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A SATISFACTORY STRATEGY OF MULTIOBJECTIVE TWO PERSON MATRIX GAMES WITH FUZZY PAYOFFS

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Abstract. The multiobjective two person matrix game problem with fuzzy payoffs is considered in this paper. It is assumed that fuzzy payoffs are triangular fuzzy numbers. The problem is converted to several multiobjective matrix game problems with interval payoffs by using the α-cuts of fuzzy payoffs. By solving these problems some α-Pareto optimal strategies with some interval outcomes are obtained. An interactive algorithm is presented to obtain a satisfactory strategy of players. Validity and applicability of the method is illustrated by a practical example.

1. Introduction

Game theory is a formal way to analyze interaction among a group of rational decision makers who behave strategically. Games are broadly classified into two major categories: cooperative and noncooperative games (for example, see [16], [26], [28]). Two person zero-sum finite games, which often are called matrix games for short, are an important kind of noncooperative games. Matrix games have been extensively studied and successfully applied to many fields such as economics, finance, business competition, voting, auctions, research and development races, cartel behavior and e-commerce as well as advertising.

Research on game theory in fuzzy environment has been accumulating since the mid 1970s. Butnariu [5], was the first to study two person noncooperative games in a fuzzy environment, claiming that all of one player's strategies are not equally possible and the grade of membership of a strategy depends on the behavior of the opponent. He also considered the case where the set of strategies of the player could be seen as a fuzzy set. Subsequently, he examined n-person noncooperative games in a fuzzy invironment and presented the concept of equilibrium solutions for such games [6]. Buckley [4] analyzed the behavior of decision makers using two person fuzzy games similar to Butnariu [5]. Billot [3] defined the individual relations of preference by a procedure different from Butnaria's definition of preference and examined equilibrium solutions of n-person noncooperative games. Campos [7] introduced two person zero-sum games with fuzzy payoffs. The problem treated by Campos was a single-objective game, and a max-min problem was formulated by using the fuzzy mathematical programming method. Bector and

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Chandra [1] proposed linear programming (LP) methods for solving fuzzy matrix games based on certain duality of LP with fuzzy parameters. Maeda [24] defined three kinds of minimax equilibrium strategies based on the fuzzy max order and proposed a bi-matrix game method for solving a particular type of fuzzy matrix games. Li ([20], [21]) proposed the two-level LP method for solving matrix games with payoffs of triangular fuzzy numbers (TFNs), which was called Li's model by Bector and Chandra [1] and Larbani [19]. Clemente and Fernandez [9] presented a new methodology for the analysis of fuzzy payoff matrix games. They provided a method to solve these problems to find Pareto optimal security strategies. Collins and Hu [10] considered interval-valued matrix games and extended the results of classical strictly determined matrix games to fuzzily determined interval matrix games. Li [22] introduced an approach to compute fuzzy values of matrix games with single objective and payoffs of triangular fuzzy numbers. However, his method does not give the optimal strategies of players. Chandra and Aggarwal [8] wrote a note on the work of Li [22] for solving the two person zero-sum games with payoffs of triangular fuzzy numbers and proposed a new methodology for solving such games. Li and Nan [23] proposed an interval-valued programming approach to matrix games with payoffs of triangular intuitionistic fuzzy numbers. Dutta and Gupta [13] extended Maeda's [24] and Cunlin and Qiang's [12] fuzzy matrix game models with symmetric and asymmetric triangular fuzzy numbers. Seikh et al. [30] presented an approach to solve matrix game with fuzzy payoffs. Also they studied matrix games with intuitionistic fuzzy payoffs [31] and presented application of intuitionistic fuzzy mathematical programming with exponential membership and quadratic non-membership functions in matrix games [32].

Games with multiple non-comparable objectives are called multiobjective games or games with vector payoffs. For multiobjective two person zero-sum games, Zeleny [33] introduced a parameter vector, a vector of weighting coefficients, which varied parametrically to analyze such games. Cook [11] also introduced a goal vector and formulated such games as goal programming problems. Fernandez and Puerto [15] showed that the set of efficient solutions of multiobjective linear programming problem derived from a zero-sum multiobjective matrix game coincides with the set of Pareto optimal security strategies for one of the players in the original game. Fahem and Radjef [14] investigated the concept of properly efficient equilibrium solution for a multicriteria noncooperative strategic game. Nishizaki and Sakawa [27] considered multiobjective two person zero-sum games with fuzzy payoffs and fuzzy goals. Kumar [18] proposed a max-min solution approach for multiobjective matrix games with fuzzy goals.

In this paper, we consider fuzzy matrix games with multiple payoffs in which players' payoffs are fuzzy numbers and players' pure and mixed strategies are crisp. The remainder of the paper is organized as follows. In section 2, some preliminaries, containing necessary notations and definitions of fuzzy sets, interval arithmetic and zero-sum games are presented. In section 3, a method is proposed for computing fuzzy values of two person zero-sum multiobjective games with fuzzy payoffs, which is generalization of the work of Seikh et al. [30], which solved the same problem in single objective case, without obtaining a unique strategy for players. In section 4, an interactive algorithm is presented to solve the mentioned problem which computes a satisfactory strategy of game for players. Finally, conclusion is made in section 5.

2. Preliminaries

2.1. **Fuzzy Sets and Interval Arithmetic.** In this subsection, we recall some definitions and preliminaries of fuzzy sets according to [29].

Definition 2.1. Let X denote a universal set. A fuzzy subset $\tilde{a}$ of X is defined by its membership function $\mu_{\tilde{a}} : X \to [0,1]$, which assigns to each element $x \in X$ a real number $\mu_{\tilde{a}}(x)$ in the interval $[0,1]$.

In the above definition, $\mu_{\tilde{a}}(x)$ is the grade of membership of x in the set $\tilde{a}$.

Definition 2.2. The support of $\tilde{a}$, denoted by $supp(\tilde{a})$, is the set of points $x \in X$ at which $\mu_{\tilde{a}}(x)$ is positive.

Definition 2.3. $\tilde{a}$ is said to be normal if there is $x \in X$ such that $\mu_{\tilde{a}}(x) = 1$.

Definition 2.4. The α-cut of the fuzzy set $\tilde{a}$, denoted by $\tilde{a}_\alpha$, is an ordinary set defined by $\tilde{a}_\alpha = \{x \,|\mu_{\tilde{a}}(x) \geq \alpha\}$ when $\alpha \in (0,1]$, and $\tilde{a}_0 = closure\,\{x \,|\mu_{\tilde{a}}(x) > 0\}$.

Definition 2.5. $\tilde{a}$ is said to be a convex fuzzy set if its α-cuts are convex.

Definition 2.6. A fuzzy number is a convex normalized fuzzy set of the real line $\mathbb{R}^1$ whose membership function is piecewise continuous.

From the definition of a fuzzy number $\tilde{a}$, it is significant to note that each α-cut $\tilde{a}_\alpha$ of a fuzzy number $\tilde{a}$ is a closed interval $[{a_\alpha}^L, {a_\alpha}^R]$.

Definition 2.7. A triangular fuzzy number (TFN) $\tilde{a} = (a^l, a^m, a^r)$ is a special fuzzy number, whose membership function is given by

$$\mu_{\tilde{a}}(x) = \begin{cases} (x - a^l)/(a^m - a^l) & a^l \leq x \leq a^m \\ (a^r - x)/(a^r - a^m) & a^m \leq x \leq a^r \\ 0 & otherwise \end{cases} \tag{1}$$

where a^m is the mean of $\tilde{a}$, and a^l and a^r are the left and right end points of $supp(\tilde{a})$, respectively.

For any triangular fuzzy number $\tilde{a} = (a^l, a^m, a^r)$, it is easily derived from (1) that $\left[a_\alpha^L, a_\alpha^R\right] = \alpha\tilde{a}_1 + (1-\alpha)\tilde{a}_0$, which means that any α-cut of a triangular fuzzy number is directly obtained from its 1-cut and 0-cut.
Let $a = \left[a^L, a^R\right]$ be an interval. The interval a can also be represented in the form

$$a = \langle a_C, a_W \rangle = \{x \in \mathbb{R} \,|a_C - a_W \leq x \leq a_C + a_W\},$$

where $a_C = \frac{1}{2}(a^R + a^L)$ and $a_W = \frac{1}{2}(a^R - a^L)$ are the center and half-width of a, respectively.
Different order relations between intervals have been presented in different researches [25]. In this paper we use the following definition, in which a and b are two intervals and x is a non-negative real variable. A brief comparison on different interval orders and features of this ordering are presented in [25].

Definition 2.8. [25]. The satisfactory crisp equivalent forms of interval inequality constraints $ax \leq_I b$ and $ax \geq_I b$ are defined as

$$ax \leq_I b \Leftrightarrow \begin{cases} a^R x \leq b^R \\ \frac{(ax)_C - b_C}{(ax)_W + b_W} \leq \beta \end{cases} \qquad , ax \geq_I b \Leftrightarrow \begin{cases} a^L x \geq b^L \\ \frac{b_C - (ax)_C}{(ax)_W + b_W} \leq \beta \end{cases}$$

where $\leq_I$ and $\geq_I$ denote the interval number inequalities and $\beta \in [0, 1]$ represents the minimal acceptance degree of the inequality constraints which may be allowed to violate.

2.2. **Solving Interval Linear Programming Problems.** In this subsection, we explain a method for solving linear programming problems with interval coefficients according to [25] and [30]. Consider the following interval linear programming problem

$$\begin{aligned} \max \ & \sum_{j=1}^{n} c_j x_j \\ \text{s.t.} \ & \sum_{j=1}^{n} a_{ij} x_j \geq_I b_i \quad i = 1, \dots, m \\ & x_j \geq 0 \quad j = 1, \dots, n \end{aligned} \tag{2}$$

where, the coefficients in the objective function and the constraints and the right hand side values are intervals. Assuming a pessimistic procedure, the problem transforms to a bi-objective mathematical programming problem. In the maximizing case, the central value and the lower bound of the interval objective function are maximized. There exists several solution methods for the obtained bi-objective problem. In this paper we use the weighted average approach which gives Pareto optimal solutions [29]. According to the mentioned method and Definition 2.8, the problem (2) is converted into the following classical linear programming problem

$$\begin{aligned} \max \ & \sum_{j=1}^{n} \left(\frac{3c_j^L + c_j^R}{4}\right) x_j \\ s.t. \ & \sum_{j=1}^{n} a_{ij}^L x_j \geq b_i^L \quad i = 1, \dots, m \\ & \sum_{j=1}^{n} \left\{(1+\beta) a_{ij}^R + (1-\beta) a_{ij}^L\right\} x_j \geq (1+\beta) b_i^L + (1-\beta) b_i^R \quad i = 1, \dots, m \\ & x_j \geq 0 \quad j = 1, \dots, n \end{aligned}$$

2.3. **Zero-Sum Game.** Let two players in a two person zero-sum game be denoted by Players I and II. Assume that $I = \{1, \dots, m\}$ and $J = \{1, \dots, n\}$ are the sets of pure strategies of Players I and II, respectively. When Player I chooses the pure strategy i and Player II chooses the pure strategy j, then a_{ij} is the payoff for Player I and $-a_{ij}$ is the payoff for Player II. The two person zero-sum matrix game G in normal form can be represented as a payoff matrix $A = [a_{ij}]_{m \times n}$. Consider the game G with no saddle point, i.e. $\max_i \min_j a_{ij} \neq \min_j \max_i a_{ij}$. To solve such a game, Neumann and Morgenstern [26] introduced the concept of mixed strategy for Players I and II. Mixed strategy spaces are denoted for Players I and II as follows, respectively:

$$X = \{x \in \mathbb{R}^m | \sum_{i=1}^{m} x_i = 1, x_i \geq 0, i = 1, \ldots, m\}, \tag{3}$$

$$Y = \{y \in \mathbb{R}^n | \sum_{j=1}^{n} y_j = 1, y_j \geq 0, j = 1, \ldots, n\}. \tag{4}$$

In fact the mixed strategies for Players I and II are the probability distributions on the sets I and J, respectively. It is conventional to assume that Player I is a maximizing player and Player II is a minimizing player. Further, for $x \in X$, $y \in Y$, the scalar $x^T Ay$ is the expected payoff to Player I and as the game G is zero sum, the expected payoff to Player II is $-x^T Ay$. Neumann and Morgenstern [26] showed that for a two person zero-sum game G with payoff matrix A, we have

$$\max_{x\in X} \min_{y\in Y} \; x^T Ay = \min_{y\in Y} \max_{x\in X} \; x^T Ay.$$

A pair of strategies (x^*, y^*) satisfying the above equation is called an equilibrium solution.

3. Multiobjective Two Person Zero-Sum Game with Fuzzy Payoffs

In this section, we consider a multiobjective game problem in fuzzy environment. We introduce fuzzy payoffs to express imprecision of information in decision making problems.

Definition 3.1. (Zero-sum game with fuzzy payoffs [27]). When Player I chooses a pure strategy $i \in I$ and Player II chooses a pure strategy $j \in J$, let $\tilde{a}_{ij}$ be a fuzzy payoff for Player I and $-\tilde{a}_{ij}$ be a fuzzy payoff for Player II. Let the fuzzy payoff $\tilde{a}_{ij}$ be represented by the triangular fuzzy numbers

$$\tilde{a}_{ij} = (a^l_{ij}, a^m_{ij}, a^r_{ij}), \tag{5}$$

The two person zero-sum fuzzy game can be represented by the fuzzy payoff matrix

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{11} & \ldots & \tilde{a}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \ldots & \tilde{a}_{mn} \end{bmatrix}. \tag{6}$$

The game defined by (6) is called a two person zero-sum game with triangular fuzzy payoffs. Note that $-\tilde{a}_{ij} = (-a^r_{ij}, -a^m_{ij}, -a^l_{ij})$ is the fuzzy payoff for Player II.

When one player chooses a strategy, his payoff is represented by a triangular fuzzy number. The outcome of the game has a zero-sum structure such that, when one player receives a gain, the other player suffers an equal loss.
Solution concepts of two person zero-sum game with fuzzy payoffs are defined in different papers (for example see [1], [9], [27]).
Assume that each player has p objectives. The following multiple fuzzy payoff matrices represent a multiobjective two person zero-sum game with fuzzy payoffs:

$$\tilde{A}^1 = \begin{bmatrix} \tilde{a}^1_{11} & \ldots & \tilde{a}^1_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{a}^1_{m1} & \ldots & \tilde{a}^1_{mn} \end{bmatrix}, \quad \ldots \quad, \tilde{A}^p = \begin{bmatrix} \tilde{a}^p_{11} & \ldots & \tilde{a}^p_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{a}^p_{m1} & \ldots & \tilde{a}^p_{mn} \end{bmatrix} \tag{7}$$

where $\tilde{A}^k$ is the payoff matrix of the game with respect to the k-th objective function, for $k = 1, \ldots, p$. The mixed strategy spaces for Players I and II are given by (3) and (4), respectively.
Let each $\tilde{a}_{ij}^k$ be a triangular fuzzy number as $(\tilde{a}_{ij}^{kl}, \tilde{a}_{ij}^{km}, \tilde{a}_{ij}^{kr})$. We use the concept of α-cuts to convert the game with fuzzy payoffs to a game with interval payoffs. For each $\alpha \in [0,1]$, the α-cuts of triangular fuzzy numbers $\tilde{a}_{ij}^k$ are the intervals $(\tilde{a}_{ij}^k)_\alpha = [(\tilde{a}_{ij}^{kL})_\alpha, (\tilde{a}_{ij}^{kR})_\alpha] = [\alpha a_{ij}^{km} + (1-\alpha)a_{ij}^{kl}, \alpha a_{ij}^{km} + (1-\alpha)a_{ij}^{kr}]$.
For a fixed value of α, we denote by $\tilde{A}_\alpha^k = \left[(\tilde{a}_{ij}^k)_\alpha\right]$, the matrix containing α-cuts of the elements of individual matrix $\tilde{A}^k$ for $k = 1, \ldots, p$. Assume that we have a fixed value of α. Choosing $x \in X$ and $y \in Y$ by Players I and II, respectively, implies that the expected payoff in level α (α-level expected payoff) of the game is

$$v_\alpha(x,y) = x^T \tilde{A}_\alpha y = [v_\alpha^1(x,y), \ldots, v_\alpha^p(x,y)] \tag{8}$$

where $\tilde{A}_\alpha = [\tilde{A}_\alpha^1, \ldots, \tilde{A}_\alpha^p]$ and

$$v_\alpha{}^k(x,y) = x^T \tilde{A}_\alpha^k y = x^T [(\tilde{a}_{ij}^k)_\alpha] y = [x^T (\tilde{a}_{ij}^{kL})_\alpha y, x^T (\tilde{a}_{ij}^{kR})_\alpha y], \; k = 1, \ldots, p.$$

It should be emphasized here that by taking any value in the intervals $(\tilde{a}_{ij}^k)_\alpha = [(\tilde{a}_{ij}^{kL})_\alpha, (\tilde{a}_{ij}^{kR})_\alpha]$, we have a (crisp) multiobjective matrix game. Naturally, Player II's payoff is $-(\tilde{a}_{ij}^k)_\alpha = [-(\tilde{a}_{ij}^{kR})_\alpha, -(\tilde{a}_{ij}^{kL})_\alpha]$.
Player I (the maximizer) has to find the maximum outcome against any strategy of Player II (the minimizer). Thus, for each strategy $x \in X$ of Player I, the security levels in level α (α-security levels) of Player I are interval payoffs which can be guaranteed against any response of Player II. Therefore, the α-security levels for Players I and II with respect to k-th objective function are defined as follows, respectively:

$$(\underline{V}^k)_\alpha(x) = \min\nolimits_{y \in Y} \; v_\alpha^k(x,y) \quad k = 1, \ldots, p, \tag{9}$$

$$(\bar{V}^k)_\alpha(y) = \max\nolimits_{x \in X} \; v_\alpha^k(x,y) \quad k = 1, \ldots, p. \tag{10}$$

For a strategy $x \in X$ and a given value of α, the k-th α-security level of Player I is given by (9), which is a linear programming problem with interval objective function. Thus, from a viewpoint of logic, the values of objective functions of these problems should be intervals as well. The α-security levels are p-tuples denoted by

$$\underline{V}_\alpha(x) = ((\underline{V}^1)_\alpha(x), \ldots, (\underline{V}^p)_\alpha(x)), \tag{11}$$

$$\bar{V}_\alpha(y) = ((\bar{V}^1)_\alpha(y), \ldots, (\bar{V}^p)_\alpha(y)). \tag{12}$$

The above concept allows us to analyze multiobjective matrix games with fuzzy payoffs under the rationale of worst case behavior of the opponent. Here, we state solution process of game for Player I. A similar process can be easily introduced for Player II. For a given value of α, Player I has to choose x such that the α-security levels of the game are maximized. Hence, Player I faces with the following multiobjective mathematical programming problem

$$\max\nolimits_{x \in X} \; \underline{V}_\alpha(x) \tag{13}$$

or equivalently,

$$\begin{aligned}&\max\,(\min_{y\in Y} v^1_\alpha(x,y),\ldots,\ \min_{y\in Y} v^p_\alpha(x,y))\\ &s.t.\quad \sum_{i=1}^{m} x_i = 1\\ &\qquad\ x_i \geq 0 \quad i=1,\ldots,m.\end{aligned} \tag{14}$$

Using the usual transformation of minimax problems to linear problems [29], the above problem is converted to the following multiobjective mathematical programming problem

$$\begin{aligned}&\max\,((\underline{v}^1)_\alpha,\ldots,(\underline{v}^p)_\alpha)\\ &s.t.\\ &\quad \sum_{j=1}^{n}\sum_{i=1}^{m} x_i(\tilde{a}^1_{ij})_\alpha y_j \geq_I (\underline{v}^1)_\alpha \quad \forall y\in Y\\ &\quad\ \vdots\\ &\quad \sum_{j=1}^{n}\sum_{i=1}^{m} x_i(\tilde{a}^p_{ij})_\alpha y_j \geq_I (\underline{v}^p)_\alpha \quad \forall y\in Y\\ &\quad \sum_{i=1}^{m} x_i = 1\\ &\quad x_i \geq 0 \quad i=1,\ldots,m.\end{aligned} \tag{15}$$

The problem (15) is a multiobjective mathematical programming problem in which each element $y \in Y$ corresponds to exactly p constraints. Thus, the problem contains an infinite number of constraints. Consider the set of extreme points of Y, that is, $S = \{(1,0,\ldots,0),(0,1,\ldots,0),\ldots,(0,0,\ldots,1)\}$. Obviously, the constraints corresponding to the elements of S are as follows:

$$\begin{aligned}&\sum_{i=1}^{m} (\tilde{a}^1_{ij})_\alpha x_i \geq_I (\underline{v}^1)_\alpha \quad j=1,\ldots,n\\ &\qquad\vdots\\ &\sum_{i=1}^{m} (\tilde{a}^p_{ij})_\alpha x_i \geq_I (\underline{v}^p)_\alpha \quad j=1,\ldots,n\end{aligned}$$

On the other hand, the other constraints (namely, the constraints corresponding to the elements of $Y\backslash S$) are redundant, because $\geq_I$ is preserved under convex combinations and consequently, each of these constraints can be written as a convex combination of the above constraints. Therefore, the problem (15) is equivalent to the following problem which have a finite number of constraints.

$$\begin{aligned}&\max\ ((\underline{v}^1)_\alpha,\ldots,(\underline{v}^p)_\alpha)\\ &s.t.\\ &\quad \sum_{i=1}^{m} (\tilde{a}^1_{ij})_\alpha x_i \geq_I (\underline{v}^1)_\alpha \quad j=1,\ldots,n\\ &\qquad\vdots\\ &\quad \sum_{i=1}^{m} (\tilde{a}^p_{ij})_\alpha x_i \geq_I (\underline{v}^p)_\alpha \quad j=1,\ldots,n\\ &\quad \sum_{i=1}^{m} x_i = 1\\ &\quad x_i \geq 0 \quad i=1,\ldots,m.\end{aligned} \tag{16}$$

To solve the above problem, we use weighted sum method as follows:

$$\begin{array}{l}\max \sum_{k=1}^{p} \lambda_k (\underline{v}^k)_\alpha \\ s.t. \\ \sum_{i=1}^{m} (\tilde{a}_{ij}^1)_\alpha x_i \geq_I (\underline{v}^1)_\alpha \quad j=1,\ldots,n \\ \vdots \\ \sum_{i=1}^{m} (\tilde{a}_{ij}^p)_\alpha x_i \geq_I (\underline{v}^p)_\alpha \quad j=1,\ldots,n \\ \sum_{i=1}^{m} x_i = 1 \\ x_i \geq 0 \quad i=1,\ldots,m \end{array} \tag{17}$$

where $\lambda \in \Lambda = \left\{ \lambda \in \mathbb{R}^p \,\middle|\, \lambda \geq 0, \sum_{k=1}^{p} \lambda_k = 1 \right\}$. The component λ_k of the vector $\lambda = (\lambda_1, ..., \lambda_p) \in \Lambda$ in the problem (17) can be interpreted as the relative importance of the k-th objective function to Player I. The problem (17) is a linear programming problem, where $(\tilde{a}_{ij}^k)_\alpha$ and $(\underline{v}^k)_\alpha, k = 1, \ldots, p$ are intervals. Using the explained method in subsection 2.2, the problem (17) is converted into the following linear programming problem (PI-α)

$$\begin{array}{l}\max \sum_{k=1}^{p} \lambda_k \left(\frac{3(\underline{v}^{kL})_\alpha + (\underline{v}^{kR})_\alpha}{4}\right) \\ s.t. \\ \sum_{i=1}^{m} (\tilde{a}_{ij}^{1L})_\alpha x_i \geq (\underline{v}^{1L})_\alpha \quad j=1,\ldots,n \\ \sum_{i=1}^{m} \left\{(1+\beta)(\tilde{a}_{ij}^{1R})_\alpha + (1-\beta)(\tilde{a}_{ij}^{1L})_\alpha\right\} x_i \geq (1+\beta)(\underline{v}^{1L})_\alpha + (1-\beta)(\underline{v}^{1R})_\alpha \quad j=1,\ldots,n \\ \vdots \\ \sum_{i=1}^{m} (\tilde{a}_{ij}^{pL})_\alpha x_i \geq (\underline{v}^{pL})_\alpha \quad j=1,\ldots,n \\ \sum_{i=1}^{m} \left\{(1+\beta)(\tilde{a}_{ij}^{pR})_\alpha + (1-\beta)(\tilde{a}_{ij}^{pL})_\alpha\right\} x_i \geq (1+\beta)(\underline{v}^{pL})_\alpha + (1-\beta)(\underline{v}^{pR})_\alpha \quad j=1,\ldots,n \\ (\underline{v}^{kL})_\alpha \leq (\underline{v}^{kR})_\alpha \quad k=1,\ldots,p \\ \sum_{i=1}^{m} x_i = 1 \\ x_i \geq 0 \quad i=1,\ldots,m \end{array}$$

where the parameters α and β are given by Player I. We take $\beta = 0$, which indicates that the inequality constraints are not allowed to violate. By solving the problem (PI-α) for a given value of α, the Pareto optimal strategy x^* in level α (we call it α-Pareto optimal strategy) and the corresponding left and right end points of α-cuts of security levels $(\underline{v}^k)_\alpha$ for Player I are obtained.

By a similar process, we can obtain the following linear programming problem (PII-α) to find an α-Pareto optimal strategy of Player II and the corresponding left and right end points of α-cuts of security levels $(\bar{v}^k)_\alpha$, for a fixed value of α.

$$\begin{array}{l}\min \sum_{k=1}^{p} \lambda_k \left(\frac{3(\bar{v}^{kR})_\alpha + (\bar{v}^{kL})_\alpha}{4}\right) \\ s.t. \\ \sum_{j=1}^{n} (\tilde{a}_{ij}^{1R})_\alpha y_j \leq (\bar{v}^{1R})_\alpha \quad i=1,\ldots,m \\ \sum_{j=1}^{n} \left\{(1+\beta)(\tilde{a}_{ij}^{1L})_\alpha + (1-\beta)(\tilde{a}_{ij}^{1R})_\alpha\right\} y_j \leq (1-\beta)(\bar{v}^{1L})_\alpha + (1+\beta)(\bar{v}^{1R})_\alpha \quad i=1,\ldots,m \\ \vdots \\ \sum_{j=1}^{n} (\tilde{a}_{ij}^{pR})_\alpha y_j \leq (\bar{v}^{pR})_\alpha \quad i=1,\ldots,m \\ \sum_{j=1}^{n} \left\{(1+\beta)(\tilde{a}_{ij}^{pL})_\alpha + (1-\beta)(\tilde{a}_{ij}^{pR})_\alpha\right\} y_j \leq (1-\beta)(\bar{v}^{pL})_\alpha + (1+\beta)(\bar{v}^{pR})_\alpha \quad i=1,\ldots,m \\ (\bar{v}^{kL})_\alpha \leq (\bar{v}^{kR})_\alpha \quad k=1,\ldots,p \\ \sum_{j=1}^{n} y_j = 1 \\ y_j \geq 0 \quad j=1,\ldots,n \end{array}$$

where $\lambda_k, k = 1, \dots, p$ can be interpreted as the relative importance of the k-th objective function to Player II.
The weights $\lambda_1, \dots, \lambda_p$ in the problems (PI-α) and (PII-α) which indicate the preference degrees of different objective functions are given by players. If players can not give the weights $\lambda_1, \dots, \lambda_p$ in problems (PI-α) and (PII-α), generally multi-attribute decision making methods such as AHP and TOPSIS [17] can be used to determine these weights. Belenson and Kapur [2] used two person zero-sum game with mixed strategies for finding weights without decision makers preferences.

Theorem 3.2. *Let $\tilde{\underline{v}}^k, k = 1, \dots, p$ be the fuzzy set defined through the following family of ordinary sets:*

$$\left\{ [(\underline{v}^{kL})_\alpha, (\underline{v}^{kR})_\alpha] \,|\alpha \in [0,1] \right\}$$

where $(\underline{v}^{kL})_\alpha$ and $(\underline{v}^{kR})_\alpha$ are obtained by solving the problem (PI-α). Then $\tilde{\underline{v}}^k$ is a fuzzy number.

Proof. Since $\min_{y \in Y} v_\alpha^{kL}(x, y) \le \min_{y \in Y} v_\alpha^{kR}(x, y)$ for each α, we have $(\underline{v}^{kL})_\alpha \le (\underline{v}^{kR})_\alpha$. Also, it is easy to verify that $(\underline{v}^{kL})_\alpha \le (\underline{v}^{kL})_{\alpha'}$ and $(\underline{v}^{kR})_{\alpha'} \le (\underline{v}^{kR})_\alpha$, for each $\alpha, \alpha^{'} \in [0,1]$ with $\alpha \le \alpha^{'}$. Therefore, $[(\underline{v}^{kL})_{\alpha'}, (\underline{v}^{kR})_{\alpha'}] \subseteq [(\underline{v}^{kL})_\alpha, (\underline{v}^{kR})_\alpha]$ if $\alpha \le \alpha^{'}$; i.e. the intervals $[(\underline{v}^{kL})_\alpha, (\underline{v}^{kR})_\alpha]$ are nested. This guarantees that the intervals $[(\underline{v}^{kL})_\alpha, (\underline{v}^{kR})_\alpha]$ can be used to construct a fuzzy number. $\square$

Remark 3.3. The proposed method in this section can also be used for multiobjective matrix game problems with interval payoffs. In fact, it is enough to solve the problem (PI-α) and (PII-α), with given interval payoffs instead of the α-cuts of fuzzy payoffs.

4. An Interactive Algorithm for Computing Satisfactory Strategy

In Section 3, we proposed a method for computing the fuzzy values of multi-objective matrix games with fuzzy payoffs. However, this method presents several strategies for Player I. Because for each $\alpha \in [0,1]$ we solve a problem, which gives a strategy. Decision analyst has to choose one strategy among them to present to Player I. In this section, we propose an interactive algorithm to choose a strategy preferred by Player I when the fuzzy payoffs are triangular fuzzy numbers.
For a given multiobjective matrix game with triangular fuzzy payoffs and an initial vector λ to show the relative importance of the objectives, suppose that we have computed α-security levels for $\alpha = 0, 1$ by solving the problem (PI-α) and also, we have constructed the fuzzy values $\tilde{\underline{v}}^k = (v^{kl}, v^{km}, v^{kr})$ by these α-security levels for $k = 1, 2, \dots, p$. When $\alpha = 0$, the security levels of the game of Player I are the widest intervals. Thus it is impossible that the α-security levels of the game for Player I falls outside of these intervals. For $\alpha = 1$, the security levels for Player I are the most likely values. If Player I is satisfied with the 1-security levels or 0-security levels, then we present the corresponding strategy to him and stop. Otherwise, we begin algorithm by the obtained solution with $\alpha = 0$. This strategy corresponds to p intervals $[(\underline{v}^{kL})_\alpha, (\underline{v}^{kR})_\alpha]$ of α-security levels for $k = 1, 2, \dots, p$. Thus $(\underline{v}^{kL})_\alpha$

and $(\underline{v}^{kR})_\alpha$ present respectively the least and most amount of α-security level to the k-th objective and obviously,

$$v^{kl} \leq (\underline{v}^{kL})_\alpha \leq v^{km}, \quad v^{km} \leq (\underline{v}^{kR})_\alpha \leq v^{kr}. \tag{18}$$

Since Player I is maximizer, we can define his satisfactory strategy as follows.

Definition 4.1. We say that a strategy x is satisfactory for Player I, if Player I is satisfied with the least amounts of all its α-security levels.

Similarly, since Player II is minimizer, we can define his satisfactory strategy as follows.

Definition 4.2. We say that a strategy y is satisfactory for Player II, if Player II is satisfied with the most amounts of all its α-security levels.

To find a satisfactory strategy x, we consider the following situations separately:

(1) Player I is satisfied with the least amounts of all α-security levels.
(2) Player I is satisfied with the most amounts of all α-security levels but is not satisfied with the least amounts of α-security levels for some $k \in \{1, 2, \ldots, p\}$. Furthermore, his desired values of these least amounts for the k-th objective belong to $[v^{kl}, v^{km}]$.
(3) Player I is not satisfied with the least and most amount of some α-security levels and he asks a new value of some $(\underline{v}^{kL})_\alpha$ and/or some $(\underline{v}^{kR})_\alpha$.

In case (1), since Player I maximizes gain floor, if he is satisfied with the least amounts of all α-security levels, then he obviously satisfied with the most amounts of them. The obtained strategy is the satisfactory strategy of Player I.
In case (2), we can improve the least amount of each α-security level by increasing the value of α. However, we cannot increase α arbitrarily. Because the most amount of each α-security level is decreased by increasing α (see Figure 1). Thus we have to choose a new value of α to satisfy Player I relatively. Let us explain how to determine the new value of α in more details. Suppose that Player I is not satisfied with $(\underline{v}^{kL})_\alpha$ for some $k \in \{1, 2, \ldots, p\}$. We denote by K the index set of such objectives. For each $k \in K$, we ask Player I to determine his desired value of $(\underline{v}^{kL})_\alpha$ denoted by $(\underline{v}^{kL})^*_\alpha$. If $(\underline{v}^{kL})^*_\alpha \notin [(\underline{v}^{kL})_\alpha, v^{km}]$ for some $k \in K$, then the third case occurs. Otherwise, achieving the desired values is possible by increasing the value of α. For a fixed $k \in K$, we increase α to $\alpha'_k = \frac{(\underline{v}^{kL})^*_\alpha - v^{kl}}{v^{km} - v^{kl}}$ which is the value of α related to the desired value $(\underline{v}^{kL})^*_\alpha$ (see Figure 1 in which the desired value is represented by $v^L_{\alpha'}$).

Finally, we set the new value of α to $\alpha' = max_{k\in K}\alpha'_k$, to achieve all desired values. Then, we inform Player I that increasing α to α', decreases the most amount $(\underline{v}^{kR})_\alpha$ to $(\underline{v}^{kR})_{\alpha'} = v^{kr} + \alpha'(v^{km} - v^{kr})$. If he does not accept, then we recommend him to decrease the desired values $(\underline{v}^{kL})^*_\alpha$'s to find a new value of α less than α'. This process is repeated until a suitable value of α is found. Then, we solve the problem (PI-α) for the obtained value of α to present the satisfactory strategy to him.
In case (3), assume that Player I asks a new value $\hat{\underline{v}}_\alpha = [\hat{\underline{v}}^L_\alpha, \hat{\underline{v}}^R_\alpha]$ for some α-security

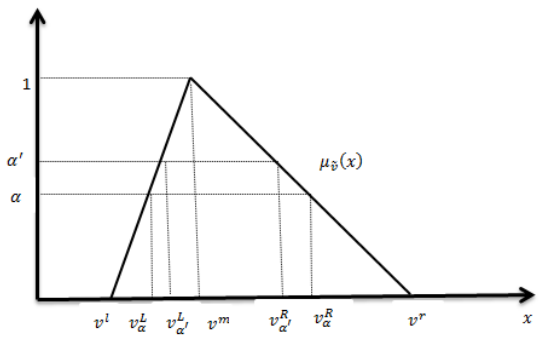

Figure 1. The Change of Lower Bound of α-security Level and Computing the New Value of α

levels. In this case, we can not modify α like case (2) because $(\underline{v}^{kL})^*_\alpha \notin [(\underline{v}^{kL})_\alpha, v^{km}]$ and/or $(\underline{v}^{kR})^*_\alpha \notin [v^{km}, v^{kr}]$ for some $k \in K$. In this situation to attain specified security levels by Player I, we propose the following goal programming problem

$$\begin{array}{l} \min D(\underline{v}_\alpha, \underline{\hat{v}}_\alpha) \\ s.t. \quad The\, constraints\, of\, problem\, (\text{PI-}\alpha) \\ \quad (\underline{v}^{kL})_\alpha = (\underline{v}^{kL})^*_\alpha \quad k \in \{1,\dots,p\} \setminus \underline{K} \\ \quad (\underline{v}^{kR})_\alpha = (\underline{v}^{kR})^*_\alpha \quad k \in \{1,\dots,p\} \setminus \bar{K} \end{array} \tag{19}$$

where $D(.)$ is an interval distance function to compute the difference between objective functions $(\underline{v})_\alpha$ and the desired values $(\underline{\hat{v}})_\alpha$, $\underline{K}$ and $\bar{K}$ are respectively the index sets of objectives whose least and most α-security levels have not satisfied Player I. Here, the distance function $D(.)$ can be considered as the maximum deviations of individual goals (one can also use norm-1 for the distance between intervals). Thus the above problem is rewritten as follows:

$$\begin{array}{l} \min \max_{k\in K} \{D((\underline{v}^k)_\alpha, (\underline{\hat{v}}^k)_\alpha)\} \\ s.t. \quad The\, constraints\, of\, problem\, (\text{PI-}\alpha) \\ \quad (\underline{v}^{kL})_\alpha = (\underline{v}^{kL})^*_\alpha \quad k \in \{1,\dots,p\} \setminus \underline{K} \\ \quad (\underline{v}^{kR})_\alpha = (\underline{v}^{kR})^*_\alpha \quad k \in \{1,\dots,p\} \setminus \bar{K} \end{array} \tag{20}$$

where

$$D((\underline{v}^k)_\alpha, (\underline{\hat{v}}^k)_\alpha) = \max\{\left|(\underline{v}^{kL})_\alpha - (\underline{\hat{v}}^{kL})_\alpha\right|, \left|(\underline{v}^{kR})_\alpha - (\underline{\hat{v}}^{kR})_\alpha\right|\}. \tag{21}$$

Therefore, the optimal solution of (20) can be obtained by solving the following problem

$$\begin{array}{l} \min \max_{k\in K} \left\{(\underline{\hat{v}}^{kL})_\alpha - (\underline{v}^{kL})_\alpha, (\underline{\hat{v}}^{kR})_\alpha - (\underline{v}^{kR})_\alpha\right\} \\ s.t. \quad The\, constraints\, of\, problem\, (\text{PI-}\alpha) \\ \quad (\underline{v}^{kL})_\alpha = (\underline{v}^{kL})^*_\alpha \quad k \in \{1,\dots,p\} \setminus \underline{K} \\ \quad (\underline{v}^{kR})_\alpha = (\underline{v}^{kR})^*_\alpha \quad k \in \{1,\dots,p\} \setminus \bar{K}. \end{array} \tag{22}$$

or equivalently

$$\begin{aligned}
&\min \gamma \\
&s.t. \quad \gamma + (\underline{v}^{kL})_\alpha \geq (\hat{v}^{kL})_\alpha \quad k \in K \\
&\qquad \gamma + (\underline{v}^{kR})_\alpha \geq (\hat{v}^{kR})_\alpha \quad k \in K \\
&\qquad The\, constraints\, of\, problem\, (22).
\end{aligned} \tag{23}$$

From the aforementioned discussion, the process of solving multiobjective two person zero-sum games with triangular fuzzy number payoffs is summarized as follows.

Interactive Algorithm for Fuzzy Multiobjective Matrix Game

Inputs:

p**:** The number of objectives

m**:** The number of strategies for Player I

n**:** The number of strategies for Player II

$\tilde{A}^k$**:** The payoff matrix corresponding to the k-th objective function of the game problem for $k = 1, 2, \ldots, p$.

$\lambda = (\lambda_1, \ldots, \lambda_p)$**:** The objectives weights (the relative importance of objective functions)

Step 1: Set $\tilde{A} = [\tilde{A}^1, \ldots, \tilde{A}^p]$ as the payoff matrix of the game problem.

Step 2: Solve the linear programming problem (PI-α) for $\alpha = 0$ and $\alpha = 1$ and hereby obtain $\underline{\tilde{v}}^k = [(\underline{v}^{kL})_\alpha, (\underline{v}^{kR})_\alpha]$ for $k = 1, \ldots, p$ and the corresponding strategies of Player I.

Step 3: If Player I is satisfied with at least one of the obtained solutions (0-security levels or 1-security levels) in Step 2, then stop. The corresponding strategy is satisfactory. Otherwise, consider the obtained solution with $\alpha = 0$ and go to Step 4.

Step 4: If Player I is satisfied with the most amounts of all α-security levels but he wants to increase the least amounts of some α-security levels to $(\underline{v}^{kL})^*_\alpha \in [v^{kl}, v^{km}]$, and is satisfied with reducing the most amounts of all the corresponding α-security levels (we denote by K the index set of such objectives), go to Step 5. Otherwise go to Step 6.

Step 5: Substitute α by $\alpha' = max_{k \in K} \alpha'_k$ where $\alpha'_k = \frac{(\underline{v}^{kL})^*_\alpha - v^{kl}}{v^{km} - v^{kl}}$. Solve linear programming problem (PI-α) for the new value of α. If Player I is satisfied with the least and most amounts of all α-security levels, then give Player I the obtained strategy. Otherwise, ask Player I the new least amounts of α-security levels and go to Step 4.

Step 6: If Player I want to change the least amounts of some objectives to $(\underline{v}^{kL})^*_\alpha$ where $(\underline{v}^{kL})^*_\alpha \notin [v^{kl}, v^{km}]$ or he want to change the most amount of some objectives to $(\underline{v}^{kR})^*_\alpha$ where $(\underline{v}^{kR})^*_\alpha \notin [v^{km}, v^{kr}]$ or he want to change both the least and the most amount of some objectives, then solve the linear programming problem (23) and give Player I a relatively satisfactory strategy and stop.

Outputs: A satisfactory strategy of Player I and the corresponding α-security levels.

The above algorithm can be used to obtain satisfactory strategy of Player II after some minor modifications.

Now, we illustrate the proposed method by a numerical example. Since the multiobjective two person matrix game with fuzzy payoffs has not been considered in previous researches, there is no numerical example with fuzzy payoffs in previous researches. So, we took the following example from [34], and changed its payoffs to triangular fuzzy numbers.

4.1. **Numerical Example.** Suppose that there are two companies I and II aiming to enhance the sales amount and market share of a product in a targeted market (a target market is a specific group of people to whom you are trying to sell your products or services.) under the circumstance that the demand amount of the product in the targeted market basically is fixed. In other words, the sales amount and market share of one company are increased while the sales amount and market share of another company are decreased. The two companies consider two different strategies to increase the sales amount and market share: strategy I (to reduce the price), strategy II (advertisement).

The above problem may be regarded as a fuzzy matrix game. Namely, the companies I and II are considered as Players I and II, respectively. Due to the lack of information, the managers are not able to evaluate the sales amount and market share of companies exactly. In order to handle such the uncertain situation, triangular fuzzy numbers are used to express the estimated values of sales amount and market share of the product. The marketing research department of company I establishes the following payoff matrices

$$\tilde{A}_1 = \begin{bmatrix} (175, 180, 190) & (150, 156, 158) \\ (80, 90, 100) & (175, 180, 190) \end{bmatrix},$$

$$\tilde{A}_2 = \begin{bmatrix} (125, 130, 135) & (120, 130, 135) \\ (120, 130, 135) & (150, 160, 170) \end{bmatrix}.$$

This problem is a biobjective two person zero-sum game.

Assume that the importance of objective functions to Player I are the same. So we set $\lambda_1 = \lambda_2 = 0.5$.

The problem (PI-α) for these data and $\beta = 0$ is as follows:

$$\begin{array}{ll} \max & \frac{3}{8}((\underline{v}^{1L})_\alpha + (\underline{v}^{2L})_\alpha) + \frac{1}{8}((\underline{v}^{1R})_\alpha + (\underline{v}^{2R})_\alpha) \\ s.t. & \\ & (175 + 5\alpha)x_1 + (80 + 10\alpha)x_2 \geq (\underline{v}^{1L})_\alpha \\ & (150 + 6\alpha)x_1 + (175 + 5\alpha)x_2 \geq (\underline{v}^{1L})_\alpha \\ & (125 + 5\alpha)x_1 + (120 + 10\alpha)x_2 \geq (\underline{v}^{2L})_\alpha \\ & (120 + 10\alpha)x_1 + (150 + 10\alpha)x_2 \geq (\underline{v}^{2L})_\alpha \\ & (365 - 5\alpha)x_1 + 180x_2 \geq (\underline{v}^{1L})_\alpha + (\underline{v}^{1R})_\alpha \\ & (308 + 4\alpha)x_1 + (365 - 5\alpha)x_2 \geq (\underline{v}^{1L})_\alpha + (\underline{v}^{1R})_\alpha \\ & 260x_1 + (255 + 5\alpha)x_2 \geq (\underline{v}^{2L})_\alpha + (\underline{v}^{2R})_\alpha \\ & (255 + 5\alpha)x_1 + 320x_2 \geq (\underline{v}^{2L})_\alpha + (\underline{v}^{2R})_\alpha \\ & (\underline{v}^{1L})_\alpha \leq (\underline{v}^{1R})_\alpha \\ & (\underline{v}^{2L})_\alpha \leq (\underline{v}^{2R})_\alpha \\ & x_1 + x_2 = 1 \\ & x_1, x_2 \geq 0 \end{array} \tag{24}$$

The upper and lower bounds of α-cut of the security levels of the game for Player I and the corresponding α-Pareto optimal strategy for different values of $\alpha \in [0, 1]$ are shown in the Table 1 (the solutions for $\alpha = 0.1, \ldots, \alpha = 0.9$ are presented for more comparison of the results by the reader).

It can be easily seen from Table 1 that when $\alpha = 0$, the α-cut of the security levels of the game for Player I are the intervals $[155.2083, 164.6667]$ and $[123.9583, 136.0417]$ respectively, which are the widest intervals. For $\alpha = 1$, the α-security levels of

α	x_1^*	x_2^*	$(\underline{v}^1)_\alpha = [(\underline{v}^{1L})_\alpha, (\underline{v}^{1R})_\alpha]$	$(\underline{v}^2)_\alpha = [(\underline{v}^{2L})_\alpha, (\underline{v}^{2R})_\alpha]$
0	0.7916667	0.2083333	[155.2083,164.6667]	[123.9583,135]
0.1	0.7914573	0.2085427	[155.7927,164.3065]	[123.9583,134.5]
0.2	0.7912458	0.2087542	[156.3771,163.9461]	[125.1650,134]
0.3	0.7910321	0.2089679	[156.9615,163.5854]	[125.9615,133.5]
0.4	0.7908163	0.2091837	[157.5459,163.2245]	[126.3724,133]
0.5	0.7905983	0.2094017	[158.1303,162.8632]	[126.9765,132.5]
0.6	0.7903780	0.2096220	[158.7148,162.5017]	[127.5808,132]
0.7	0.79015	0.2098446	[159.2992,162.1399]	[128.1852,131.5]
0.8	0.7899306	0.2100694	[159.8837,161.7778]	[128.7899,131]
0.9	0.7897033	0.2102967	[160.4682,161.4154]	[129.3949,130.5]
1	0.7894737	0.2105263	[161.0526,161.0526]	[130,130]

TABLE 1. The Mixed Strategies and α-security Levels for Different Values of α in the Numerical Example

the game for Player I are 161.0526 and 130, respectively, which are the most likely values. Therefore, the fuzzy values of the game for Player I are obtained as follows:

$$\tilde{\underline{v}}^1 = ((\underline{v}^{1L})_0, (\underline{v}^{1R})_1, (\underline{v}^{1R})_0) = (155.2083, 161.0526, 164.6667)$$
$$\tilde{\underline{v}}^2 = ((\underline{v}^{2L})_0, (\underline{v}^{2R})_1, (\underline{v}^{2R})_0) = (123.9583, 130, 135)$$

which are triangular fuzzy numbers.

Assume that Player I is not satisfied with the obtained 0-security levels and 1-security levels. We begin algorithm with $\alpha = 0$. For $\alpha = 0$, the α-Pareto optimal strategy for Player I is $x^* = (0.7916667, 0.2083333)$ and 0-security levels are the intervals $[155.2083, 164.6667]$ and $[123.9583, 135]$. Assume that Player I is not satisfied with the lower bounds of 0-security levels and wants to increase them from 155.2083 and 123.9583 to 160 and 125, respectively. Then, as discussed in the algorithm, we have $\alpha = \max\{0.82, 0.17\}$. Hence, we solve the problem (24) for $\alpha = 0.82$. The new security levels of the game for Player I are $(\tilde{\underline{v}}^1)_\alpha = [160, 161.705]$ and $(\tilde{\underline{v}}^2)_\alpha = [128.911, 130.9]$. If Player I satisfied with the obtained solutions stop. Assume that Player I is not satisfied with the obtained security levels and he present the new intervals $[163, 170]$ and $[135, 140]$ for two objective functions, respectively. Note that $163 \notin [160, 161.705]$ and $135 \notin [128.911, 130.9]$. Therefore, according to the algorithm (Step 6), in this case we solve the following problem:

$$\begin{aligned}
&\min \gamma \\
&s.t. \\
&\quad \gamma + (\underline{v}^{1L})_\alpha \geq 163 \\
&\quad \gamma + (\underline{v}^{1R})_\alpha \geq 170 \\
&\quad \gamma + (\underline{v}^{2L})_\alpha \geq 135 \\
&\quad \gamma + (\underline{v}^{2R})_\alpha \geq 140 \\
&\quad (175 + 5\alpha)x_1 + (80 + 10\alpha)x_2 \geq (\underline{v}^{1L})_\alpha \\
&\quad (150 + 6\alpha)x_1 + (175 + 5\alpha)x_2 \geq (\underline{v}^{1L})_\alpha \\
&\quad (125 + 5\alpha)x_1 + (120 + 10\alpha)x_2 \geq (\underline{v}^{2L})_\alpha \\
&\quad (120 + 10\alpha)x_1 + (150 + 10\alpha)x_2 \geq (\underline{v}^{2L})_\alpha \\
&\quad (365 - 5\alpha)x_1 + 180x_2 \geq (\underline{v}^{1L})_\alpha + (\underline{v}^{1R})_\alpha \\
&\quad (308 + 4\alpha)x_1 + (365 - 5\alpha)x_2 \geq (\underline{v}^{1L})_\alpha + (\underline{v}^{1R})_\alpha \\
&\quad 260x_1 + (255 + 5\alpha)x_2 \geq (\underline{v}^{2L})_\alpha + (\underline{v}^{2R})_\alpha \\
&\quad (255 + 5\alpha)x_1 + 320x_2 \geq (\underline{v}^{2L})_\alpha + (\underline{v}^{2R})_\alpha \\
&\quad (\underline{v}^{1L})_\alpha \leq (\underline{v}^{1R})_\alpha \\
&\quad (\underline{v}^{2L})_\alpha \leq (\underline{v}^{2R})_\alpha \\
&\quad x_1 + x_2 = 1 \\
&\quad x_1, x_2 \geq 0
\end{aligned} \tag{25}$$

Using the simplex method, we obtain $x^* = (0.86, 0.14)$ and stop. This means that Player I by choosing his first strategy with about 86% chance can get the specified intervals of gain-floor for the objectives.

Similarly, for Player II, assume that the importance of objective functions are the same and $\lambda_1 = \lambda_2 = 0.5$. The problem (PII-$\alpha$) for these data and $\beta = 0$ is as follows:

$$\begin{array}{ll} \min & \frac{3}{8}((\bar{v}^{1R})_\alpha + (\bar{v}^{2R})_\alpha) + \frac{1}{8}((\bar{v}^{1L})_\alpha + (\bar{v}^{2L})_\alpha) \\ s.t. & \\ & (190-10\alpha)y_1 + (158-2\alpha)y_2 \le (\bar{v}^{1R})_\alpha \\ & (100-10\alpha)y_1 + (190-10\alpha)y_2 \le (\bar{v}^{1R})_\alpha \\ & (135-5\alpha)y_1 + (135-5\alpha)y_2 \le (\bar{v}^{2R})_\alpha \\ & (135-5\alpha)y_1 + (170-10\alpha)y_2 \le (\bar{v}^{2R})_\alpha \\ & (365-5\alpha)y_1 + (308+4\alpha)y_2 \le (\bar{v}^{1L})_\alpha + (\bar{v}^{1R})_\alpha \\ & 180y_1 + (365-5\alpha)y_2 \le (\bar{v}^{1L})_\alpha + (\bar{v}^{1R})_\alpha \\ & 260y_1 + (255+5\alpha)y_2 \le (\bar{v}^{2L})_\alpha + (\bar{v}^{2R})_\alpha \\ & (260+5\alpha)y_1 + 320y_2 \le (\bar{v}^{2L})_\alpha + (\bar{v}^{2R})_\alpha \\ & (\bar{v}^{1L})_\alpha \le (\bar{v}^{1R})_\alpha \\ & (\bar{v}^{2L})_\alpha \le (\bar{v}^{2R})_\alpha \\ & y_1 + y_2 = 1 \\ & y_1, y_2 \ge 0 \end{array} \tag{26}$$

For $\alpha = 0$, the α-Pareto optimal strategy for Player II is $y^* = (1, 0)$ and 0-security levels are the intervals $[175, 190]$ and $[125, 135]$ respectively. When $\alpha = 1$, the α-Pareto optimal strategy for Player II is $y^* = (0.92, 0.08)$ and 1-security levels are 178.15 and 132.31, respectively. Therefore, the fuzzy values of the game for Player II are obtained as follows:

$$\tilde{\bar{v}}^1 = ((\bar{v}^{1L})_0, (\bar{v}^{1R})_1, (\bar{v}^{1R})_0) = (175, 178.15, 190)$$
$$\tilde{\bar{v}}^2 = ((\bar{v}^{2L})_0, (\bar{v}^{2R})_1, (\bar{v}^{2R})_0) = (125, 132.31, 135)$$

which are triangular fuzzy numbers. Thus, we can use interactive algorithm from the point of view Player II.
The above problems have been solved by Lingo software.

5. Conclusion

The problem of zero-sum multiobjective game with fuzzy payoffs has not been considered in previouse researches, based on the best knowledge of the authors. In this paper, a method is presented to find a satisfactory strategy and security levels in such problems. The fuzzy payoffs in this research are considered to be triangular fuzzy numbers. The problem is converted to a multiobjective game problem with interval payoffs by considering the concept of α-cuts, and its solutions are obtained by solving an interval multiobjective linear programming problem. The obtained strategy for a given α is called α-Pareto optimal strategy, and its corresponding objective values are called α-security levels of players. It is shown that the security levels for players are triangular fuzzy numbers. Finally, an interactive algorithm is proposed to compute a strategy that players are satisfied with its corresponding α-security levels. The main advantage of this method is that it does not require any defuzzification. Note that, when a defuzzification is used, the fuzzy aspect of payoffs are actually lost, which is not desirable. The proposed method in this paper can be applied to interval-valued matrix games simply. The major limitation in ours interactive approach is considering triangular fuzzy number payoffs. Applying the proposed method to other kinds of fuzzy number payoffs needs more researches, which can be our future work.

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