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FIXED POINTS OF FUZZY GENERALIZED CONTRACTIVE MAPPINGS IN FUZZY METRIC SPACES

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Abstract. In this paper, we introduce a new concept of fuzzy generalized contraction and give a fixed point result for such mappings in the setting of fuzzy M-complete metric spaces. We also give an affirmative partial answer to a question posed by Wardowski [D. Wardowski, Fuzzy contractive mappings and fixed points in fuzzy metric spaces, Fuzzy Set Syst., **222**(2013), 108-114]. Some examples are also given to support our main result.

1. Introduction

Kramosil and Michalek [8] introduced the notion of fuzzy metric spaces. The conditions which they formulated were modified later by George and Veeramani [1] in order to obtain a Hausdorff topology in fuzzy metric spaces. The paper of Grabiec [2] started the study of fixed point theory in fuzzy metric spaces. In [4] Hadžić and Pap studied fixed point theory for multivalued mappings in probabilistic metric spaces and applied their result in fuzzy metric spaces in the sense of Kaleva and Seikkala [6]. In [3] Gregori and Sapena extended the Banach fixed point theorem to fuzzy contractive mappings of complete fuzzy metric spaces. In [17] the authors discussed the unique existence of fixed points for mappings in fuzzy metric spaces in the sense of Kaleva and Seikkala. For more on fixed point theory for contraction type mappings in fuzzy metric spaces, see [4, 9, 10, 11, 13, 15] and references therein.

Recently, Wardowski [16] introduced a new concept of a fuzzy $\mathcal{H}$-contractive mappings, as a generalization of the fuzzy contraction due to Gregori and Sapena [3], and formulated the conditions guaranteeing the convergence of a fuzzy $\mathcal{H}$-contractive sequence to a unique fixed point in a fuzzy M-complete metric space.

In the present paper, motivated by the work of Wardowski [16], we introduce a new concept of fuzzy generalized contraction, as a generalization of the fuzzy $\mathcal{H}$-contractive, by replacing the constant k by a function α and then give a fixed point result for such mappings in the setting of fuzzy M-complete metric spaces. Some examples are given to support our main result. We also give an affirmative partial answer to a question posed by Wardowski [16].

Now we recall some basic definitions and the properties about fuzzy metric spaces.

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Definition 1.1. [12] A binary operation $*: [0,1]^2 \to [0,1]$ is called a continuous t-norm if the following conditions are satisfied

(1) $*$ is continuous;
(2) $a * b = b * a$;
(3) $a * b \le c * d$ for $a \le c, b \le d$;
(4) $(a * b) * c = a * (b * c)$;
(5) $a * 0 = 0$, $a * 1 = a$;

for all $a, b, c, d \in [0,1]$.

For $a_1, a_2, ..., a_n \in [0,1]$ and $n \in \mathbb{N}$, the product $a_1 * a_2 * ... * a_n$ will be denoted by $\prod_{i=1}^n a_i$. Some typical examples of continuous t-norms are ab, $\min\{a,b\}$ and $\max\{a+b-1, 0\}$. For details concerning t-norms the reader is referred to [7].

Definition 1.2. [1] A triple $(X, M, *)$ is called a fuzzy metric space if X is a nonempty set, $*$ is a continuous t-norm and $M: X^2 \times (0,\infty) \to [0,1]$ is a fuzzy set satisfying the following conditions:

(GV1) $M(x,y,t) > 0$;
(GV2) $M(x,y,t) = 1 \Leftrightarrow x = y$;
(GV3) $M(x,y,t) = M(y,x,t)$;
(GV4) $M(x,z,t+s) \ge M(x,y,t) * M(y,z,s)$;
(GV5) $M(x,y,t): (0,\infty) \to [0,1]$ is a continuous mapping;

for all $x, y, z \in X$ and $s, t > 0$.

Definition 1.3. [1] Let $(X, M, *)$ be a fuzzy metric space. A sequence $\{x_n\}$ is called an M-Cauchy sequence if $\lim_{m,n\to\infty} M(x_m, x_n, t) = 1$, for each $t > 0$.

Definition 1.4. [1] Let $(X, M, *)$ be a fuzzy metric space, $\{x_n\}$ be a sequence in X and let $x \in X$. Then

$$\lim_{n\to\infty} x_n = x \Leftrightarrow \lim_{n\to\infty} M(x_n, x, t) = 1 \ \forall\ t > 0.$$

Definition 1.5. An M-complete fuzzy metric space is a fuzzy metric space in which every M-Cauchy sequence is convergent.

Denote by $\mathcal{H}$ the family of all onto and strictly decreasing mappings $\eta: (0,1] \to [0,\infty)$. Note that if $\eta \in \mathcal{H}$, then $\eta(1) = 0$, η and η^{-1} are continuous.
In [16] the author introduced a new type of contraction in a fuzzy metric space as a generalization of the fuzzy contraction due to Gregori and Sapena [3]. We can read it as follows:

Definition 1.6. [16] Let $(X, M, *)$ be a fuzzy metric space. A mapping $T: X \to X$ is said to be fuzzy $\mathcal{H}$-contractive with respect to $\eta \in \mathcal{H}$ if there exists $k \in (0,1)$ satisfying

$$\eta(M(Tx, Ty, t)) \le k\eta(M(x,y,t)),\ \forall\ x, y \in X\ \forall\ t > 0.$$

Definition 1.7. [16] Let $(X, M, *)$ be a fuzzy metric space. A sequence $\{x_n\}$ in X is called fuzzy $\mathcal{H}$-contractive with respect to $\eta \in \mathcal{H}$ if the following holds:

$$\eta(M(x_{n+1}, x_{n+2}, t)) \le k\eta(M(x_n, x_{n+1}, t)),\ \forall\ n \in \mathbb{N}\ \forall\ t > 0,$$

where $k \in (0,1)$.

Now we recall the main result of Wardowski [16].

Theorem 1.8. [16] *Let $(X, M, *)$ be an M-complete fuzzy metric space and let $T : X \to X$ be a fuzzy $\mathcal{H}$-contractive mapping with respect to $\eta \in \mathcal{H}$ such that:*

(a) $\prod_{i=1}^{k} M(x, Tx, t_i) \neq 0$, *for all* $x \in X$, $k \in \mathbb{N}$ *and any sequence* $\{t_n\} \subseteq (0, \infty)$, $t_n \downarrow 0$;
(b) $r * s > 0 \Rightarrow \eta(r * s) \leq \eta(r) + \eta(s)$, *for all* $r, s \in \{M(x, Tx, t) : x \in X, t > 0\}$;
(c) $\{\eta(M(x, Tx, t_i)) : i \in \mathbb{N}\}$ *is bounded for any* $x \in X$ *and any sequence* $\{t_n\} \subseteq (0, \infty)$, $t_n \downarrow 0$.

Then T has a unique fixed point $x^* \in X$ and for each $x_0 \in X$ the sequence $\{T^n x_0\}$ converges to x^*.

The following example shows that the conditions of Theorem 1.8 must be strengthened.

Example 1.9. Let $X = [0, \infty)$, $a*b = ab$ for all $a, b > 0$ and let $M(x, y, t) = e^{\frac{-|x-y|}{t}}$ for all $x, y \in X$ and each $t > 0$. Then $(X, M, *)$ is an M-complete fuzzy metric space. Define a map $T : X \to X$ by $Tx = \frac{x}{2}$. Now consider a mapping $\eta \in \mathcal{H}$ of the form $\eta(t) = -\ln t$ for each $t \in (0, 1]$. Then it is easy to see that T is fuzzy $\mathcal{H}$-contractive with respect to η with $k = \frac{1}{2}$ and obviously 0 is a fixed point of T but the condition (c) of Theorem 1.8 does not hold and so we cannot invoke Theorem 1.8 to show the existence of a fixed point for T.
Now let $X = [0, \infty)$, $a * b = ab$ for all $a, b > 0$ and let $M(x, y, t) = e^{-|x-y|}$ for all $x, y \in X$ and each $t > 0$. Let $T : X \to X$ be given by $Tx = \ln(1 + x)$. Then T has a fixed point, the conditions (a), (b) and (c) of Theorem 1.8 hold but T is not a fuzzy $\mathcal{H}$-contractive with respect to any $\eta \in \mathcal{H}$ which has a non-zero left derivative at 1. To show the claim, on the contrary, assume that there exists $k \in (0, 1)$ such that

$$\eta(e^{-|\ln(1+x)-\ln(1+y)|}) \leq k\eta(e^{-|x-y|}),$$

for each $x, y \in [0, \infty)$. Letting $y = 0$, we get

$$\eta(e^{-\ln(1+x)}) \leq k\eta(e^{-x}),$$

for each $x \in [0, \infty)$ and so

$$\frac{\eta(1) - \eta(\frac{1}{1+x})}{1 - \frac{1}{1+x}} \geq k \frac{1 - e^{-x}}{1 - \frac{1}{1+x}} \frac{\eta(1) - \eta(e^{-x})}{1 - e^{-x}}.$$

Then by letting $x \to 0^+$, we obtain $\eta'_-(1) \geq k\eta'_-(1)$, a contradiction (note that $\eta'_-(1) < 0$).

2. Main Results

To set up our main result in this section, we first give some definitions.

Definition 2.1. Let $(X, M, *)$ be a fuzzy metric space and let $A \subseteq X$, We say that A is bounded if for each $t > 0$ there exists $r \in (0, 1)$ such that $M(x, y, t) \geq 1 - r$ for each $x, y \in A$.

Definition 2.2. Let $(X, M, *)$ be a fuzzy metric space. We say that $M(x, y, .)$ is continuous uniformly for $x, y \in X$ if for each $t_0 > 0$ and each $\epsilon > 0$ there exists $\delta > 0$ such that

$$t > 0,\ |t - t_0| \leq \delta \Rightarrow |M(x, y, t) - M(x, y, t_0)| < \epsilon, \text{ for each } x, y \in X.$$

Example 2.3. Let (X, d) be a metric space and let $a * b = ab$ for each $a, b \in [0, 1]$. For each $t > 0$, define $M(x, y, t) = \frac{t}{t+d(x,y)}$ for all $x, y \in X$. Then $M(x, y, .)$ is continuous uniformly for $x, y \in X$.

Example 2.4. Let $X = (0, \infty)$, $a * b = ab$ and $M(x, y, t) = \frac{\min\{x,y\}}{\max\{x,y\}}$ for each $x, y \in X$ and any $t > 0$. Then, $M(x, y, .)$ is continuous uniformly for $x, y \in X$.

Denote by $\mathcal{S}$ the family of all functions $\alpha : [0, \infty) \to [0, 1)$ such that

$$\limsup_{s \to t} \alpha(s) < 1, \text{ for each } t > 0.$$

Now we introduce fuzzy generalized $\mathcal{H}$-contractive mappings as a generalization of fuzzy $\mathcal{H}$-contractions by replacing the constant k by a function α.

Definition 2.5. Let $(X, M, *)$ be a fuzzy metric space. A mapping $T : X \to X$ is said to be fuzzy generalized $\mathcal{H}$-contractive with respect to $\eta \in \mathcal{H}$ and $\alpha \in \mathcal{S}$ if T satisfying

$$\eta(M(Tx, Ty, t)) \leq \alpha(\eta(M(x, y, t)))\eta(M(x, y, t)), \tag{1}$$

for all $x, y \in X$ and any $t > 0$.

If $\alpha(t) = k$ for each $t \in [0, \infty)$, where $k \in [0, 1)$ is a constant, then T is a fuzzy $\mathcal{H}$-contractive map [16].
Now we are ready to state our main result.

Theorem 2.6. *Let $(X, M, *)$ be an M-complete fuzzy metric space such that $M(x, y, .)$ is continuous uniformly for $x, y \in X$. Let $T : X \to X$ be a fuzzy generalized $\mathcal{H}$-contractive mapping with respect to $\eta \in \mathcal{H}$ and $\alpha \in \mathcal{S}$. Assume that for each $x \in X$, $\mathcal{O}(x) = \{x, Tx, T^2x, ..., T^nx, ...\}$ is bounded. Then T has a unique fixed point $x^* \in X$ and for each $x_0 \in X$ the sequence $\{T^nx_0\}$ converges to x^*.*

Proof. Let $x_0 \in X$. Define the sequence $\{x_n\}$ in X by $x_n = T^nx_0$, $n \in \mathbb{N}$. From (1), we have

$$\begin{aligned}\eta(M(x_{n+1}, x_n, t)) &= \eta(M(Tx_n, Tx_{n-1}, t)) \\ &\leq \alpha(\eta(M(x_n, x_{n-1}, t)))\eta(M(x_n, x_{n-1}, t)) \leq \eta(M(x_n, x_{n-1}, t)),\end{aligned} \tag{2}$$

for each $n \in \mathbb{N}$ and $t > 0$. From (2), we deduce that $\{\eta(M(x_n, x_{n-1}, t))\}_n$ is a non-negative non-increasing sequence of real numbers and so is convergent to $r \geq 0$. We show that $r = 0$. On the contrary, assume that $r > 0$. Since

$$\limsup_{n \to \infty} \alpha(\eta(M(x_n, x_{n-1}, t))) \leq \limsup_{s \to r} \alpha(s) < 1,$$

then there exist $n_0 \in \mathbb{N}$ and $k \in (0, 1)$ such that

$$\alpha(\eta(M(x_n, x_{n-1}, t))) \leq k, \text{ for } n \geq n_0. \tag{3}$$

From (2) and (3), we get

$$\eta(M(x_{n+1}, x_n, t)) \leq k\eta(M(x_n, x_{n-1}, t)) \text{ for } n \geq n_0$$

and by taking the limit we have $r \leq kr < r$, a contradiction. Thus

$$\lim_{n\to\infty} \eta(M(x_n, x_{n-1}, t)) = 0$$

and hence

$$\lim_{n\to\infty} M(x_n, x_{n-1}, t) = 1, \ \forall\ t > 0 \tag{4}$$

Now, we show that $\{x_n\}$ is an M-Cauchy sequence. On the contrary, assume that there exist subsequences $\{x_{m_k}\}$ and $\{x_{n_k}\}$ such that

$$0 < l = \lim_{k\to\infty} M(x_{m_k}, x_{n_k}, t) < 1, \tag{5}$$

notice that since $\mathcal{O}(x_0)$ is bounded, then $l > 0$. Since

$$\limsup_{k\to\infty} \alpha(\eta(M(x_{m_k}, x_{n_k}, t))) \leq \limsup_{s\to\eta(l)} \alpha(s) < 1,$$

then there exists $0 < b < 1$ such that

$$\alpha(\eta(M(x_{m_k}, x_{n_k}, t))) \leq b \text{ for sufficiently large } k. \tag{6}$$

Since $M(x, y, .)$ is continuous uniformly for $x, y \in X$, then for each $\epsilon > 0$ there exists $\delta > 0$ such that

$$|M(x, y, t) - M(x, y, t+\delta)| < \epsilon, \ \forall\ x, y \in X. \tag{7}$$

Then from (1), (6) and (7), we get

$$\begin{aligned} M(x_{m_k}, x_{n_k}, t) &\geq M(x_{m_k}, x_{n_k}, t+\delta) - \epsilon \\ &\geq M(x_{m_k}, x_{m_k+1}, \frac{\delta}{2}) * M(x_{m_k+1}, x_{n_k+1}, t) * M(x_{n_k}, x_{n_k+1}, \frac{\delta}{2}) - \epsilon \\ &= M(x_{m_k}, x_{m_k+1}, \frac{\delta}{2}) * M(Tx_{m_k}, Tx_{n_k}, t) * M(x_{n_k}, x_{n_k+1}, \frac{\delta}{2}) - \epsilon \\ &\geq M(x_{m_k}, x_{m_k+1}, \frac{\delta}{2}) * \eta^{-1}(\alpha(\eta(M(x_{m_k}, x_{n_k}, t))) \\ &\quad \eta(M(x_{m_k}, x_{n_k}, t))) * M(x_{n_k}, x_{n_k+1}, \frac{\delta}{2}) - \epsilon \\ &\geq M(x_{m_k}, x_{m_k+1}, \frac{\delta}{2}) * \eta^{-1}(b\eta(M(x_{m_k}, x_{n_k}, t))) * M(x_{n_k}, x_{n_k+1}, \frac{\delta}{2}) - \epsilon, \end{aligned}$$

for sufficiently large k. Letting $k \to \infty$, from (4) and the above, we obtain

$$l \geq \eta^{-1}(b\eta(l)) - \epsilon. \tag{8}$$

Since $\epsilon > 0$ is arbitrary, (8) yields $l \geq \eta^{-1}(b\eta(l))$ and so $\eta(l) \leq b\eta(l)$. Hence $\eta(l) = 0$, a contradiction (note that $l < 1$). Thus $\{x_n\}$ is a Cauchy sequence. By the M-completeness of X there exists $x^* \in X$ such that $\lim_{n\to\infty} x_n = x^*$. Then $\lim_{n\to\infty} \eta(M(x^*, x_n, t)) = 0$. Since

$$\eta(M(Tx^*, x_{n+1}, t)) \leq \alpha(\eta(M(x^*, x_n, t)))\eta(M(x^*, x_n, t))$$

we get $\lim_{n\to\infty} \eta(M(Tx^*, x_{n+1}, t)) = 0$ and so $x^* = \lim_{n\to\infty} x_{n+1} = Tx^*$.
To prove the uniqueness, suppose that there exists $y^* \in X$, $y^* \neq x^*$ such that $Ty^* = y^*$. Then

$$\eta(M(x^*, y^*, t)) = \eta(M(Tx^*, Ty^*, t)) \leq \alpha(\eta(M(x^*, y^*, t)))\eta(M(x^*, y^*, t)),$$

which gives $\eta(M(x^*, y^*, t)) = 0$. So, $M(x^*, y^*, t) = 1$, and hence $x^* = y^*$, a contradiction. □

In [16], Wardowski asked if the condition (a) in Theorem 1.8 can be omitted for nilpotent norms? The following corollary shows that if $M(x, y, .)$ is continuous uniformly for $x, y \in X$ and $\mathcal{O}(x)$ is bounded for each $x \in X$ we can omit conditions (a), (b) and (c) in Theorem 1.8.

Corollary 2.7. *Let $(X, M, *)$ be an M-complete fuzzy metric space such that $M(x, y, .)$ is continuous uniformly for $x, y \in X$. Let $T : X \to X$ be a fuzzy $\mathcal{H}$-contractive mapping with respect to $\eta \in \mathcal{H}$. Assume that for each $x \in X$, $\mathcal{O}(x)$ is bounded. Then T has a unique fixed point $x^* \in X$ and for each $x_0 \in X$ the sequence $\{T^n x_0\}$ converges to x^*.*

Proof. Let $\alpha(t) = k$, for $t \in [0, \infty)$, where $k \in [0, 1)$ is a constant, and apply Theorem 2.6. □

Now we illustrate our main result by the following examples.

Example 2.8. Let $X = [-1, 1]$, $a * b = ab$ for all $a, b > 0$ and let $M(x, y, t) = (\frac{t}{t+1})^{|x-y|}$ for all $x, y \in X$ and each $t > 0$. Then $(X, M, *)$ is an M-complete fuzzy metric space. Define a map $T : X \to X$ by $Tx = \frac{x^2}{4} - 1$. Let $x = 0$ and let $\{t_n\}$ be a sequence of positive numbers with $\lim_{n\to\infty} t_n = 0$. Then

$$\lim_{n\to\infty} M(x, Tx, t_n) = \lim_{n\to\infty} M(0, -1, t_n) = \lim_{n\to\infty} \frac{t_n}{t_n + 1} = 0,$$

and so $\lim_{n\to\infty} \eta(M(x, Tx, t_n)) = \lim_{t\to 0} \eta(t) = \infty$. Thus, the assumption (c) of the above mentioned theorem of Wardowski does not hold and so we cannot invoke Theorem 1.8 to show that the mapping T has a fixed point.
Now we show that the conditions of Corollary 2.7 are fulfilled. To show the claim notice first that for each $x, y \in X$, we have $M(x, y, t) = (\frac{t}{t+1})^{|x-y|} \geq (\frac{t}{t+1})^2$ and so $\mathcal{O}(x)$ is bounded for each $x \in X$. It is straightforward to show that $M(x, y, .)$ is continuous uniformly for $x, y \in X$. Let $\eta(t) = -\ln t$ and let $k = \frac{1}{2}$. We show that T is a fuzzy $\mathcal{H}$-contractive mapping with respect to η. Let $x, y \in X$ then

$$\eta(M(Tx, Ty, t)) = |\frac{x^2 - y^2}{4}| \ln(\frac{t+1}{t})$$

$$= |\frac{x+y}{4}||x - y| \ln(\frac{t+1}{t}) \leq k|x - y| \ln(\frac{t+1}{t}) = k\eta(M(x, y, t)).$$

It follows from Corollary 2.7 that T has a unique fixed point ($2 - \sqrt{8}$ is the unique fixed point of T).

Example 2.9. Let $X = [0, 1]$, $a * b = ab$ for all $a, b > 0$ and let $M(x, y, t) = \frac{t+1}{t+1+|x-y|}$ for any $x, y \in X$ and $t > 0$. It is easy to see that $(X, M, *)$ is a complete fuzzy metric space. Define a mapping $T : X \to X$ by $Tx = \ln(1 + x)$ for each $x \in [0, 1]$. For each $x, y \in X$, we have $M(x, y, t) \geq \frac{t+1}{t+2}$ and so $\mathcal{O}(x)$ is bounded for each $x \in X$. It is straightforward to show that $M(x, y, .)$ is continuous uniformly for $x, y \in X$. We first show that T is not a fuzzy $\mathcal{H}$-contractive map with respect to any $\eta \in \mathcal{H}$ which has nonzero left derivative at $t = 1$. On the contrary, assume that there exists $\eta \in \mathcal{H}$ and $k \in (0, 1)$, such that

$$\eta(M(Tx, Ty, t)) \leq k\eta(M(x, y, t)), \ \forall \ x, y \geq 0 \ \forall \ t > 0.$$

Letting $t \to 0^+$ and taking $y = 0$, from the above, we get

$$\eta(\frac{1}{1 + \ln(1 + x)}) \leq k\eta(\frac{1}{1 + x}) \ \forall \ x > 0.$$

Then

$$1 = \frac{\eta'_-(1)}{\eta'_-(1)} = \lim_{x\to 0^+} \frac{\frac{\eta(\frac{1}{1+\ln(1+x)})}{\frac{1}{1+\ln(1+x)}-1}}{\frac{\eta(\frac{1}{1+x})}{\frac{1}{1+x}-1}}$$

$$= \lim_{x\to 0^+} \frac{x}{\ln(1 + x)} \frac{\eta(\frac{1}{1+\ln(1+x)})}{\eta(\frac{1}{1+x})} = \lim_{x\to 0^+} \frac{\eta(\frac{1}{1+\ln(1+x)})}{\eta(\frac{1}{1+x})} \leq k,$$

a contradiction.
Now we show that T satisfies the conditions of Theorem 2.6. Let

$$\alpha(t) = \begin{cases} \frac{\ln(1+t)}{t} & \text{if } 0 < t, \\ 1, & \text{if } t = 0 \end{cases}$$

and let $\eta(s) = \frac{1}{s} - 1$ for $s \in (0, 1]$. Then for any $x \neq y$, we have (note that the function $\frac{\ln(1+t)}{t}$ is strictly decreasing in $(0, 1)$)

$$\eta(M(Tx, Ty, t)) \leq \frac{|\ln(1 + x) - \ln(1 + y)|}{t + 1}$$

$$\leq \frac{\ln(1 + |x - y|)}{t + 1} = \frac{\ln(1 + |x - y|)}{|x - y|} \frac{|x - y|}{t + 1}$$

$$\leq \frac{\ln(1 + \frac{|x-y|}{t+1})}{\frac{|x-y|}{t+1}} \frac{|x - y|}{t + 1}$$

$$= \alpha(\frac{|x - y|}{t + 1}) \frac{|x - y|}{t + 1} = \alpha(\eta(M(x, y, t)))\eta(M(x, y, t)).$$

It follows from Theorem 2.6 that T has a unique fixed point (0 is the unique fixed point of T).

3. Conclusions

In this work we introduce a new notion of fuzzy generalized $\mathcal{H}$-contraction, which is a generalization of fuzzy $\mathcal{H}$-contraction recently introduced by Wardowski. Then we give a fixed point result for such mappings in the setting of fuzzy M-complete metric spaces. We also give an affirmative partial answer to a question posed by Wardowski concerning the fixed point property of $\mathcal{H}$-contractive mappings.

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نقاط ثابت نگاشتهای انقباضی فازی تعمیم یافته در فضاهای متری فازی

چکیده. در این مقاله، یک مفهوم نو از یک نگاشت انقباضی تعمیم یافته فازی معرفی می کنیم و یک قضیه نقطه ثابت برای چنین نگاشتهایی در یک فضای متری فازی M-کامل به دست می دهیم. ما همچنین یک پاسخ جزیی به پرسشی که واردوسکی
[D. Wardowski, Fuzzy contractive mappings and fixed points in fuzzy metric spaces, Fuzzy Set Syst., 222(2013), 108-114]
مطرح کرد خواهیم داد. چندین مثال برای پشتیبانی از نتایج به دست آمده آورده شده اند.