{32}^{ij} = -N_2^T + T_2^T - T_3 + M_3 A_{i2},$$
$$\hat{H}_{33}^{ij} = -Q - N_3^T - N_3 + T_3 + T_3^T,$$
$$\hat{H}_{41}^{ij} = P + N_4 + M_4 A_{i1} + M_4 B_i K_j - M_1^T,$$
$$\hat{H}_{42}^{ij} = -T_4 + M_4 A_{i2} - M_2^T,$$
$$\hat{H}_{43}^{ij} = -N_4 + T_4 - M_3^T,$$
$$\hat{H}_{44}^{ij} = \alpha R_1 + 2\beta R_2 - M_4 - M_4^T.$$

Proof. Replacing $\tilde{A}_{i1}$, $\tilde{A}_{i2}$ and $\tilde{B}_i$ with $A_{i1} + U_i F_i(t) E_{i1}$, $A_{i2} + U_i F_i(t) E_{i2}$ and $B_i + U_i F_i(t) E_i$ in (17) (18), respectively, in terms of (2), (3) and Lemma 3.2 and using Schur complements, we can obtain (19) and (20). □

Since (19) and (20) are not LMIs, we need to convert them into LMIs before using the MATLAB LMI Toolbox to solve them . The following theorem gives a sufficient condition for the existence of a robust H_∞ controller and the state feedback controller gains are derived by solving pertinent LMIs.

Theorem 3.9. *For a prescribed scalar* $\gamma > 0$ *and given scalars* $\delta_l (l = 2, 3, 4)$, $\delta_4 \neq 0$, $h_m \geq 0$ *and* $h_M \geq 0$, *system (7) is stable and satisfies* $\|\tilde{z}(t)\|_2 < \gamma \|\omega(t)\|_2$ *for all nonzero* $\omega(t) \in L_2[0, \infty)$ *and any* $h(t)$ *satisfying (8), if there exist* $\tilde{P} > 0$, $\tilde{Q} > 0$, $\tilde{R}_1 > 0$, $\tilde{R}_2 > 0$, X, Y_j $(j = 1, 2, \cdots, n)$, $\tilde{N}_k$ *and* $\tilde{T}_k$ $(k = 1, 2, 3, 4)$ *of appropriate dimensions and positive constant* η_{ij} *such that the following LMIs simultaneously* *hold.*

$$\begin{bmatrix} \Xi_{11}^{ii} & * & * & * \\ \Xi_{21}^{ii} & \Xi_{22} & * & * \\ \Xi_{31} & 0 & \Xi_{33} & * \\ \Xi_{41}^{ii} & 0 & 0 & \Xi_{44}^{ii} \end{bmatrix} < 0, \qquad 1 \le i \le n, \tag{21}$$

$$\begin{bmatrix} \Xi_{11}^{ij} + \Xi_{11}^{ji} & * & * & * & * & * \\ \Xi_{21}^{ij} & \Xi_{22} & * & * & * & * \\ \Xi_{21}^{ji} & 0 & \Xi_{22} & * & * & * \\ \Xi_{31} & 0 & 0 & \frac{1}{2}\Xi_{33} & * & * \\ \Xi_{41}^{ij} & 0 & 0 & 0 & \Xi_{44}^{ij} & * \\ \Xi_{41}^{ji} & 0 & 0 & 0 & 0 & \Xi_{44}^{ji} \end{bmatrix} < 0, \qquad 1 \le i < j \le n, \tag{22}$$

where

$$\Xi_{11}^{ij} = \begin{bmatrix} \Pi_{11}^{ij} & * & * & * & * \\ \Pi_{21}^{ij} & \Pi_{22}^{ij} & * & * & * \\ \Pi_{31}^{ij} & \Pi_{32}^{ij} & \Pi_{33}^{ij} & * & * \\ \Pi_{41}^{ij} & \Pi_{42}^{ij} & \Pi_{43}^{ij} & \Pi_{44}^{ij} & * \\ \Pi_{51}^{ij} & \Pi_{52}^{ij} & \Pi_{53}^{ij} & \Pi_{54}^{ij} & \Pi_{55}^{ij} \end{bmatrix}, \quad \Xi_{21}^{ij} = [C_i X^T + D_i Y_j \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\Xi_{22} = -I, \quad \Xi_{31} = \begin{bmatrix} \alpha\tilde{N}_1^T & \alpha\tilde{N}_2^T & \alpha\tilde{N}_3^T & \alpha\tilde{N}_4^T & 0 \\ \beta\tilde{T}_1^T & \beta\tilde{T}_2^T & \beta\tilde{T}_3^T & \beta\tilde{T}_4^T & 0 \end{bmatrix}, \quad \Xi_{33} = \begin{bmatrix} -\alpha\tilde{R}_1 & * \\ 0 & -\beta\tilde{R}_2 \end{bmatrix},$$

$$\Xi_{41}^{ij} = \begin{bmatrix} E_{i1}X^T & 0 & 0 & 0 & 0 \\ 0 & E_{i2}X^T & 0 & 0 & 0 \\ E_i Y_j & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \Xi_{44} = diag\{-\eta_{ij}I \quad -\eta_{ij}I \quad -\eta_{ij}I\},$$

$$\Pi_{11}^{ij} = \tilde{Q} + \tilde{N}_1 + \tilde{N}_1^T + A_{i1}X^T + B_iY_j + XA_{i1}^T + Y_j^T B_i^T + 3\eta_{ij}U_iU_i^T,$$

$$\Pi_{21}^{ij} = \tilde{N}_2 - \tilde{T}_1 + \delta_2 A_{i1}X^T + \delta_2 B_i Y_j + XA_{i2}^T + 3\eta_{ij}\delta_2 U_i U_i^T,$$

$$\Pi_{22}^{ij} = -\tilde{T}_2 - \tilde{T}_2^T + \delta_2 A_{i2}X^T + \delta_2 XA_{i2}^T + 3\eta_{ij}\delta_2^2 U_iU_i^T,$$

$$\Pi_{31}^{ij} = -\tilde{N}_1^T + \tilde{T}_1^T + \tilde{N}_3 + \delta_3 A_{i1}X^T + \delta_3 B_i Y_j + 3\eta_{ij}\delta_3 U_iU_i^T,$$

$$\Pi_{32}^{ij} = -\tilde{N}_2^T + \tilde{T}_2^T - \tilde{T}_3 + \delta_3 A_{i2}X^T + 3\eta_{ij}\delta_2\delta_3 U_iU_i^T,$$

$$\Pi_{33}^{ij} = -\tilde{Q} - \tilde{N}_3^T - \tilde{N}_3 + \tilde{T}_3 + \tilde{T}_3^T + 3\eta_{ij}\delta_3^2 U_iU_i^T,$$

$$\Pi_{41}^{ij} = \tilde{P} + \tilde{N}_4 + \delta_4 A_{i1}X^T + \delta_4 B_i Y_j - X + 3\eta_{ij}\delta_4 U_iU_i^T,$$

$$\Pi_{42}^{ij} = -\tilde{T}_4 + \delta_4 A_{i2}X^T - \delta_2 X + 3\eta_{ij}\delta_2\delta_4 U_iU_i^T,$$

$$\Pi_{43}^{ij} = -\tilde{N}_4 + \tilde{T}_4 - \delta_3 X + 3\eta_{ij}\delta_3\delta_4 U_iU_i^T,$$

$$\Pi_{44}^{ij} = \alpha\tilde{R}_1 + 2\beta\tilde{R}_2 - \delta_4 X^T - \delta_4 X + 3\eta_{ij}\delta_4^2 U_iU_i^T,$$

$$\Pi_{51}^{ij} = B_{\omega i}^T, \quad \Pi_{52}^{ij} = \delta_2 B_{\omega i}^T, \quad \Pi_{53}^{ij} = \delta_3 B_{\omega i}^T, \quad \Pi_{54}^{ij} = \delta_4 B_{\omega i}^T, \quad \Pi_{55}^{ij} = -\gamma^2 I.$$

Moreover, the state feedback controller gains of (6) are given by $K_j = Y_j X^{-T}$ *for* $j = 1, 2, \cdots, n$.

Proof. By Remark 3.7, we know when (19) and (20) hold, system (7) with $\omega(t) = 0$ is asymptotically stable. Denote $M_2 = \delta_2 M_1$, $M_3 = \delta_3 M_1$, $M_4 = \delta_4 M_1$. It can be seen that $\delta_4 \neq 0$ and M_1 is nonsingular from (19) and (20). Now define $X = M_1^{-1}$, $\tilde{P} = XPX^T$, $\tilde{Q} = XQX^T$, $\tilde{R}_1 = XR_1X^T$, $\tilde{R}_2 = XR_2X^T$, $\tilde{N}_k = XN_kX^T$, $\tilde{T}_k = XT_kX^T$ $(k = 1, 2, 3, 4)$, $Y_j = K_jX^T$ $(j = 1, 2, \cdots, n)$ and $\eta_{ij} = \varepsilon_{ij}^{-1}$. Then pre and post-multiplying both sides of (19) by $diag\{X\ X\ X\ X\ I\ X\ X\ I\ I\ I\}$ and its transpose, pre and post-multiplying both sides of (20) by $diag\{X\ X\ X\ X\ I\ X\ X\ I\ I\ I\ I\ I\}$ and its transpose, we obtain (21) and (22) using Schur complements. □

Remark 3.10. From the proofs of Theorem 3.5 and Theorem 3.6, one can see that no restriction on the derivative of the time-varying delay is needed(In[1, 6, 9], $h(t)$ should satisfy $\dot{h}(t) \leq h_0 < 1$), which means that a fast time-varying delay is allowed.

Remark 3.11. Theorem 3.9 helps us calculate the upper bound of $h(t)$ as follows. First, given α or h_m(if h_m is given, replace α and β with $\frac{1}{2}(h_M+h_m)$ and $\frac{1}{2}(h_M-h_m)$ in (21) and (22)). Second, find the maximum allowable value of β or h_M satisfying (21), (22) by setting the proper values for δ_2, δ_3 and δ_4, then solve the corresponding feedback gains $K_j = Y_jX^{-T}$. Finally calculate β_{max} or h_{Mmax}.

4. A Numerical Example

In this section, we apply the proposed method to design a delay-dependent robust H_∞ controller for an uncertain nonlinear delay system. Consider an uncertain nonlinear time-delay system as follows:

$$\begin{cases} \dot{x}_1(t) = -x_1(t)(3+\cos^2 x_2(t)) + x_2(t) - x_1(t-h(t)) - x_2(t-h(t))(2+\sin^2 x_2(t)) \\ \qquad\qquad + c(t)x_2(t)\sin^2 x_2(t) + c(t)x_1(t)\cos^2 x_2(t) + (1+\sin^2 x_2(t))\omega(t), \\ \dot{x}_2(t) = 0.5x_1(t)(1+\cos^2 x_2(t)) - x_2(t) - x_2(t-h(t))(1+\sin^2 x_2(t)) + u(t), \end{cases} \tag{23}$$

where $c(t)$ is an uncertain parameter satisfying $c(t) \in [-0.2,\ 0.2]$. If we select the membership function as $M_1(x_2(t)) = \sin^2(x_2(t))$ and $M_2(x_2(t)) = \cos^2(x_2(t))$, then the nonlinear time-delay system (23) can be represented by the following uncertain time-varying delay T–S model

Plant Rule 1:

If $\quad x_2(t)$ is M_1,

Then

$$\begin{cases} \dot{x}(t) = & (A_{11} + \Delta A_{11}(t))x(t) + A_{12}x(t-h(t)) + B_1u(t) + B_{\omega 1}\omega(t), \\ \tilde{z}(t) = & C_1x(t) + D_1u(t); \end{cases}$$

Plant Rule 2:

If $\quad x_2(t)$ is M_2,

Then

$$\begin{cases} \dot{x}(t) = & (A_{21} + \Delta A_{21}(t))x(t) + A_{22}x(t-h(t)) + B_2u(t) + B_{\omega 2}\omega(t), \\ \tilde{z}(t) = & C_2x(t) + D_2u(t), \end{cases} \tag{24}$$

where

$$A_{11} = \begin{bmatrix} -3 & 1 \\ 0.5 & -1 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -1 & -3 \\ 0 & -2 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} -4 & 1 \\ 1 & -1 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix},$$

α	β_{max}	Feedback gain K_1	Feedback gain K_2
0.7	0.2781	[-0.7602 -7.7387]	[-1.2089 -10.3102]
0.8	0.2765	[-0.7682 -7.6759]	[-1.2139 -10.1248]
1.0	0.2749	[-0.7766 -7.6550]	[-1.2202 -10.0355]

TABLE 1. The maximum allowable bound β_{max} and the corresponding state-feedback gains K_j

h_m	h_{Mmax}	Feedback gain K_1	Feedback gain K_2
0	0.61	[-0.6543 -8.9925]	[-1.1295 -11.5260]
0.4	0.95	[-0.7591 -7.1916]	[-1.1609 -9.0167]
0.8	1.34	[-0.7815 -7.0236]	[-1.1672 -8.4982]
1.4	1.94	[-0.7844 -7.2100]	[-1.1846 -8.7907]

TABLE 2. The maximum allowable bound $h_M max$ and the corresponding state-feedback gains K_j

$$B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_{\omega 1} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad B_{\omega 2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_1 = C_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$D_1 = D_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad U_1 = U_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_{11} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix}, \quad E_{21} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix}.$$

By Remark 3.11, we choose the H_∞ performance level $\gamma = 1$, $\delta_2 = 0.1$, $\delta_3 = -0.5$, $\delta_4 = 0.3$. For different α, using the MATLAB LMI Toolbox to solve the LMIs (21) and (22), we get the results shown in Table 1.

From Table 1, we can see that the maximum allowable bound β_{max} may be obtained for several values of α. We can also get the corresponding state-feedback gains K_j of H_∞ controller by solving the equation "$K_j = Y_j X^{-T}$". In this case, for any $h(t) \in [\alpha - \beta_{max},\ \alpha + \beta_{max}]$, the fuzzy system (24) is asymptotically stable and satisfies $\|\tilde{z}(t)\|_2 < \gamma \|\omega(t)\|_2$ under controller (6).

Analogously, we can get the results for different h_m, as shown in Table 2.

5. **Conclusion**

In this paper, we have studied the delay-dependent robust H_∞ controller design for a class of T–S fuzzy-model-based systems with time-varying delay and norm-bounded parameter uncertainty. A sufficient condition for the existence of a robust H_∞ controller has been obtained in an LMI form by introducing free-weighting matrices and using the Lyapunov-Krasovskii functional approach. No restriction on the derivative of the time-varying delay is needed. Finally, a numerical example is used to illustrate the effectiveness of the proposed robust H_∞ control design method.

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