SHAPLEY FUNCTION BASED INTERVAL-VALUED INTUITIONISTIC FUZZY VIKOR TECHNIQUE FOR CORRELATIVE MULTI-CRITERIA DECISION MAKING PROBLEMS

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Abstract. Interval-valued intuitionistic fuzzy set (IVIFS) has developed to cope with the uncertainty of imprecise human thinking. In the present communication, new entropy and similarity measures for IVIFSs based on exponential function are presented and compared with the existing measures. Numerical results reveal that the proposed information measures attain the higher association with the existing measures, which demonstrate their efficiency and reliability. To deal with the interactive characteristics among the elements in a set, Shapley weighted similarity measure based on proposed similarity measure for IVIFSs is discussed via Shapley function. Thereafter, the linear programming model for optimal fuzzy measure is originated for incomplete information about the weights of the criteria and thus, the optimal weight vector is obtained in terms of Shapley values. Further, the VIKOR technique is discussed for correlative multi-criteria decision making problems under interval-valued intuitionistic fuzzy environment. Finally, an example of investment problem is presented to exemplify the application of the proposed technique under incomplete and uncertain information situation.

1. Introduction

Due to uncertain and partial information, ambiguity is omnipresent in every decisions. To manage the ambiguity occurred in real life problems, Zadeh [63] originated the idea of fuzzy set theory, which is characterized by the degree of membership. To express the opinions of the decision makers in an interval, Zadeh [64] introduced the concept of inverval-valued fuzzy set (IVFS). However, fuzzy sets (FSs) have been applied in various fields of science and technology but it shows some limitations in dealing with vagueness of two or more sources simultaneously. To evade the limitations of FSs, Atanassov [3] developed the idea of intuitionistic fuzzy set (IFS) which is characterized by the degree of membership and non-membership. As the generalization of FSs, IFSs are more appropriate in the representation of inadequate knowledge of many decision making problems and have been implemented by various authors for different purposes [1, 38, 40]. Due to time complexity and

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lack of hesitant information in IFSs, Atanassov and Gargov [2] evaluated the notion of interval-valued intuitionistic fuzzy set (IVIFS) which combines the concept of IFSs and IVFSs to deal with imprecision and uncertainty of an information in terms of an intervals rather than real numbers. Nowadays, various decision makers extended the topic of IVIFSs in decision making problems. For instance, Hashemi et al. [14] presented a novel multiple attribute group decision-making (MAGDM) model based on compromise ratio method under interval-valued intuitionistic fuzzy environment and applied it to reservoir flood control operation. Ebrahimnejad et al. [9] proposed an interval-valued intuitionistic fuzzy multiple-criteria group decision making approach to select the best outsourcing provider.

Information measures (entropy and similarity measures) play a substantial role in the fuzzy set theory and have been explored by numerous researchers from different point of view. To deal with the measure of ambiguity between FSs, De Luca and Termini [8] defined an axiomatic definition of fuzzy entropy based on Shannon's entropy [44]. Thereafter, various entropy measures on FSs have been studied [16, 15, 28, 29, 30, 31, 43]. Further, Burillo and Bustince [5] generated the idea of entropy on IVFSs and IFSs to measure the degree of intuitionism. Szmidt and Kacprzyk [50] explained the axioms of De Luca and Termini's [8] entropy on IFSs with the numerical explanation of IFSs. After that, several researchers generalized the entropy measures on IFSs [17, 27, 32, 33, 34, 54]. To avoid the scarcity occurred in information accessibility, Liu et al. [24] extended the axioms of Szmidt and Kacprzyk's [50] entropy on IFSs and anticipated a set of axiomatic requirements on IVIFSs. Nowadays, various entropy measures on IVIFSs have been discussed in the literature [7, 26, 55, 56].

The similarity measure for IFSs is accustomed to assess the degree of similarity between two IFSs. The relationship between entropy and similarity measures on IFSs have widely been associated in decision making and pattern recognition problems. Firstly, Li and Cheng [21] proposed a similarity measure for IFSs and then applied in pattern recognition problems. Liang and Shi [22] developed the similarity measures to discriminate the IFSs and presented the relationships between these measures. Furthermore, various literatures discussed the development of similarity measures for IFSs [18, 19, 48, 49, 59]. Xu [59, 60] developed the notion of degree of similarity between IVIFSs and extended various distance based similarity measures of IFSs to IVIFSs and applied it to pattern recognition problems through interval-valued intuitionistic fuzzy information.

In most of the situations, the elements in a set are usually correlated. To deal with the correlative elements in a set, Sugeno [47] initiated the notion of fuzzy measure that has effectively used for modeling the interaction between elements in the real life problems. In correlated decision making problems, the Shapley value measures the importance of a criterion in all coalitions with identical position probability. The Shapley value has been applied by various authors in the literature [26, 41].

Multi-criteria decision making (MCDM) is the procedure of selecting the best option from a distinct set of feasible alternatives with respect to a finite set of criteria. Generally, it is not possible for an alternative to satisfies all the conflicting criteria simultaneously. Due to increasing complexity and inconsistency occurred in real life decision making problems, FSs and its extension have received more attention from decision makers in MCDM literature as the FSs are more capable than crisp numbers to deal with imprecise human thinking [11, 12, 13, 25, 35, 36, 39]. To cope with complex MCDM problems, numerous techniques such as Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [27, 33], VlseKriterijuska Optimizacija I Komoromisno Resenje (VIKOR) [4, 10], Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) [6, 53], Elimination Et Choice Translating Reality (ELECTRE) [9, 52] etc. have been developed in different fields.

The idea of VIKOR has been pioneered by Opricovic [42], to find the optimal solution of MCDM problems with inconsistent and non-commensurable decisive factors which can assist the judges to reach a final decision. The main characteristic of VIKOR technique is to establish a compromise solution that maintains the maximum group utility for the majority and the minimum individual regret for the opponent. This technique establishes a conciliation grading index on the distance of alternative to the ideal solution as in TOPSIS technique, but the fundamental principle of the TOPSIS is that the preferred option should be the nearest to the positive ideal solution (PIS) and furthermost to the negative ideal solution (NIS) [65]. Nowadays, the VIKOR technique has been applied by various authors in different fields such as problem of material handling equipment [37], selection problem for sustainable manufacturing [46], contractor selection problem in construction industry [51], strategy supplier selection in nuclear power industry [58].

Due to complexity of socio-economic environment, time pressure and lack of hesitant information in IFSs, the decision information may be provided with IVIFSs. IVIFSs are more prominent than IFSs to tackle the major part in the hesitant system and establish more attention from researchers in the field of MCDM problems. Nowadays, there are few researches on entropy and similarity measures for IVIFSs. Thus, it is very interesting and important to generalize the entropy and similarity measures of IFSs to IVIFSs. Some authors [43, 54] mentioned that the exponential entropy has an advantage over Shannon's [44] entropy, therefore, the exponential entropy of IFSs to IVIFSs is generalized in this paper. Up to now, in various decision making problems, there is an interaction between elements in a set. Therefore, to study the overall interaction of elements, the Shapley values are evaluated. The normalized values in VIKOR technique do not depend on the assessment part of criterion, hence, this technique is appropriate for those decisions in which the decision expert wants to utmost profit and the risk of choice is less essential [65].

Inspired by the aforementioned works, a new entropy measure for IVIFSs based on exponential function, which is the generalized form of Intuitionistic fuzzy entropy measure [54], is introduced and compared with existing entropy measures for IVIFSs. A new similarity measure based on exponential function, which is the generalized form of similarity measure for IFSs [18], is developed and compared with existing similarity measures for IVIFSs. Shapley values are calculated for correlated characteristics of criteria that can determine the weight vector of the single criterion contribution on the basis of different combinations of sub-criteria. Shapley weighted similarity measure is developed and applied in the field of pattern recognition problem. Corresponding to proposed entropy measure, an interval-valued intuitionistic fuzzy VIKOR (IVIF-VIKOR) technique is generated and applied in the selection of investment problem. Finally, the comparison between IVIF-VIKOR and IVIF-TOPSIS is discussed and demonstrated in the example.

2. **Preliminaries**

In this section, fundamental ideas of IFSs, IVIFSs, entropy and similarity measures for IVIFSs, fuzzy measure, Shapley function and the VIKOR technique are discussed.

2.1. **Some Basic Concepts.**

Definition 2.1. Let X be the universe of discourse. An IFS A in X is an object having the form

$$A = \{\langle x_i,\, \mu_A(x_i),\, \nu_A(x_i)\rangle \,:\, x_i \in X\},$$

where $\mu_A : X \to [0,\ 1]$ and $\nu_A : X \to [0,\ 1]$ such that

$$0 \le \mu_A(x_i) + \nu_A(x_i) \le 1,\ \ \forall\, x_i \in X.$$

The numbers $\mu_A(x_i)$ and $\nu_A(x_i)$ denote the degree of membership and non-membership of the element $x_i \in X$, respectively.
For convenience of notations, the term $IFS(X)$ denotes the set of all IFSs in X. For each IFS A in X, the intuitionistic fuzzy index (hesitancy degree) of an element $x_i \in X$ in A is defined as

$$\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i).$$

It is evident that $0 \le \pi_A(x_i) \le 1, \forall\, x_i \in X$. (Atanassov [3])
FSs can also be defined using the notation of IFSs. An FS A defined on X can be represented as the following IFS:

$$A = \{\langle x_i,\ \mu_A(x_i),\ 1 - \mu_A(x_i)\rangle \ :\ x_i \in X\},$$

with $\pi_A(x_i) = 0,\ \forall\, x_i \in X$.
The complementary set A^c of A is defined as

$$A^c = \{\langle x_i,\, \nu_A(x_i),\, \mu_A(x_i)\rangle \,:\, x_i \in X\}.$$

Definition 2.2. Let X be the universe of discourse and $int\,(0,\,1)$ denotes all closed subintervals of the interval $[0,\,1]$. An IVIFS A in X is defined as [2]

$$A = \{\langle x_i,\, \mu_A(x_i),\, \nu_A(x_i)\rangle \,:\, x_i \in X\},$$

where $\mu_A : X \to int\,(0,\ 1)$ and $\nu_A : X \to int\,(0,\ 1)$, with the condition

$$0 \le \sup\,(\mu_A(x_i)) + \sup\,(\nu_A(x_i)) \le 1,\, \forall\, x_i \in X.$$

The numbers $\mu_A(x_i)$ and $\nu_A(x_i)$ denote the degree of membership and non-membership for each element x_i to A, respectively.
For convenience, if $\mu_A(x_i) = \left[\mu_A^-(x_i),\, \mu_A^+(x_i)\right],\ \nu_A(x_i) = \left[\nu_A^-(x_i),\, \nu_A^+(x_i)\right]$, then

$$A = \left\{\left\langle x_i,\ \left[\mu_A^-(x_i),\, \mu_A^+(x_i)\right],\ \left[\nu_A^-(x_i),\, \nu_A^+(x_i)\right]\right\rangle \,:\, x_i \in X\right\}$$

such that
$$0 \leq \mu_A^+(x_i) + \nu_A^+(x_i) \leq 1, \forall x_i \in X.$$
The term $IVIFS(X)$ denotes the set of all IVIFSs in X. Now, the interval
$$\left[1 - \mu_A^-(x_i) - \nu_A^-(x_i),\, 1 - \mu_A^+(x_i) - \nu_A^+(x_i)\right]$$
abridged by $\left[\pi_A^-(x_i),\, \pi_A^+(x_i)\right]$ and symbolized by $\pi_A(x_i)$, the interval-valued intuitionistic fuzzy index (hesitancy degree) of x_i in A.
Clearly, if $\mu_A(x_i) = \mu_A^-(x_i) = \mu_A^+(x_i)$ and $\nu_A(x_i) = \nu_A^-(x_i) = \nu_A^+(x_i)$, then the given IVIFS A is reduced to an ordinary IFS.

Definition 2.3. Let X be the universe of discourse and $A,\, B \in IVIFS(X)$ defined by [2]
$$A = \left\{\left\langle x_i,\, \left[\mu_A^-(x_i),\, \mu_A^+(x_i)\right],\, \left[\nu_A^-(x_i),\, \nu_A^+(x_i)\right]\right\rangle : x_i \in X\right\}$$
and
$$B = \left\{\left\langle x_i,\, \left[\mu_B^-(x_i),\, \mu_B^+(x_i)\right],\, \left[\nu_B^-(x_i),\, \nu_B^+(x_i)\right]\right\rangle : x_i \in X\right\},$$
then the operations on IVIFSs are defined as follows:

(1) $A \subseteq B$ if and only if $\mu_A^-(x_i) \leq \mu_B^-(x_i)$, $\mu_A^+(x_i) \leq \mu_B^+(x_i)$, $\nu_A^-(x_i) \geq \nu_B^-(x_i)$, and $\nu_A^+(x_i) \geq \nu_B^+(x_i)$, for each $x_i \in X$;
(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;
(3) $A^c = \left\{\left\langle x_i,\, \left[\nu_A^-(x_i),\, \nu_A^+(x_i)\right],\, \left[\mu_A^-(x_i),\, \mu_A^+(x_i)\right]\right\rangle : x_i \in X\right\}$;
(4) $A \cup B = \left\{\left\langle x_i,\, \left[\mu_A^-(x_i) \vee \mu_B^-(x_i),\, \mu_A^+(x_i) \vee \mu_B^+(x_i)\right],\right.\right.$
$\left.\left.\left[\nu_A^-(x_i) \wedge \nu_B^-(x_i),\, \nu_A^+(x_i) \wedge \nu_B^+(x_i)\right]\right\rangle : x_i \in X\right\}$.

Definition 2.4. A real valued function $E : IVIFS(X) \to [0,\ 1]$ is said to be an entropy measure for IVIFSs, if it satisfies the following axiomatic requirements: [24]

(P1). $E(A) = 0$ iff A is a crisp set;
(P2). $E(A) = 1$ iff $\left[\mu_A^-(x_i),\ \mu_A^+(x_i)\right] = \left[\nu_A^-(x_i),\ \nu_A^+(x_i)\right]$ for each $x_i \in X$;
(P3). $E(A) = E(A^c)$;
(P4). $E(A) \leq E(B)$ if $A \subseteq B$ when $\mu_B^-(x_i) \leq \nu_B^-(x_i)$ and $\mu_B^+(x_i) \leq \nu_B^+(x_i)$ for each $x_i \in X$ or $B \subseteq A$ when $\mu_B^-(x_i) \geq \nu_B^-(x_i)$ and $\mu_B^+(x_i) \geq \nu_B^+(x_i)$ for each $x_i \in X$.

Definition 2.5. A real-valued function $Sim : IVIFS(X) \times IVIFS(X) \to [0,\ 1]$ is called a similarity measure on $IVIFS(X)$, if it satisfies the following axiomatic requirements: [59]

(S1). $0 \leq Sim\,(A,\, B) \leq 1$;
(S2). $Sim(A,\, B) = 1 \Leftrightarrow A\, =\, B$;
(S3). $Sim\,(A,\, B) = Sim(B,\, A)$;
(S4). For all $A,\, B,\, C\, \in IVIFS\,(X)$, if $A \subseteq B \subseteq C$, then $Sim\,(A,\, C) \leq Sim\,(A,\, B)$ and $Sim\,(A,\, C) \leq Sim\,(B,\, C)$.

Sugeno [47] developed the concept of fuzzy measure to model the interactions between elements in various real life circumstances, which is defined as

Definition 2.6. A fuzzy measure g on a finite universal set $X = \{1, 2, ..., n\}$ is a set function $g : P(X) \to [0, 1]$ which satisfying the following axioms:

(A1). $g(\varphi)\, =\, 0,\ g(X)\, =\, 1.$

(A2). If $A, B \in P(X)$ and $A \subseteq B$, then $g(A) \leq g(B)$. Here, $P(X)$ denotes the power set of X.

In decision making problems, $g(A)$ can be viewed as the significance degree of the decisive factor set A. If the fuzzy measure is additive, then it is hold that

$$g(A) = \sum_{j \in A} g(\{j\}), \forall A \subseteq X,$$

where $g(\{j\})$ is the weight or the significance degree of the element $j \in A$ and the values of the fuzzy measure can be evaluated from the values of X.

If the elements in a set are usually correlated, then the importance of each element is not only determined by itself but also influenced by other elements. Therefore, to study the overall contribution of an element based on different combinations of sub-criteria, the Shapley values can be computed for fuzzy measures and determine the importance of each singleton. Shapley function is one of the most important tool for studying the interaction between elements, which is defined as follows [45]:

Definition 2.7. Let g be the fuzzy measure on the set $X = \{1, 2, ..., n\}$ and $|X| = n$. Then, the Shapley value of the element $j \in X$ with respect to ϕ_j is

$$\phi_j(g, X) = \sum_{H \subset X \setminus \{j\}} \frac{(n - |H| - 1)!\ |H|!}{n!} \left(g(H \bigcup \{j\}) - g(H)\right), \ \forall j \in X. \tag{1}$$

The vector $\phi(g) = [\phi_1, \phi_2, ..., \phi_n]$ is the Shapley value of the fuzzy measure g. The Shapley value determines the expected marginal contribution of the particular element to the set X.

2.2. **Classical Concepts of VIKOR Method.** Opricovic [42] introduced VIKOR means multi-criteria optimization and compromise solution, is an efficient technique to determine a compromise solution of the MCDM problems with conflicting and non-commensurable criteria. A compromise solution for the MCDM problems with a set of incompatible criteria is a feasible solution which is closest to the ideal solution. The VIKOR technique provides a compromise ranking list by using multi-criteria ranking approach based on the particular measure of "closeness" to the "ideal" solution. The multi-criteria measure for compromise ranking is developed for L_p- metric used as an aggregation function in the compromise programming method [23].

A MCDM problem with m alternative $U_i\,(i = 1, 2, ..., m)$ and n criteria $V_j\,(j = 1, 2, ..., n)$ is given as in Table 1.

In Table 1, $\xi_{ij}\,(i = 1, 2, ..., m;\ j = 1, 2, ..., n)$ represents the assessment values of the alternatives $U_i\,(i = 1, 2, ..., m)$ over the criteria $V_j\,(j = 1, 2, ..., n)$. The expansion of the VIKOR technique is started with the discrete form of Shapley value based L_p-metric over the alternatives $U_i\,(i = 1, 2, ..., m)$ in the compromise programming, which is given as follows:

$$L_{p,i} = \left(\sum_{j=1}^{n} \left(\phi_j \frac{\xi_j^+ - \xi_{ij}}{\xi_j^+ - \xi_j^-}\right)^p\right)^{1/p},\ 1 \leq p \leq \infty,\ i = 1, 2, ..., m,$$

	V_1	V_2	...	...	...	V_n
U_1	ξ_{11}	ξ_{12}	...	...	...	ξ_{1n}
U_2	ξ_{21}	ξ_{22}	...	...	...	ξ_{2n}
...	...	...	...	...	...	...
...	...	...	...	...	...	...
...	...	...	...	...	...	...
U_m	ξ_{m1}	ξ_{m2}	...	...	...	ξ_{mn}

TABLE 1. MCDM Problem with $'m'$ Alternatives w.r.t. $'n'$ Criteria

where $\phi_j\,(j\,=1,\,2,\,...,\,n)$ denotes the Shapley value of the criteria, $\xi_i^+\,=\,\max_i\,\xi_{ij}$ and $\xi_i^-\,=\,\min_i\,\xi_{ij}$ are the peak and dip values of the alternatives $U_i\,(i\,=1,\,2,\,...,\,m)$ over the benefit-type criteria $V_j\,(j\;=\;1,\,2,...,\,n)$, respectively. This technique presents a maximum "overall satisfactory" for the majority of criteria and a minimum "sacrifice" of each individual criterion which are formulated by the metrics $L_{1,i}$ and $L_{\infty,i}$, respectively. The compromise ranking procedure of the classical VIKOR technique has the following steps:

Step 1: Find the peak value ξ_j^+ and dip value ξ_j^- of the criteria.

Step 2: Compute the values of G_i and $I_i(i\,=1,\,2,\,...,m)$ over the alternatives $U_i\,(i\,=1,\,2,\,...,\,m)$ which represent the group utility and individual regret of the opponent, with the following relations:

$$G_i\;=\;L_{1,i}\;=\;\sum_{j=1}^{J}\phi_j\,\frac{(\xi_j^+\,-\xi_{ij})}{(\xi_j^+\,-\xi_j^-)} \tag{2}$$

and

$$I_i\;=\;L_{\infty,i}=\max_j\,\left(\phi_j\,\frac{\xi_j^+\,-\xi_{ij}}{\xi_j^+\,-\xi_j^-}\right). \tag{3}$$

To find the compromise solution, we have to minimize the above two objective functions. Minimization of G_i implies maximum group utility and minimization of I_i implies minimum individual regret of the opponent.

Step 3: Calculate the values of $\Upsilon_i(i\,=\,1,\,2,...,\,m)$ by the relation

$$\Upsilon_i\;=\;\tau\,\frac{(G_i\,-G^+)}{(G^-\,-G^+)}\;+\;(1-\tau)\,\frac{(I_i\,-I^+)}{(I^-\,-\,I^+)}, \tag{4}$$

where $G^+=\min_i\,G_i$, $G^-=\max_i\,G_i$, $I^+=\min_i\,I_i$, $I^-=\max_i\,I_i$ and τ is the weight of the strategy of the majority of criteria (majority of attribute or maximum group utility), whereas $(1-\tau)$ is the weight of individual regret.

Step 4: Grade the alternatives according to the values of G_i, I_i and Υ_i. The smaller value of Υ_i denotes the optimal alternative.

Step 5: Find out the compromise solution. The following conditions prove the uniqueness of the final alternative:
(C1). Acceptable advantage:

$$\Upsilon(U^{(2)})-\Upsilon(U^{(1)})\geq\frac{1}{(m-1)},$$

where m is the number of alternatives, $U^{(1)}$ and $U^{(2)}$ are the alternatives with the first and second locations in the ranking list, respectively.
(C2). Adequate stability: The alternative $U^{(1)}$ must also be ranked by G_i and I_i. This compromise solution is stable within a decision making process which can be selected through "voting by majority rule ($\tau > 0.5$)" or "by consensus ($\tau \approx 0.5$)" or "by veto ($\tau < 0.5$)" .
If the condition (C1) is not fulfilled, then the maximum value of M is examined by the following relation:

$$\Upsilon(U^{(M)}) - \Upsilon(U^{(1)}) < \frac{1}{(m-1)}.$$

Thus, all the alternatives $U^{(i)}$ ($i = 1, 2, ..., M$) are the compromise solutions.
The alternatives $U^{(1)}$ and $U^{(2)}$ are compromise solutions in case of condition (C2) is not satisfied.

3. Information Measures for IVIFSs

In this section, new entropy and similarity measures are developed for IVIFSs.

3.1. **New Entropy Measure for IVIFSs.** For each $A \in IVIFS(X)$, entropy measure for IVIFS A is denoted by $E(A)$ and defined as

$$E(A) = \frac{1}{n\left(\sqrt{e}-1\right)} \sum_{i=1}^{n} \left[\left(\frac{\left(\mu_A^-(x_i) + \mu_A^+(x_i)\right) + 2 - \left(\nu_A^-(x_i) + \nu_A^+(x_i)\right)}{4} \right) e^{\left(\frac{\left(\nu_A^-(x_i) + \nu_A^+(x_i)\right) + 2 - \left(\mu_A^-(x_i) + \mu_A^+(x_i)\right)}{4} \right)} + \left(\frac{\left(\nu_A^-(x_i) + \nu_A^+(x_i)\right) + 2 - \left(\mu_A^-(x_i) + \mu_A^+(x_i)\right)}{4} \right) . e^{\left(\frac{\left(\mu_A^-(x_i) + \mu_A^+(x_i)\right) + 2 - \left(\nu_A^-(x_i) + \nu_A^+(x_i)\right)}{4} \right)} - 1 \right]. \tag{5}$$

Theorem 3.1. *The mapping $E(A)$, defined by equation(5), is an entropy measure for IVIFSs.*

Proof. In order for equation(5) to be qualified as a sensible measure of interval-valued intuitionistic fuzzy entropy measure, it must satisfy the conditions (P1)-(P4) in Definition 2.4.
(P1). Let A be a crisp set. Then, we have

$$\left[\mu_A^-(x_i),\ \mu_A^+(x_i)\right] = [1, 1],\ \left[\nu_A^-(x_i),\ \nu_A^+(x_i)\right] = [0, 0]$$

or

$$\left[\mu_A^-(x_i),\ \mu_A^+(x_i)\right] = [0,\, 0],\ \left[\nu_A^-(x_i),\ \nu_A^+(x_i)\right] = [1,\, 1],$$

for each $x_i \in X$. From equation(5), we obtain that $E(A) = 0$.

Suppose that

$$\frac{\left(\mu_A^-(x_i) + \mu_A^+(x_i)\right) + 2 - \left(\nu_A^-(x_i) + \nu_A^+(x_i)\right)}{4} = \varphi_A(x_i). \tag{6}$$

$$E(A) = \frac{1}{n\,(\sqrt{e}-1)} \sum_{i=1}^{n} \left[\varphi_A(x_i)\, e^{(1-\varphi_A(x_i))} + (1-\varphi_A(x_i))\, e^{\varphi_A(x_i)} - 1\right]. \tag{7}$$

From [43], we know that equation(7) becomes zero if and only if $\varphi_A(x_i) = 0$ or 1, $\forall\, x_i \in X$, *i.e.*,

$$\frac{\left(\mu_A^-(x_i) + \mu_A^+(x_i)\right) + 2 - \left(\nu_A^-(x_i) + \nu_A^+(x_i)\right)}{4} = 0\,,\ \forall\, x_i \in X \tag{8}$$

or

$$\frac{\left(\mu_A^-(x_i) + \mu_A^+(x_i)\right) + 2 - \left(\nu_A^-(x_i) + \nu_A^+(x_i)\right)}{4} = 1,\ \forall\, x_i \in X. \tag{9}$$

This set of equations implies that A is a crisp set.

(P2). Let $\left[\mu_A^-(x),\ \mu_A^+(x)\right] = \left[\nu_A^-(x),\ \nu_A^+(x)\right]$, *i.e.*, $\mu_A^-(x) = \nu_A^-(x)$ and $\mu_A^+(x) = \nu_A^+(x)$ for each $x_i \in X$. Applying this condition to equation(5) yields $E(A) = 1$.

From equation(7), we obtain

$$E(A) = \frac{1}{n} \sum_{i=1}^{n} f\left(\varphi_A(x_i)\right),$$

where

$$f\left(\varphi_A(x_i)\right) = \left[\frac{\varphi_A(x_i)\, e^{(1-\varphi_A(x_i))} + (1-\varphi_A(x_i))\, e^{\varphi_A(x_i)} - 1}{(\sqrt{e}-1)}\right],\ \forall\, x_i \in X. \tag{10}$$

Let us suppose that $E(A) = 1$, *i.e.*, $\frac{1}{n}\sum_{i=1}^{n} f\left(\varphi_A(x_i)\right) = 1$.

$$f\left(\varphi_A(x_i)\right) = 1,\ \forall\, x_i \in X. \tag{11}$$

Differentiating equation(11) with respect to $\varphi_A(x_i)$ and equating to zero, we have

$$\frac{\partial f}{\partial\left(\varphi_A(x_i)\right)} = e^{(1-\varphi_A(x_i))} - \varphi_A(x_i) e^{(1-\varphi_A(x_i))} - e^{\varphi_A(x_i)} + (1-\varphi_A(x_i))\, e^{\varphi_A(x_i)} = 0,$$

it implies

$$(1-\varphi_A(x_i))\, e^{(1-\varphi_A(x_i))} = \varphi_A(x_i) e^{\varphi_A(x_i)},\ \ \forall\ x_i \in X. \tag{12}$$

Using the fact that $f(x) = x\, e^x$ is a bijection function, we can write

$$\begin{aligned}(1-\varphi_A(x_i)) &= \varphi_A(x_i),\\ \varphi_A(x_i) &= 0.5,\ \forall\, x_i \in X.\end{aligned} \tag{13}$$

And find

$$\left[\frac{\partial^2 f}{\partial\left(\varphi_A(x_i)\right)^2}\right]_{\varphi_A(x_i)=0.5} < 0,\ \forall\, x_i \in X. \tag{14}$$

Hence $f(\varphi_A(x_i))$ is a concave function and has a global maximum at $\varphi_A(x_i) = 0.5$. Since $E(A) = \frac{1}{n}\sum_{i=1}^{n} f(\varphi_A(x_i))$, so $E(A)$ attains the maximum value when $\varphi_A(x_i) = 0.5$, *i.e.*, $\left[\mu_A^-(x),\, \mu_A^+(x)\right] = \left[\nu_A^-(x),\, \nu_A^+(x)\right]$ for each $x_i \in X$.

(P3). For $A^c = \left\{\left\langle x,\ \left[\nu_A^-(x),\, \nu_A^+(x)\right], \left[\mu_A^-(x),\, \mu_A^+(x)\right]\right\rangle : x \in X\right\}$, we can easily get that

$$E(A) = E(A^c).$$

(P4). In order to show that equation(5) fulfils (P4), it suffices to prove that the function

$$h(x,\ y) = \left[\left(\frac{x+2-y}{4}\right) e^{\left(\frac{y+2-x}{4}\right)} + \left(\frac{y+2-x}{4}\right) e^{\left(\frac{x+2-y}{4}\right)} - 1\right], \tag{15}$$

where $x,\, y \in [0,\, 1]$ is increasing with respect to x and decreasing with respect to y. Taking the partial derivatives of h with respect to x and y, respectively, yields

$$\frac{\partial h}{\partial x} = \frac{1}{4}\left[\left(\frac{y+2-x}{4}\right)\ e^{\left(\frac{y+2-x}{4}\right)} - \left(\frac{x+2-y}{4}\right) e^{\left(\frac{x+2-y}{4}\right)}\right], \tag{16}$$

$$\frac{\partial h}{\partial y} = \frac{1}{4}\left[\left(\frac{x+2-y}{4}\right) e^{\left(\frac{x+2-y}{4}\right)} - \left(\frac{y+2-x}{4}\right) e^{\left(\frac{y+2-x}{4}\right)}\right]. \tag{17}$$

In order to find the critical point of h, we set $\frac{\partial h}{\partial x} = 0$ and $\frac{\partial h}{\partial y} = 0$.
This gives

$$x = y. \tag{18}$$

From equation(16) and equation(18), we obtain

$$\frac{\partial h}{\partial x} \geq 0, \text{ when } \ x \leq y$$

and

$$\frac{\partial h}{\partial x} \leq 0, \text{ when } \ x \geq y,$$

for any $x,\, y \in [0,\, 1]$.

Thus, $h(x,\, y)$ is increasing with respect to x for $x \leq y$ and decreasing when $x \geq y$. Similarly, we obtain that

$$\frac{\partial h}{\partial y} \leq 0, \text{ when } \ x \leq y$$

and

$$\frac{\partial h}{\partial y} \geq 0, \text{ when } \ x \geq y.$$

If $A \subseteq B$ with $\mu_B^-(x_i) \leq \nu_B^-(x_i)$ and $\mu_B^+(x_i) \leq \nu_B^+(x_i)$ for each $x_i \in X$, then it follows that $\mu_A^-(x_i) \leq \nu_A^-(x_i)$ and $\mu_A^+(x_i) \leq \nu_A^+(x_i)$. Therefore, we have

$$E(A) \leq E(B).$$

Similarly, when $B \subseteq A$ with $\mu_B^-(x_i) \geq \nu_B^-(x_i)$ and $\mu_B^+(x_i) \geq \nu_B^+(x_i)$ for each $x_i \in X$ and thus, it can be prove that

$$E(A) \leq E(B).$$

□

Remark 3.2. On the off chance that an IVIFS diminishes to be an IFS, then the entropy measure defined by equation(5) reduces to the intuitionistic fuzzy entropy measure defined by [54].

Remark 3.3. It is also interesting to notice that if an IFS is an ordinary FS, *i.e.*, for all $x_i \in X$, $\nu_A(x_i) = 1 - \mu_A(x_i)$, then the exponential intuitionistic fuzzy entropy measure reduces to exponential fuzzy entropy measure given by [43].

Example 3.4. Let $X = \{x_1, x_2, ..., x_n\}$ be the universe of discourse. Let $A = \{\langle x_i, [0.2, 0.2]\,,\, [0.2, 0.3]\rangle : x_i \in X\}$ and $B = \{\langle x_i, [0.2, 0.3]\,,\, [0.4, 0.6]\rangle :\ x_i \in X\}$ be two IVIFSs in X.

Instinctively, we observe that A is fuzzier than B. Now, calculate the entropy measure on these IVIFSs by using the formula equation(5) and thus, the computed entropy measures $E(A)$ and $E(B)$ are as follows:

$$E(A) \approx 0.9972 > \ E(B) \approx \ 0.7754$$

which indicate that $E(A) > E(B)$ and thus, it is consistent with our perception.

3.2. **Effectiveness of the Proposed Entropy Measure.** In this subsection, the effectiveness of proposed entropy measure equation(5) is expressed by comparing it with some existing entropy measures in [7, 26, 55, 56].

Let $A = \{x_i,\, [\mu_A^-(x_i),\, \mu_A^+(x_i)],\, [\nu_A^-(x_i),\, \nu_A^+(x_i)] \,:\, x_i \,\in\, X\,\}$ be an IVIFS in X. Let us recall the following existing entropy measures:
Chen et al. [7]:

$$E_{CX}(A) = -\frac{1}{n\,\ln 4}\sum_{i=1}^{n}\begin{bmatrix} \mu_A^-(x_i)\,\ln\mu_A^-(x_i)\,+\,\mu_A^+(x_i)\,\ln\mu_A^+(x_i) \\ +\nu_A^-(x_i)\,\ln\nu_A^-(x_i) + \nu_A^+(x_i)\,\ln\nu_A^+(x_i) \\ -(1-\pi_A^-(x_i))\,\ln(1-\pi_A^-(x_i)) - \pi_A^-(x_i)\ln 2 \\ -(1-\pi_A^+(x_i))\,\ln(1-\pi_A^+(x_i)) - \pi_A^+(x_i)\ln 2 \end{bmatrix}. \tag{19}$$

Wei et al. [55]:

$$E_{WWZ}(A) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\min\{\mu_A^-(x_i),\nu_A^-(x_i)\} + \min\{\mu_A^+(x_i),\nu_A^+(x_i)\} + \pi_A^-(x_i) + \pi_A^+(x_i)}{\max\{\mu_A^-(x_i),\nu_A^-(x_i)\} + \max\{\mu_A^+(x_i),\nu_A^+(x_i)\} + \pi_A^-(x_i) + \pi_A^+(x_i)}\right). \tag{20}$$

Wei and Zhang [56]:

$$E_{WZ}(A) = \frac{1}{n}\sum_{i=1}^{n}\cos\left\{\frac{\left|\mu_A^-(x_i) - \nu_A^-(x_i)\right| + \left|\mu_A^+(x_i) - \nu_A^+(x_i)\right|}{2(2+\pi_A^+(x_i)\,+\pi_A^-(x_i))}\pi\right\}. \tag{21}$$

Meng and Chen [26]:

$$E_{MC}(A) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\min\{\mu_A^-(x_i),\,\nu_A^-(x_i)\} + \min\{\mu_A^+(x_i),\,\nu_A^+(x_i)\}}{\max\{\mu_A^-(x_i),\,\nu_A^-(x_i)\} + \max\{\mu_A^+(x_i),\,\nu_A^+(x_i)\}}\right). \tag{22}$$

Example 3.5. Let us compute entropy measures for the following IVIFSs:

$$A_1 = \{\langle x_i,\, [0.1,\, 0.2],\, [0.5,\, 0.6]\rangle : x_i \in X\},$$
$$A_2 = \{\langle x_i,\, [0.2,\, 0.2],\, [0.2,\, 0.3]\rangle : x_i \in X\},$$
$$A_3 = \{\langle x_i,\, [0.2,\, 0.3],\, [0.4,\, 0.6]\rangle : x_i \in X\}$$

and

$$A_4 = \{\langle x_i,\, [0.3,\, 0.4],\, [0.5,\, 0.6]\rangle : x_i \in X\}.$$

	E_{CX}	E_{WWZ}	E_{WZ}	E_{MC}	E
A_1	0.8195	0.5294	0.8855	0.2727	0.8463
A_2	0.9927	0.9373	0.9987	0.8000	0.9977
A_3	0.9387	0.6667	0.9511	0.5000	0.9403
A_4	0.9846	0.6923	0.9595	0.6364	0.9618

Table 2. Values of Different Entropy Measures

The above mentioned entropy measures equation(19)-equation(22) satisfy the set of requirements in Definition 2.4. Table 2 represents the values of the different entropy measures.

It can be interpreted that the closer the membership degree to the non-membership degree, the higher the value of interval-valued intuitionistic fuzzy entropy. And hence, from Table 2, it can be construed that the measures are satisfying the following order:

$$E_{CX}(A_1) < E_{CX}(A_3) < E_{CX}(A_4) < E_{CX}(A_2),$$
$$E_{WWZ}(A_1) < E_{WWZ}(A_3) < E_{WWZ}(A_4) < E_{WWZ}(A_2),$$
$$E_{WZ}(A_1) < E_{WZ}(A_3) < E_{WZ}(A_4) < E_{WZ}(A_2),$$
$$E_{MC}(A_1) < E_{MC}(A_3) < E_{MC}(A_4) < E_{MC}(A_2)$$

and

$$E(A_1) < E(A_3) < E(A_4) < E(A_2).$$

Thus, the obtained result is in accordance with existing measures which shows the effectiveness of the proposed interval-valued intuitionistic fuzzy entropy measure.

3.3. **Similarity Measure for IVIFSs.** Let $X = \{x_1, x_2, ..., x_n\}$ be a finite universe of discourse and $A,\ B \in IVIFS(X)$. Corresponding to [18], a new exponential-type similarity measure based on the distance measure between A and B is defined as follows:

$$S(A,\ B) = 1 - \frac{1 - \exp\left[-\frac{1}{4}\sum_{i=1}^{n}\begin{pmatrix} \left|\sqrt{\mu_A^-(x_i)} - \sqrt{\mu_B^-(x_i)}\right| \\ +\left|\sqrt{\mu_A^+(x_i)} - \sqrt{\mu_B^+(x_i)}\right| \\ +\left|\sqrt{\nu_A^-(x_i)} - \sqrt{\nu_B^-(x_i)}\right| \\ +\left|\sqrt{\nu_A^+(x_i)} - \sqrt{\nu_B^+(x_i)}\right| \end{pmatrix}\right]}{1 - \exp(-n)}. \tag{23}$$

It is well known that an exponential operation is extremely valuable to deal with the classical Shannon entropy [22] and cluster analysis [57]. Therefore, we implement the exponential operation to the Hamming distance and observe that

$$d(A,\ B) = \frac{1 - \exp\left[-\frac{1}{4}\sum_{i=1}^{n}\begin{pmatrix} \left|\sqrt{\mu_A^-(x_i)} - \sqrt{\mu_B^-(x_i)}\right| \\ +\left|\sqrt{\mu_A^+(x_i)} - \sqrt{\mu_B^+(x_i)}\right| \\ +\left|\sqrt{\nu_A^-(x_i)} - \sqrt{\nu_B^-(x_i)}\right| \\ +\left|\sqrt{\nu_A^+(x_i)} - \sqrt{\nu_B^+(x_i)}\right| \end{pmatrix}\right]}{1 - \exp(-n)} \tag{24}$$

as a normalized exponential-type modified Hamming distance. Thus, the similarity measure $S(A,\ B)$ between A and B given in equation(23) is constructed based on the Hamming distance and an exponential function.

Lemma 3.6. *Let* $f(\xi) = 1 - \frac{1-\exp(-\xi)}{1-\exp(-n)}$, *then*

$$\max_{\xi \in [0,\, n]} f(\xi) = f(0) = 1$$

and

$$\min_{\xi \in [0,\, n]} f(\xi) = f(n) = 0.$$

Proof. Since $f'(\xi) = -\frac{\exp(-\xi)}{1-\exp(-n)} < 0,\ \forall\, \xi \in [0,\ n]$, then $f(\xi)$ is decreasing in $[0,\ n]$. □

Theorem 3.7. *The mapping $S(A,\ B)$, defined by equation(23), is a similarity measure on $IVIFS(X)$.*

Proof. In order for equation(23) to be qualified as a sensible measure of IVIFSs, it must satisfy the conditions (S1)-(S4) in Definition 2.5.

(S1). Let $A,\ B \in IVIFS(X)$ and

$$\xi = \frac{1}{4}\sum_{i=1}^{n}\begin{pmatrix} \left|\sqrt{\mu_A^-(x_i)} - \sqrt{\mu_B^-(x_i)}\right| \\ +\left|\sqrt{\mu_A^+(x_i)} - \sqrt{\mu_B^+(x_i)}\right| \\ +\left|\sqrt{\nu_A^-(x_i)} - \sqrt{\nu_B^-(x_i)}\right| \\ +\left|\sqrt{\nu_A^+(x_i)} - \sqrt{\nu_B^+(x_i)}\right| \end{pmatrix}.$$

Since $0 \le \xi \le n$, then $S(A,\ B) = f(\xi)$, thus, by Lemma 3.6, we obtain $0 \le S(A,\ B) \le 1$.

(S2). Let $A,\ B \in IVIFS(X)$ and $A = B$, *i.e.*, $\left[\mu_A^-(x_i),\ \mu_A^+(x_i)\right] = \left[\nu_A^-(x_i),\ \nu_A^+(x_i)\right]$, then $S(A,\ B) = 1$. Suppose that $S(A,\ B) = 1$, then, from equation(23), we obtain that

$$1 - \frac{1-\exp\left[-\frac{1}{4}\sum_{i=1}^{n}\begin{pmatrix} \left|\sqrt{\mu_A^-(x_i)} - \sqrt{\mu_B^-(x_i)}\right| \\ +\left|\sqrt{\mu_A^+(x_i)} - \sqrt{\mu_B^+(x_i)}\right| \\ +\left|\sqrt{\nu_A^-(x_i)} - \sqrt{\nu_B^-(x_i)}\right| \\ +\left|\sqrt{\nu_A^+(x_i)} - \sqrt{\nu_B^+(x_i)}\right| \end{pmatrix}\right]}{1-\exp(-n)} = 1.$$

It follows that

$$\begin{pmatrix} \left|\sqrt{\mu_A^-(x_i)} - \sqrt{\mu_B^-(x_i)}\right| \\ +\left|\sqrt{\mu_A^+(x_i)} - \sqrt{\mu_B^+(x_i)}\right| \\ +\left|\sqrt{\nu_A^-(x_i)} - \sqrt{\nu_B^-(x_i)}\right| \\ +\left|\sqrt{\nu_A^+(x_i)} - \sqrt{\nu_B^+(x_i)}\right| \end{pmatrix} = 0, \ \forall\, x_i \in X.$$

Thus, $A = B$. It implies that $S(A,\, B)$ satisfies (S2).
(S3). It is obvious that $S(A,\, B) = S(B,\, A)$.
(S4). Since $A \subseteq B \subseteq C$, therefore, we have

$$\mu_A^-(x_i) \le \mu_B^-(x_i) \le \mu_C^-(x_i),$$

$$\mu_A^+(x_i) \le \mu_B^+(x_i) \le \mu_C^+(x_i),$$

$$\nu_A^-(x_i) \ge \nu_B^-(x_i) \ge \nu_C^-(x_i)$$

and

$$\nu_A^+(x_i) \ge \nu_B^+(x_i) \ge \nu_C^+(x_i), \forall\, x_i \in X.$$

Let

$$\xi_1 = \frac{1}{4}\sum_{i=1}^{n} \begin{pmatrix} \left|\sqrt{\mu_A^-(x_i)} - \sqrt{\mu_B^-(x_i)}\right| \\ +\left|\sqrt{\mu_A^+(x_i)} - \sqrt{\mu_B^+(x_i)}\right| \\ +\left|\sqrt{\nu_A^-(x_i)} - \sqrt{\nu_B^-(x_i)}\right| \\ +\left|\sqrt{\nu_A^+(x_i)} - \sqrt{\nu_B^+(x_i)}\right| \end{pmatrix}$$

$$\le \xi_2 = \frac{1}{4}\sum_{i=1}^{n} \begin{pmatrix} \left|\sqrt{\mu_A^-(x_i)} - \sqrt{\mu_C^-(x_i)}\right| \\ +\left|\sqrt{\mu_A^+(x_i)} - \sqrt{\mu_C^+(x_i)}\right| \\ +\left|\sqrt{\nu_A^-(x_i)} - \sqrt{\nu_C^-(x_i)}\right| \\ +\left|\sqrt{\nu_A^+(x_i)} - \sqrt{\nu_C^+(x_i)}\right| \end{pmatrix}.$$

Now, by using Lemma 3.6, we obtain that $S(A,\, B) = f(\xi_1) \ge f(\xi_2) = S(A,\, C)$. Similarly, $S(B,\, C) \ge S(A,\, C)$. Thus, $S(A,\, B)$ satisfies (S4). □

Let $X = \{x_1, x_2, ..., x_n\}$ be a finite universe of discourse and $A,\, B \in IVIFS(X)$. Now, compare the proposed similarity measure with some existing similarity measures. First, we recall the following similarity measures:
Xu [59]:

$$S_{X_1}(A,\,B) = 1 - \left[\frac{1}{4n}\sum_{i=1}^{n}\left(\begin{array}{l}\left|\mu_A^-(x_i)-\mu_B^-(x_i)\right|^p\\+\left|\mu_A^+(x_i)-\mu_B^+(x_i)\right|^p\\+\left|\nu_A^-(x_i)-\nu_B^-(x_i)\right|^p\\+\left|\nu_A^+(x_i)-\nu_B^+(x_i)\right|^p\end{array}\right)\right]^{1/p},\ p>0, \tag{25}$$

$$S_{X_2}(A,\,B) = 1 - \left[\frac{1}{n}\sum_{i=1}^{n}\max\left(\begin{array}{l}\left|\mu_A^-(x_i)-\mu_B^-(x_i)\right|^p,\\\left|\mu_A^+(x_i)-\mu_B^+(x_i)\right|^p,\\\left|\nu_A^-(x_i)-\nu_B^-(x_i)\right|^p,\\\left|\nu_A^+(x_i)-\nu_B^+(x_i)\right|^p\end{array}\right)\right]^{1/p},\ p>0. \tag{26}$$

If $p \to +\infty$ in equation(25) and $p = 1$ in equation(26), then these are reduced to the following formulae, respectively:

$$S_{X_1}^{\infty}(A,\,B) = 1 - \left[\max_i\left(\begin{array}{l}\left|\mu_A^-(x_i)-\mu_B^-(x_i)\right|,\\\left|\mu_A^+(x_i)-\mu_B^+(x_i)\right|,\\\left|\nu_A^-(x_i)-\nu_B^-(x_i)\right|,\\\left|\nu_A^+(x_i)-\nu_B^+(x_i)\right|\end{array}\right)\right], \tag{27}$$

$$S_{X_2}^{1}(A,\,B) = 1 - \left[\frac{1}{n}\sum_{i=1}^{n}\max\left(\begin{array}{l}\left|\mu_A^-(x_i)-\mu_B^-(x_i)\right|,\\\left|\mu_A^+(x_i)-\mu_B^+(x_i)\right|,\\\left|\nu_A^-(x_i)-\nu_B^-(x_i)\right|,\\\left|\nu_A^+(x_i)-\nu_B^+(x_i)\right|\end{array}\right)\right]. \tag{28}$$

Wei et al. [55]:

$$S_w(A,\,B) = \frac{1}{n}\sum_{i=1}^{n}\frac{2-\min\left\{\mu_i^-,\,\nu_i^-\right\}-\min\left\{\mu_i^+,\,\nu_i^+\right\}}{2-\max\left\{\mu_i^-,\,\nu_i^-\right\}-\max\left\{\mu_i^+,\,\nu_i^+\right\}}, \tag{29}$$

where $\mu_i^- = \left|\mu_A^-(x_i)-\mu_B^-(x_i)\right|$, $\nu_i^- = \left|\nu_A^-(x_i)-\nu_B^-(x_i)\right|$, $\mu_i^+ = \left|\mu_A^+(x_i)-\mu_B^+(x_i)\right|$ and $\nu_i^+ = \left|\nu_A^+(x_i)-\nu_B^+(x_i)\right|$.

Example 3.8. Let

$$A = \{\langle x_i,\,[0.2,\,0.3],\,[0.4,\,0.6]\rangle\,:\,x_i\in X\},$$
$$B = \{\langle x_i,\,[0.3,\,0.4],\,[0.3,\,0.5]\rangle : x_i\in X\}$$

and

$$C = \{\langle x_i,\,[0.3,\,0.4],\,[0.4,\,0.6]\rangle\,:\,x_i\in X\}$$

be three IVIFSs.

Comparing A and C for each $x_i \in X$, A and C have the equal degree of non-membership but differ in degree of membership, while comparing A and B for each $x_i \in X$, A and B have different degree of membership and non-membership. Hence, it can be concluded that A is more similar to C than to B.

Now, evaluating the similarity measures equation(23) and equation(29), we have $S(A,\,B) = 0.7401$, $S_w(A,\,B) = 0.8182$ and $S(A,\,C) = 0.8668$, $S_w(A,\,C) = 0.9091$ which indicates that IVIFS A is more similar to IVIFS C than IVIFS B and thus, it is consistent with our intuition.

However, if we apply the formulae equation(27) and equation(28) to calculate the similarity measures, then $S^{\infty}_{X_1}(A,\,B) = S^{\infty}_{X_1}(A,\,C) = S^{1}_{X_2}(A,\,B) = S^{1}_{X_2}(A,\,C) = 0.9$, which is not reasonable. Therefore, the similarity measures equation(23) and equation(29) are demonstrated to be more reasonable than $S^{\infty}_{X_1}(A,\,B),\ S^{\infty}_{X_1}(A,\,C)$, $S^{1}_{X_2}(A,\,B)$ and $S^{1}_{X_2}(A,\,C)$ in some cases.

4. Shapley Weighted Similarity Measure and Its Application

In this section, Shapley weighted similarity measure based on above mentioned similarity measure equation(23) is developed to pact with the correlated elements in a set and applied in the field of pattern recognition.

4.1. Shapley Weighted Similarity Measure. Let X be a set of correlated elements and the fuzzy measure of each combination is given in the power set $P(X)$ of X. Then, Shapley-weighted similarity measure for IVIFSs A and B is defined as follows:

$$S^S(A,\,B) = 1 - \frac{1-\exp\left[-\frac{1}{4}\sum_{i=1}^{n}\phi_i(g,\,X)\begin{pmatrix} \left|\sqrt{\mu_A^-(x_i)} - \sqrt{\mu_B^-(x_i)}\right| \\ +\left|\sqrt{\mu_A^+(x_i)} - \sqrt{\mu_B^+(x_i)}\right| \\ +\left|\sqrt{\nu_A^-(x_i)} - \sqrt{\nu_B^-(x_i)}\right| \\ +\left|\sqrt{\nu_A^+(x_i)} - \sqrt{\nu_B^+(x_i)}\right| \end{pmatrix}\right]}{1-\exp(-n)}. \tag{30}$$

where $\phi_j(g,\,X)$ denotes the Shapley value of the element $j\,(j \in X)$ with respect to the fuzzy measure g.

The corresponding weighted similarity measure is obtained in case of no interaction between elements of a set X.

4.2. Model for Optimal Measures. By taking entropy measure into deliberation, if the information about the weights of criteria is partially known or completely unknown, then the sum of the entropy measure of the criteria $V_j(j = 1,\,2,\,3,...,n)$ is given as $\sum_{i=1}^{m} E(\xi_{ij})$, where ξ_{ij} is the interval-valued intuitionistic fuzzy value of the alternative U_i with respect to the criterion V_j. In the theory of entropy measure, if the entropy values of a criteria are small across the alternatives, then it can afford valuable information to the decision makers. Therefore, the criterion should be assigned a bigger weight, otherwise, these criterion will be refereed as irrelevant by most of the decision makers. Furthermore, the optimal fuzzy measure makes superior inclusive value for each preferable alternative.

If the information about the weight of criteria is completely unknown, then the following linear programming model is established for the optimal fuzzy measure on criteria set V :

$$\begin{aligned} &\min \sum_{j=1}^{n}\sum_{i=1}^{m} E(\xi_{ij})\,\phi_j(g,\,X) \\ &s.\,t. \begin{cases} g(\varphi) = 0,\ g(X) = 1, \\ g(K) \le g(L),\ \forall\, K,\, L \subseteq X,\ K \subseteq L, \end{cases} \end{aligned} \tag{31}$$

where $\phi_j(g,\,X)$ is the Shapley value of the criteria $V_j(j = 1,\,2,\,...,n)$.

If the information about the weight of criteria is partially known, then the following linear programming model is constructed for the optimal fuzzy measure on the criteria set V :

$$\min \sum_{j=1}^{n} \sum_{i=1}^{m} E(\xi_{ij})\, \phi_j(g,\, X)$$
$$s.t. \begin{cases} g(\varphi) = 0,\ g(X) = 1, \\ g(V_j) \in S_j,\ j = 1, 2, ..., n, \\ g(K) \leq g(L),\ \forall\, K,\, L \subseteq X,\ K \subseteq L, \end{cases} \tag{32}$$

where $\phi_j(g,\, X)$ is the Shapley value of the criteria set $V_j (j = 1,\, 2,\, ...,n)$ and $S_j = \left[s_j^+,\, s_j^-\right]$ is its range set.

4.3. **Application of Shapley-Weighted Similarity Measures in Pattern Recognition.** If the fuzzy measure of each combination in the criteria set is given, then the following decision procedure is involved to the patterns under interval-valued intuitionistic fuzzy environment:

Step 1: Let $U = \{U_1,\, U_2, ..., U_n\}$ be the set of patterns and $V = \{V_1,\, V_2, ..., V_n\}$ be the set of criteria determined by the decision makers. The evaluation value of each patterns U_i is determined with respect to the criteria V_j and it is given as an interval-valued intuitionistic fuzzy value.

In addition, assume that there is a sample θ denoted by an interval-valued intuitionistic fuzzy set that has to be identified.

Step 2: Utilize the model equation(31)[or equation(32)] with respect to the entropy measure E to compute the optimal fuzzy measures of all the combinations in criteria set V.

Step 3: Calculate the Shapley value of each criterion by using equation(1).

Step 4: Compute the Shapley-weighted similarity measure between $U_i(i = 1, 2, ..., m)$ and θ using equation(30).

Step 5: Select the best one.

Example 4.1. Suppose that there are four kinds of minerals $U = \{U_1,\, U_2,\, U_3,\, U_4\}$ and a recognized sample θ, which are represented by IVIF values in the criteria set $V = \{V_1,\, V_2,\, V_3\}$.

The importance of criteria is given by $[0.35,\, 0.5]$, $[0.25,\, 0.55]$ and $[0.4,\, 0.65]$. Now, the foremost purpose is to determine that the recognized sample belongs to which kind of minerals. Therefore, the following steps are involved to the patterns under interval-valued intuitionistic fuzzy environment.

Step 1: The evaluation value of each pattern $U_i(i = 1,\, 2, ..., m)$ is listed with respect to the criteria set $V_j(j = 1, 2, ..., n)$, which is given as follows:

$$U_1 = \left\{ \begin{array}{c} \langle V_1,\, [0.2,\, 0.4]\,, [0.4,\, 0.5]\rangle\,, \\ \langle V_2,\, [0.2,\, 0.3]\,, [0.4,\, 0.6]\rangle\,, \\ \langle V_3,\, [0.6,\, 0.7]\,, [0.2,\, 0.3]\rangle \end{array} \right\},$$

$$U_2 = \left\{ \begin{array}{c} \langle V_1,\, [0.2,\, 0.3]\,, [0.4,\, 0.6]\rangle\,, \\ \langle V_2,\, [0.3,\, 0.4]\,, [0.3,\, 0.5]\rangle\,, \\ \langle V_3,\, [0.3,\, 0.4]\,, [0.4,\, 0.6]\rangle \end{array} \right\},$$

$$U_3 = \left\{ \begin{array}{c} \langle V_1,\, [0.1,\, 0.4]\,, [0.5,\, 0.6]\rangle\,, \\ \langle V_2,\, [0.2,\, 0.4]\,, [0.4,\, 0.5]\rangle\,, \\ \langle V_3,\, [0.2,\, 0.3]\,, [0.4,\, 0.6]\rangle \end{array} \right\},$$

	V_1	V_2	V_3
$E(U_1)$	0.9785	0.9403	0.8463
$E(U_2)$	0.9403	0.9977	0.9785
$E(U_3)$	0.9138	0.9785	0.9403
$E(U_4)$	0.9617	0.9785	0.9138

TABLE 3. Entropy Values with Respect to $'E'$

$$U_4 = \left\{ \begin{array}{c} \langle V_1,\, [0.2,\, 0.4]\,, [0.4,\, 0.6] \rangle\,, \\ \langle V_2,\, [0.3,\, 0.4]\,, [0.4,\, 0.6] \rangle\,, \\ \langle V_3,\, [0.4,\, 0.7]\,, [0.2,\, 0.3] \rangle \end{array} \right\}.$$

A sample θ is given as follows:

$$\theta = \left\{ \begin{array}{c} \langle V_1,\, [0.2,\, 0.3]\,, [0.4,\, 0.5] \rangle\,, \\ \langle V_2,\, [0.2,\, 0.3]\,, [0.3,\, 0.5] \rangle\,, \\ \langle V_3,\, [0.3,\, 0.4]\,, [0.2,\, 0.5] \rangle \end{array} \right\}.$$

Step 2: In this step, compute the fuzzy measures of each combinations in the criteria set V. For this, using equation(5) to calculate the entropy measure of the pattern $U_i (i = 1, 2, 3, 4)$ with respect to the criterion V_j $(j = 1,\, 2,\, 3)$. From Table 3, the linear programming model equation(32) is obtained as follows:

$$\begin{array}{l} Min\,[-0.0067\,\{g(V_1) - g(V_2,\, V_3)\} \\ \quad +0.0520\,\{g(V_2) - g(V_1,\, V_3)\} \\ \quad -0.0453\,\{g(V_3) - g(V_1,\, V_2)\} + 4.7695] \end{array}$$

such that

$$\left\{ \begin{array}{l} g(V_1,\, V_2) \leq 1,\, g(V_1,\, V_3) \leq 1,\, g(V_2,\, V_3) \leq 1, \\ g(V_1) \leq g(V_1,\, V_2),\, g(V_2) \leq g(V_1,\, V_2), \\ g(V_1) \leq g(V_1,\, V_3),\, g(V_3) \leq g(V_1,\, V_3) \\ g(V_2) \leq g(V_2,\, V_3),\, g(V_3) \leq g(V_2,\, V_3), \\ g(V_1) \in [0.35,\, 0.5]\,,\, g(V_2) \in [0.25,\, 0.55]\,,\, g(V_3) \in [0.4,\, 0.65] \end{array} \right. . \tag{33}$$

Solving equation(33) by using MATHEMATICA, the following fuzzy measure on criteria set V is obtained:

$$g(V_1) = 0.35 = g(V_1, V_2),$$

$$g(V_2) = 0.25, g(V_3) = 0.65 = g(V_2,\, V_3),$$

$$g(V_1,\, V_3) = g(V_1,\, V_2,\, V_3) = 1.$$

Step 3: The calculated Shapley values are

$$\phi^E_{V_1}(g,\, X) = 0.3084,\, \phi^E_{V_2}(g,\, X) = 0.0833,\, \phi^E_{V_3}(g,\, X) = 0.6083.$$

Step 4: By using equation(30), the calculated Shapley-weighted similarity measure between $U_i (i = 1,\, 2,\, 3,\, 4)$ and θ is given as follows:

$$S^S(U_1,\, \theta) = 0.8990,\; S^S(U_2,\, \theta) = 0.9458,$$

$$S^S(U_3,\, \theta) = 0.9192,\; S^S(U_4,\, \theta) = 0.9061.$$

Step 5: From step 4, we concluded that the sample θ belongs to second kind of minerals.

To show the effectiveness of the proposed Shapley weighted similarity measure, let us remind some existing similarity measures based on Shapley value.
Xu's measures [60]:

$$S_{X1}^{S}(A,\,B) = 1 - \frac{1}{4}\sum_{i=1}^{n}\phi_i(g,\,X)\begin{pmatrix} \left|\mu_A^-(x_i) - \mu_B^-(x_i)\right| \\ + \left|\mu_A^+(x_i) - \mu_B^+(x_i)\right| \\ + \left|\nu_A^-(x_i) - \nu_B^-(x_i)\right| \\ + \left|\nu_A^+(x_i) - \nu_B^+(x_i)\right| \end{pmatrix}. \tag{34}$$

$$S_{X2}^{S}(A,\,B) = 1 - \left[\frac{1}{4}\sum_{i=1}^{n}\phi_i(g,\,X)\begin{pmatrix} (\mu_A^-(x_i) - \mu_B^-(x_i))^2 \\ +(\mu_A^+(x_i) - \mu_B^+(x_i))^2 \\ +(\nu_A^-(x_i) - \nu_B^-(x_i))^2 \\ +(\nu_A^+(x_i) - \nu_B^+(x_i))^2 \end{pmatrix}\right]^{1/2}. \tag{35}$$

$$S_{X3}^{S}(A,\,B) = 1 - \sum_{i=1}^{n}\phi_i(g,\,X)\left(\max\begin{Bmatrix} \left|\mu_A^-(x_i) - \mu_B^-(x_i)\right|, \\ \left|\mu_A^+(x_i) - \mu_B^+(x_i)\right|, \\ \left|\nu_A^-(x_i) - \nu_B^-(x_i)\right|, \\ \left|\nu_A^+(x_i) - \nu_B^+(x_i)\right| \end{Bmatrix}\right). \tag{36}$$

$$S_{X4}^{S}(A,\,B) = 1 - \left[\sum_{i=1}^{n}\phi_i(g,\,X)\left(\max\begin{Bmatrix} (\mu_A^-(x_i) - \mu_B^-(x_i))^2, \\ (\mu_A^+(x_i) - \mu_B^+(x_i))^2, \\ (\nu_A^-(x_i) - \nu_B^-(x_i))^2, \\ (\nu_A^+(x_i) - \nu_B^+(x_i))^2 \end{Bmatrix}\right)\right]^2. \tag{37}$$

Ye's measure [62]:

$$S_Y^S(A,\,B) = \frac{1}{n}\sum_{i=1}^{n}\phi_i(g,\,X)\left(\frac{2\,(\mu_A(x_i)\,\mu_B(x_i) + \nu_A(x_i)\,\nu_B(x_i))}{(\mu_A(x_i))^2 + (\mu_B(x_i))^2 + (\nu_A(x_i))^2 + (\nu_B(x_i))^2}\right). \tag{38}$$

Meng and Chen's measures [26]:

$$S_{MC1}^S(A,B) = \frac{1}{n}\sum_{i=1}^{n}\phi_i(g,X)\left(\frac{4 - \min(\mu_{AB}^-(x_i),\nu_{AB}^-(x_i)) - \min(\mu_{AB}^+(x_i),\nu_{AB}^+(x_i))}{4 + \max(\mu_{AB}^-(x_i),\nu_{AB}^-(x_i)) + \max(\mu_{AB}^+(x_i),\nu_{AB}^+(x_i))}\right). \tag{39}$$

$$S_{MC2}^S(A,B) = \frac{1}{n}\sum_{i=1}^{n}\phi_i(g,X)\left(\frac{2 - \max(\mu_{AB}^-(x_i),\nu_{AB}^-(x_i)) - \max(\mu_{AB}^+(x_i),\nu_{AB}^+(x_i))}{2 + \min(\mu_{AB}^-(x_i),\nu_{AB}^-(x_i)) + \min(\mu_{AB}^+(x_i),\nu_{AB}^+(x_i))}\right). \tag{40}$$

$$S_{MC3}^S(A,\,B) = \frac{1}{n}\sum_{i=1}^{n}\phi_i(g,\,X)\left(\frac{4 - \underline{\alpha}_i^2 - \bar{\alpha}_i^2}{4 + \underline{\alpha}_i + \bar{\alpha}_i}\right). \tag{41}$$

$$S_{MC4}^S(A,\,B) = \frac{1}{n}\sum_{i=1}^{n}\phi_i(g,\,X)\left(\frac{2 - \underline{\alpha}_i - \bar{\alpha}_i}{2 + \underline{\alpha}_i^2 + \bar{\alpha}_i^2}\right). \tag{42}$$

Here,

$$\underline{\alpha}_i = \mu_{AB}^-(x_i) \wedge \nu_{AB}^-(x_i) \wedge \mu_{AB}^+(x_i) \wedge \nu_{AB}^+(x_i),$$
$$\bar{\alpha}_i = \mu_{AB}^-(x_i) \vee \nu_{AB}^-(x_i) \vee \mu_{AB}^+(x_i) \vee \nu_{AB}^+(x_i),$$
$$\mu_{AB}^-(x_i) = \left|\mu_A^-(x_i) - \mu_B^-(x_i)\right|,$$
$$\mu_{AB}^+(x_i) = \left|\mu_A^+(x_i) - \mu_B^+(x_i)\right|,$$
$$\nu_{AB}^-(x_i) = \left|\nu_A^-(x_i) - \nu_B^-(x_i)\right|$$

and

$$\nu_{AB}^+(x_i) = \left|\nu_A^+(x_i) - \nu_B^+(x_i)\right|.$$

Shapley weighted similarity measure	Ranking order
Xu's measures [60]	$S_{X1}(U_2,\theta) > S_{X1}(U_3,\theta) > S_{X1}(U_4,\theta) > S_{X1}(U_1,\theta)$ $S_{X2}(U_2,\theta) > S_{X2}(U_3,\theta) > S_{X2}(U_4,\theta) > S_{X2}(U_1,\theta)$ $S_{X3}(U_2,\theta) = S_{X3}(U_3,\theta) > S_{X3}(U_1,\theta) = S_{X3}(U_4,\theta)$ $S_{X4}(U_2,\theta) = S_{X4}(U_3,\theta) > S_{X4}(U_1,\theta) = S_{X4}(U_4,\theta)$
Ye's measure [62]	$S_Y(U_2,\theta) \succ S_Y(U_3,\theta) \succ S_Y(U_4,\theta) \succ S_Y(U_1,\theta)$
Meng and Chen measures [26]	$S_{MC1}(U_2,\theta) > S_{MC1}(U_3,\theta) > S_{MC1}(U_4,\theta) > S_{MC1}(U_1,\theta)$ $S_{MC2}(U_2,\theta) > S_{MC2}(U_3,\theta) > S_{MC2}(U_4,\theta) > S_{MC2}(U_1,\theta)$ $S_{MC3}(U_2,\theta) > S_{MC3}(U_3,\theta) = S_{MC3}(U_4,\theta) > S_{MC3}(U_1,\theta)$ $S_{MC4}(U_2,\theta) > S_{MC4}(U_1,\theta) > S_{MC4}(U_4,\theta) > S_{MC4}(U_3,\theta)$
Proposed measure	$S(U_2,\theta) > S(U_3,\theta) > S(U_4,\theta) > S(U_1,\theta)$

TABLE 4. Ranking Order for Different Shapley Weighted Similarity Measures

In this example, the ranking results with respect to existing Shapley weighted similarity measures equation(34)-equation(42) and proposed measure are shown in Table 4.

From Table 4, we observe that the ranking values with respect to various Shapley similarity measures are different but all the ranking results show that the sample θ belongs to the second kind of minerals and hence, this example reveals the effectiveness of the proposed Shapley similarity measure.

5. Interval-Valued Intuitionistic Fuzzy VIKOR (IVIF-VIKOR) Technique for Multiple Criteria Decision Making (MCDM) Problems Based on Shapley Value

The fundamental objective of VIKOR technique is to appraise the compromise solution. Actually, the compromise solution is a meticulous efficient solution, which is closest to the optimal solution derived from particular measure. The relation between efficient and compromise solution are depicted in Figure 1. From Figure 1, Θ^+ denotes the optimal solution and Θ denotes the compromise solution, which is a special point in the curve of efficient solution and is nearest to the optimal point Θ^+.

Now, an extended VIKOR technique for group decision making with interval-valued intuitionistic fuzzy numbers is anticipated in the following steps. Further, an example of investment selection problem under incomplete and uncertain information is validated to determine the efficiency and reliability of the proposed technique:

Step 1: Determine the most important criteria. A set of alternatives $U = \{U_1, U_2, ..., U_m\}$ and a set of criteria $V = \{V_1, V_2, ..., V_n\}$ determined by the decision makers. Because of the deficient and uncertain information about the alternatives, decision maker assigns an interval-valued intuitionistic fuzzy number to estimate his/her judgment on the alternatives U_i with respect to criteria V_j. For a better decision, we need to specify the importance of the decision of each decision maker. Therefore, we determine the weight of each criterion.

Step 2: Determination of Shapley value of the criteria. For interdependent and interactive characteristics among elements, the weight vector of the criterion is computed in terms of Shapley values.

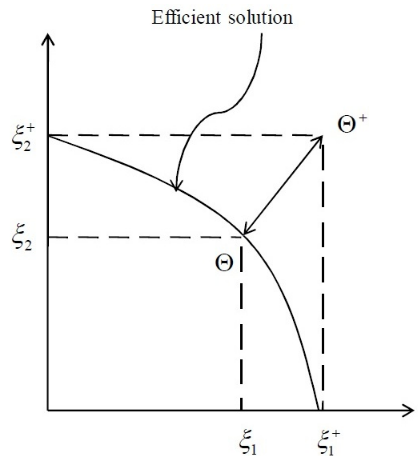

Figure 1. Efficient and Compromise Solutions

Step 3: Determine the peak and dip value. The fundamental concept of VIKOR technique is to find the peak and dip value of the alternative $U_i\,(i = 1, 2, ..., m)$ which can define here in terms of the IVIF positive-ideal solution (PIS) and the IVIF negative-ideal solution (NIS). The PIS is the solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the NIS is the solution that minimizes the benefit criteria and maximizes the cost criteria. Now, the positive-ideal solution U^+ and the negative-ideal solution U^- can be defined as follows:

$$U^+ = \left\{\left\langle\left[\mu_{1+}^- \; \mu_{1+}^+\right],\; \left[\nu_{1+}^- \, \nu_{1+}^+\right]\right\rangle,\; ...,\; \left\langle\left[\mu_{n+}^- \; \mu_{n+}^+\right],\; \left[\nu_{n+}^- \, \nu_{n+}^+\right]\right\rangle\right\} \tag{43}$$

$$U^- = \left\{\left\langle\left[\mu_{1-}^- \; \mu_{1-}^+\right],\; \left[\nu_{1-}^- \, \nu_{1-}^+\right]\right\rangle,\; ...,\; \left\langle\left[\mu_{n-}^- \; \mu_{n-}^+\right],\; \left[\nu_{n-}^- \, \nu_{n-}^+\right]\right\rangle\right\} \tag{44}$$

where for each $j = 1, 2, ..., n$.

$$\left\langle\left[\mu_{j+}^- \; \mu_{j+}^+\right],\; \left[\nu_{j+}^- \, \nu_{j+}^+\right]\right\rangle = \left\langle\left[\max_i \mu_{ij}^-,\; \max_i \mu_{ij}^+\right],\; \left[\min_i \nu_{ij}^-,\; \min_i \nu_{ij}^+\right]\right\rangle,$$

$$\left\langle\left[\mu_{j-}^- \; \mu_{j-}^+\right],\; \left[\nu_{j-}^- \, \nu_{j-}^+\right]\right\rangle = \left\langle\left[\min_i \mu_{ij}^-,\; \min_i \mu_{ij}^+\right],\; \left[\max_i \nu_{ij}^-,\; \max_i \nu_{ij}^+\right]\right\rangle.$$

Step 4: Calculate the values of G_i, I_i and Υ_i for different values of alternatives $U_i\,(i = 1, 2, ..., m)$ and sorting them in decreasing order. The values of group utility, individual regret and compromise measure are essential in selecting the best option among a set of alternatives. By using equations (2),(3),(4) and (24), we get the calculated values of G_i, I_i and Υ_i (without loss of generality, $\tau = 0.5$). Now, sorting the values of G_i, I_i and Υ_i in decreasing order and rank the choices according to the computed values of Υ_i.

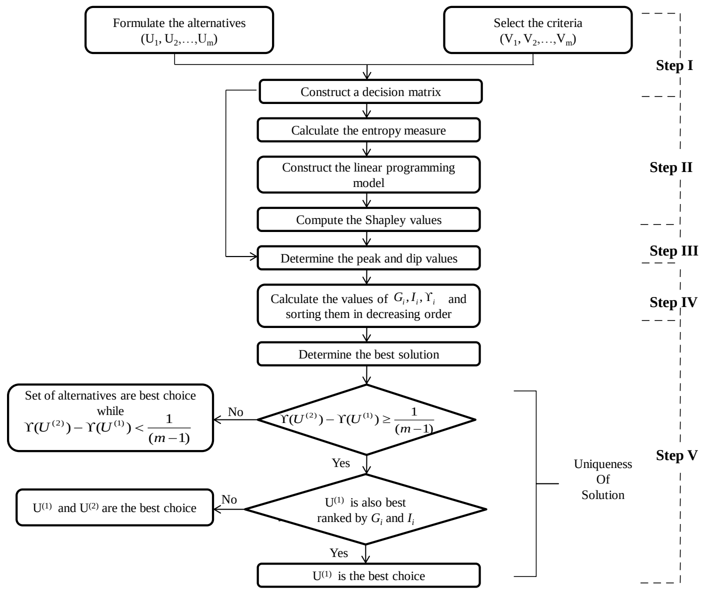

FIGURE 2. Flowchart of Shapley Value Based VIKOR Technique for MCDM Problems

Step 5: Determine the compromise solution. In this step, the best alternative are determined regarding the smallest value of Υ_i and the worst alternative as the greatest value of Υ_i. The concluded alternative is said to be best if the conditions (C1) and (C2) are satisfied. If the condition (C1) is not satisfied, then all the alternatives $U^{(i)}$ $(i = 1, 2, ..., m)$ are the compromise solutions. If the condition (C2) is not satisfied, then the alternatives U^1 and U^2 are compromise solutions. Flow chart of extended VIKOR technique is depicted in Figure 2, which as

Example 5.1. Consider the decision-making problem discussed in [20]. There is an investment company which wants to invest a sum of money in the peak option. There is a panel with four possible alternatives to invest the money: (i) U_1 is a vehicle company; (ii) U_2 is a food company; (iii) U_3 is an arms company; (iv) U_4 is a computer company. The investment company must take a decision according to the following three criteria: (i) V_1 is risk analysis; (ii) V_2 is growth analysis; (iii) V_3 is environmental impact analysis.

Step 1: Determine the most important criteria.
Let $U = \{U_1, U_2, ..., U_4\}$ be a set of alternatives and $V = \{V_1, V_2, V_3\}$ be a set of criteria. The evaluation value of criteria on alternatives U_i is represented by the following IVIFS:

$$U_1 = \left\{ \begin{array}{c} \langle V_1, \, [0.4, 0.5], [0.3, 0.4] \rangle, \\ \langle V_2, \, [0.4, 0.6], [0.2, 0.4] \rangle, \\ \langle V_3, \, [0.1, 0.3], [0.5, 0.6] \rangle \end{array} \right\},$$

	V_1	V_2	V_3
$E(U_1)$	0.9904	0.9617	0.8825
$E(U_2)$	0.8463	0.9463	0.8051
$E(U_3)$	0.9904	0.9617	0.9403
$E(U_4)$	0.6508	0.8051	0.9617

Table 5. Entropy Values with Respect to $'E'$

$$U_2 = \left\{ \begin{array}{c} \langle V_1, [0.6, 0.7], [0.2, 0.3] \rangle, \\ \langle V_2, [0.6, 0.7], [0.2, 0.3] \rangle, \\ \langle V_3, [0.4, 0.8], [0.1, 0.2] \rangle \end{array} \right\},$$

$$U_3 = \left\{ \begin{array}{c} \langle V_1, [0.3, 0.6], [0.3, 0.4] \rangle, \\ \langle V_2, [0.5, 0.6], [0.3, 0.4] \rangle, \\ \langle V_3, [0.4, 0.5], [0.1, 0.3] \rangle \end{array} \right\},$$

$$U_4 = \left\{ \begin{array}{c} \langle V_1, [0.7, 0.8], [0.1, 0.2] \rangle, \\ \langle V_2, [0.6, 0.7], [0.1, 0.3] \rangle, \\ \langle V_3, [0.3, 0.4], [0.1, 0.2] \rangle \end{array} \right\}.$$

The importance of criteria is given by $[0.4,\, 0.6]$, $[0.3,\, 0.5]$ and $[0.6,\, 0.8]$.

Step 2: Determination of Shapley value of the criteria.
Using equation(5), the entropy measures of the patterns $U_i\,(i = 1, 2, ..., m)$ with respect to the criteria $V_j\,(j = 1, 2, ..., n)$ are listed in Table 5. From Table 5, the linear programming model equation(32) is constructed as follows:

$$\begin{array}{l} Min\,[-0.0348\,\{g(V_1) - g(V_2, V_3)\} \\ \quad +0.0137\,\{g(V_2) - g(V_1, V_3)\} \\ \quad +0.0211\,\{g(V_3) - g(V_1, V_2)\} + 3.5474] \end{array}$$

such that

$$\left\{ \begin{array}{l} g(V_1, V_2) \le 1,\, g(V_1, V_3) \le 1,\, g(V_2, V_3) \le 1, \\ g(V_1) \le g(V_1, V_2),\, g(V_2) \le g(V_1, V_2), \\ g(V_1) \le g(V_1, V_3),\, g(V_3) \le g(V_1, V_3) \\ g(V_2) \le g(V_2, V_3),\, g(V_3) \le g(V_2, V_3), \\ g(V_1) \in [0.4,\, 0.6],\, g(V_2) \in [0.3,\, 0.5],\, g(V_3) \in [0.6,\, 0.8] \end{array} \right. .$$

Solving Linear programming model by using MATHEMATICA, then the following fuzzy measure on criteria set V is obtained:

$$g(V_1) = 0.6 = g(V_2, V_3) = g(V_3), g(V_2) = 0.3, g(V_1, V_2) = 1 = g(V_1, V_3) = g(V_1, V_2, V_3).$$

The calculated Shapley values are

$$\phi^E_{V_1}(g, X) = 0.5167,\ \phi^E_{V_2}(g, X) = 0.1667,\ \phi^E_{V_3}(g, X) = 0.3166.$$

Thus, the Shapley values of all the decision attributes are obtained as

$$W = \left(\phi^{E_1}_{V_1}(g, X),\, \phi^{E_1}_{V_2}(g, X),\, \phi^{E_1}_{V_3}(g, X) \right)^T = (0.5167, 0.1667, 0.3166)^T.$$

Step 3: By using equations(43)-(44), IVIF peak and dip values are evaluated as in Table 6.

	IVIF Peak values	IVIF dip values
V_1	$[0.7, 0.8]$, $[0.1, 0.2]$	$[0.3,\ 0.6]$, $[0.3,\ 0.4]$
V_2	$[0.6,\ 0.7]$, $[0.1,\ 0.3]$	$[0.5,\ 0.6]$, $[0.3,\ 0.4]$
V_3	$[0.4,\ 0.8]$, $[0.1,\ 0.2]$	$[0.1,\ 0.3]$, $[0.5,\ 0.6]$

Table 6. Interval-valued Intuitionistic Fuzzy Peak and Dip Values

G_i	I_i	Υ_i
1.0000	0.5167	1.0000
0.2400	0.2067	0.2073
0.7626	0.5167	0.8683
0.0989	0.0989	0.0000

Table 7. Group Utility, Individual Regret and Compromise Measure

Step 4: Calculate the values of G_i, I_i and Υ_i for different values of alternatives $U_i\,(i = 1, 2, ..., m)$ and sorting them in decreasing order.
Using equations (2),(3),(4) and (24), we obtain the calculated values of G_i, I_i and Υ_i, which are given as in Table 7. Now, sorting the values of G_i, I_i and Υ_i in decreasing order, which are given as follows:

$$G_4 \succ G_2 \succ G_3 \succ G_1,$$

$$I_4 \succ I_2 \succ I_3 = I_1$$

and

$$\Upsilon_4 \succ \Upsilon_2 \succ \Upsilon_3 \succ \Upsilon_1.$$

Rank the alternatives according to the values of $\Upsilon_i\,(i = 1, 2, 3, 4)$. Minimum value of Υ_i represents the best alternative, therefore, the computer company U_4 is the best alternative.

Step 5: Determine the compromise solution.
The computer company U_4 is the optimal or compromise solution if the following conditions are satisfied:

(C1). Here, $\Upsilon(U^{(2)}) - \Upsilon(U^{(1)}) = 0.2073 < \frac{1}{(4-1)} = 0.3333$. Therefore, the condition (C1) is not satisfied.
(C2). The choice is also ranked according to the values of G_i and I_i. Here, $G_4 \succ G_2 \succ G_3 \succ G_1$ and $I_4 \succ I_2 \succ I_3 = I_1$.
If the condition (C1) is not fulfilled, then the maximum value of M is evaluated by the following relation:

$$\Upsilon(U^{(M)}) - \Upsilon(U^{(1)}) < \frac{1}{(m-1)}.$$

Thus, the alternatives U_2 and U_4 are compromise solutions. The ranking of the alternatives and compromise solution are given as in Table 8.

Measures	Ranking	Compromise Solution
$G_i\,(i = 1, 2, 3, 4)$	$U_4 \succ U_2 \succ U_3 \succ U_1$	U_4
$I_i\,(i = 1, 2, 3, 4)$	$U_4 \succ U_2 \succ U_3 = U_1$	U_4
$\Upsilon_i\,(i = 1, 2, 3, 4)$	$U_4 \succ U_2 \succ U_3 \succ U_1$	U_2, U_4

Table 8. Ranking and the Compromise Solution

Technique	Ranking	Best alternative(s)
Wei et al. technique [55]	$U_2 \succ U_4 \succ U_1 \succ U_3$	U_2
Ye technique [61]	$U_4 \succ U_2 \succ U_3 \succ U_1$	U_4
Proposed Shapley value based VIKOR technique	$U_4 \succ U_2 \succ U_3 \succ U_1$ (G) $U_4 \succ U_2 \succ U_3 = U_1$ (I) $U_4 \succ U_2 \succ U_3 \succ U_1$ (Υ)	U_2, U_4

Table 9. Comparison of Ranking Order for Different Techniques

5.1. **Comparison and Discussion.** To present a better investigation of the evaluation results, the ranking of the companies obtained from proposed technique is compared with other techniques in Table 9 and graphically depicted in Figure 3.

From Table 9, we can see that the ranking of the companies obtained from our proposed VIKOR technique is $U_4 \succ U_2 \succ U_3 \succ U_1$, which is similar to Ye [61] technique but different from Wei et al. [55] technique.

Thus, the obtained result from proposed technique indicates that the companies U_2 and U_4 are the best. Now, the main difference between proposed and Ye [61] techniques are given as follows:

(1) The proposed IVIF-VIKOR is developed for correlative MCDM problems with incomplete information about the weights of the criteria. In this technique, the weights of the criteria is evaluated in terms of Shapley values. While in [61], the optimal decision-making method is presented for MCDM problems in which there is an independent characteristics among the criteria and information about the weights of the criteria is already known.

(2) In the proposed technique, the ranking of the alternatives is determined by the compromise measure. While in [61], the ranking of the alternatives is concluded on the basis of weights of the alternatives.

(3) The highest ranked alternative by proposed technique is nearest to the optimal solution. While the highest ranked alternative by [61] is the best in terms of the ranking index but not always the nearest to the optimal solution.

In order to obtain the best alternative, the performances of four companies with respect to each criterion are computed and revealed in Figure 4. It can be seen from Figure 4, company 2 (food company) and company 4 (computer company) perform relatively better than the other two companies under most of the criteria, and are closer to IVIF peak values as compared to the other companies.

On the other hand, company 1 (vehicle company) performs relatively worse as in comparison to the other three companies under most of the criteria and is closer to IVIF dip values than remaining three companies. And hence, the company 2 and company 4 should be selected as the best alternative.

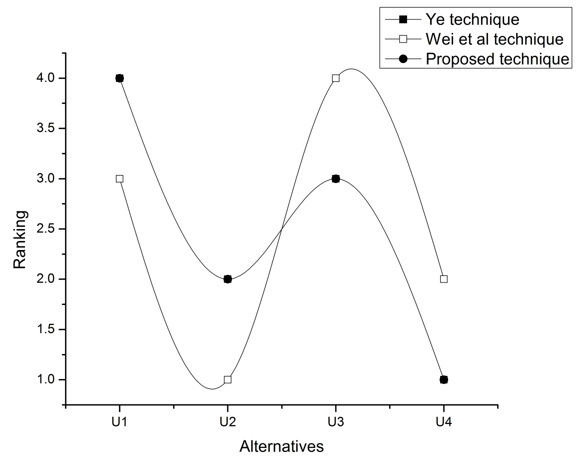

Figure 3. Comparison of Proposed Technique with Some Existing Techniques

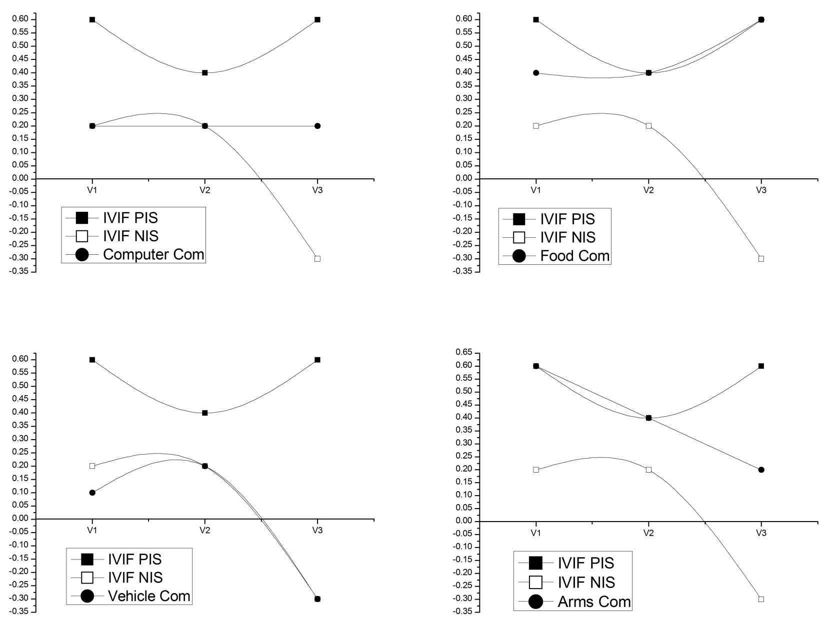

Figure 4. Comparison Between Each Company and IVIF Peak and IVIF Dip Values

6. Conclusions

In this communication, new entropy and similarity measures are developed for IVIFSs based on exponential function. Numerical results are presented to show the efficiency and reliability of the proposed entropy and similarity measures.

On the basis of proposed similarity measure, Shapley-weighted similarity measure is discussed for inter-dependent elements of a set. In order to deal with the situation where information about the weight of the criteria is incomplete, the linear programming model for optimal fuzzy measure based on entropy measure is developed which utilizes the Shapley values as their weight vector.

Further, an extended IVIF-VIKOR technique is established for a multi-criteria decision making problems with correlated characteristics among criteria and applied in the field of investment problem. In the proposed technique, Shapley values are utilized as the weights of the criteria. Later, the IVIF peak and dip values are anticipated. Thereafter, the ranking of the alternatives are estimated on the basis of compromise measure and selected the most desirable alternative satisfying the conditions of acceptable advantage and adequate inequality.

Finally, the comparison of the ranking index obtained by proposed and previously existing techniques are explained. The graphs of the investment problem depict that the proposed IVIF-VIKOR technique is in accordance with existing technique and the comparisons expose the validity of the proposed technique over others.

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SHAPLEY FUNCTION BASED INTERVAL-VALUED INTUITIONISTIC FUZZY VIKOR TECHNIQUE FOR CORRELATIVE MULTI-CRITERIA DECISION MAKING PROBLEMS

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تکنیک ویکر (VIKOR) فازی شهودی بازه – مقدار بر مبنای تابع شپلی (Shapley) برای مسائل تصمیم گیری چند معیاره وابسته

چکیده. مجموعه فازی شهودی بازه–مقدار برای واقعیت بخشیدن به عدم قطعیت تفکرات مبهم بشری گسترش داده شده است. در ارایه ارتباط ، واحد اندازه گیری جدید، واندازه های تشابه برای IVIFSs بر اساس تابع نمایی ارایه گردیده و با اندازه های موجود مقایسه شده اند . نتایج عددی آشکار می سازد که اندازه های اطلاع پیشنهادی با اندازه های موجود به وابستگی بالاتری دست می یابد، که کارایی و اعتبار آنها را اثبات می کند . برای رسیدگی به مشخصه های متقابل در میان عناصر یک مجموعه ، اندازه تشابه وزن دار شپلی بر اساس اندازه تشابه پیشنهادی برای IVIFSs از طریق تابع شپلی مورد بحث قرار گرفته است . بعد از آن ، برای اندازه فازی بهینه مدل برنامه نویسی خطی برای اطلاعات ناتمام وزن های محک آغاز شده ، و از اینرو ، بردار وزن بهینه براساس مقادیر شپلی بدست آمده است . بعلاوه ، برای مسایل تصمیم گیری چند معیاره وابسته به هم تحت محیط فازی شهودی بازه – مقدار تکنیک VIKOR مورد بررسی قرار گرفته است . بالاخره ، مثالی از مسئله سرمایه گذاری ارایه گردیده تا کاربرد تکنیک پیشنهادی تحت شرایط اطلاعات ناتمام و عدم اطمینان نشان داده شود .