INTUITIONISTIC FUZZY INFORMATION MEASURES WITH APPLICATION IN RATING OF TOWNSHIP DEVELOPMENT

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Abstract. Predominantly in the faltering atmosphere, the precise value of some factors is difficult to measure. Though, it can be easily approximated by intuitionistic fuzzy linguistic term in the real-life world problem. To deal with such situations, in this paper two information measures based on trigonometric function for intuitionistic fuzzy sets, which are a generalized version of the fuzzy information measures are introduced. Based on it new trigonometric similarity measure is developed. Mathematical illustration displays reasonability and effectiveness of the information measures for IFSs by comparing it with the existing information measures. Corresponding to information and similarity measures for IFSs, two new methods: (1) Intuitionistic Fuzzy Similarity Measure Weighted Average Operator (IFSMWAO) method for township development and (2) TOPSIS method for multiple criteria decision making (MCDM) (investment policies) problems have been developed. In the existing methods the authors have assumed the weight vectors, while in the proposed method it has been calculated using intuitionistic fuzzy information measure. This enhances the authenticity of the proposed method.

1. Introduction

Zadeh introduced fuzzy set theory [49], since then various generalization forms have been proposed and studied to deal with imprecision and uncertainty [1, 2, 5, 8, 10, 12, 27, 42]. Information and similarity measures for fuzzy sets (FSs) have been explored extensively by many researchers as vital topics in the FS theory. The entropy of a fuzzy set describes the fuzziness degree of the fuzzy set. De Luca and Termini [9] introduced some axioms to describe the fuzziness degree of a fuzzy set. Kauffman [20] proposed a method for measuring the fuzziness degree of a fuzzy set by a metric distance between its membership function and the membership function of its nearest crisp set. Yager [46] suggested the information measure can be expressed as a distance between a fuzzy set and its complement. Zeng and Li [51] demonstrated that similarity and information measures of fuzzy sets can be altered to each other through axiomatic definitions.

Further, Atanassov proposed the notions of intuitionistic fuzzy sets (IFSs) [2] and interval-valued intuitionistic fuzzy sets (IVIFSs) [1]. Corresponding to these important numerical indexes in the fuzzy set theory, many researchers extended the concepts to the IVFS theory and IFS theory and investigated their related

Received: April 2015; Revised: February 2016; Accepted: April 2016

Key words and phrases: Intuitionistic fuzzy set, Intuitionistic fuzzy information, Similarity measure, Township development, TOPSIS.

topics from different points of view [4, 6, 11, 18, 50, 52, 53]. Burrillo and Bustince [4] introduced the notions of entropy for IVFSs and IFSs to measure the degree of intuitionism of an IVFS or an IFS. Szmidt and Kacprzyk [34] proposed a non probabilistic type entropy measure with a geometric interpretation for IFSs. Hung and Yang [16] gave their axiomatic definitions of entropies for IFSs and IVFSs by exploiting the concept of probability. Farhadinia [11] generalized some results on the entropy of IVFSs based on the intuitionistic distance and its relationship with similarity measure. After that, many authors also proposed different entropy formulae for IFSs [16, 35, 36, 43, 47], IVFSs [35, 52] and vague sets [54].

Vlochos and Sergiadis [36] exposed sensitive and mathematical association among the information measures for FSs and IFSs in terms of fuzziness and intuitionism. It has been noticed that information measures for FSs is certainly a measure of fuzziness, while for IFSs, information measures can measure both fuzziness and intuitionism. It is known that the fuzziness is occupied by the difference between membership degree and non membership degree, and the intuitionism is dominated by the hesitation degree. Hence, it is very interesting to construct entropy formulae measuring both fuzziness and intuitionism.

The similarity measure is a paramount concept in the fuzzy set theory; it betokens the amount of harmonized characteristic of two fuzzy sets. Wang [37] initiated the principle of the similarity measure and introduced a computation procedure for it. Thereafter, the similarity measure of FSs has been explored. Relevantly, Wang et al. [38] performed a comparative study of similarity measures. Liang and Shi [24] and Mitchell [31] enhanced the similarity measure of IFS developed by [21]; Szmidt and Kacprzyk [33] applied the similarity measure of IFS in group decision making. Many authors fixate on the similarity measure and the information measure for IFS [21, 22, 24, 33, 34] and the relationship between them, particularly on the efficient transformation of the information measure into the similarity measure for IFS and vice versa, predicated on their definitions based on axioms.

The information and similarity measures of IFSs have been applied widely in decision making [7, 32, 33] and pattern recognition [21, 31, 45]. However, in these applications, due to the growing complication of the social-economic environment and a lack of knowledge or data about the problem domains, the decision information may be provided with IFSs, which are characterized by membership functions and non-membership functions whose values are real numbers.

In multi-criteria decision making (MCDM) problems, the decision makers rank options after qualitative or quantitative assessment of a finite set of mutually dependent or autonomous conditions. Desirable alternative can be chosen by providing predilection information in terms of exact numerical value or interval. This predilection information in real life situation can be considered in a qualitative way with vague or imprecise value.

Various researchers like [17] developed the TOPSIS method for order predilection by similarity measure to an ideal solution; Joshi and Kumar [19] proposed intuitionistic fuzzy information and distance measure predicated TOPSIS method for multi-criteria decision making (MCDM). However, sundry authors introduced IFS with TOPSIS to give a hybrid method for MCDM problems. The methods for

MCDM problems based on IFS utilizing information measure weights and linear programming were given by [15, 25, 26, 28, 48].

Motivated by the above-mentioned works, we propose two information measures based on trigonometric function for intuitionistic fuzzy sets, which are a generalized version of the fuzzy information measures in [14, 29] and a complementarity of existing information measures for intuitionistic fuzzy sets and compared with existing information measures. We also develop the similarity measure based on trigonometric function for IFSs. Corresponding to information and similarity measures for IFSs, we develop two methods: (1) New IFSMWAO method for township development and (2) New TOPSIS method for multiple criteria decision making (MCDM) (investment policies) problems. In the previous methods [15, 39, 40], authors have assumed the weight vectors, while in the proposed method we have calculated the weight vectors using intuitionistic fuzzy information measure. This enhances the authenticity of the proposed method.

2. **Prerequisite**

In this section, we discuss some fundamental conceptions or preliminaries related to fuzzy sets (FSs) and intuitionistic fuzzy sets (IFSs):

Definition 2.1. Let $X = \{x_1,\ x_2,\ ...,\ x_n\}$ be universe of discourse and let $\tilde{A} \subset X$, then fuzzy set $\tilde{A}$ is defined by [49]

$$\tilde{A} = \{(x_i, \mu_{\tilde{A}}(x_i)) : \mu_{\tilde{A}}(x_i) \in [0, 1];\ \forall x_i \in X\},$$

where $\mu_{\tilde{A}} : X \to [0, 1]$ is membership function of $\tilde{A}$. The number $\mu_{\tilde{A}}(x_i)$ shows the degree of membership of $x_i \in X$ to $\tilde{A}$.

Definition 2.2. An intuitionistic fuzzy set (IFS) A in a finite universe of discourse $X = \{x_1,\ x_2,\ ...,\ x_n\}$ is defined as [2]

$$A = \{\langle x,\ \mu_A(x),\ \nu_A(x)\rangle\ :\ x \in X\},$$

where $\mu_A : X \to [0,\ 1]$ is the degree of membership and $\nu_A : X \to [0,\ 1]$ is the degree of non- membership of $x \in X$ in A, respectively, such that

$$0 \le \mu_A(x) + \nu_A(x) \le 1,\ \ \forall x \in X.$$

The intuitionistic index (or hesitancy degree) of an element $x \in X$ in A is as

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

It implies $0 \le \pi_A(x) \le 1,\ \ \forall x \in X$ [2, 3].
If $\pi_A(x) = 0$ then IFSs can formed FSs, i. e.,
$A = \{\langle x,\ \mu_A(x),\ 1 - \mu_A(x)\rangle\ :\ x \in X\}$ with $\pi_A(x) = 0,\ \forall\ x \in X$.
The complement set of A is A^c and defined as

$$A^c = \{\langle x,\ \nu_A(x),\ \mu_A(x)\rangle\ :\ x \in X\}.$$

Definition 2.3. Let $X = \{x_1,\ x_2,\ ...,\ x_n\}$ be universe of discourse and $A, B \in IFSs(X)$ defined by [2, 3]

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,|\, x \in X\}\ \text{ and }\ B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \,|\, x \in X\},$$

then operations on IFSs are defined as follows:
(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)\ \forall x \in X$;
(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
(c) $A \cup B = \{\langle x, (\mu_A(x) \vee \mu_B(x)), (\mu_A(x) \wedge \mu_B(x))\rangle \,|\, x \in X\}$;
(d) $A \cap B = \{\langle x, (\mu_A(x) \wedge \mu_B(x)), (\mu_A(x) \vee \mu_B(x))\rangle \,|\, x \in X\}$.

For easement, the pair $(\mu_A(x), \nu_A(x))$ is called intuitionistic fuzzy number (IFN) [44] and is denoted by $\alpha = \langle \mu_{\alpha_i}, \nu_{\alpha_i}\rangle$ such that $0 \leq \mu_{\alpha_i} \leq 1$, $0 \leq \nu_{\alpha_i} \leq 1$ and $\mu_{\alpha_i} + \nu_{\alpha_i} \leq 1$.

Definition 2.4. Let $\alpha = \langle \mu_{\alpha_i}, \nu_{\alpha_i}\rangle$ and $\beta = \langle \mu_{\beta_i}, \nu_{\beta_i}\rangle$ be two IFNs, two intuitionistic fuzzy aggregation operators are defined as [44]

$$\alpha \oplus \beta = \langle \mu_{\alpha_i} + \mu_{\beta_i} - \mu_{\alpha_i}\mu_{\beta_i}, \nu_{\alpha_i}\nu_{\beta_i}\rangle \text{ and } w\alpha = \left\langle 1 - (1 - \mu_{\alpha_i})^w, \nu_{\alpha_i}^w \right\rangle, w > 0. \quad (1)$$

The other measure of IFS, the similarity measure, which plays an important role in many fields such as decision making, risk analysis, pattern recognition, cluster analysis, approximate reasoning and so on.

Definition 2.5. A real function $Sim : IFS(X) \times IFS(X) \to [0, 1]$ is called the similarity measure on $IFS(X)$, if Sim satisfies the following properties: [22]
(S1) $Sim\,(A,\ A^c) = 0$ if A is a crisp set;
(S2) $Sim\,(A, B) = 1$ iff $A = B$;
(S3) $Sim\,(A, B) = Sim\,(B, A)$;
(S4) $Sim(A, C) \leq Sim(A, B)$ and $Sim(A, C) \leq Sim(B, C)$, for all $A, B, C \in IFS(X)$, if $A \subseteq B \subseteq C$.

Method for Transforming IFSs into FSs: Li et al. [23] introduced a method for transforming 'intuitionistic fuzzy sets' into 'fuzzy sets' by distributing hesitation degree equally with membership and non membership.

Definition 2.6. Let $A \in IFS$, then the fuzzy membership function $\mu_{\tilde{A}}(x)$ to $\tilde{A}$ ($\tilde{A}$ be the fuzzy set corresponding to intuitionistic fuzzy set A) is defined as [23]

$$\mu_{\tilde{A}}(x) = \mu_A(x) + \frac{\pi_A(x)}{2} = \frac{\mu_A(x) + 1 - \nu_A(x)}{2}. \quad (2)$$

3. Similarity Measure for IFSs

In this section, we construct trigonometric similarity measure for IFSs.

Theorem 3.1. *Let* $X = \{x_1,\ x_2,\ ...,\ x_n\}$, $A = \{\langle x_i,\ \mu_A(x_i),\ \nu_A(x_i)\rangle : x_i \in X\}$ *and* $B = \{\langle x_i, \mu_B(x_i), \nu_B(x_i)\rangle : x_i \in X\}$ *defines*

$$Sim\,(A, B) = 1 - \frac{1}{n}\sum_{i=1}^{n} \sin\left[\left\{\frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{4\,(1 + |\pi_A(x_i) - \pi_B(x_i)|)}\right\}\pi\right]. \quad (3)$$

Then, $Sim\,(A, B)$ *is similarity measure on* $IFS(X)$.

Proof. Let $A,\ B,\ C \in IFS(X),\ A \subset B \subset C$, then
$\mu_A(x_i) \leq \mu_B(x_i) \leq \mu_C(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i) \geq \nu_C(x_i)$.

Therefore, $0 \leq |\mu_A(x_i) - \mu_B(x_i)| \leq |\mu_A(x_i) - \mu_C(x_i)| \leq 1$ and
$0 \leq |\nu_A(x_i) - \nu_B(x_i)| \leq |\nu_A(x_i) - \nu_C(x_i)| \leq 1$, thus

$$\sin\left\{\frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{4\,(1 + |\pi_A(x_i) - \pi_B(x_i)|)}\right\}\pi$$

$$\leq \sin\left\{\frac{|\mu_A(x_i) - \mu_C(x_i)| + |\nu_A(x_i) - \nu_C(x_i)|}{4\,(1 + |\pi_A(x_i) - \pi_C(x_i)|)}\right\}\pi.$$

Therefore, $Sim(A, C) \leq Sim(A, B)$.
Similarly, we can show that $Sim(A, C) \leq Sim(B, C)$.
From the above result, we get that Sim satisfies (S4). It is obvious that Sim satisfies (S1), (S2) and (S3). Therefore, Sim is similarity measure on $IFS(X)$. $\square$

Proposition 3.2. *For $A, B \in IFSs(X)$ and if they satisfy that for any $x_i \in X$, either $A \subseteq B$ or $B \subseteq A$, then*

(1) $Sim(A, B) = Sim(A \cup B, A \cap B)$;
(2) $Sim(A, B) = Sim(A^c, B^c)$;
(3) $Sim(A^c, B) = Sim(A, B^c)$.

Proposition 3.3. *For $A, B, C \in IFSs(X)$, then*

(1) $Sim(A \cup B, C) \leq Sim(A, C) + Sim(B, C)$;
(2) $Sim(A \cap B, C) \leq Sim(A, C) + Sim(B, C)$;
(3) $Sim(A \cup B, C) + Sim(A \cap B, C) \leq Sim(A, C) + Sim\,(B, C)$.

Taking into consideration that the elements in the universe of discourse may have different utilities or importance in the given scenario, we may assign the weight to it as
Let $w = (w_1, w_2, ..., w_1)^T$ be a weight vector of the elements $x_i \in X; i = 1(1)n$. Then we can define the weighted similarity measure as

$$Sim\,(A, B) = 1 - \sum_{i=1}^{n} w_i \sin\left[\left\{\frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{4\,(1 + |\pi_A(x_i) - \pi_B(x_i)|)}\right\}\pi\right], \quad (4)$$

where $w_i \geq 0$ and $\sum_{i=1}^{n} w_i = 1$. If $w = \left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)^T$, then measure (4) reduces to (3).

4. Trigonometric Intuitionistic Fuzzy Information Measures

In this section, we generate two new trigonometric intuitionistic fuzzy information measures.

Let $A \in IFS(X)$, where $X = \{x_1, x_2, ..., x_n\}$. Then, trigonometric intuitionistic fuzzy information measure $h_1(A)$ of A can be defined as follows:

$$h_1(A) = \frac{1}{n} \sum_{i=1}^{n} \left[1 - \sin\left\{\frac{(\mu_A(x_i) \sim \nu_A(x_i))}{2\,(1 + \pi_A(x_i))}\right\}\pi\right]. \quad (5)$$

Again to construct a new information measure for intuitionistic fuzzy set, intuitionistic fuzzy set can be altered to a fuzzy set by taking the substitution

$\mu_{\tilde{A}}(x_i) = (\mu_A(x_i) + 1 - \nu_A(x_i))/2$. Considering this substitution a new trigonometric intuitionistic fuzzy information measure $h_2(A)$ of A is introduced as

$$h_2(A) = \frac{1}{2n}\sum_{i=1}^{n}\left[\sin\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\pi + \sin\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\pi\right]. \quad (6)$$

Theorem 4.1. *The trigonometric intuitionistic fuzzy information measures $h_1(A)$ and $h_2(A)$ satisfy the following axiomatic requirements* [34]*:*

(P1): $h_1(A) = h_2(A) = 0$ *(minimum)* $\Leftrightarrow$ *A is a crisp set;*

(P2): $h_1(A) = h_2(A) = 1$ *(maximum)* $\Leftrightarrow \mu_A(x_i) = \nu_A(x_i)$ *for all* $x_i \in X$*;*

(P3): $h_1(A) \le h_1(B)$ *and* $h_2(A) \le h_2(B)$ *if A is less fuzzy than B, i.e.,* $\mu_A(x_i) \le \mu_B(x_i)$ *and* $\nu_A(x_i) \ge \nu_B(x_i)$ *for* $\mu_B(x_i) \le \nu_B(x_i)$ *or* $\mu_A(x_i) \ge \mu_B(x_i)$ *and* $\nu_A(x_i) \le \nu_B(x_i)$ *for* $\mu_B(x_i) \ge \nu_B(x_i)$ *for any* $x_i \in X$*;*

(P4): $h_1(A) = h_1(A^c)$ *and* $h_2(A) = h_2(A^c)$*.*

Proof. Let us consider $f(A)$ and $g(A)$ as follows:

$$f(A) = 1 - \sin\left\{\frac{\mu_A(x_i) \sim \nu_A(x_i)}{2\,(1+\pi_A(x_i))}\right\}\pi, \quad (7)$$

$$g(A) = \frac{1}{2}\left\{\sin\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\pi + \sin\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\pi\right\}. \quad (8)$$

For $x_i \in X = \{x_1, x_2, ..., x_n\}$, we have $0 \le f(A) \le 1$ and $0 \le g(A) \le 1$.

(P1): Let A be a crisp set, *i.e.*, $\mu_A(x_i) = 0,\ \nu_A(x_i) = 1$ or $\mu_A(x_i) = 1,\ \nu_A(x_i) = 0$ for all $x_i \in X$, then $f(A) = g(A) = 0$.
Hence $h_1(A) = h_2(A) = 0$.

(P2): For $\mu_A(x_i) = \nu_A(x_i)$, where $x_i \in X$. From (7) and (8), we get $f(A) = g(A) = 1$. Applying it in (5) and (6), we get $h_1(A) = h_2(A) = 1$.

(P3): To show that (5) and (6) fulfil the constraints of (P3), let us assume the following functions:

$$f_1(x,\ y) = 1 - \sin\left\{\frac{x \sim y}{2\,(2-x-y)}\right\}\pi, \quad (9)$$

$$g_1(x,\ y) = \frac{1}{2}\left\{\sin\left(\frac{x+1-y}{2}\right)\pi + \sin\left(\frac{y+1-x}{2}\right)\pi\right\}, \quad (10)$$

where $x, y \in [0, 1]$ and two functions $f_1(x, y)$ and $g_1(x, y)$ are increasing with respect to its first argument x and decreasing for y. Taking the partial derivative of $f_1(x, y)$ and $g_1(x, y)$ with respect to x and y, respectively, yields

$$\frac{\partial f_1(x,\,y)}{\partial x} = -\cos\left(\frac{x \sim y}{2\,(2-x-y)}\pi\right)\left\{\frac{2\,(2-x-y)\,\frac{\partial}{\partial x}\,(x \sim y) + 2\,(x \sim y)}{\{2\,(2-x-y)\}^2}\pi\right\}, \quad (11)$$

$$\frac{\partial f_1(x,\,y)}{\partial y} = -\cos\left(\frac{x \sim y}{2\,(2-x-y)}\pi\right)\left\{\frac{2\,(2-x-y)\,\frac{\partial}{\partial y}\,(x \sim y) + 2\,(x \sim y)}{\{2\,(2-x-y)\}^2}\pi\right\}, \quad (12)$$

and

$$\frac{\partial g_1(x,\,y)}{\partial x} = \frac{\pi}{4}\left[\cos\left(\frac{x+1-y}{2}\right)\pi - \cos\left(\frac{y+1-x}{2}\right)\pi\right], \quad (13)$$

$$\frac{\partial g_1(x,\, y)}{\partial y} = \frac{\pi}{4}\left[-\cos\left(\frac{x+1-y}{2}\right)\pi + \cos\left(\frac{y+1-x}{2}\right)\pi\right]. \tag{14}$$

To find the critical point of $f_1(x,\, y)$ and $g_1(x,\, y)$, we set $\frac{\partial f_1(x,\, y)}{\partial x} = 0$, $\frac{\partial f_1(x,\, y)}{\partial y} = 0$, $\frac{\partial g_1(x,\, y)}{\partial x} = 0$ and $\frac{\partial g_1(x,\, y)}{\partial y} = 0$. Solving for the critical point x_{cp}, we get

$$x_{cp} = y. \tag{15}$$

From (11), (13) and (15), we have

$$\frac{\partial f_1(x,\, y)}{\partial x} \geq 0 \quad \text{for} \quad x \leq y \quad \text{and} \quad \frac{\partial f_1(x,\, y)}{\partial x} \leq 0 \quad \text{for} \quad x \geq y, \tag{16}$$

$$\frac{\partial g_1(x,\, y)}{\partial x} \geq 0 \quad \text{for} \quad x \leq y \quad \text{and} \quad \frac{\partial g_1(x,\, y)}{\partial x} \leq 0 \quad \text{for} \quad x \geq y. \tag{17}$$

For any $x,\ y \in [0,\ 1]$, $f_1(x,\ y)$ and $g_1(x,\ y)$ are increasing with respect to x for $x \leq y$ and decreasing when $x \geq y$. Similarly, we obtain

$$\frac{\partial f_1(x,\, y)}{\partial y} \leq 0 \quad \text{for} \quad x \leq y \quad \text{and} \quad \frac{\partial f_1(x,\, y)}{\partial y} \geq 0 \quad \text{for} \quad x \geq y, \tag{18}$$

$$\frac{\partial g_1(x,\, y)}{\partial y} \leq 0 \quad \text{for} \quad x \leq y \quad \text{and} \quad \frac{\partial g_1(x,\, y)}{\partial x} \geq 0 \quad \text{for} \quad x \geq y. \tag{19}$$

Let us consider $A \leq B$ and the finite universe of discourse X is partitioned into two disjoint sets X_1 and X_2 with $X_1 \cup X_2 = X$. Further, we assume that $x_i \in X_1$, $\mu_A(x_i) \leq \mu_B(x_i) \leq \nu_B(x_i) \leq \nu_A(x_i)$ and $x_i \in X_2$, $\mu_A(x_i) \geq \mu_B(x_i) \geq \nu_B(x_i) \geq \nu_A(x_i)$. Then, from the monotonicity of $f_1(x,\ y)$ and $g_1(x,\ y)$, we obtain $h_1(A) \leq h_1(B)$ and $h_2(A) \leq h_2(B)$ when $A \leq B$.
The closer the $\mu_A(x_i)$ to $\nu_A(x_i)$ for $x_i \in X$, the greater the value of $h_1(A)$ and $h_2(A)$, and at $\mu_A(x_i)$ equals to $\nu_A(x_i)$ for $x_i \in X$, the value reaches its maximum, *i. e.*, $h_1(A) = h_2(A) = 1$.
(P4): It is clear that $A^c = \{\langle x_i,\ \nu_A(x_i),\ \mu_A(x_i)\rangle : x_i \in X\}$ for $x_i \in X$, *i.e.*, $\mu_{A^c}(x_i) = \nu_A(x_i)$ and $\nu_{A^c}(x_i) = \mu_A(x_i)$. From (7) and (8), we get $f(A) = f(A^c)$ and $g(A) = g(A^c)$.
Hence $h_1(A) = h_1(A^c)$ and $h_2(A) = h_2(A^c)$. □

Furthermore, let $w = (w_1, w_2, ..., w_1)^T$ be a weight vector of the elements $x_i \in X; i = 1(1)n$, based on (5) and (6), the weighted information measures are defined as

$$h_1(A) = \sum_{i=1}^{n} w_i \left[1 - \sin\left\{\frac{(\mu_A(x_i) \sim \nu_A(x_i))}{2\,(1 + \pi_A(x_i))}\right\}\pi\right],$$

and

$$h_2(A) = \frac{1}{2}\sum_{i=1}^{n} w_i \left[\sin\left(\frac{\mu_A(x_i) + 1 - \nu_A(x_i)}{2}\right)\pi + \sin\left(\frac{\nu_A(x_i) + 1 - \mu_A(x_i)}{2}\right)\pi\right].$$

5. Mathematical Illustration

In this section, the effectiveness of the proposed information measures for IFSs are illustrated mathematically by its comparison with the existing information measues in [30], [36], [41] and [47].

Let $A \in IFSs(X)$, where $X = \{x_1, x_2, ..., x_n\}$. Vlochos and Sergiadis [36] introduced a measure of intuitionistic fuzzy information E_{ln}. Ye [47] suggested two measures of intuitionistic fuzzy entropy E_1 and E_2. Wei et al [41] defined a measure of intuitionistic fuzzy entropy E_{WGG}. Mishra et al. [30] proposed an exponential intuitionistic fuzzy information measure h_e.
Here, the list of intuitionistic fuzzy information measures as follows:

$$E_{\ln}(A) = -\frac{1}{n \ln 2} \sum_{i=1}^{n} [\mu_A(x_i) \ln \mu_A(x_i) + \nu_A(x_i) \ln \nu_A(x_i) \\ -(1 - \pi_A(x_i)) \ln(1 - \pi_A(x_i)) - \pi_A(x_i) \ln 2], \quad (20)$$

$$E_1(A) = \frac{1}{n} \sum_{i=1}^{n} \left[\frac{\left\{ \sin \frac{\pi(1+\mu_A(x_i)-\nu_A(x_i))}{4} + \sin \frac{\pi(1+\nu_A(x_i)-\mu_A(x_i))}{4} - 1 \right\}}{\sqrt{2} - 1} \right], \quad (21)$$

$$E_2(A) = \frac{1}{n} \sum_{i=1}^{n} \left[\frac{\left\{ \cos \frac{\pi(1+\mu_A(x_i)-\nu_A(x_i))}{4} + \cos \frac{\pi(1+\nu_A(x_i)-\mu_A(x_i))}{4} - 1 \right\}}{\sqrt{2} - 1} \right], \quad (22)$$

$$E_{WGG}(A) = \frac{1}{n} \sum_{i=1}^{n} \left[\left\{ \sqrt{2} \cos \ \pi \left(\frac{\mu_A(x_i) - \nu_A(x_i)}{4} \right) - 1 \right\} \times \frac{1}{\sqrt{2} - 1} \right], \quad (23)$$

$$h_e(A) = \frac{1}{n\sqrt{e}\,(\sqrt{e} - 1)} \sum_{i=1}^{n} \left[e - \left(\frac{\mu_A(x_i) + 1 - \nu_A(x_i)}{2} \right) e^{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2} \right)} \\ - \left(\frac{\nu_A(x_i) + 1 - \mu_A(x_i)}{2} \right) e^{\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2} \right)} \right]. \quad (24)$$

Example 5.1. Let us consider the universe $X = \{x\}$ and calculate the information measure for the following intuitionistic fuzzy sets:

$A_1 = \{\langle x, 0.2, 0.5\rangle\}$, $A_2 = \{\langle x, 0.3, 0.5\rangle\}$, $A_3 = \{\langle x, 0.4, 0.5\rangle\}$, $A_4 = \{\langle x, 0.5, 0.5\rangle\}$.

From these results, we can interpret that the closer the membership degree to the non-membership degree, the higher the value of intuitionistic fuzzy information ($h_1(A_1) \leq h_1(A_2) \leq h_1(A_3) \leq h_1(A_4)$ and $h_2(A_1) \leq h_2(A_2) \leq h_2(A_3) \leq h_2(A_4)$). And when the membership degree is equal to the non membership degree, *i.e.*, $A_4 = \{\langle x, 0.5,\ 0.5\rangle\}$, the intuitionistic fuzzy information measures reaches their

IFSs	h_1	h_2	h_e	$E_{\ln}$	E_1	E_2	E_{WGG}
A_1	0.6454	0.8910	0.9130	0.9042	0.9057	0.9057	0.9056
A_2	0.7412	0.9511	0.9614	0.9635	0.9580	0.9580	0.9580
A_3	0.8577	0.9877	0.9904	0.9920	0.9896	0.9896	0.9896
A_4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 1. Comparison with Existing Information Measures for A_1, A_2, A_3 and A_4

maximum. Thus, it can easily be interpreted that the measures of the intuitionistic fuzzy information have the following order:

$$h(A_1) \leq h(A_2) \leq h(A_3) \leq h(A_4).$$

Hence effectiveness of the proposed measures $h_1(A)$ and $h_2(A)$ can be deduced from the Table 1.

6. Applications

Information measures for FSs and IFSs have applications in various fields viz., pattern recognition, image processing, etc., a few among them are listed below:

6.1. Rating of Township Development.

Embryonic the township and the firm have to keep in mind various factors with different probabilities. Predominantly in the stumbling atmosphere, the precise value of these factors is difficult to measure. Though, it can be easily approximated by intuitionistic fuzzy linguistic term in the real-life world problem. By intuitionistic fuzzy linguistic term, we can expediently represent the possibility and the tenderness of these factors. Throughout the fuzzy exploration, we can precisely envisage the rating of the township development.

In broad, by survey and statistical analysis from some field specialists and accomplishment administrators we can effortlessly get some essential factors of township development. The factors that incur the township development mainly include the security, power accessibility, 24 hour water accessibility, connectivity from railway station and airport and to the hospitals, hygiene, shopping complex, recreation zone, school, transportation, swimming pool, fitness centre, health spa and resort, parking, as well as the emergency facility availability, etc. Let $P = \{p_1, p_2, ..., p_m\}$ be the set of all factors mandatory for township development. Usually, the factors are intuitionistic fuzzy concept. The accurate values of the requirement and the occurring probability of each factors are difficult to measure in faltering setting. On the contrary, government administrators and related field specialists tend to assess the possibility and the requirement of the above uncertain factors in developing township by using intuitionistic fuzzy language terms like $C=$ { very high, High, medium, low, very low} and $F=$ {excellent, significant, appreciable, substantial, considerable} rather than by using exact real numbers.

In a lucid manner of treatment of judgement expression, a unified set of linguistic variables is predetermined in present communication, which can be personalized to all the factors in developing a township from the satisfactory perception as shown in Table 2, where each linguistic term is assigned as an intuitionistic fuzzy number

Linguistic terms	IFNs
Very high (VH)	(0.9, 0.1)
high (H)	(0.7, 0.2)
Medium (M)	(0.5, 0.4)
Low (L)	(0.3, 0.6)
Very Low (VL)	(0.1, 0.9)

Table 2. Linguistic Terms for Grading the Factors in Developing Township

Linguistic terms	IFNs
Excellent (E)	(0.9, 0.1)
Significant (S)	(0.8, 0.2)
Appreciable (A)	(0.6, 0.3)
Substantial (Su)	(0.4, 0.5)
Considerable (C)	(0.2, 0.7)

Table 3. Linguistic Terms for Rating the Township Development

(IFN), for example, $W = (0.3, 0.6)$ represents the membership is 0.3 and non membership is 0.6, indicating the degree of effectiveness lies in interval [0.3, 0.4].

To determine the grading of township development and to provide healthy living atmosphere, we should first place the factors grading. For convenience, the five factors rating of township development are pre-established and categorized by the following intuitionistic fuzzy linguistic terms as listed in Table 3. Here, we denote all the five factors grading by the set

$$R = \{R_1(E),\, R_2(S),\, R_3(A),\, R_4(Su),\, R_5(C)\}\,.$$

We give the intuitionistic fuzzy ample factors assessment process for the township development involved intuitionistic fuzzy early vigilance assessment value under uncertain environment.

Step 1: Let $P = \{p_1, p_2, ..., p_m\}$ be the set of all fuzzy factors of township development and $Q = \{q_1,\, q_2,\, ...,\, q_n\}$ be the judgment set for factors occurring probability, q_j $(j = 1(1)n)$ be the different probability grading of the occurrence of each factor. Let $\tilde{D} = (\tilde{r}_{i\,j})_{m\times n}$ be the fuzzy judgment matrix, $\tilde{r}_{i\,j}$ is the intuitionistic fuzzy membership value of factor p_i with respect to the judgment criteria q_j, which can be given by the facts and knowledge of field specialists.

Step 2: By using information measure formula (6), we can easily compute the information of each intuitionistic fuzzy value in the intuitionistic fuzzy judgment matrix and get the information matrix of this judgment matrix as $D = (\delta_{i\,j})_{m\times n}$, where $\delta_{i\,j} = h_2\,(\tilde{r}_{i\,j})$.

Step 3: Normalize the information values in the above decision matrix by using the equation

$$\overline{\delta}_{i\,j} = \frac{\delta_{i\,j}}{\max \delta_{i\,j}},\ j = 1(1)n;\ i = 1(1)m. \tag{25}$$

and expressed it as $D = \left(\overline{\delta}_{i\,j}\right)_{m\times n}$.

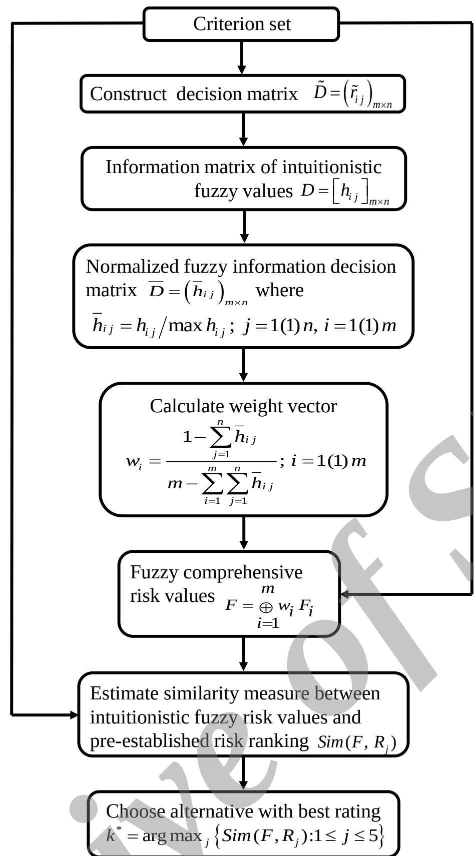

Figure 1. Implementation flow chart of IFSMWAO method

Step 4: Compute the occurring probability of each factor p_i by applying formula

$$w_i=\frac{1-\sum_{j=1}^{n}\bar{\delta}_{i\,j}}{m-\sum_{i=1}^{m}\sum_{j=1}^{n}\bar{\delta}_{i\,j}},\ i=1(1)m. \tag{26}$$

Step 5: Estimate the fuzzy ample factors value according to the above probability of each factor in township development by the following formula:

$$F=\overset{m}{\underset{i=1}{\oplus}} w_i\,F_i \tag{27}$$

where F_i is the probability of the factor p_i.

Step 6: Determine the similarity measure $Sim\,(F,\ R_j)$ between the intuitionistic fuzzy ample factor value R and each pre-established rating R_j, where R_j the j^{th} rank in the pre-established factor grade is

$$R=\{R_1(E),\,R_2(S),\,R_3(A),\,R_4(Su),\,R_5(C)\}\,.$$

Early vigilance rating	Grading
R_1 (Excellent)	Marked by Five star
R_2 (Significant)	Marked by Four star
R_3 (Appreciable)	Marked by Three star
R_4 (Substantial)	Marked by Two star
R_5 (Considerable)	Marked by One star

TABLE 4. Decision Mechanism for Early Vigilance Rating of Township Development

Step 7: Estimate the factor grading of the township development.

By using the similarity measure, we can determine the factor of the township development. If $k^* = \arg\max_j \{Sim(F, R_j) : 1 \leq j \leq 5\}$, then the unexpected township development should belong to the given factor grading R_{k^*}.

Step 8: The approximated early vigilance rating, we design the decision mechanism and adopt the corresponding strategy to deal with the occurrence of township development as following Table 4.

In recent times, developers and field professionals usually tend to employ intuitionistic fuzzy values to evaluate the constraints for township development with respect to various earning indexes. In this subsection, we illustrate the application of the proposed intuitionistic fuzzy information measure and similarity measure mathematically in early vigilance degree evaluation and decision making for uncertain township developers.

Example 6.1. Suppose the developers try to design a decision mechanism and carry out some developing measures, it requires the management monitor to all the index information of possible township development and evaluates the comprehensive factor value of township development. Assume the set of factors $P = \{p_1$ (water accessibility, power accessibility), p_2 (connectivity from railway station and airport and to the hospitals), p_3 (hygiene, shopping complex, recreation zone, school,), p_4 (fitness centre, health spa and resort)$\}$ must be taken into account for the township development. And the occurring probability of each factor is unknown, but may be evaluated by the intuitionistic fuzzy judgment constraints in Q. $\{q_1$(Very Low), q_2(Low), q_3(Medium), q_4(High), q_5(Very High)$\}$. The evaluating results are expressed by an intuitionistic fuzzy comprehensive judgment matrix, where q_j $(1 \leq j \leq 5)$ denotes the different occurring possibility of each factor of township development as in Table 5. And the outcome of each factor is given by intuitionistic fuzzy linguistic term from the related management experts.

Assume that the outcome of the above four factors of township development as very high, high, medium, low, very low. Our main task is to determine the early vigilance rating of the township development involved intuitionistic fuzzy value. That is to decide the factor grade, out of the five grades R_1, R_2, R_3, R_4, R_5, the township development belongs to. We employ the intuitionistic fuzzy information measure to calculate the occurring probability of each fuzzy factor and the total factor value of township development and then help the related township development authority to adopt the corresponding decision mechanism to organize and enhance

factors	q_1	q_2	q_3	q_4	q_5
p_1	(0.1, 0.8)	(0.3, 0.5)	(0.5, 0.4)	(0.8, 0.1)	(0.6, 0.3)
p_2	(0.7, 0.2)	(0.6, 0.2)	(0.8, 0.1)	(0.5, 0.3)	(0.9, 0.1)
p_3	(0.5, 0.4)	(0.8, 0.1)	(0.1, 0.5)	(0.25, 0.65)	(0.2, 0.5)
p_4	(0.6, 0.3)	(0.4, 0.6)	(0.3, 0.7)	(0.2, 0.6)	(0.3, 0.4)

Table 5. Intuitionistic Fuzzy Judgment of the Occurring Probability of Factors in Township Development

the ample interests of township development. First, we regard Table 5 as the intuitionistic fuzzy ample judgment matrix $\tilde{D} = (\tilde{r}_{i\,j})_{4\times 5}$ of factors with respect to all the judgment criteria, where $\tilde{r}_{i\,j}$ represents the intuitionistic membership value of factor p_i with respect to probability grade q_j, for example, $\tilde{r}_{i\,j} = (0.8,\,0.1)$ represents the true membership and the false membership of factor u_3 belong to the occurring probability grade v_2 are 0.8 and 0.1, respectively.

By using the information measure formula (6), we can compute the information measure of each intuitionistic fuzzy value in the above judgment matrix and get the following information measure matrix

$$D = (\delta_{i\,j})_{4\times 5} = \begin{bmatrix} 0.4540 & 0.9511 & 0.9877 & 0.4540 & 0.8910 \\ 0.7071 & 0.8090 & 0.4540 & 0.9511 & 0.3090 \\ 0.9877 & 0.4540 & 0.8090 & 0.8090 & 0.8910 \\ 0.8910 & 0.9511 & 0.8090 & 0.8090 & 0.9877 \end{bmatrix}.$$

With formula (25), we transform the above information measure matrix to the normalized information measure matrix below.

$$\overline{D} = (\overline{\delta}_{ij})_{4\times 5} = \begin{bmatrix} 0.4596 & 0.9629 & 1.0000 & 0.4596 & 0.9021 \\ 0.7435 & 0.8506 & 0.4773 & 1.0000 & 0.3249 \\ 1.0000 & 0.4596 & 0.8191 & 0.8191 & 0.9021 \\ 0.9021 & 0.9621 & 0.8191 & 0.8191 & 1.0000 \end{bmatrix}.$$

Then, by the formula (26) we compute the occurring probability of each factor of the township development by the formula

$$w_i = \frac{1 - \sum_{j=1}^{n} \overline{\delta}_{i\,j}}{m - \sum_{i=1}^{m}\sum_{j=1}^{n} \overline{\delta}_{i\,j}},\ i = 1(1)m.$$

Thus, the probability vector of all the factors of township developers are obtained as $W = (0.2383,\, 0.2051,\, 0.2568,\, 0.2998)^T$.
Next, according to the given consequences' of each factors, we get

$$\begin{aligned}(F_1,\ F_2,\ F_3,\ F_4) &= \{\text{High}, \text{Medium}, \text{Low}, \text{Very High}\} \\ &= \{(0.7,\ 0.2),\ (0.5,\ 0.4),\ (0.3,\ 0.6),\ (0.9,\ 0.1)\}.\end{aligned}$$

From the previous formulae (1) and (27), we calculate the intuitionistic fuzzy ample factor value by

$$\begin{aligned}F = \overset{m}{\underset{i=1}{\oplus}} w_i\,F_i &= 0.2383R_1 \oplus 0.2051R_2 \oplus 0.2568R_3 \oplus 0.2998R_4 \\ &= (0.7021,\, 0.2484).\end{aligned}$$

Method	Ranking	Best alternative
TOPSIS proposed by [13]	$R_2 \succ R_3 \succ R_4 \succ R_1 \succ R_5$	R_2
Fuzzy TOPSIS proposed by [15]	$R_2 \succ R_3 \succ R_1 \succ R_4 \succ R_5$	R_2
Intuitionistic fuzzy TOPSIS proposed by [19]	$R_2 \succ R_3 \succ R_1 \succ R_4 \succ R_5$	R_2
Proposed IFSMWAO method	$R_2 \succ R_3 \succ R_1 \succ R_4 \succ R_5$	R_2

TABLE 6. Ranking Order of Alternative for Different Methods

Then, according to the similarity formula (3) between intuitionistic fuzzy values, we calculate the similarity measure between the calculated fuzzy comprehensive factor value and each given factor grade in $R = \{R_1(E), R_2(S), R_3(A), R_4(Su), R_5(C)\}$ as follows:
$Sim\,(F, R_1) = 0.7437$, $Sim(F, R_2) = 0.9157$, $Sim(F, R_3) = 0.8853$, $Sim(F, R_4) = 0.5978$ and $Sim(F, R_5) = 0.3459$. Since

$$Sim\,(F, R_2) \geq Sim\,(F, R_3) \geq Sim\,(F, R_1) \geq Sim\,(F, R_4) \geq Sim\,(F, R_5),\ i.e.,$$

$k^* = \arg\max_j \{Sim\,(F,\, R_j) : R_j \in R\}$, then the ample factors of township developers should belong to R_2 rating and the factors of this township developers may be "Significant or four star". The descending order relations of relative closeness coefficient for various alternatives are shown in Table 6.

Descending order relation of relative closeness coefficient is in accordance with the already existing methods. There is no conflict in choosing R_2 as the best alternative corresponding to the proposed methods. Comparison of the proposed method with the already existing methods is displayed in Table 6. The comparison reveals that the proposed method is reasonable and authentic as compared with the other methods.

6.2. **TOPSIS Method for Company Investment Policies.** In many practical decision making problems, such as the supplier selection or selection of a partner for an enterprize in the field of supply chain management, military system efficiency evaluation, and so on, decision makers usually need to provide their preferences over alternatives. Consider as the socioeconomic environment becomes more complex, the preference information provided by decision makers is becomes imprecise. In such cases, it is suitable and convenient to express the decision makers' preferences in intuitionistic fuzzy sets (IFSs). Here, new TOPSIS method for solving a multiple criteria decision making (MCDM) quandaries with weights is developed.

Let $O = \{O_1, O_2, ..., O_m\}$ be the set of options, $C = \{C_1, C_2, ..., C_n\}$ be a set of criteria.

Step 1: Construction of Decision Matrix for IFSs

First of all, intuitionistic fuzzy decision matrix $D = [d_{ij}]_{n\times m}$ of intuitionistic fuzzy value $d_{ij} = (\mu_{ij}, \nu_{ij})$ is constructed. Let μ_{ij} and ν_{ij} be the degrees of membership and non-membership of the alternatives E_i satisfying the criterion G_j. The intuitionistic index $\pi_{ij} = 1 - \mu_{ij} - \nu_{ij}$ denotes the decision maker's hesitation of the alternatives E_i with respect to criterion G_j.

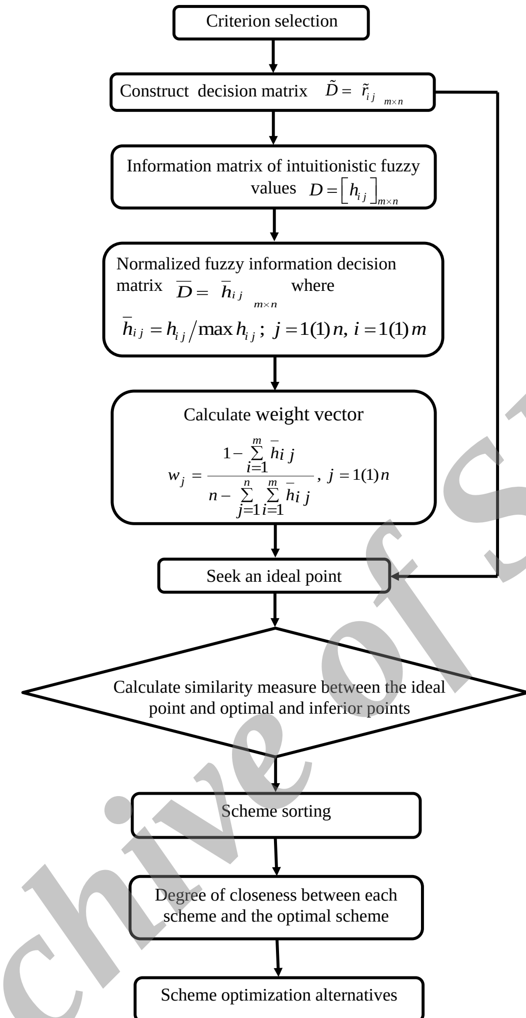

Figure 2. Implementation Flow Chart of New TOPSIS Method

Step 2: Obtain Information and Normalized Information Matrix

From (5), compute the information of each intuitionistic fuzzy value in the intuitionistic fuzzy judgment matrix and get the information matrix of this judgment matrix as $D = (h_{i\,j})_{n\times m}$, where $h_{i\,j} = h_1(\tilde{r}_{i\,j})$.

Normalize the information values in the above decision matrix by

$$\overline{h}_{i\,j} = \frac{h_{i\,j}}{\max h_{i\,j}},\ j = 1(1)m;\ \ i = 1(1)n. \tag{28}$$

And the normalized information matrix is expressed as $\overline{D} = \left(\overline{h}_{i\,j}\right)_{n\times m}$.

Step 3: Determination of Weights of Criteria

Let $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$, be weight vector, where $\omega_j \geq 0$ and $\sum_{j=1}^{n} \omega_j = 1$, and w_j indicates the relative importance of criterion G_j. These weights are expressed as intuitionistic fuzzy numbers. In order to obtain ω, we must gather the decision maker's opinions to get the aggregated intuitionistic fuzzy weight of criteria. Compute the weight vector by applying the given formula

$$w_j = \frac{1 - \sum_{i=1}^{m} \overline{h}_{i\,j}}{n - \sum_{j=1}^{n} \sum_{i=1}^{m} \overline{h}_{i\,j}}, \; j = 1(1)n. \tag{29}$$

Step 4: Compute the Positive-ideal Solution (PIS) and Negative-ideal Solution (NIS)

According to intuitionistic fuzzy theory and the doctrine of the conventional TOPSIS technique, positive-ideal O^+ and negative-ideal solution O^- can be defined as follows:

$$O^+ = \left\{ \left(\max_j \mu_{ij} |\; i \in C, \; \min_j \; \nu_{ij} \mid i \in C \right) : \; j = 1, 2, ..., n \right\}, \tag{30}$$

$$O^- = \left\{ \left(\min_j \mu_{ij} \mid i \in C, \; \max_j \; \nu_{ij} \mid i \in C \right) : \; j = 1, 2, ..., n \right\}, \tag{31}$$

where for each $i = 1(1)m$.

Step 5: Calculation of Similarity Measures from Positive and Negative Ideal Solution

From (4), calculate the weighted similarity measure $S(O_i, O^+)$ among the options $O_i (i = 1, 2, ..., m)$ and the positive-ideal solution O^+ and the similarity measure $S(O_i, O^-)$ among the options $O_i (i = 1, 2, ..., m)$ and the negative-ideal solution O^-.

Step 6: Calculation of Relative Closeness Coefficient (CC)

At last, relative closeness coefficient of each alternative with respect to intuitionistic fuzzy ideal solutions can be computed by using the following expression:

$$C_c(O_i) = \frac{S(O_i, O^+)}{S(O_i, O^+) + S(O_i, O^-)}, \quad i = 1(1)m. \tag{32}$$

Step 7: Choose the optimal value $C_c(O_k)$ (say) among the values $C_c(O_i), \; i = 1(1)m$. And hence O_k is the optimal choice.

In the process of MADM with intuitionistic fuzzy information, sometimes, the attribute values take the form of intuitionistic fuzzy numbers and the information about attribute weights are incompletely known or completely unknown. In such cases, it is suitable and convenient to express the decision makers' preferences in an intuitionistic fuzzy number (IFN). Therefore, it is necessary and interesting to pay attention to the group decision-making problems with interval-valued intuitionistic preference information. Therefore, it is necessary to pay attention to this issue.

Example 6.2. In order to illustrate the proposed method, suppose that there is an investment company to invest a sum of money in the best option. Assume that there is a panel with five alternatives to invest the money: a car company O_1,

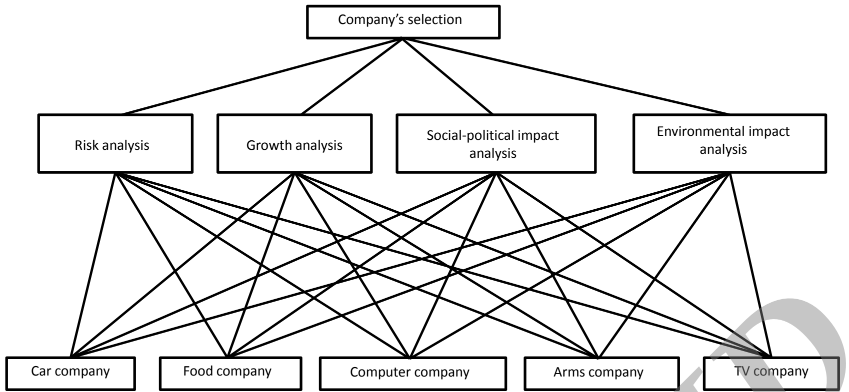

FIGURE 3. The Hierarchical Structure

a food company O_2, a computer company O_3, an arms company O_4, and a TV company O_5. The investment company needs to take a decision according to the following four attributes: A_1 is the risk analysis; A_2 is the growth analysis; A_3 is the social-political impact analysis; A_4 is the environmental impact analysis. The five possible alternatives $O_i\,(i=1,2,3,4,5)$ are to be evaluated by the decision maker under the above four attributes in the following form:

$$\begin{array}{c} \\ O_1 \\ O_2 \\ O_3 \\ O_4 \\ O_5 \end{array} \begin{array}{c} \begin{array}{cccc} \quad A_1 \quad & \quad A_2 \quad & \quad A_3 \quad & \quad A_4 \quad \end{array} \\ \begin{bmatrix} (0.5,\,0.4) & (0.6,\,0.3) & (0.3,\,0.6) & (0.2,\,0.7) \\ (0.7,\,0.3) & (0.7,\,0.2) & (0.7,\,0.2) & (0.4,\,0.5) \\ (0.6,\,0.4) & (0.5,\,0.4) & (0.5,\,0.3) & (0.6,\,0.3) \\ (0.8,\,0.1) & (0.6,\,0.3) & (0.3,\,0.4) & (0.2,\,0.6) \\ (0.6,0.2) & (0.4,\,0.3) & (0.7,\,0.1) & (0.5,\,0.3) \end{bmatrix} \end{array}$$

Step 2: By using the information measure (5), authors can evaluate the information measure of each intuitionistic fuzzy value in the above judgment matrix and get the following information matrix:

$$D = (h_{i\,j})_{5\times\,4} = \begin{bmatrix} 0.9877 & 0.8910 & 0.8910 & 0.7071 \\ 0.8090 & 0.7071 & 0.7071 & 0.9877 \\ 0.9511 & 0.9877 & 0.9511 & 0.8910 \\ 0.4540 & 0.8910 & 0.9877 & 0.8090 \\ 0.8090 & 0.9877 & 0.5878 & 0.9511 \end{bmatrix}.$$

Using (28), the information matrix is transform into the normalized information matrix :

$$\overline{D} = \left(\overline{h}_{i\,j}\right)_{5\,\times\,4} = \begin{bmatrix} 1.0000 & 0.9021 & 0.9021 & 0.7159 \\ 0.8191 & 0.7159 & 0.7159 & 1.0000 \\ 0.9629 & 1.0000 & 0.9629 & 0.9021 \\ 0.4597 & 0.9021 & 1.0000 & 0.8191 \\ 0.8191 & 1.0000 & 0.5951 & 0.9629 \end{bmatrix}.$$

Alternatives	O_j^+	O_j^-	Closeness coefficient	Ranking
O_1	0.5451	0.9381	0.3675	5
O_2	0.8482	0.6231	0.5765	1
O_3	0.7545	0.7177	0.5125	3
O_4	0.6912	0.7754	0.4713	4
O_5	0.8450	0.6361	0.5705	2

Table 7. Intuitionistic Similarity Measures of Each Alternatives from IF PIS and IF NIS

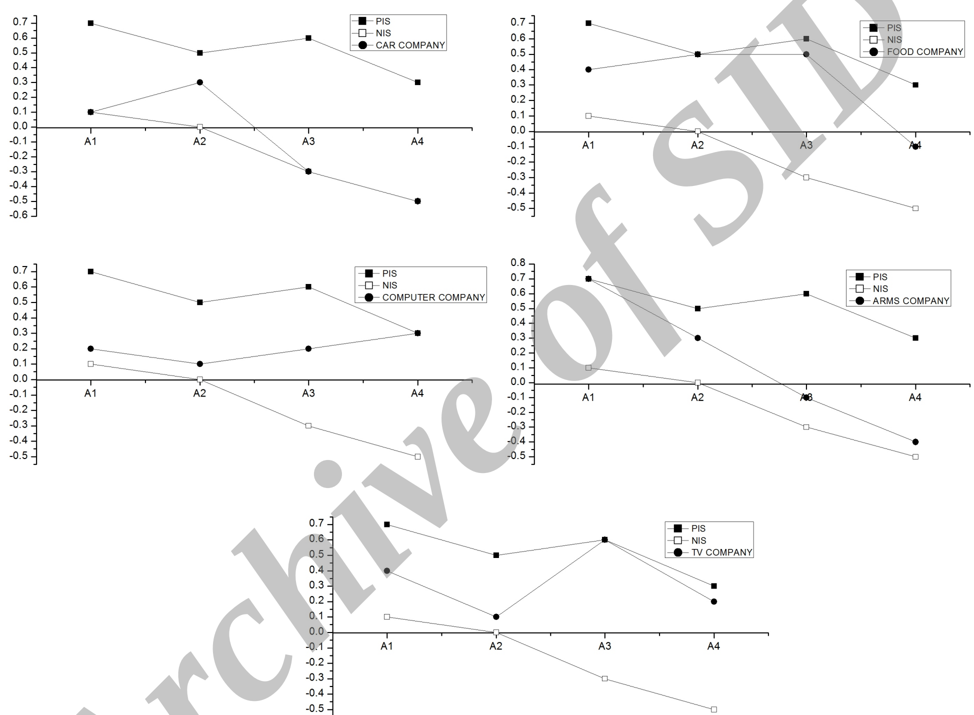

Figure 4. Comparison Between Each Company, IF PIS and IF NIS

Step 3: Compute criterion weight vector, utilize the normalized information matrix and by the formula (29), we have

$$w_j = \frac{1 - \sum_{i=1}^{m} \overline{h}_{i\,j}}{n - \sum_{j=1}^{n} \sum_{i=1}^{m} \overline{h}_{i\,j}},\ i = 1(1)n.$$

Thus, the weight vector of all the decision attributes are obtained as

$$W = (0.2326,\ 0.2676,\ 0.2414,\ 0.2584)^T.$$

Method	Ranking	Best alternative
Fuzzy TOPSIS method proposed by [15]	$O_5 \succ O_2 \succ O_3 \succ O_4 \succ O_1$	O_5
TOPSIS method proposed by [40]	$O_5 \succ O_2 \succ O_3 \succ O_4 \succ O_1$	O_5
Intuitionistic fuzzy TOPSIS proposed by [19]	$O_2 \succ O_5 \succ O_4 \succ O_3 \succ O_1$	O_2
Proposed new TOPSIS method	$O_2 \succ O_5 \succ O_3 \succ O_4 \succ O_1$	O_2

Table 8. Ranking Order of Alternative for Different Methods

Step 4: Compute the Positive-ideal Solution (PIS) and Negative-ideal Solution (NIS)

According to intuitionistic fuzzy theory and the doctrine of the conventional TOPSIS technique, positive-ideal O^+ and negative-ideal solution O^- can be defined as follows:

$$O^+ = \{(0.8, 0.1), (0.7, 0.2), (0.7, 0.1), (0.6, 0.3)\},$$
$$O^- = \{(0.5, 0.4), (0.4, 0.4), (0.3, 0.6), (0.2, 0.7)\}.$$

Step 5: Intuitionistic fuzzy separation measure of each alternative from the positive and negative-ideal solution are computed by using (4) and are given by Table 7.
Accordance with the descending order of relative closeness coefficients values four alternatives are ranked as $O_2 \succ O_5 \succ O_3 \succ O_4 \succ O_1$. Comparisons of the proposed method with the existing methods are depicted in Table 8.
There is no conflict in choosing O_2 as the best alternative corresponding to the proposed methods.

In this subsection, the descending order ranking of the companies using IF PIS and IF NIS is drawn graphically in Figure 4. And we have found that the graphical interpretation is in accordance with the proposed method. As can be seen from Figure 4 food company (O_2) performs relatively better than the other four companies under most of the criteria, and is closer to IF PIS than other companies. On the other hand, car company (O_1) performs relatively worse than the other four companies under most of the criteria and is closer to NIS than the other companies. To summarize, food company (O_2) should be selected for cooperation.

7. Conclusion

During the recent years, to determine suitable alternatives in the multiple criteria decision making problems has become a key strategic consideration. In MCDM process the alternatives information and performances are usually incomplete and uncertain. Therefore, the decision makers are unable (or unwilling) to express their judgment on the alternatives with exact and crisp values and the evaluations are very often expressed in linguistic terms. In such situation intuitionistic fuzzy set theory is an appropriate tool to deal with this kind of problems. While many information measures have been developed, still there is a scope that parametrically generalized measures can be developed, which will find better relevance in a diverse fields.

In this paper, similarity measure for intuitionistic fuzzy sets is developed and trigonometric information measures for intuitionistic fuzzy sets are introduced. The two measures for IFSs are reasonable and effective is illustrated mathematically. Corresponding to the intuitionistic fuzzy information and similarity measures, two methods for multiple criteria decision making problems are developed. And weight of each criterion is calculated using intuitionistic fuzzy information measure. Two real case studies are discussed to rank the township development and investment policies problems.

In the proposed approaches firstly the proposed information measure is used to construct intuitionistic fuzzy decision matrix and calculate the weight of each criteria. Then, (1) the ample factor for each factor in township development, (2) the IF PIS and the IF NIS are estimated.

Based on the proposed similarity measures, we calculate (1) similarity between intuitionistic fuzzy risk values and pre-established risk ranking and choose the most enviable one(s), (2) the relative closeness of each alternative to the IF PIS and rank the alternative according to the relative closeness and select the most desirable one(s).

Further, the ranking obtained by proposed methods with the already existing methods are compared in Tables 6 and Table 8. Comparisons reveal the authenticity of the proposed method over others. Reliability of the proposed methods are also enhanced by calculating the weight vector, which was previously assumed by few of the authors.

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INTUITIONISTIC FUZZY INFORMATION MEASURES WITH APPLICATION IN RATING OF TOWNSHIP DEVELOPMENT

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اندازه های داده فازی شهودی و کاربرد آنها در رتبه بندی توسعه شهری

چکیده. عمدتاً در شرایط شک وتردید، اندازه گیری مقدار دقیق بعضی از عوامل مشکل است. در صورتیکه در مسائل جهان واقعی با زبان فازی شهودی به آسانی قابل تقریب زدن می باشند. در این مقاله برای مواجه با چنین شرایطی دو اندازه داده برای مجموعه های فازی شهودی بر اساس تابع مثلثاتی معرفی شده اند که تعمیمی از اندازه های داده فازی می باشند. بر این اساس اندازه تشابه مثلثاتی جدید گسترش داده شده است. شرح ریاضی، معتبر بودن و مؤثر بودن آنها را برای IFSها در مقایسه با اندازه های موجود نشان می دهد. متناظر با داده و اندازه های تشابه برای IFSها دو روش جدید بسط داده شده است. (۱)روش عملگر میانگین موزون اندازه تشابه فازی شهودی (IFSMWAO) برای توسعه شهری و (۲)روش TOPSIS برای مسائل تصمیم گیری چند معیاره (MCDM) (سیاست سرمایه گذاری) . در روشهای موجود محققین بردارهای وزن را مفروض دارند، در صورتیکه در روش پیشنهادی استفاده اندازه داده فازی شهودی محاسبه شده است. این موضوع اعتبار روش پیشنهادی را افزایش می دهد.