

A COMPROMISE RATIO RANKING METHOD OF TRIANGULAR INTUITIONISTIC FUZZY NUMBERS AND ITS APPLICATION TO MADM PROBLEMS

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ABSTRACT. Triangular intuitionistic fuzzy numbers (TIFNs) is a special case of intuitionistic fuzzy (IF) set and the ranking of TIFNs is an important problem. The aim of this paper is to develop a new methodology for ranking TIFNs by using multiattribute decision making methods (MADM). In this methodology, the value and ambiguity indices of TIFNs may be considered as the attributes and the TIFNs in comparison are seen as the alternatives. A compromise ratio method for fuzzy MADM is developed based on the concept that larger TIFN should close to the maximum value index and is far away from the minimum ambiguity index simultaneously. The proposed ranking method is applied to solve multiattribute decision making problems in which the ratings of alternatives on attributes are expressed by using TIFNs. Numerical examples are examined to demonstrate the implementation process and applicability of the proposed method in this paper. Furthermore, a comparison analysis of the proposed method is conducted to show its advantages over other methods.

1. Introduction

Under some conditions, crisp data are inadequate or insufficient to model the ratings of alternatives on attributes in real-life decision making problems due to a lack of information. Therefore, fuzzy number provides an efficient tool to solve the decision making problems with fuzzy information [5, 9, 10, 17]. However, in some situations, due to a lack of information or imprecision of the available information, decision makers are not able to exactly forecast the ratings of alternatives on attributes and could only estimate these ratings of alternatives on attributes approximately with some imprecise degrees. But it is possible that they are not so sure about it. In other words, there may be a hesitation degree about the approximate the ratings of alternatives on attributes for decision makers. However, fuzzy number is no means to incorporate the hesitation degree. As a generalization of fuzzy numbers [5], an intuitionistic fuzzy number (IFN) seems to suitably describe such uncertainty situation. It is more abundant and flexible for the IFNs to express and describe information than the fuzzy numbers when uncertain information is involved. Recently, there exist little literatures involving IFNs and its application [2, 4, 6-8, 11-16, 19-21]. Mitchell [13] interpreted an IFN as an ensemble of ordinary

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fuzzy numbers and introduced a ranking method. Nayagam et al. [15] described a type of IFNs and introduced a method of intuitionistic fuzzy scoring for ranking of IFNs. However, these existing definitions of IFNs are complicated and the ranking methods of IFNs have tedious calculations. Wang and Zhang [20, 21] defined an intuitionistic trapezoidal fuzzy number and an expected value of the intuitionistic trapezoidal fuzzy number as well as a ranking method. Wang and Zhang [19] transformed the ranking of intuitionistic trapezoidal fuzzy numbers to that of interval numbers. As far as we know, the ranking of interval numbers is a difficult problem. Triangular intuitionistic fuzzy numbers (TIFNs) are special IFNs and are important for fuzzy multiattribute decision making (MADM) problems, in that TIFNs have appealing interpretations and can be easily specified and implemented by the decision maker. However, there exists little investigation on the ranking and application of TIFNs to MADM problems. Extending the two characteristics of fuzzy numbers introduced by Delgado et al. [3], Li et al. [8, 11] defined the value and ambiguity of a TIFN and given the lexicographic ranking method and the ratio ranking method for TIFNs, respectively, which are applied to MADM problems in which the ratings of alternatives on attributes are expressed by using TIFNs. It is easily seen that the lexicographic ranking method given by Li et al. [11] is essentially a single-index approach. The reason is that it is mainly considered value index, which is not always feasible and effective for TIFNs with bigger value index and bigger ambiguity index. The ratio ranking method of TIFNs developed by Li [8] does not guarantee to have the bigger value index and smaller ambiguity index simultaneously. Furthermore, the relative importance of the value index and ambiguity index of TIFNs is not considered in both the lexicographic ranking method of TIFNs [11] and the ratio ranking method of TIFNs [8]. Considering that the value index and ambiguity index of the TIFN are in conflict. Thus, a ranking approach should be developed on the basis of the value and ambiguity two indices by using MADM methods, i.e., the value and ambiguity indices of TIFNs may be considered as the attributes and the TIFNs in comparison are seen as the alternatives. On the other hand, the relative importance of the value index and ambiguity index, which is a major concern in ranking of TIFNs, need to be considered. Thus this paper extends the compromise ratio method for fuzzy MADM given by Li [9] to rank the TIFNs. Moreover, the concept of TIFN is considered as a representation for these uncertain factors in real-life decision problems and we study MADM problems in which the ratings of alternatives on attributes are expressed by using TIFNs.

The rest of this paper is organized as follows. In section 2, the basic concepts of TIFNs and the compromise ratio method for fuzzy MADM are introduced. Section 3 introduces the definitions and properties of value-index and ambiguity-index of TIFNs. Hereby, a compromise ratio ranking method for TIFNs is developed on the value-index and ambiguity-index of TIFNs. Furthermore, an example is examined to demonstrate the implementation process and effectiveness of the method proposed in this paper. Section 4 presents MADM problems in which the ratings of alternatives on attributes are expressed with TIFNs, which are solved by the extended additive weighted method. A numerical example and a comparison analysis are given in section 5. This paper concludes in section 6.

2. Basic Definitions

In this section, TIFNs and their operations are defined as follows.

Definition 2.1. [11] A TIFN $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a special IF-set on the real number set R of real numbers, whose membership function and non-membership function are defined as follows.

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-\underline{a}}{a-\underline{a}}w_{\tilde{a}} & \text{if } \underline{a} \leq x < a \\ w_{\tilde{a}} & \text{if } x = a \\ \frac{\bar{a}-x}{\bar{a}-a}w_{\tilde{a}} & \text{if } a \leq x < \bar{a} \\ 0 & \text{if } x < \underline{a} \text{ or } x > \bar{a} \end{cases} \tag{1}$$

and

$$v_{\tilde{a}}(x) = \begin{cases} \frac{a-x+u_{\tilde{a}}(x-\underline{a})}{a-\underline{a}} & \text{if } \underline{a} \leq x < a \\ u_{\tilde{a}} & \text{if } x = a \\ \frac{x-a+u_{\tilde{a}}(\bar{a}-x)}{\bar{a}-a} & \text{if } a \leq x < \bar{a} \\ 1 & \text{if } x < \underline{a} \text{ or } x > \bar{a} \end{cases} \tag{2}$$

respectively.

In Figure 1, we report the relationship between the membership function and the non-membership function of the TIFN.

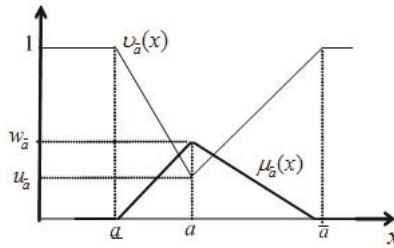


FIGURE 1. Triangular Intuitionistic Fuzzy Number \tilde{a}

The values $w_{\tilde{a}}$ and $u_{\tilde{a}}$ respectively represent the maximum membership degree and the minimum non-membership degree which satisfy the conditions, that is, $0 \leq w_{\tilde{a}} \leq 1$, $0 \leq u_{\tilde{a}} \leq 1$ and $0 \leq w_{\tilde{a}} + u_{\tilde{a}} \leq 1$.

Let

$$\chi_{\tilde{a}}(x) = 1 - \mu_{\tilde{a}}(x) - v_{\tilde{a}}(x), \tag{3}$$

which is called as an Atanassov's intuitionistic fuzzy index of an element x in \tilde{a} . It is the degree of indeterminacy membership of the element x to \tilde{a} .

Two parameters $w_{\tilde{a}}$ and $u_{\tilde{a}}$ are introduced in Definition 2.1 to reflect the confidence level and non-confidence level of the TIFN $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$, respectively. If $\underline{a} \geq 0$ and one of the three values \underline{a} , a and \bar{a} is not equal to 0, then the TIFN $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is called as a positive TIFN, denoted by $\tilde{a} > 0$. Likewise, if $\bar{a} \leq 0$ and one of the three values \underline{a} , a and \bar{a} is not equal to 0, then the TIFN $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is called as a negative TIFN, denoted by $\tilde{a} < 0$.

In a similar way to the arithmetic operations of the triangular fuzzy numbers [4], the arithmetic operations of TIFNs may be defined as follows.

Definition 2.2. [8, 11] Let $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (\underline{b}, b, \bar{b}); w_{\tilde{b}}, u_{\tilde{b}} \rangle$ be two TIFNs and γ be a real number. The arithmetic operations are defined as follows.

$$\tilde{a} + \tilde{b} = \langle (\underline{a} + \underline{b}, a + b, \bar{a} + \bar{b}); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \rangle \quad (4)$$

$$\tilde{a} - \tilde{b} = \langle (\underline{a} - \bar{b}, a - b, \bar{a} - b); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \rangle \quad (5)$$

$$\tilde{a}\tilde{b} = \begin{cases} \langle (\underline{ab}, ab, \bar{a}\bar{b}); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \rangle & \text{if } \tilde{a} > 0 \text{ and } \tilde{b} > 0 \\ \langle (\underline{a}\bar{b}, ab, \bar{a}\bar{b}); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \rangle & \text{if } \tilde{a} < 0 \text{ and } \tilde{b} > 0 \\ \langle (\bar{a}\bar{b}, ab, \underline{ab}); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \rangle & \text{if } \tilde{a} < 0 \text{ and } \tilde{b} < 0 \end{cases} \quad (6)$$

$$\tilde{a}/\tilde{b} = \begin{cases} \langle (\underline{a}/\bar{b}, a/b, \bar{a}/b); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \rangle & \text{if } \tilde{a} > 0 \text{ and } \tilde{b} > 0 \\ \langle (\bar{a}/\bar{b}, a/b, \underline{a}/\bar{b}); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \rangle & \text{if } \tilde{a} < 0 \text{ and } \tilde{b} > 0 \\ \langle (\bar{a}/b, a/b, \underline{a}/\bar{b}); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \rangle & \text{if } \tilde{a} < 0 \text{ and } \tilde{b} < 0 \end{cases} \quad (7)$$

$$\gamma\tilde{a} = \begin{cases} \langle (\gamma\underline{a}, a, \gamma\bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle & \text{if } \gamma > 0 \\ \langle (\gamma\bar{a}, a, \gamma\underline{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle & \text{if } \gamma < 0 \end{cases} \quad (8)$$

$$\tilde{a}^{-1} = \langle (1/\bar{a}, 1/a, 1/\underline{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle \text{ if } \tilde{a} > 0 \quad (9)$$

According to the cut sets of the IF-set defined in [1], the cut sets of a TIFN can be defined as follows (see [8, 11]).

Definition 2.3. A (α, β) -cut set of $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a crisp subset of R which is defined as

$$\tilde{a}_{\alpha\beta}^{\alpha} = \{x | \mu_{\tilde{a}}(x) \geq \alpha, v_{\tilde{a}}(x) \leq \beta\}, \quad (10)$$

where $0 \leq \alpha \leq w_{\tilde{a}}, u_{\tilde{a}} \leq \beta \leq 1$ and $0 \leq \alpha + \beta \leq 1$.

Definition 2.4. [2] A α -cut set of $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a crisp subset of R which is defined as

$$\tilde{a}^{\alpha} = \{x | \mu_{\tilde{a}}(x) \geq \alpha\}, \quad (11)$$

where $0 \leq \alpha \leq w_{\tilde{a}}$.

Using equation (1) and Definition 2.4, \tilde{a}^{α} is a closed interval, denoted by $\tilde{a}^{\alpha} = [L^{\alpha}(\tilde{a}), R^{\alpha}(\tilde{a})]$, which can be calculated as follows

$$[L^{\alpha}(\tilde{a}), R^{\alpha}(\tilde{a})] = \left[\underline{a} + \frac{\alpha}{w_{\tilde{a}}}(a - \underline{a}), a - \frac{\alpha}{w_{\tilde{a}}}(\bar{a} - a) \right]. \quad (12)$$

Definition 2.5. [2] A β -cut set of $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a crisp subset of R which is defined as

$$\tilde{a}_{\beta} = \{x | v_{\tilde{a}}(x) \leq \beta\}, \quad (13)$$

where $u_{\tilde{a}} \leq \beta \leq 1$.

Using equation (2) and Definition 2.5, \tilde{a}_β is a closed interval, denoted by $\tilde{a}_\beta = [L_\beta(\tilde{a}), R_\beta(\tilde{a})]$, which can be calculated as follows

$$\tilde{a}_\beta = \left[\frac{(1 - \beta)a + (\beta - u_{\tilde{a}})\underline{a}}{1 - u_{\tilde{a}}}, \frac{(1 - \beta)a + (\beta - u_{\tilde{a}})\bar{a}}{1 - u_{\tilde{a}}} \right]. \tag{14}$$

The value and ambiguity of a TIFN are defined similarly to those of a fuzzy number introduced by Delgado et al. [5].

Definition 2.6. [8, 11] Let \tilde{a}^α and \tilde{a}_β be any α -cut set and β -cut set of $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$, respectively. Then the values of the membership function $\mu_{\tilde{a}}$ and the non-membership function $\nu_{\tilde{a}}$ for the TIFN \tilde{a} are defined as follows

$$V_\mu(\tilde{a}) = \int_0^{w_{\tilde{a}}} \frac{L^\alpha(\tilde{a}) + R^\alpha(\tilde{a})}{2} f(\alpha) d\alpha \tag{15}$$

and

$$V_\nu(\tilde{a}) = \int_{u_{\tilde{a}}}^1 \frac{L_\beta(\tilde{a}) + R_\beta(\tilde{a})}{2} g(\beta) d\beta, \tag{16}$$

respectively.

The function $f(\alpha)$ is a non-negative and non-decreasing function on the interval $[0, w_{\tilde{a}}]$ with $f(0) = 0$ and $\int_0^{w_{\tilde{a}}} f(\alpha) d\alpha = w_{\tilde{a}}$. In fact, the function $f(\alpha)$ gives different weight to different α -cut set, where $\alpha \in [0, w_{\tilde{a}}]$. The larger α -cut set has the bigger weight $f(\alpha)$. Similarly, the function $g(\beta)$ is a non-negative and non-increasing function on the interval $[u_{\tilde{a}}, 1]$ with $g(1) = 0$ and $\int_{u_{\tilde{a}}}^1 g(\beta) d\beta = 1 - u_{\tilde{a}}$. The function $g(\beta)$ has also the effect of weighting on the different β -cut sets, where $\beta \in [u_{\tilde{a}}, 1]$. The larger β -cut sets has the smaller weight $g(\beta)$.

In actual applications, $f(\alpha)$ and $g(\beta)$ can be chosen according to real-life situations. In order to calculate simply, here we chose $f(\alpha)$ and $g(\beta)$ as follows.

$$f(\alpha) = \frac{2\alpha}{w_{\tilde{a}}} (\alpha \in [0, w_{\tilde{a}}]) \tag{17}$$

and

$$g(\beta) = \frac{2(1 - \beta)}{1 - u_{\tilde{a}}} (\beta \in [u_{\tilde{a}}, 1]), \tag{18}$$

respectively.

According to equations (12), (15) and (17), the value of the membership function of a TIFN \tilde{a} is calculated as follows.

$$V_\mu(\tilde{a}) = \frac{(\underline{a} + 4a + \bar{a})w_{\tilde{a}}}{6} \tag{19}$$

and

$$V_\nu(\tilde{a}) = \frac{(\underline{a} + 4a + \bar{a})(1 - u_{\tilde{a}})}{6}. \tag{20}$$

Due to the condition that $0 \leq w_{\tilde{a}} + u_{\tilde{a}} \leq 1$, it is directly derived from equations (19) and (20) that $\frac{(\underline{a} + 4a + \bar{a})w_{\tilde{a}}}{6} \leq \frac{(\underline{a} + 4a + \bar{a})(1 - u_{\tilde{a}})}{6}$, i.e., $V_\mu(\tilde{a}) \leq V_\nu(\tilde{a})$, if \tilde{a} is a non-negative TIFN. Thus, in this case the values of the membership and non-membership functions of a TIFN \tilde{a} may be concisely expressed as an interval $[V_\mu(\tilde{a}), V_\nu(\tilde{a})]$. Suppose that the TIFNs are non-negative throughout the paper unless otherwise stated.

Definition 2.7. [8, 11] Let $\tilde{\alpha}^\alpha$ and $\tilde{\alpha}_\beta$ be any α -cut set and β -cut set of $\tilde{a} = \langle (a, a, \bar{a}); w_{\bar{a}}, u_{\bar{a}} \rangle$, respectively. Then the ambiguities of the membership function $\mu_{\bar{a}}$ and the non-membership function $v_{\bar{a}}$ for the TIFN \tilde{a} are defined as

$$A_\mu(\tilde{a}) = \int_0^{w_{\bar{a}}} (R^\alpha(\tilde{a}) - L^\alpha(\tilde{a}))f(\alpha)d\alpha \quad (21)$$

and

$$A_v(\tilde{a}) = \int_{u_{\bar{a}}}^1 (R_\beta(\tilde{a}) - L_\beta(\tilde{a}))g(\beta)d\beta, \quad (22)$$

respectively.

Obviously, $A_\mu(\tilde{a}) \geq 0$ and $A_v(\tilde{a}) \geq 0$. According to equations (12), (17) and (21), the ambiguity of the membership function of the TIFN \tilde{a} is calculated as follows.

$$A_\mu(\tilde{a}) = \frac{(\bar{a} - a)w_{\bar{a}}}{3} \quad (23)$$

Similarly, according to equations (14), (18) and (22), the ambiguity of the non-membership function of the TIFN \tilde{a} is calculated as follows.

$$A_v(\tilde{a}) = \frac{(\bar{a} - a)(1 - u_{\bar{a}})}{3}. \quad (24)$$

Due to the condition that $0 \leq w_{\bar{a}} + u_{\bar{a}} \leq 1$, it is directly derived from equations (23) and (24) that $A_\mu(\tilde{a}) \leq A_v(\tilde{a})$. Thus, in this case, the ambiguities of the membership and non-membership functions of a TIFN \tilde{a} may be concisely expressed as an interval $[A_\mu(\tilde{a}), A_v(\tilde{a})]$.

The compromise ratio method for fuzzy MADM developed by Li [9] focuses on ranking and selecting from a set of alternatives in the presence of conflicting attributes. The basic principle of the compromise ratio method for fuzzy MADM is that the chosen alternative should have the distance from the ideal solution as short as possible and the distance from the negative-ideal solution as far as possible simultaneously. Furthermore, the relative importance of these distances is considered in [9]. Let $X = (x_1, x_2, \dots, x_n)$ be n alternatives. $D(x_j, x^+)$ and $D(x_j, x^-)$ represent the distances of each alternative x_j ($j = 1, 2, \dots, n$) from the ideal solution x^+ and the negative-ideal solution x^- , respectively. However, in some situations it may not always guarantee to have both the shortest distance from the ideal solution and the longest distance from the negative-ideal solution. So, Li [9] defined a compromise ratio for every alternative as follows.

$$\xi(x_j) = \varepsilon \frac{D_1(x^+) - D(x_j, x^+)}{D_1(x^+) - D_2(x^+)} + (1 - \varepsilon) \frac{D(x_j, x^-) - D_2(x^-)}{D_1(x^-) - D_2(x^-)}$$

where, $D_1(x^+) = \max_{1 \leq j \leq n} \{D(x_j, x^+)\}$, $D_2(x^+) = \min_{1 \leq j \leq n} \{D(x_j, x^+)\}$, $D_1(x^-) = \max_{1 \leq j \leq n} \{D(x_j, x^-)\}$ and $D_2(x^-) = \min_{1 \leq j \leq n} \{D(x_j, x^-)\}$. Here, $\varepsilon \in [0, 1]$ indicates the attitudinal factor of the decision maker. The preference of order of the alternative set X is generated only by the distances $D(x_j, x^+)$ if $\varepsilon = 1$. The preference of order of the alternative set X is generated only by the distances $D(x_j, x^-)$ if $\varepsilon = 0$. The preference of order of the alternative set X is generated by considering equally importance of both the distances $D(x_j, x^+)$ and $D(x_j, x^-)$ if $\varepsilon = 1/2$.

The index $\xi(x_j) \in [0, 1]$ measures the compromise extent that the alternative $x_j (j = 1, 2, \dots, n)$ closes to the ideal solution x^+ and is far away from the negative-ideal solution x^- . The bigger $\xi(x_j)$, the better x_j .

3. The Compromise Ratio Ranking Method for TIFNs and a Comparative Analysis

Based on the above value and ambiguity of a TIFN, a compromise ratio ranking method of TIFNs is proposed in this section.

Definition 3.1. [8, 11] Let $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ be a TIFN. A value-index and an ambiguity-index for \tilde{a} are defined as

$$V_\lambda(\tilde{a}) = V_\mu(\tilde{a}) + \lambda(V_\nu(\tilde{a}) - V_\mu(\tilde{a})) \tag{25}$$

and

$$A_\lambda(\tilde{a}) = A_\nu(\tilde{a}) - \lambda(A_\nu(\tilde{a}) - A_\mu(\tilde{a})), \tag{26}$$

respectively, where $\lambda \in [0, 1]$ is a weight which represents the decision maker's preference information. $\lambda \in [0, 1/2)$ shows that the decision maker prefers to the value of the membership function and the ambiguity of non-membership function, $\lambda \in (1/2, 1]$ shows that the decision maker prefers to the value of the non-membership function and the ambiguity of the membership function, and $\lambda = 1/2$ shows that decision maker is indifferent to between membership functions and non-membership functions for the values and the ambiguities, respectively. Therefore, the value-index and the ambiguity-index may reflect the decision maker's subjective attitudes to the values and ambiguities of the membership function and the non-membership function of the TIFN, respectively.

The properties of $V_\lambda(\tilde{a})$ and $A_\lambda(\tilde{a})$ are summarized as in Theorem 3.2.

Theorem 3.2. [8] $V_\lambda(\tilde{a})$ and $A_\lambda(\tilde{a})$ are continuous non-decreasing and non-increasing functions of the parameter $\lambda \in [0, 1]$, respectively, if $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a non-negative TIFN.

If $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a non-negative TIFN, then from Theorem 3.2, equations (25) and (26) are derived as

$$\max\{V_\lambda(\tilde{a})\} = V_\nu(\tilde{a}) \tag{27}$$

and

$$\min\{A_\lambda(\tilde{a})\} = A_\mu(\tilde{a}). \tag{28}$$

It is noted that the larger the value index $V_\lambda(\tilde{a})$ and the smaller the ambiguity index $A_\lambda(\tilde{a})$, the bigger the TIFN $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$. However, a TIFN $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ with a large value index may not always guarantee to have a small ambiguity index simultaneously. So, in this case the compromise ratio method for fuzzy MADM problems given by Li [9] is used to rank the TIFNs.

In the following, the compromise ratio method of MADM will be extended to rank the TIFNs.

Let $\tilde{a}_i = \langle (\underline{a}_i, a_i, \bar{a}_i); w_{\tilde{a}_i}, u_{\tilde{a}_i} \rangle$ ($i = 1, 2, \dots, m$) be m TIFNs. Denote $I = \{i = 1, 2, \dots, m\}$, a compromise ratio ranking method based on the value index and the ambiguity index for the TIFNs \tilde{a}_i ($i \in I$) is defined as follows.

$$\xi(\tilde{a}_i, \lambda, \varepsilon) = \varepsilon \frac{V_\lambda(\tilde{a}_i) - \min_{i \in I}\{V_\lambda(\tilde{a}_i)\}}{\max_{i \in I}\{V_\lambda(\tilde{a}_i)\} - \min_{i \in I}\{V_\lambda(\tilde{a}_i)\}} + (1 - \varepsilon) \frac{\max_{i \in I}\{A_\lambda(\tilde{a}_i)\} - A_\lambda(\tilde{a}_i)}{\max_{i \in I}\{A_\lambda(\tilde{a}_i)\} - \min_{i \in I}\{A_\lambda(\tilde{a}_i)\}} \quad (29)$$

where, $\varepsilon \in [0, 1]$ is a compromise coefficient which represents the decision maker's preference information. $\varepsilon \in [1/2, 1]$ indicates that the decision maker gives the more importance to closing to the maximum value index, and $\varepsilon \in [0, 1/2]$ indicates that the decision maker is interested in the farthest distance from the maximum ambiguity index. And equally importance to both closing to the maximum value index and the farthest distance from the maximum ambiguity index will be given when $\varepsilon = 1/2$. The index $\xi(\tilde{a}_i, \lambda, \varepsilon) \in [0, 1]$ measures the compromise extent that the value index of a TIFN \tilde{a}_i closes to the maximum value index $\max\{V_\lambda(\tilde{a}_i)\}$ and is far away from the maximum ambiguity index $\max\{A_\lambda(\tilde{a}_i)\}$ simultaneously. The bigger $\xi(\tilde{a}_i, \lambda, \varepsilon)$, the larger \tilde{a}_i . The ranking order of TIFNs $\tilde{a}_i = \langle (\underline{a}_i, a_i, \bar{a}_i); w_{\tilde{a}_i}, u_{\tilde{a}_i} \rangle$ ($i = 1, 2, \dots, m$) is generated according to $\xi(\tilde{a}_i, \lambda, \varepsilon)$, the biggest TIFN satisfies $\xi(\tilde{a}_i^*, \lambda, \varepsilon) = \max_{i \in I}\{\xi(\tilde{a}_i, \lambda, \varepsilon)\}$.

In the compromise ratio method, the maximum value of $\xi(\tilde{a}_i, \lambda, \varepsilon)$ ($i \in I$) defined in equation (29) has the intention to maximize the value index and minimize the ambiguity index simultaneously. Furthermore, the relative importance of the value index and the ambiguity index, which could be a major concern in ranking of TIFNs, is considered by the compromise ratio method.

If $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ ($i \in I$) is a non-negative TIFN, according to equations (27) and (28), equation (29) can be transformed into

$$\xi(\tilde{a}_i, \lambda, \varepsilon) = \varepsilon \frac{V_\lambda(\tilde{a}_i) - \min_{i \in I}\{V_\mu(\tilde{a}_i)\}}{\max_{i \in I}\{V_\nu(\tilde{a}_i)\} - \min_{i \in I}\{V_\mu(\tilde{a}_i)\}} + (1 - \varepsilon) \frac{\max_{i \in I}\{A_\nu(\tilde{a}_i)\} - A_\lambda(\tilde{a}_i)}{\max_{i \in I}\{A_\nu(\tilde{a}_i)\} - \min_{i \in I}\{A_\mu(\tilde{a}_i)\}} \quad (30)$$

Theorem 3.3. $\xi(\tilde{a}_i, \lambda, \varepsilon)$ is a continuous non-decreasing function of the parameter $\lambda \in [0, 1]$, if $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ ($i \in I$) is a non-negative TIFNs.

Proof. It is easily seen that $\xi(\tilde{a}_i, \lambda, \varepsilon)$ is a linear function of the variable $\lambda \in [0, 1]$. Hence, it is continuous on $\lambda \in [0, 1]$.

If $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a non-negative TIFNs ($i \in I$), then using equations (25) and (26), for $\lambda \in [0, 1]$, $\xi(\tilde{a}_i, \lambda, \varepsilon)$ can be rewritten as

$$\xi(\tilde{a}_i, \lambda, \varepsilon) = \varepsilon \frac{V_\mu(\tilde{a}_i) + \lambda(V_\nu(\tilde{a}_i) - V_\mu(\tilde{a}_i)) - \min_{i \in I}\{V_\mu(\tilde{a}_i)\}}{\max_{i \in I}\{V_\nu(\tilde{a}_i)\} - \min_{i \in I}\{V_\mu(\tilde{a}_i)\}} + (1 - \varepsilon) \frac{\max_{i \in I}\{A_\nu(\tilde{a}_i)\} - A_\nu(\tilde{a}_i) + \lambda(A_\nu(\tilde{a}_i) - A_\mu(\tilde{a}_i))}{\max_{i \in I}\{A_\nu(\tilde{a}_i)\} - \min_{i \in I}\{A_\mu(\tilde{a}_i)\}} \quad \cdot$$

The partial derivative of $\xi(\tilde{a}_i, \lambda, \varepsilon)$ with respect to $\lambda \in [0, 1]$ can be calculated as follows.

$$\frac{\partial \xi(\tilde{a}_i, \lambda, \varepsilon)}{\partial \lambda} = \varepsilon \frac{V_\nu(\tilde{a}_i) - V_\mu(\tilde{a}_i)}{\max_{i \in I}\{V_\nu(\tilde{a}_i)\} - \min_{i \in I}\{V_\mu(\tilde{a}_i)\}} + (1 - \varepsilon) \frac{A_\nu(\tilde{a}_i) - A_\mu(\tilde{a}_i)}{\max_{i \in I}\{A_\nu(\tilde{a}_i)\} - \min_{i \in I}\{A_\mu(\tilde{a}_i)\}} \quad (31)$$

Due to $V_v(\tilde{a}_i) \geq V_\mu(\tilde{a}_i)$ and $A_v(\tilde{a}_i) \geq A_\mu(\tilde{a}_i)$, it is directly derived from equation (31) that $\frac{\partial \xi(\tilde{a}_i, \lambda, \varepsilon)}{\partial \lambda} \geq 0$. Therefore, we have proven that $\xi(\tilde{a}_i, \lambda, \varepsilon)$ is a continuous non-decreasing function of the parameter $\lambda \in [0, 1]$ if $\tilde{a} = \langle (\underline{a}, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a non-negative TIFNs ($i \in I$). \square

Let $\tilde{a}_i = \langle (\underline{a}_i, a_i, \bar{a}_i); w_{\tilde{a}_i}, u_{\tilde{a}_i} \rangle$ ($i = 1, 2, \dots, m$) be m TIFNs. A compromise ratio ranking procedure based on equation (29) can be developed for ranking the TIFNs $\tilde{a}_i = \langle (\underline{a}_i, a_i, \bar{a}_i); w_{\tilde{a}_i}, u_{\tilde{a}_i} \rangle$ ($i = 1, 2, \dots, m$). It is summarized as follows.

Step 1: Compute $V_\mu(\tilde{a}_i)$, $V_v(\tilde{a}_i)$, $A_\mu(\tilde{a}_i)$ and $A_v(\tilde{a}_i)$ ($i = 1, 2, \dots, m$) by using equations (19), (20), (23) and (24).

Step 2: Compute $V_\lambda(\tilde{a}_i)$ and $A_\lambda(\tilde{a}_i)$ ($i = 1, 2, \dots, m$) by using equations (25) and (26).

Step 3: Compute $\xi(\tilde{a}_i, \lambda, \varepsilon)$ ($i = 1, 2, \dots, m$) for the same given $\lambda \in [0, 1]$ and $\varepsilon \in [0, 1]$ by using equation (29).

Step 4: Rank the TIFNs $\tilde{a}_i = \langle (\underline{a}_i, a_i, \bar{a}_i); w_{\tilde{a}_i}, u_{\tilde{a}_i} \rangle$ ($i = 1, 2, \dots, m$), according to non-increasing order of the ratios $\xi(\tilde{a}_i, \lambda, \varepsilon)$ ($i = 1, 2, \dots, m$). The maximum TIFN is the one with the largest compromise ratio, i.e., $\max_{i \in I} \{\xi(\tilde{a}_i, \lambda, \varepsilon)\}$.

A numerical example is presented to show an applicability of the aforementioned ranking method and the comparative analysis to the ratio ranking method of TIFNs given by Li [8].

Example 3.4. Let us consider two TIFNs $\tilde{a} = \langle (0.3, 0.5, 0.7); 0.6, 0.3 \rangle$ and $\tilde{b} = \langle (0.1, 0.5, 0.9); 0.5, 0.2 \rangle$. For the same given $\lambda \in [0, 1]$ and $\varepsilon \in [0, 1]$, the ranking results obtained by the Li's method [8] and the compromise ratio method proposed in this paper are given in the Table from 1 to 3 as follows.

λ	$\xi(\tilde{a})$	$\xi(\tilde{b})$	$R(\tilde{a})$	$R(\tilde{b})$	Li's method	the proposed method
0.1	0.530	0.088	0.279	0.220	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.2	0.556	0.176	0.284	0.234	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.3	0.582	0.264	0.289	0.248	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.4	0.609	0.352	0.294	0.262	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.5	0.635	0.440	0.299	0.277	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.6	0.661	0.528	0.304	0.292	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.7	0.688	0.616	0.309	0.307	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.8	0.714	0.704	0.314	0.322	$\tilde{a} > \tilde{b}$	$\tilde{a} < \tilde{b}$
0.9	0.740	0.792	0.319	0.337	$\tilde{a} > \tilde{b}$	$\tilde{a} < \tilde{b}$
1	0.767	0.880	0.324	0.353	$\tilde{a} > \tilde{b}$	$\tilde{a} < \tilde{b}$

TABLE 1. Ranking Results Obtained by Li's Method and the Proposed Methods with $\varepsilon = 0.7$

From Table 1, if $\varepsilon = 0.7$, i.e., the decision maker gives the more importance to the large value index, then for the same $\lambda \in [0, 1]$ the ranking results of TIFNs \tilde{a} and \tilde{b} obtained by the proposed method and Li's method are almost the same

λ	$\xi(\tilde{a})$	$\xi(\tilde{b})$	$R(\tilde{a})$	$R(\tilde{b})$	Li's method	the proposed method
0.1	0.638	0.080	0.279	0.220	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.2	0.660	0.160	0.284	0.234	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.3	0.682	0.240	0.289	0.248	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.4	0.703	0.320	0.294	0.262	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.5	0.725	0.400	0.299	0.277	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.6	0.747	0.480	0.304	0.292	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.7	0.768	0.560	0.309	0.307	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.8	0.790	0.640	0.314	0.322	$\tilde{a} > \tilde{b}$	$\tilde{a} < \tilde{b}$
0.9	0.812	0.720	0.319	0.337	$\tilde{a} > \tilde{b}$	$\tilde{a} < \tilde{b}$
1	0.833	0.800	0.324	0.353	$\tilde{a} > \tilde{b}$	$\tilde{a} < \tilde{b}$

TABLE 2. Ranking Results Obtained by Li's Method and the Proposed Methods with $\varepsilon = 0.7$

λ	$\xi(\tilde{a})$	$\xi(\tilde{b})$	$R(\tilde{a})$	$R(\tilde{b})$	Li's method	the proposed method
0.1	0.801	0.068	0.279	0.220	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.2	0.816	0.136	0.284	0.234	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.3	0.831	0.204	0.289	0.248	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.4	0.845	0.272	0.294	0.262	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.5	0.860	0.340	0.299	0.277	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.6	0.875	0.408	0.304	0.292	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.7	0.889	0.476	0.309	0.307	$\tilde{a} > \tilde{b}$	$\tilde{a} > \tilde{b}$
0.8	0.904	0.544	0.314	0.322	$\tilde{a} > \tilde{b}$	$\tilde{a} < \tilde{b}$
0.9	0.919	0.612	0.319	0.337	$\tilde{a} > \tilde{b}$	$\tilde{a} < \tilde{b}$
1	0.933	0.680	0.324	0.353	$\tilde{a} > \tilde{b}$	$\tilde{a} < \tilde{b}$

TABLE 3. Ranking Results Obtained by Li's Method and the Proposed Methods with $\varepsilon = 0.2$

expect for $\lambda = 0.8$. This shows that the proposed method is effective. However, if $\varepsilon = 0.2$ or $\varepsilon = 0.5$, i.e., the decision maker gives the more importance to the small ambiguity index or equally importance to both the large value index and the small ambiguity index, then for the same given $\lambda \in [0.8, 1]$ the ranking results of TIFNs \tilde{a} and \tilde{b} obtained by the Li's method [8] and the proposed method are different. This shows that the ranking results are related to the parameter $\varepsilon \in [0, 1]$. That is to say, the relative importance of the value index and the ambiguity index plays an important role in the ranking order of TIFNs. Thus, it is necessary to consider the relative importance of the value index and the ambiguity index.

4. An Extended MADM Method Based on the Compromise Ratio Ranking Method of TIFNs

The ranking of TIFNs is important in MADM with TIFNs information. In this section, the additive weighting method is extended to solve MADM problems

in which the ratings of alternatives on attributes are expressed by using TIFNs. Sometimes such MADM problems are called as MADM problems with TIFNs for short. According to the additive weighting method and the operation on TIFNs defined in Definition 2.2, the weighted comprehensive values of alternatives are TIFNs with a preference of decision makers, which is important in real-life decision making. On the other hand, since decision makers with different preference information have different choose for the weighted comprehensive values of alternatives. However, this result cannot be reflected in the lexicographic ranking method and the ratio ranking method given by Li et al. [11] and Li [8] (see the numerical example in section 5. Thus, we will apply the proposed compromise ratio ranking method of TIFNs to rank the weighted comprehensive values of alternatives to generate a ranking order of alternatives.

Suppose that there exists an alternative set $A = \{A_1, A_2, \dots, A_m\}$, which consists of m non-inferior alternatives from which the most preferred alternative has to be selected. Each alternative is assessed on n attributes. Denote the set of all attributes by $X = \{X_1, X_2, \dots, X_n\}$. Assume that the ratings of alternatives on attributes are given by using TIFNs. Namely, the rating of any alternative $A_i \in A$ ($i = 1, 2, \dots, m$) on each attribute $X_j \in X$ ($j = 1, 2, \dots, n$) is a TIFN $\tilde{a}_{ij} = \langle (\underline{a}_{ij}, a_{ij}, \bar{a}_{ij}); w_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}} \rangle$. Thus a MADM problem with TIFNs can be expressed concisely in the matrix format as $(\tilde{a}_{ij})_{m \times n}$.

Due to the fact that different attributes may have different importance. Assume that the relative weight of the attribute X_j is ω_j ($j = 1, 2, \dots, n$), satisfying the normalization conditions, that is, $\omega_j \in [0, 1]$ and $\omega_1 + \omega_2 + \dots + \omega_n = 1$. Let $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the relative weight vector of all attributes, which are known a priori.

The extended additive weighted method for the MADM problem with TIFNs can be summarized as follows.

(i) Normalize the TIFN decision matrix. In order to eliminate the effect of different physical dimensions on the results of the final decision making, the normalized TIFN decision matrix can be calculated using the following formulae

$$\tilde{r}_{ij} = \langle (\frac{a_{ij}}{a_j^+}, \frac{a_{ij}}{a_j^+}, \frac{\bar{a}_{ij}}{a_j^+}); w_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}} \rangle \quad (i = 1, 2, \dots, m; j \in B) \tag{32}$$

and

$$\tilde{r}_{ij} = \langle (\frac{\underline{a}_{ij}}{a_j^-}, \frac{a_{ij}}{a_j^-}, \frac{\bar{a}_{ij}}{a_j^-}); w_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}} \rangle \quad (i = 1, 2, \dots, m; j \in C), \tag{33}$$

respectively, where B and C are the subscript sets of benefit attributes and cost attributes,

$$a_j^+ = \max\{\bar{a}_{ij}\} \quad (i = 1, 2, \dots, m) \quad (j \in B)$$

and

$$a_j^- = \min\{\underline{a}_{ij}\} \quad (i = 1, 2, \dots, m) \quad (j \in C).$$

(ii) Construct the weighted normalized TIFN decision matrix. Using equation (8), the weighted normalized TIFN decision matrix can be calculated as $(\tilde{u}_{ij})_{m \times n}$, where

$$\tilde{u}_{ij} = \omega \tilde{r}_{ij}. \tag{34}$$

(iii) Calculate the weighted comprehensive values of alternatives. Using equation (4), the weighted comprehensive values of alternatives $A_i \in A$ ($i = 1, 2, \dots, m$) are calculated as follows.

$$\tilde{S}_i = \sum_{j=1}^n \tilde{u}_{ij}. \quad (35)$$

Obviously, \tilde{S}_i ($i = 1, 2, \dots, m$) are TIFNs.

(iv) Rank all alternatives. The ranking order of the alternatives A_i can be generated according to the compromise ratio ranking method proposed in section 3.

5. A Numerical Example

We will analyze a personnel selection problem in this section. Suppose that a software company desires to hire a system analyst. After preliminary screening, three candidates A_1 , A_2 and A_3 remain for further evaluation. The decision making committee assesses the three candidates based on five attributes, including emotional steadiness (X_1), oral communication skill (X_2), personality (X_3), past experience (X_4) and self-confidence (X_5). Assume that the total mark of each attribute is 10. Using statistical methods, the ratings of the candidates with respect to the attributes are given as in the TIFN decision matrix \tilde{A} as follows

$$\tilde{A} = \begin{bmatrix} \langle (5.7, 7.7, 9.3); 0.7, 0.2 \rangle & \langle (5, 7, 9); 0.6, 0.3 \rangle & \langle (5.7, 7.7, 9); 0.8, 0.1 \rangle \\ \langle (6.5, 8.6, 10); 0.4, 0.5 \rangle & \langle (8, 9, 10); 0.6, 0.3 \rangle & \langle (8.3, 9.7, 10); 0.7, 0.2 \rangle \\ \langle (6.5, 8.2, 9.3); 0.8, 0.1 \rangle & \langle (7, 9, 10); 0.7, 0.2 \rangle & \langle (7, 9, 10); 0.5, 0.2 \rangle \end{bmatrix}$$

$$\dots \rightarrow \begin{bmatrix} \langle (8.33, 9.67, 10); 0.6, 0.4 \rangle & \langle (3, 5, 7); 0.6, 0.3 \rangle \\ \langle (8, 9, 10); 0.6, 0.3 \rangle & \langle (7, 9, 10); 0.6, 0.2 \rangle \\ \langle (6, 8, 9); 0.6, 0.2 \rangle & \langle (6.3, 8.3, 9.7); 0.7, 0.2 \rangle \end{bmatrix}$$

where $\langle (5.7, 7.7, 9.3); 0.7, 0.2 \rangle$ in \tilde{A} is a TIFN which indicates that the mark of the candidate A_1 with respect to the attribute X_1 is about 7.7 with the maximum satisfaction degree is 0.7, while the minimum non-satisfaction degree is 0.2. In other words, the hesitation degree is 0.1. Other TIFNs in \tilde{A} are explained similarly.

Assuming each attribute weight ω_j ($j = 1, 2, 3, 4, 5$) can be given as

$$\omega_1 = 0.14, \omega_2 = 0.33, \omega_3 = 0.12, \omega_4 = 0.3, \omega_5 = 0.14.$$

Since the five attributes are the benefit attributes, according to equations (32) and (34), the weighted normalized TIFN decision matrix is obtained as follows.

$$\hat{A} = \begin{bmatrix} \langle (0.08, 0.11, 0.13); 0.7, 0.2 \rangle & \langle (0.15, 0.21, 0.27); 0.6, 0.3 \rangle & \langle (0.06, 0.09, 0.11); 0.8, 0.1 \rangle \\ \langle (0.09, 0.12, 0.14); 0.4, 0.5 \rangle & \langle (0.24, 0.27, 0.3); 0.6, 0.3 \rangle & \langle (0.1, 0.12, 0.12); 0.7, 0.2 \rangle \\ \langle (0.09, 0.12, 0.13); 0.8, 0.1 \rangle & \langle (0.21, 0.27, 0.3); 0.7, 0.2 \rangle & \langle (0.08, 0.11, 0.12); 0.5, 0.2 \rangle \end{bmatrix}$$

$$\dots \rightarrow \begin{bmatrix} \langle (0.25, 0.290, 0.3); 0.6, 0.4 \rangle & \langle (0.042, 0.07, 0.098); 0.6, 0.3 \rangle \\ \langle (0.24, 0.27, 0.3); 0.6, 0.3 \rangle & \langle (0.098, 0.126, 0.14); 0.6, 0.2 \rangle \\ \langle (0.18, 0.24, 0.27); 0.6, 0.2 \rangle & \langle (0.088, 0.116, 0.136); 0.7, 0.2 \rangle \end{bmatrix}$$

Using equation (35) and \hat{A} , the weighted comprehensive values of the candidates, A_i ($i = 1, 2, 3$) can be obtained as follows. $\hat{S}_1 = \langle (0.592, 0.774, 0.910); 0.6, 0.4 \rangle$, $\hat{S}_2 = \langle (0.769, 0.903, 1); 0.4, 0.5 \rangle$ and $\hat{S}_3 = \langle (0.653, 0.849, 0.956); 0.5, 0.2 \rangle$, respectively.

To rank the TIFNs \tilde{S}_1 , \tilde{S}_2 and \tilde{S}_3 according to the compromise ratio ranking method proposed in Section 3. The ranking procedure is summarized as follows.

Step 1: Compute $V_\mu(\tilde{S}_i), V_\nu(\tilde{S}_i), A_\mu(\tilde{S}_i)$ and $A_\nu(\tilde{S}_i)$ ($i = 1, 2, 3$) by using equations (19), (20), (23) and (24).

According to equations (19) and (20), the values of the membership functions and non-membership functions of \tilde{S}_1 , \tilde{S}_2 and \tilde{S}_3 can be calculated as follows.

$$V_\mu(\tilde{S}_1) = 0.4596, V_\nu(\tilde{S}_1) = 0.4596,$$

$$V_\mu(\tilde{S}_2) = 0.3588, V_\nu(\tilde{S}_2) = 0.4485$$

and

$$V_\mu(\tilde{S}_3) = 0.4170, V_\nu(\tilde{S}_3) = 0.6672,$$

respectively.

Step 2: Compute $V_\lambda(\tilde{S}_i)$ and $A_\lambda(\tilde{S}_i)$ ($i = 1, 2, 3$) by using equations (25) and (26). Using equation (25), the value-indices of \tilde{S}_1 , \tilde{S}_2 and \tilde{S}_3 can be obtained as follows.

$$V_\lambda(\tilde{S}_1) = 0.4596, V_\lambda(\tilde{S}_2) = 0.3588 + 0.0897\lambda, V_\lambda(\tilde{S}_3) = 0.4170 + 0.2502\lambda. \quad (36)$$

Similarly, according to equations (23) and (24), the ambiguities of the membership functions and non-membership functions of \tilde{S}_1 , \tilde{S}_2 and \tilde{S}_3 can be calculated as follows.

$$A_\mu(\tilde{S}_1) = 0.0636, A_\nu(\tilde{S}_1) = 0.0636,$$

$$A_\mu(\tilde{S}_2) = 0.0308, A_\nu(\tilde{S}_2) = 0.0385$$

and

$$A_\mu(\tilde{S}_3) = 0.0505, A_\nu(\tilde{S}_3) = 0.0808,$$

respectively.

Using equation (26), the ambiguity indexes of \tilde{S}_1 , \tilde{S}_2 and \tilde{S}_3 can be obtained as follows.

$$A_\lambda(\tilde{S}_1) = 0.0636, A_\lambda(\tilde{S}_2) = 0.0385 - 0.0077\lambda, A_\lambda(\tilde{S}_3) = 0.0808 - 0.0303\lambda. \quad (37)$$

Step 3: Compute $\xi(\tilde{S}_i, \lambda, \varepsilon)$ ($i = 1, 2, 3$) for the same given $\lambda \in [0, 1]$ and $\varepsilon \in [0, 1]$ by using equation (30).

According to equation (30), the compromise ratios of the TIFNs \tilde{S}_1 , \tilde{S}_2 and \tilde{S}_3 can be obtained as follows.

$$\xi(\tilde{S}_1, \lambda, \varepsilon) = 0.344 - 0.017\varepsilon,$$

$$\xi(\tilde{S}_2, \lambda, \varepsilon) = 0.846 - 0.846\varepsilon + 0.154\lambda + 0.137\lambda\varepsilon$$

and

$$\xi(\tilde{S}_3, \lambda, \varepsilon) = 0.189\varepsilon + 0.606\lambda + 0.205\lambda\varepsilon,$$

respectively.

Step 4: Rank the TIFNs \tilde{S}_1 , \tilde{S}_2 and \tilde{S}_3 according to the non-increasing order of the ratios $\xi(\tilde{S}_i, \lambda, \varepsilon)$ ($i = 1, 2, 3$). The maximum TIFN is the one with the largest compromise ratio, i.e., $\max\{\xi(\tilde{S}_i, \lambda, \varepsilon)\}$ ($i = 1, 2, 3$).

For the given $\varepsilon \in [0, 1]$ and $\lambda = 1/2$, the ranking orders of the TIFNs \tilde{S}_1 , \tilde{S}_2 and \tilde{S}_3 are generated as Table 4.

ε	λ	$\xi(\tilde{S}_1)$	$\xi(\tilde{S}_2)$	$\xi(\tilde{S}_3)$	ranking results
0.1	0.5	0.342	0.845	0.332	$\tilde{S}_2 > \tilde{S}_1 > \tilde{S}_3$
0.2	0.5	0.341	0.767	0.361	$\tilde{S}_2 > \tilde{S}_3 > \tilde{S}_1$
0.3	0.5	0.339	0.690	0.390	$\tilde{S}_2 > \tilde{S}_3 > \tilde{S}_1$
0.4	0.57	0.337	0.612	0.420	$\tilde{S}_2 > \tilde{S}_3 > \tilde{S}_1$
0.5	0.5	0.335	0.534	0.449	$\tilde{S}_2 > \tilde{S}_3 > \tilde{S}_1$
0.6	0.5	0.334	0.456	0.478	$\tilde{S}_3 > \tilde{S}_2 > \tilde{S}_1$
0.7	0.5	0.332	0.379	0.507	$\tilde{S}_3 > \tilde{S}_2 > \tilde{S}_1$
0.8	0.5	0.330	0.301	0.536	$\tilde{S}_3 > \tilde{S}_1 > \tilde{S}_2$
0.9	0.5	0.329	0.223	0.565	$\tilde{S}_3 > \tilde{S}_1 > \tilde{S}_2$
1	0.5	0.327	0.145	0.594	$\tilde{S}_3 > \tilde{S}_1 > \tilde{S}_2$

TABLE 4. Ranking Results obtained by the Proposed Methods with $\varepsilon \in [0, 1]$ and $\lambda = 1/2$

For the given $\lambda \in [0, 1]$ and $\varepsilon = 1/2$, the ranking orders of the TIFNs \tilde{S}_1 , \tilde{S}_2 and \tilde{S}_3 are generated as Table 5.

ε	λ	$\xi(\tilde{S}_1)$	$\xi(\tilde{S}_2)$	$\xi(\tilde{S}_3)$	ranking results
0.1	0.5	0.335	0.445	0.165	$\tilde{S}_2 > \tilde{S}_1 > \tilde{S}_3$
0.2	0.5	0.335	0.467	0.236	$\tilde{S}_2 > \tilde{S}_1 > \tilde{S}_3$
0.3	0.5	0.335	0.490	0.307	$\tilde{S}_2 > \tilde{S}_1 > \tilde{S}_3$
0.4	0.5	0.335	0.512	0.378	$\tilde{S}_2 > \tilde{S}_3 > \tilde{S}_1$
0.5	0.5	0.335	0.534	0.449	$\tilde{S}_2 > \tilde{S}_3 > \tilde{S}_1$
0.6	0.5	0.335	0.556	0.520	$\tilde{S}_2 > \tilde{S}_3 > \tilde{S}_1$
0.7	0.5	0.335	0.579	0.590	$\tilde{S}_3 > \tilde{S}_2 > \tilde{S}_1$
0.8	0.5	0.335	0.601	0.661	$\tilde{S}_3 > \tilde{S}_2 > \tilde{S}_1$
0.9	0.5	0.335	0.623	0.732	$\tilde{S}_3 > \tilde{S}_2 > \tilde{S}_1$
1	0.5	0.335	0.645	0.803	$\tilde{S}_3 > \tilde{S}_2 > \tilde{S}_1$

TABLE 5. Ranking Results obtained by the Proposed Methods with $\lambda \in [0, 1]$ and $\varepsilon = 1/2$

It is easily seen that from the Table 5, $\xi(\tilde{S}_i, \lambda, \varepsilon) (i = 1, 2, 3)$ are continuous non-decreasing functions with respect to the parameter $\lambda \in [0, 1]$, which are consistent with Theorem 2. It is also easily derived from the Tables 4 and 5 that the ranking results obtained by the compromise ratio ranking method proposed in this paper are related to the attitude parameters $\lambda \in [0, 1]$ and $\varepsilon \in [0, 1]$, which may reflect the decision maker's subjective attitudes to the TIFNs. From the Table 4, $\varepsilon \in [0, 1/2]$ indicates that the decision maker is interested in the farthest distance from the maximum ambiguity index. That is to say, the decision maker prefers to the TIFN with the small ambiguity index. It is derived from the equation (37) that $\tilde{S}_2 = \langle (0.769, 0.903, 1); 0.4, 0.5 \rangle$ has the smaller ambiguity index than the one of TIFN $\tilde{S}_3 = \langle (0.653, 0.849, 0.956); 0.5, 0.2 \rangle$. Thus, in this case the decision maker

will choose $\tilde{S}_2 = \langle (0.769, 0.903, 1); 0.4, 0.5 \rangle$. Similarly, $\varepsilon \in [1/2, 1]$ indicates that the decision maker gives the more importance to closing to the maximum value index. That is to say, the decision maker prefers to the TIFN with the bigger value index. From equation (36), it is not difficult to see that the TIFN \tilde{S}_3 has the bigger value index than the one of TIFN \tilde{S}_2 , so that the decision maker will choose $\tilde{S}_3 = \langle (0.653, 0.849, 0.956); 0.5, 0.2 \rangle$. Therefore, if $\lambda = 0.5$ and $\varepsilon \in [0, 1/2]$, then the ranking order of the three candidates is generated as $\tilde{S}_2 > \tilde{S}_3 > \tilde{S}_1$, and for $\lambda = 0.5$ and $\varepsilon \in [1/2, 1]$, the ranking order of the three candidates is generated as $\tilde{S}_3 > \tilde{S}_2 > \tilde{S}_1$, which shows that the compromise ratio ranking method proposed in this paper is reasonable and can respect different decision makers' preference information.

Since the TIFN \tilde{S}_3 has the bigger value index than the one of TIFN \tilde{S}_2 for any $\lambda \in [0, 1]$, the ranking result $\tilde{S}_2 > \tilde{S}_3$ can not be generated by using the lexicographic ranking method of TIFNs given by Li et al. [11]. Similarly, using the ratio ranking method of TIFNs given by Li [8], the ranking result $\tilde{S}_2 > \tilde{S}_3$ is not also obtained. Intuitively, it is more reasonable to choose $\tilde{S}_2 = \langle (0.769, 0.903, 1); 0.4, 0.5 \rangle$ instead of $\tilde{S}_3 = \langle (0.653, 0.849, 0.956); 0.5, 0.2 \rangle$ and $\tilde{S}_1 = \langle (0.592, 0.774, 0.910); 0.6, 0.4 \rangle$, i.e., the candidate A_2 is the best selection for a risk-prone decision maker because $\tilde{S}_2 = \langle (0.769, 0.903, 1); 0.4, 0.5 \rangle$ may express an ill-known quantify which is approximately equal to 0.903. However, this result cannot be obtained by using the lexicographic ranking method and the ratio ranking method given by Li et al. [11] and Li [8], respectively. This shows that the relative importance of the value index and ambiguity index of TIFNs need to be considered because it is important in the ranking of TIFNs. Therefore, the compromise ratio ranking method proposed in this paper is more feasible and effective than the ranking methods developed by Li et al. [11] and Li [8].

6. Conclusion

This paper extends the fuzzy MADM method to rank the TIFNs. The compromise ratio ranking method for TIFNs is developed on the basis of the value and ambiguity indexes of TIFNs, which is based on an aggregating function representing some balances between the shortest distance from the maximum value index and the farthest distance from the maximum ambiguity index. The relative importance of the distances from the maximum value index and the maximum ambiguity index, which has been considered in the compromise ratio ranking method of TIFNs, is a major concern in the ranking of TIFN. In contrast, the lexicographic ranking method given by Li et al. [11] is essentially a single-index approach with the value index, which is not always feasible and effective for TIFNs with the bigger value index and bigger ambiguity index. The ratio ranking method of TIFNs developed by Li [8] does not guarantee to have the bigger value index and smaller ambiguity index simultaneously. Furthermore, the relative importance of the value index and ambiguity index of TIFNs is not considered in the lexicographic ranking method of TIFNs [11] and the ratio ranking method of TIFNs [8]. Obviously, the compromise ratio ranking method of TIFNs proposed in this paper is significantly different from other ranking methods of TIFNs [8, 11].

The computation procedure of the compromise ratio ranking method of TIFNs is described in detail in this paper. Furthermore, the proposed ranking method is applied to solve MADM problems with TIFNs because this method is easily implemented and has a natural interpretation. It is easily seen that the proposed ranking method can be extended to the more general IFNs in a straightforward manner.

It is very key and difficult to determine the attitude parameters of the decision maker, i.e. λ and ε , when the method proposed in this paper is applied to the ranking of TIFNs. Some methods for obtaining the attitude of the decision maker by using fuzzy systems modeling were proposed in the references [18, 22]. Some more effective methods will be investigated on determining (learning or extracting) the parameters λ and ε in the near future.

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