ON SOMEWHAT FUZZY AUTOMATA CONTINUOUS FUNCTIONS IN FUZZY AUTOMATA TOPOLOGICAL SPACES

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Abstract. In this paper, the concepts of somewhat fuzzy automata continuous functions and somewhat fuzzy automata open functions in fuzzy automata topological spaces are introduced and some interesting properties of these functions are studied. In this connection, the concepts of fuzzy automata resolvable spaces and fuzzy automata irresolvable spaces are also introduced and their properties are studied.

1. Introduction

Zadeh [25] innovated the concept of a fuzzy set in 1965 and then it has invaded almost all branches of mathematics. The notion of an automaton was first fuzzified by Wee [24]. Later, the concepts of fuzzy subsystems and strong fuzzy subsystems of a fuzzy finite state machine (ffsm) were introduced and studied by Malik and Mordeson [15]. In [4, 18, 19], it is shown that certain topological and fuzzy topological concepts can be used in fuzzy automata theory to throw light on the structure of such fuzzy automata, particularly, to obtain certain results pertaining to their connectivity and separation properties. Zhihui, Ping and Yongming [14] discussed the relationships among several types of fuzzy automata. In [7, 9, 13, 11, 12, 22, 21], the researchers began to work on fuzzy automata with membership values in complete residuated lattice, lattice ordered monoid and some kind of lattices. Ignjatovic, Ciric and Simovic [10] studied the concepts of subsystems, reverse subsystems and double subsystems of a fuzzy automaton in terms of fuzzy relation inequalities and equations. Tiwari, Singh, Sharan and Yadav [23] introduced and studied the concept of bifuzzy core inducing a bifuzzy topology on the state-set of fuzzy automaton. In classical topology, the class of somewhat continuous functions was introduced and studied by Gentry and Hoyle [6]. Later, the concept of somewhat continuous functions in classical topology has been extended to fuzzy topological spaces. Somewhat fuzzy continuous functions and somewhat open functions in fuzzy topological spaces were introduced and studied by Thangaraj and Balasubramanian [20]. Hewitt [8] introduced the concepts of resolvability and irresolvability in topological spaces. In this paper, the concepts of somewhat fuzzy automata continuous functions and somewhat fuzzy automata open functions in fuzzy automata topological

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spaces are introduced and some interesting properties of these functions are studied. In this connection, the concepts of fuzzy automata resolvable spaces and fuzzy automata irresolvable spaces are also introduced and their properties are studied.

2. Preliminaries

Definition 2.1. [18] A fuzzy automaton is a triple $M = (Q, X, \delta)$, where Q is a set (of states of M), X is a monoid(the input monoid of M), whose identity shall be denoted as e, and δ is a fuzzy subset of $Q \times X \times Q$, *i.e.*, a map $\delta : Q \times X \times Q \to [0, 1]$, such that $\forall q, p \in Q$, $\forall x, y \in X$.
(i) $\delta(q, e, p) = 1$ or 0, according as $q = p$ or $q \neq p$,
(ii) $\delta(q, xy, p) = \vee\{\delta(q, x, r) \wedge \delta(r, x, p) : r \in Q\}$.

Notation. For any non-empty set of states Q, I^Q denotes the collection of all functions from Q into I, where I is the unit interval $[0, 1]$.

Definition 2.2. [16] $\lambda \in I^Q$ is called a fuzzy subsystem of (Q, X, δ) if
$\lambda(q) \geq \lambda(p) \wedge \delta(p, x, q)$, $\forall p, q \in Q$, $x \in X$.

Proposition 2.3. [4] *The function* $c : I^Q \to I^Q$ *defined as*
$c(\lambda)(q) = \bigvee \{ \bigvee \{ \lambda(p) \wedge \delta(p, x, q) : x \in X \} : p \in Q \}$, $\forall \lambda \in I^Q$, $\forall q \in Q$.
is a kuratowski saturated fuzzy closure operator on Q.

This proposition shows that c is a fuzzy closure operator on I^Q. Then c induces a fuzzy topology τ on Q. The fuzzy topology τ is called the fuzzy topology associated with the fuzzy automaton M.

Proposition 2.4. [4] $\lambda \in I^Q$ *is a fuzzy subsystem of* (Q, X, δ) *iff* $c(\lambda) = \lambda$. *(i.e., iff* λ *is closed with respect to the fuzzy topology induced by* c *on* Q*)*

Definition 2.5. [4] A fuzzy subset λ of Q is said to be a generating fuzzy set of M if $c(\lambda) = 1$.

Definition 2.6. [3] Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let f be a function from the fuzzy topological space (X, T) to the fuzzy topological space (Y, S). Let λ be a fuzzy set in (Y, S). The inverse image of λ under f written as $f^{-1}(\lambda)$ is the fuzzy set in (X, T) defined by $f^{-1}(\lambda)(x) = \lambda(f(x))$, for all $x \in X$. Also the image of λ in (X, T) under f written as $f(\lambda)$ is the fuzzy set in (Y, S) defined by

$$f(\lambda)(y) = \begin{cases} sup_{x \in f^{-1}(y)} \lambda(x), & \text{if } f^{-1}(y) \text{ is non-empty, for each } y \in Y. \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.7. [3] Let $f : (X, T) \longrightarrow (Y, S)$ be a mapping. For fuzzy sets λ and μ of (X, T) and (Y, S) respectively, the following statements hold.
(1) $ff^{-1}(\mu) \leq \mu$;
(2) $f^{-1}f(\lambda) \geq \lambda$;
(3) $f(1 - \lambda) \geq 1 - f(\lambda)$;
(4) $f^{-1}(1 - \mu) = 1 - f^{-1}(\mu)$;
(5) If f is injective, then $f^{-1}f(\lambda) = \lambda$;

(6) If f is surjective, then $ff^{-1}(\mu) = \mu$;
(7) If f is bijective, then $f(1 - \lambda) = 1 - f(\lambda)$.

Lemma 2.8. [1] *Let $f : (X, T) \longrightarrow (Y, S)$ be a mapping and $\{\lambda_\alpha\}$ be a family of fuzzy sets of Y. Then*
(a) $f^{-1}(\cup_\alpha \lambda_j) = \cup_\alpha f^{-1}(\lambda_j)$,
(b)$f^{-1}(\cap_\alpha \lambda_j) = \cap_\alpha f^{-1}(\lambda_j)$.

Lemma 2.9. [5] *Let $f : (X, T) \longrightarrow (Y, S)$ be a mapping and $\{A_j\}$, $j \in J$ be a family of fuzzy sets of X. Then*
(a) $f(\cup_{j\in J} A_j) = \cup_{j\in J} f(A_j)$,
(b)$f(\cap_{j\in J} A_j) \leq \cap_{j\in J} f(A_j)$.

Lemma 2.10. [1] *Let $g : X \longrightarrow X \times Y$ be the graph of a function $f : X \longrightarrow Y$ defined by $g(x) = (x, f(x))$. If λ is a fuzzy set of X and μ is a fuzzy set of Y, then $g^{-1}(\lambda \times \mu) = \lambda \wedge f^{-1}(\mu)$.*

Definition 2.11. [17] Two fuzzy sets μ and γ of X are said to be disjoint if they do not intersect at any point of X. That is, $\mu(x) + \gamma(x) \leq 1$, for all $x \in X$.

Definition 2.12. [20] A mapping $f : X \longrightarrow Y$ is somewhat fuzzy continuous if there exists a fuzzy open set $\mu \neq 0_X$ on X such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any fuzzy open set ν on Y.

It is clear that every fuzzy continuous mapping is a somewhat fuzzy continuous mapping. But the converse is not true in general.

Definition 2.13. [20] A mapping $f : X \longrightarrow Y$ is somewhat fuzzy open if there exists a fuzzy open set $\nu \neq 0_Y$ on Y such that $\nu \leq f(\mu) \neq 0_Y$ for any fuzzy open set μ on X.

Note that every fuzzy open mapping is a somewhat fuzzy open mapping but the converse is not true in general.

Definition 2.14. [2] The product $\lambda \times \mu$ of a fuzzy set λ of X and a fuzzy set μ of Y is a fuzzy set of $X \times Y$, defined by $(\lambda \times \mu)(x, y) = min(\lambda(x), \mu(y))$, for each $(x, y) \in X \times Y$.

Definition 2.15. [2] The product $f_1 \times f_2 : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ of mappings $f_1 : X_1 \longrightarrow Y_1$ and $f_2 : X_2 \longrightarrow Y_2$, is defined by $(f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2))$, for each $(x_1, x_2) \in X_1 \times X_2$.

3. On Somewhat Fuzzy Automata Continuous Functions

In this section, the concept of somewhat fuzzy automata continuous functions is introduced. Some interesting properties and characterizations are discussed with necessary examples.

Definition 3.1. Let $M = (Q, X, \delta)$ be a fuzzy automaton, where Q is a set (of states of M). For all $\lambda \in I^Q$ and $q \in Q$, $c(\lambda)(p) = \bigvee_{q\in Q} \{ \bigvee_{x\in X} \{ \lambda(q) \wedge \delta(q, x, p) \} \}$ is a fuzzy closure operator on Q. Let $\tau(Q) = \{ \lambda : c(\lambda) = \lambda \}$ be the collection of fuzzy subsystems which satisfies the following axioms:

(i) $0_Q, 1_Q \in \tau(Q)$,
(ii) If $\lambda, \mu \in \tau(Q)$, then $\lambda \wedge \mu \in \tau(Q)$,
(iii) If $\lambda_i \in \tau(Q)$ for each $i \in \mathfrak{J}$, then $\vee\lambda_i \in \tau(Q)$.

Then, the ordered pair $(Q, \tau(Q))$ is said to be a fuzzy automata topological space (in short, $FATS$) iff there exists a fuzzy automaton (Q, X, δ) such that $\tau(Q)$ is a fuzzy topology associated with (Q, X, δ). Moreover, members of $\tau(Q)$ are said to be the fuzzy automata open subsystems and their complements are said to be the fuzzy automata closed subsystems.

Notation. Throughout this paper, 0_Q denotes $\mu_{0_Q}(q) = 0$, for all $q \in Q$ and 1_Q denotes $\mu_{1_Q}(q) = 1$, for all $q \in Q$.

Example 3.2. Let $M = (Q, X, \delta)$ be a fuzzy automaton where $Q = X = \{0, 1, 2, ...\}$ and $\delta : Q \times X \times Q \to [0, 1]$ is given by

$$\delta(q, 0, p) = \begin{cases} 1, & \text{if } q = p \\ 0, & \text{if } q \neq p \end{cases}$$

with $\delta(q, x_0, p) = 0.7$, $\delta(q, x_0, q) = 0.65$, $\delta(p, x_0, p) = 0.6$, $\delta(p, x_0, q) = 0.8$ for fixed $x_0 \in X (x_0 \neq 0)$ and for fixed $p, q \in Q$. For other $p, q \in Q$ and $x \in X$, $\delta(p, x, q) = 0$. Let $\lambda, \mu \in I^Q$ be defined as follows : $\lambda(p) = 0.55$, $\lambda(q) = 0.6$, $\mu(p) = 0.22$, $\mu(q) = 0.25$ and for other $r \in Q$, $\lambda(r) = 0$, $\mu(r) = 0$. The Kuratowski saturated fuzzy closure operator $c : I^Q \to I^Q$ on Q is defined as

$$c(\lambda)(q) = \bigvee \Big\{ \bigvee \{\lambda(p) \wedge \delta(p, x, q) : x \in X\} : p \in Q \Big\}, \text{ for all } \lambda \in I^Q \text{ and } q \in Q.$$

It is clear that $c(\lambda) = \lambda$, $c(\mu) = \mu$, $c(0_Q) = 0_Q$ and $c(1_Q) = 1_Q$.
Then, $\tau(Q) = \{ 0_Q, 1_Q, \lambda, \mu \}$ is a fuzzy automata topology on Q and hence the ordered pair $(Q, \tau(Q))$ is a fuzzy automata topological space.

Definition 3.3. Let $(Q, \tau(Q))$ be a $FATS$. For any $\lambda \in I^Q$,
(i) $FAint(\lambda) = \vee\{ \mu \mid \mu \leq \lambda, \ \mu \in \tau(Q) \}$ is said to be the fuzzy automata interior (in short, $FAint(\lambda)$) of λ,
(ii) $FAcl(\lambda) = \wedge\{ \mu \mid \lambda \leq \mu, \ 1_Q - \mu \in \tau(Q) \}$ is said to be the fuzzy automata closure (in short, $FAcl(\lambda)$) of λ.

Definition 3.4. Let $M = (Q, X, \delta)$ and $N = (R, X, \mu)$ be any two fuzzy automata and let $(Q, \tau(Q))$ and $(R, \tau(R))$ be any two fuzzy automata topological spaces. Any function $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ is said to be fuzzy automata continuous if for each fuzzy automata open subsystem $\lambda \in I^R$ in $(R, \tau(R))$ the inverse image $f^{-1}(\lambda) \in I^Q$ is a fuzzy automata open subsystem in $(Q, \tau(Q))$.

Definition 3.5. Let $M = (Q, X, \delta)$ and $N = (R, X, \mu)$ be any two fuzzy automata and let $(Q, \tau(Q))$ and $(R, \tau(R))$ be any two fuzzy automata topological spaces. A function $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ is said to be somewhat fuzzy automata continuous if $\lambda \in \tau(R)$ and $f^{-1}(\lambda) \neq 0_Q$ implies that there exists a fuzzy automata open subsystem $\mu \in I^Q$ in $(Q, \tau(Q))$ such that $\mu \neq 0_Q$ and $\mu \leq f^{-1}(\lambda)$. That is, $FAcl(1_Q - f^{-1}(\lambda)) \neq 1_Q$ or $FAint(f^{-1}(\lambda)) \neq 0_Q$.

It is clear from Definition 3.4 and Definition 3.5 that every fuzzy automata continuous function is a somewhat fuzzy automata continuous function. But the converse is not true as shown in Example 3.6.

Example 3.6. Let $M = (Q, X, \delta)$ be a fuzzy automaton, where $Q = R = X = \{0, 1, 2, ...\}$ and $\delta : Q \times X \times Q \to [0, 1]$ is given by

$$\delta(q, 0, p) = \begin{cases} 1, & \text{if } q = p \\ 0, & \text{if } q \neq p \end{cases}$$

with $\delta(q, x_0, p) = 0.75$, $\delta(q, x_0, q) = 0.7$, $\delta(p, x_0, p) = 0.65$, $\delta(p, x_0, q) = 0.6$ for fixed $x_0 \in X (x_0 \neq 0)$ and for fixed $p, q \in Q$. For other $p, q \in Q$ and $x \in X$, $\delta(p, x, q) = 0$. Let $\lambda_1 \in I^Q$ and $\lambda_2 \in I^R$ be defined as follows : $\lambda_1(p) = 0.2$, $\lambda_1(q) = 0.3$, $\lambda_2(p) = 0.6$, $\lambda_2(q) = 0.7$ and for other $r \in Q$, $\lambda_1(r) = 0$, $\lambda_2(r) = 0$. The Kuratowski saturated fuzzy closure operator $c : I^Q \to I^Q$ on Q is defined as

$$c(\lambda)(q) = \bigvee \Big\{ \bigvee \{\lambda(p) \wedge \delta(p, x, q) : x \in X\} : p \in Q \Big\}, \text{ for all } \lambda \in I^Q \text{ and } q \in Q.$$

It is clear that $c(\lambda_1) = \lambda_1$, $c(\lambda_2) = \lambda_2$, $c(0_Q) = 0_Q$ and $c(1_Q) = 1_Q$.
Then, $\tau(Q) = \{ 0_Q, 1_Q, \lambda_1 \}$ and $\tau(R) = \{ 0_R, 1_R, \lambda_2 \}$ are the respective fuzzy automata topologies on Q and R and the ordered pairs $(Q, \tau(Q))$ and $(R, \tau(R))$ are the fuzzy automata topological spaces respectively. Let $f : (Q, \tau(Q)) \to (R, \tau(R))$ be a fuzzy automata identity function. Now, for $\lambda_2 \in \tau(R)$ and $f^{-1}(\lambda_2) \neq 0_Q$ there exists a fuzzy automata open subsystem $\lambda_1 \in \tau(Q)$ such that $\lambda_1 \neq 0_Q$ and $\lambda_1 \leq f^{-1}(\lambda_2) = \lambda_2$. That is, $FAcl(1_Q - f^{-1}(\lambda_2)) \neq 1_Q$. Hence f is **a somewhat fuzzy automata continuous function**. But $f^{-1}(\lambda_2) = \lambda_2$ is not a fuzzy automata open subsystem in $(Q, \tau(Q))$. Therefore, f is **not a fuzzy automata continuous function**.

Definition 3.7. A fuzzy automata subsystem $\lambda \in I^Q$ in $(Q, \tau(Q))$ is said to be a generating fuzzy automata subsystem if $FAcl(\lambda) = 1_Q$.

Example 3.8. In Example 3.2, let the fuzzy automata subsystem $\nu \in I^Q$ be defined as follows:

$$\nu(p) = 0.8, \ \ \nu(q) = 0.8$$

and for other $r \in Q$, $\nu(r) = 0$. Then $FAcl(\nu) = 1_Q$. Therefore, ν is a generating fuzzy automata subsystem in $(Q, \tau(Q))$.

Proposition 3.9. *Let $M = (Q, X, \delta)$ and $N = (R, X, \mu)$ be any two fuzzy automata and let $(Q, \tau(Q))$ and $(R, \tau(R))$ be any two fuzzy automata topological spaces. If $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ is a somewhat fuzzy automata continuous function, then $f^{-1}(1_R - \mu)$ is not a generating fuzzy automata subsystem in $(Q, \tau(Q))$, for any fuzzy automata open subsystem $\mu \in I^R$ in $(R, \tau(R))$.*

Proof. Let $0_R \neq \mu \in I^R$ be a fuzzy automata open subsystem in $(R, \tau(R))$. Since f is a somewhat fuzzy automata continuous function, there exists a fuzzy automata open subsystem $0_Q \neq \lambda \in I^Q$ in $(Q, \tau(Q))$ such that $\lambda \leq f^{-1}(\mu)$. That is $FAcl(1_Q - f^{-1}(\mu)) \neq 1_Q$ and hence $FAcl(f^{-1}(1_R - \mu)) \neq 1_Q$. Therefore, $f^{-1}(1_R - \mu)$ is not a generating fuzzy automata subsystem in $(Q, \tau(Q))$. □

Proposition 3.10. *Let $(Q, \tau(Q))$ and $(R, \tau(R))$ be any two fuzzy automata topological spaces. Let $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ be a one-to-one and onto function. Then the following statements are equivalent:*

(i) *f is somewhat fuzzy automata continuous.*
(ii) *If $\mu \in I^R$ is a fuzzy automata closed subsystem in $(R, \tau(R))$ such that $f^{-1}(\mu) \neq 1_Q$, then there exists a fuzzy automata closed subsystem $\lambda \neq 1_Q \in I^Q$ in $(Q, \tau(Q))$ such that $\lambda \geq f^{-1}(\mu)$.*
(iii) *If $\lambda \in I^Q$ is a generating fuzzy automata subsystem in $(Q, \tau(Q))$, then $f(\lambda) \in I^R$ is a generating fuzzy automata subsystem in $(R, \tau(R))$.*

Proof. (i)⇒(ii). Let f be a somewhat fuzzy automata continuous function and $\mu \in I^R$ be a fuzzy automata closed subsystem in $(R, \tau(R))$ such that $f^{-1}(\mu) \neq 1_Q$. Clearly, $1_R - \mu \in \tau(R)$ and $f^{-1}(1_R - \mu) = 1_Q - f^{-1}(\mu) \neq 0_Q$ (since $f^{-1}(\mu) \neq 1_Q$). By (i), there exists a fuzzy automata open subsystem $0_Q \neq \eta \in I^Q$ in $(Q, \tau(Q))$ such that $\eta \leq f^{-1}(1_R - \mu)$. Then $\eta \leq 1_Q - f^{-1}(\mu)$ and hence $f^{-1}(\mu) \leq 1_Q - \eta$. Clearly, $1_Q - \eta$ is a fuzzy automata closed subsystem in $(Q, \tau(Q))$. By replacing $1_Q - \eta = \lambda$, $\lambda \geq f^{-1}(\mu)$. Hence (i)⇒(ii).

(ii)⇒(iii). Let $\lambda \in I^Q$ be a generating fuzzy automata subsystem in $(Q, \tau(Q))$ and suppose $f(\lambda)$ is not a generating fuzzy automata subsystem in $(R, \tau(R))$. Then there exists a fuzzy closed subsystem $0_R \neq \mu \in I^R$ in $(R, \tau(R))$ such that $f(\lambda) < \mu < 1_R$. Since $\mu < 1_R$ and $f^{-1}(\mu) \neq 1_Q$, there exists a fuzzy automata closed subsystem $1_Q \neq \delta \in I^Q$ in $(Q, \tau(Q))$ such that $\lambda \leq f^{-1}(f(\lambda)) < f^{-1}(\mu) \leq \delta$ implies $\lambda \leq \delta$. This contradicts the assumption that λ is a generating fuzzy automata subsystem in $(Q, \tau(Q))$. Hence $f(\lambda)$ is a generating fuzzy automata subsystem in $(R, \tau(R))$.

(iii) ⇒ (i). Let $\lambda \in I^R$ be a fuzzy automata open subsystem in $(R, \tau(R))$ with $f^{-1}(\lambda) \neq 0_Q$. Suppose that there exists no fuzzy automata open subsystem $0_Q \neq \gamma \in I^Q$ in $(Q, \tau(Q))$ such that $\gamma \leq f^{-1}(\lambda)$. That is, $FAcl(1_Q - f^{-1}(\lambda)) = 1_Q$. Then $1_Q - f^{-1}(\lambda)$ is a generating fuzzy automata subsystem in $(Q, \tau(Q))$. Then by (iii) $f(1_Q - f^{-1}(\lambda))$ is a generating fuzzy automata subsystem in $(R, \tau(R))$. That is, $FAcl(f(1_Q - f^{-1}(\lambda))) = 1_R$. But $f(1_Q - f^{-1}(\lambda)) = f(f^{-1}(1_R - \lambda)) \leq 1_R - \lambda < 1_R$. Now, $f(f^{-1}(1_R - \lambda)) \leq 1_R - \lambda$ implies that $FAcl(f(f^{-1}(1_R - \lambda))) \leq FAcl(1_R - \lambda) < FAcl(1_R) = 1_R$ and then, $1_R < 1_R$. This is a contradiction to the fact that $f(1_Q - f^{-1}(\lambda))$ is a generating fuzzy automata subsystem in $(R, \tau(R))$. Hence $FAcl(1_Q - f^{-1}(\lambda)) \neq 1_Q$. Consequently, f is somewhat fuzzy automata continuous. □

Proposition 3.11. *Let $(Q, \tau(Q))$ and $(R, \tau(R))$ be any two fuzzy automata topological spaces. If the function $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ is a somewhat fuzzy automata continuous, one-to-one and onto function and if $FAcl(1_Q - \mu) = 1_Q$, for any fuzzy automata subsystem $\mu \in I^Q$ and $\mu \neq 0_Q$ in $(Q, \tau(Q))$, then $FAcl(1_R - f(\mu)) = 1_R$ in $(R, \tau(R))$.*

Proof. Let $0_Q \neq \mu \in I^Q$ be a fuzzy automata subsystem in $(Q, \tau(Q))$ such that $FAcl(1_Q - \mu) = 1_Q$. Since f is a somewhat fuzzy automata continuous function and $(1_Q - \mu)$ is a generating fuzzy automata subsystem in $(Q, \tau(Q))$, by Proposition

3.10, $f(1_Q - \mu)$ is a generating fuzzy automata subsystem in $(R, \tau(R))$. That is, $FAcl(f(1_Q - \mu)) = 1_R$. Since f is one-to-one and onto, $f(1_Q - \mu) = 1_R - f(\mu)$. Then $FAcl(1_R - f(\mu)) = 1_R$. Hence $FAcl(1_R - f(\mu)) = 1_R$ in $(R, \tau(R))$. □

Proposition 3.12. *Let $M = (Q, X, \delta)$, $M_1 = (Q_1, X, \delta_1)$ and $M_2 = (Q_2, X, \delta_2)$ be any three fuzzy automata and let $(Q, \tau(Q))$, $(Q_1, \tau(Q_1))$ and $(Q_2, \tau(Q_2))$ be any three fuzzy automata topological spaces and $\mathfrak{p}_i : (Q_1, \tau(Q_1)) \times (Q_2, \tau(Q_2)) \longrightarrow (Q_i, \tau(Q_i))$ $(i = 1, 2)$ be any fuzzy automata continuous functions. If $f : (Q, \tau(Q)) \to (Q_1, \tau(Q_1)) \times (Q_2, \tau(Q_2))$ is a somewhat fuzzy automata continuous function, then $\mathfrak{p}_i \circ f$ is also a somewhat fuzzy automata continuous function for $i = 1, 2$.*

Proof. For any fuzzy automata open subsystem $0_{Q_i} \neq \mu \in I^{Q_i}$ in $(Q_i, \tau(Q_i))$ for $i = 1, 2$, we have $(\mathfrak{p}_i \circ f)^{-1}(\mu) = f^{-1}(\mathfrak{p}_i^{-1}(\mu))$. Now $\mathfrak{p}_i^{-1}(\mu) \neq 0_{Q_1 \times Q_2}$ (since $\mu \neq 0_{Q_i}$). Since $\mathfrak{p}_i$ is a fuzzy automata continuous function, $\mathfrak{p}_i^{-1}(\mu)$ is a fuzzy automata open subsystem and since f is a somewhat fuzzy automata continuous function, there exists a fuzzy automata open subsystem $0_Q \neq \lambda \in I^Q$ in $(Q, \tau(Q))$ such that $\lambda \leq f^{-1}(\mathfrak{p}_i^{-1}(\mu))$. That is, $\lambda \leq (\mathfrak{p}_i \circ f)^{-1}(\mu)$. Hence $FAcl(1_Q - (\mathfrak{p}_i \circ f)^{-1}(\mu)) \neq 1_Q$. Therefore, $\mathfrak{p}_i \circ f$ is a somewhat fuzzy automata continuous function for $i = 1, 2$. □

Proposition 3.13. *Let $M_1 = (Q_1, X, \delta_1), M_2 = (Q_2, X, \delta_2)$ and $M_3 = (Q_3, X, \delta_3)$ be any three fuzzy automata and let $(Q_1, \tau(Q_1))$, $(Q_2, \tau(Q_2))$ and $(Q_3, \tau(Q_3))$ be any three fuzzy automata topological spaces. If $f : (Q_1, \tau(Q_1)) \longrightarrow (Q_2, \tau(Q_2))$ is a somewhat fuzzy automata continuous function and $g : (Q_2, \tau(Q_2)) \longrightarrow (Q_3, \tau(Q_3))$ is a fuzzy automata continuous function, then $g \circ f : (Q_1, \tau(Q_1)) \longrightarrow (Q_3, \tau(Q_3))$ is a somewhat fuzzy automata continuous function.*

Proof. Let $0_{Q_3} \neq \lambda \in I^{Q_3}$ be a fuzzy automata open subsystem in $(Q_3, \tau(Q_3))$. Since g is a fuzzy automata continuous function, $g^{-1}(\lambda) \neq 0_{Q_2}$ is a fuzzy automata open subsystem in $(Q_2, \tau(Q_2))$. Since f is a somewhat fuzzy automata continuous function and $g^{-1}(\lambda)$ is a fuzzy automata open subsystem in $(Q_2, \tau(Q_2))$ and $g^{-1}(\lambda) \neq 0_{Q_2}$, there exists a fuzzy automata open subsystem $0_{Q_1} \neq \mu \in I^{Q_1}$ in $(Q_1, \tau(Q_1))$ such that $\mu \leq f^{-1}(g^{-1}(\lambda))$. That is, $\mu \leq (g \circ f)^{-1}(\lambda)$. Hence $FAcl(1_{Q_1} - ((g \circ f)^{-1})(\lambda)) \neq 1_{Q_1}$. Therefore, $g \circ f$ is a somewhat fuzzy automata continuous function. □

Proposition 3.14. *Let $M_1 = (Q_1, X, \delta_1), M_2 = (Q_2, X, \delta_2)$ and $M_3 = (Q_3, X, \delta_3)$ be any three fuzzy automata and let $(Q_1, \tau(Q_1))$, $(Q_2, \tau(Q_2))$ and $(Q_3, \tau(Q_3))$ be any three fuzzy automata topological spaces. If $f : (Q_1, \tau(Q_1)) \longrightarrow (Q_2, \tau(Q_2))$ is a fuzzy automata continuous function and $g : (Q_2, \tau(Q_2)) \longrightarrow (Q_3, \tau(Q_3))$ is a somewhat fuzzy automata continuous function, then $g \circ f : (Q_1, \tau(Q_1)) \longrightarrow (Q_3, \tau(Q_3))$ is a somewhat fuzzy automata continuous function.*

Proof. Let $0_{Q_3} \neq \lambda \in I^{Q_3}$ be a fuzzy automata open subsystem in $(Q_3, \tau(Q_3))$. Since g is a somewhat fuzzy automata continuous function, there exists a fuzzy automata open subsystem $0_{Q_2} \neq \mu \in I^{Q_2}$ in $(Q_2, \tau(Q_2))$ such that $\mu \leq g^{-1}(\lambda)$. Then, $f^{-1}(\mu) \leq f^{-1}(g^{-1}(\lambda))$. That is, $f^{-1}(\mu) \leq (g \circ f)^{-1}(\lambda)$. Again since f is a fuzzy automata continuous function and μ is a fuzzy automata open subsystem in $(Q_2, \tau(Q_2))$, $f^{-1}(\mu)$ is a fuzzy automata open subsystem in $(Q_1, \tau(Q_1))$. Hence

$FAcl(1_{Q_1} - ((g \circ f)^{-1})(\lambda)) \neq 1_{Q_1}$. Therefore, $g \circ f$ is a somewhat fuzzy automata continuous function. □

Proposition 3.15. *Let $M_1 = (Q_1, X, \delta_1), M_2 = (Q_2, X, \delta_2)$ and $M_3 = (Q_3, X, \delta_3)$ be any three fuzzy automata and let $(Q_1, \tau(Q_1))$, $(Q_2, \tau(Q_2))$ and $(Q_3, \tau(Q_3))$ be any three fuzzy automata topological spaces. If $f : (Q_1, \tau(Q_1)) \longrightarrow (Q_2, \tau(Q_2))$ and $g : (Q_2, \tau(Q_2)) \longrightarrow (Q_3, \tau(Q_3))$ are somewhat fuzzy automata continuous functions, then $g \circ f : (Q_1, \tau(Q_1)) \longrightarrow (Q_3, \tau(Q_3))$ is a somewhat fuzzy automata continuous function.*

Proof. Let $0_{Q_3} \neq \lambda \in I^{Q_3}$ be a fuzzy automata open subsystem in $(Q_3, \tau(Q_3))$. Since g is a somewhat fuzzy automata continuous function, there exists a fuzzy automata open subsystem $0_{Q_2} \neq \mu \in I^{Q_2}$ in $(Q_2, \tau(Q_2))$ such that $\mu \leq g^{-1}(\lambda)$. Then, $f^{-1}(\mu) \leq f^{-1}(g^{-1}(\lambda))$. That is $f^{-1}(\mu) \leq (g \circ f)^{-1}(\lambda)$. Again since f is a somewhat fuzzy automata continuous function and $\mu \neq 0_{Q_2}$ is a fuzzy automata open subsystem in $(Q_2, \tau(Q_2))$, there exists a fuzzy automata open subsystem $0_{Q_1} \neq \gamma \in I^{Q_1}$ in $(Q_1, \tau(Q_1))$ such that $\gamma \leq f^{-1}(\mu)$. This implies that $0_{Q_1} \neq \gamma \leq f^{-1}(\mu) \leq f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$. Hence $FAcl(1_{Q_1} - ((g \circ f)^{-1})(\lambda)) \neq 1_{Q_1}$. Therefore, $g \circ f$ is a somewhat fuzzy automata continuous function. □

Proposition 3.16. *Let $M_1 = (Q_1, X, \delta_1), M_2 = (Q_2, X, \delta_2)$, $N_1 = (R_1, X, \mu_1)$ and $N_2 = (R_2, X, \mu_2)$ be any four fuzzy automata and let $(Q_1, \tau(Q_1))$, $(Q_2, \tau(Q_2))$, $(R_1, \tau(R_1))$ and $(R_2, \tau(R_2))$ be any four fuzzy automata topological spaces. If $f_1 : (Q_1, \tau(Q_1)) \longrightarrow (R_1, \tau(R_1))$ and $f_2 : (Q_2, \tau(Q_2)) \longrightarrow (R_2, \tau(R_2))$ are somewhat fuzzy automata continuous, then the product $f_1 \times f_2 : (Q_1, \tau(Q_1)) \times (Q_2, \tau(Q_2)) \longrightarrow (R_1, \tau(R_1)) \times (R_2, \tau(R_2))$ is also somewhat fuzzy automata continuous.*

Proof. Let $\eta = \lambda \times \mu$ be a fuzzy automata open subsystem in $(R_1, \tau(R_1)) \times (R_2, \tau(R_2))$ where $\lambda \in I^{R_1}$ and $\mu \in I^{R_2}$ are the fuzzy automata open subsystems in $(R_1, \tau(R_1))$ and $(R_2, \tau(R_2))$ respectively. Then $(f_1 \times f_2)^{-1}(\eta) = (f_1 \times f_2)^{-1}(\lambda \times \mu) = f_1^{-1}(\lambda) \times f_2^{-1}(\mu)$. Since f_1 is somewhat fuzzy automata continuous, there exists a fuzzy automata open subsystem $0_{Q_1} \neq \delta \in I^{Q_1}$ in $(Q_1, \tau(Q_1))$ such that $\delta \leq f_1^{-1}(\lambda)$ with $f_1^{-1}(\lambda) \neq 0_{Q_1}$. And since f_2 is somewhat fuzzy automata continuous, there exists a fuzzy automata open subsystem $0_{Q_2} \neq \gamma \in I^{Q_2}$ in $(Q_2, \tau(Q_2))$ such that $\gamma \leq f_2^{-1}(\mu)$ with $f_2^{-1}(\mu) \neq 0_{Q_2}$. Now, $\delta \times \gamma \leq f_1^{-1}(\lambda) \times f_2^{-1}(\mu) = (f_1 \times f_2)^{-1}(\lambda \times \mu)$ and $\delta \times \gamma \neq 0_{Q_1 \times Q_2}$. Hence $\delta \times \gamma$ is a fuzzy automata open subsystem in $(Q_1, \tau(Q_1)) \times (Q_2, \tau(Q_2))$. Moreover, $(\delta \times \gamma) \neq 0_{Q_1 \times Q_2}$ is a fuzzy automata open subsystem in $(Q_1, \tau(Q_1)) \times (Q_2, \tau(Q_2))$ such that $(\delta \times \gamma) \leq (f_1^{-1}(\lambda) \times f_2^{-1}(\mu)) = (f_1 \times f_2)^{-1}(\lambda \times \mu) = (f_1 \times f_2)^{-1}(\eta) \neq 0_{Q_1 \times Q_2}$. Hence $FAcl(1_{Q_1 \times Q_2} - (f_1 \times f_2)^{-1}(\eta)) \neq 1_{Q_1 \times Q_2}$. Therefore, $f_1 \times f_2$ is somewhat fuzzy automata continuous. □

Lemma 3.17. *Let $f : Q \longrightarrow R$ be a function. The graph function $g : Q \longrightarrow Q \times R$ of f is defined by $g(q) = (q, f(q))$. If $\lambda \in I^Q$ is a fuzzy subset of Q and $\mu \in I^R$ is a fuzzy subset of R, then $g^{-1}(\lambda \times \mu) = \lambda \wedge f^{-1}(\mu)$.*

Proof. For each $q \in Q$,

$$\begin{aligned} g^{-1}(\lambda \times \mu)(q) &= (\lambda \times \mu)g(q) \\ &= (\lambda \times \mu)(q, f(q)) \\ &= (\lambda \wedge f^{-1}(\mu))(q). \end{aligned}$$

Hence $g^{-1}(\lambda \times \mu) = \lambda \wedge f^{-1}(\mu)$. □

Proposition 3.18. *Let $(Q, \tau(Q))$ and $(R, \tau(R))$ be any two fuzzy automata topological spaces. If $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ is a function and the graph function $g : Q \longrightarrow Q \times R$ of f is somewhat fuzzy automata continuous, then f is also somewhat fuzzy automata continuous.*

Proof. Let $0_R \neq \lambda \in I^R$ be a fuzzy automata open subsystem in $(R, \tau(R))$. Then $1_Q \times \lambda$ is a fuzzy automata open subsystem in $(Q, \tau(Q)) \times (R, \tau(R))$. Since g is somewhat fuzzy automata continuous, then $FAcl(1_Q - (g^{-1}(1_Q \times \lambda))) \neq 1_Q$. But $f^{-1}(\lambda) = 1_Q \wedge f^{-1}(\lambda) = g^{-1}(1_Q \times \lambda)$. This implies that $FAcl(1_Q - f^{-1}(\lambda)) \neq 1_Q$. Therefore, f is somewhat fuzzy automata continuous. □

Definition 3.19. Let $(Q, \tau(Q))$ be any fuzzy automata topological space and A be an ordinary subset of Q. Then $\tau(Q)/A = \{ \lambda/A : \lambda \in \tau(Q) \}$ is a fuzzy automata topology on A and is called the induced or relative fuzzy automata topology. The pair $(A, \tau(Q)/A)$ is called a fuzzy automata subspace of $(Q, \tau(Q))$.

Proposition 3.20. *Let $(Q, \tau(Q))$ and $(R, \tau(R))$ be any two fuzzy automata topological spaces and $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ be a somewhat fuzzy automata continuous function. Let $A \subset Q$ be such that $1_A \wedge \lambda \neq 0_A$ for all $\lambda \neq 0_Q$ where λ is a fuzzy automata open subsystem in $(Q, \tau(Q))$. Let $\tau(Q)/A$ be an induced fuzzy automata topology on A. Then the function $f/A : (A, \tau(Q)/A) \longrightarrow (R, \tau(R))$ is somewhat fuzzy automata continuous.*

Proof. Let $\lambda \in I^R$ be a fuzzy automata open subsystem in $(R, \tau(R))$ such that $f^{-1}(\lambda) \neq 0_Q$. Since f is somewhat fuzzy automata continuous, there exists a fuzzy automata open subsystem $0_Q \neq \eta \in I^Q$ in $(Q, \tau(Q))$ such that $\eta \leq f^{-1}(\lambda)$. Now clearly, η/A is a fuzzy automata open subsystem in $(A, \tau(Q)/A)$ and $\eta/A \neq 0_A$, since $1_A \wedge \lambda \neq 0_A$ for all λ where λ is a fuzzy automata open subsystem in $(Q, \tau(Q))$. Also, $(f/A)^{-1}(\lambda)(x) = \lambda(f/A)(x) = \lambda f(x) \geq \eta(x) = (\eta/A)(x)$, for all $x \in A$. That is, $\eta/A \leq (f/A)^{-1}(\lambda)$. Hence $FAcl(1_A - (f/A)^{-1}(\lambda)) \neq 1_A$. This shows that f/A is somewhat fuzzy automata continuous. □

Proposition 3.21. *Let $(Q, \tau(Q))$ and $(R, \tau(R))$ be any two fuzzy automata topological spaces and let $Q = A \cup B$ where A and B are such that 1_A and 1_B are the fuzzy automata open subsystems in $(Q, \tau(Q))$. Let $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ be such that $f/A : (A, \tau(Q)/A) \longrightarrow (R, \tau(R))$ and $f/B : (B, \tau(Q)/B) \longrightarrow (R, \tau(R))$ are somewhat fuzzy automata continuous. Then f is somewhat fuzzy automata continuous.*

Proof. Let $\lambda \in I^R$ be a fuzzy automata open subsystem in $(R, \tau(R))$ such that $f^{-1}(\lambda) \neq 0_Q$. Now, consider $(f/A)^{-1}(\lambda)$ and $(f/B)^{-1}(\lambda)$. Since $f^{-1}(\lambda) \neq 0_Q$, atleast $(f/A)^{-1}(\lambda) \neq 0_A$ or $(f/B)^{-1}(\lambda) \neq 0_B$. Then by assumption, there exists a fuzzy automata open subsystem $0_Q \neq \gamma \in I^Q$ in $(Q, \tau(Q))$ such that $\gamma/A \leq (f/A)^{-1}(\lambda)$ and $\gamma/B \leq (f/B)^{-1}(\lambda)$. Then $\gamma \leq f^{-1}(\lambda)$. Hence $FAcl(1_Q - f^{-1}(\lambda)) \neq 1_Q$. This proves that f is somewhat fuzzy automata continuous. □

4. **On Somewhat Fuzzy Automata Open Functions**

In this section, the concept of somewhat fuzzy automata open functions is introduced. Some interesting properties and characterizations are discussed with necessary examples.

Definition 4.1. Let $M = (Q, X, \delta)$ and $N = (R, X, \mu)$ be any two fuzzy automata and let $(Q, \tau(Q))$ and $(R, \tau(R))$ be any two fuzzy automata topological spaces. Any function $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ is said to be a fuzzy automata open function if for each fuzzy automata open subsystem $\lambda \in I^Q$ in $(Q, \tau(Q))$ the image $f(\lambda) \in I^R$ is a fuzzy automata open subsystem in $(R, \tau(R))$.

Definition 4.2. Let $M = (Q, X, \delta)$ and $N = (R, X, \mu)$ be any two fuzzy automata and let $(Q, \tau(Q))$ and $(R, \tau(R))$ be any two fuzzy automata topological spaces. A function $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ is said to be a somewhat fuzzy automata open function if $\lambda \in \tau(Q)$ and $\lambda \neq 0_Q$ implies that there exists a fuzzy automata open subsystem $\mu \in I^R$ in $(R, \tau(R))$ such that $\mu \neq 0_R$ and $\mu \leq f(\lambda)$. That is, $FAcl(1_R - f(\lambda)) \neq 1_R$.

It is clear from Definition 4.1 and Definition 4.2 that every fuzzy automata open function is a somewhat fuzzy automata open function. But the converse is not true as shown in Example 4.3.

Example 4.3. Let $M = (Q, X, \delta)$ be a fuzzy automaton, where $Q = R = X = \{0, 1, 2, ...\}$ and $\delta : Q \times X \times Q \to [0, 1]$ is given by

$$\delta(q, 0, p) = \begin{cases} 1, & \text{if } q = p \\ 0, & \text{if } q \neq p \end{cases}$$

with $\delta(q, x_0, p) = 0.8$, $\delta(q, x_0, q) = 0.6$, $\delta(p, x_0, p) = 0.65$, $\delta(p, x_0, q) = 0.7$ for fixed $x_0 \in X (x_0 \neq 0)$ and for fixed $p, q \in Q$. For other $p, q \in Q$ and $x \in X$, $\delta(p, x, q) = 0$. Let $\lambda_1 \in I^Q$ and $\lambda_2 \in I^R$ be defined as follows : $\lambda_1(p) = 0.5$, $\lambda_1(q) = 0.7$, $\lambda_2(p) = 0.3$, $\lambda_2(q) = 0.4$. and for other $r \in Q$, $\lambda_1(r) = 0$, $\lambda_2(r) = 0$. The Kuratowski saturated fuzzy closure operator $c : I^Q \to I^Q$ on Q is defined as

$$c(\lambda)(q) = \bigvee \Big\{ \bigvee \{\lambda(p) \wedge \delta(p, x, q) : x \in X\} : p \in Q \Big\}, \text{ for all } \lambda \in I^Q \text{ and } q \in Q.$$

It is clear that $c(\lambda_1) = \lambda_1$, $c(\lambda_2) = \lambda_2$, $c(0_Q) = 0_Q$ and $c(1_Q) = 1_Q$.
Then, $\tau(Q) = \{ 0_Q, 1_Q, \lambda_1 \}$ and $\tau(R) = \{ 0_R, 1_R, \lambda_2 \}$ are the respective fuzzy automata topologies on Q and R and the ordered pairs $(Q, \tau(Q))$ and $(R, \tau(R))$ are the fuzzy automata topological spaces respectively. Let $f : (Q, \tau(Q)) \to (R, \tau(R))$ be a fuzzy automata identity function. Now, for $\lambda_1 \in \tau(Q)$ and $\lambda_1 \neq 0_Q$ there exists a fuzzy automata open subsystem $\lambda_2 \in \tau(R)$ such that $\lambda_2 \neq 0_R$ and $\lambda_2 \leq f(\lambda_1) = \lambda_1$. Hence f is a **somewhat fuzzy automata open function**. But

$f(\lambda_1) = \lambda_1$ is not a fuzzy automata open subsystem in $(R, \tau(R))$. Therefore, f is **not a fuzzy automata open function**.

Proposition 4.4. *Let $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ be a one-to-one and onto function. Then the following conditions are equivalent:*

(i) *f is a somewhat fuzzy automata open function.*

(ii) *If $\lambda \in I^Q$ is a fuzzy automata closed subsystem in $(Q, \tau(Q))$ such that $f(\lambda) \neq 1_R$, then there exists a fuzzy automata closed subsystem $\mu \neq 1_Q$ and $\mu \in I^Q$ in $(Q, \tau(Q))$ such that $f(\lambda) \leq \mu$.*

Proof. (i)⇒(ii). Let f be a somewhat fuzzy automata open function and $\mu \in I^Q$ be a fuzzy automata closed subsystem in $(Q, \tau(Q))$ such that $f(\mu) \neq 1_R$. Clearly $1_Q - \mu \in \tau(Q)$ and $f(1_Q - \mu) = 1_R - f(\mu) \neq 0_R$ (since f is one-to-one and onto). By (i), there exists a fuzzy automata open subsystem $0_R \neq \eta \in I^R$ in $(R, \tau(R))$ such that $\eta \leq f(1_Q - \mu)$. Then $\eta \leq 1_R - f(\mu)$ and hence $f(\mu) \leq 1_R - \eta$. Clearly $1_R - \eta$ is a fuzzy automata closed subsystem in $(R, \tau(R))$. By taking $1_R - \eta = \lambda$, $f(\mu) \leq \lambda$.

(ii)⇒(i). Let $0_Q \neq \mu \in I^Q$ be a fuzzy automata open subsystem in $(Q, \tau(Q))$ such that $f(\mu) \neq 0_R$. Then, $1_Q - \mu$ is a fuzzy automata closed subsystem in $(Q, \tau(Q))$ such that $f(1_Q - \mu) = 1_R - f(\mu) \neq 1_R$ (since f is one-to-one and onto, $f(1_Q - \mu) = 1_R - f(\mu)$). By hypothesis, there exists a fuzzy automata closed subsystem $1_R \neq \lambda \in I^R$ in $(R, \tau(R))$ such that $\lambda \geq f(1_Q - \mu)$. Then $\lambda \geq 1_R - f(\mu)$. Hence $1_R - \lambda \leq f(\mu)$, where $1_R - \lambda$ is a fuzzy automata open subsystem in $(R, \tau(R))$. Therefore, f is a somewhat fuzzy automata open function. □

Proposition 4.5. *Let $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ be a function from $(Q, \tau(Q))$ into $(R, \tau(R))$. Then the following conditions are equivalent:*

(i) *f is a somewhat fuzzy automata open function.*

(ii) *If $\gamma \in I^R$ is a generating fuzzy automata subsystem in $(R, \tau(R))$, then $f^{-1}(\gamma) \in I^Q$ is a generating fuzzy automata subsystem in $(Q, \tau(Q))$.*

Proof. (i)⇒(ii). Let f be a somewhat fuzzy automata open function from $(Q, \tau(Q))$ into $(R, \tau(R))$ and $\gamma \in I^R$ be a generating fuzzy automata subsystem in $(R, \tau(R))$. Suppose that $f^{-1}(\gamma)$ is not a generating fuzzy automata subsystem in $(Q, \tau(Q))$. Then there exists a fuzzy automata closed subsystem $0_Q \neq \eta \in I^Q$ in $(Q, \tau(Q))$ such that $f^{-1}(\gamma) < \eta < 1_Q$. This implies that $1_Q - \eta < 1_Q - f^{-1}(\gamma) = f^{-1}(1_R - \gamma)$. Now, $1_Q - \eta$ is a fuzzy automata open subsystem in $(Q, \tau(Q))$. Since $\eta < 1_Q, 1_R - \gamma \neq 0_R$. Since f is a somewhat fuzzy automata open function, there exists a fuzzy automata open subsystem $0_R \neq \mu \in I^R$ in $(R, \tau(R))$ such that $\mu \leq f(1_Q - \eta)$ and hence $\mu \leq f(f^{-1}(1_R - \gamma)) \leq 1_R - \gamma$ implies $\gamma < 1_R - \mu < 1_R$ and $1_R - \mu$ is a fuzzy automata closed subsystem in $(R, \tau(R))$, implies that γ is not a generating fuzzy automata subsystem in $(R, \tau(R))$. This is a contradiction. Hence $f^{-1}(\gamma)$ is a generating fuzzy automata subsystem in $(Q, \tau(Q))$.

(ii)⇒(i). Let $0_Q \neq \lambda \in I^Q$ be a fuzzy automata open subsystem in $(Q, \tau(Q))$ and $f(\lambda) \neq 0_R$. Suppose there exists no fuzzy automata open subsystem $0_R \neq \mu \in I^R$ in $(R, \tau(R))$ such that $\mu \leq f(\lambda)$. That is, $FAcl(1_R - f(\lambda)) = 1_R$. This means that $1_R - f(\lambda)$ is a generating fuzzy automata subsystem in $(R, \tau(R))$. Thus $f^{-1}(f(1_Q -$

λ)) is a generating fuzzy automata subsystem in $(Q, \tau(Q))$. Now $f^{-1}(1_R - f(\lambda)) = 1_Q - f^{-1}(f(\lambda)) \leq 1_Q - \lambda < 1_Q$ (since $\lambda \neq 1_Q$). That is, $FAcl(f^{-1}(1_R - f(\lambda))) < 1_Q$. Then, $FAcl(f^{-1}(1_R - f(\lambda))) < FAcl(1_Q) = 1_Q$. This implies that $FAcl(f^{-1}(1_R - f(\lambda))) \neq 1_Q$. This is a contradiction to the fact that $f^{-1}(1_R - f(\lambda))$ is a generating fuzzy automata subsystem in $(Q, \tau(Q))$. Hence $FAcl(1_R - f(\lambda)) \neq 1_R$. Therefore, f is a somewhat fuzzy automata open function. □

Proposition 4.6. *Let $(Q, \tau(Q))$ and $(R, \tau(R))$ be any two fuzzy automata topological spaces. If the function $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ is a somewhat fuzzy automata open function and if $FAcl(1_R - \lambda) = 1_R$, for any fuzzy automata subsystem $\lambda \neq 0_R$ in $(R, \tau(R))$, then $FAcl(1_Q - f^{-1}(\lambda)) = 1_Q$ in $(Q, \tau(Q))$.*

Proof. Let $0_R \neq \lambda \in I^R$ be a fuzzy automata subsystem in $(R, \tau(R))$ such that $FAcl(1_R - \lambda) = 1_R$. Since f is a somewhat fuzzy automata open function and $1_R - \lambda$ is a generating fuzzy automata subsystem in $(R, \tau(R))$, by Proposition 4.4, $f^{-1}(1_R - \lambda)$ is a generating fuzzy automata subsystem in $(Q, \tau(Q))$. That is $FAcl(f^{-1}(1_R - \lambda)) = 1_Q$. Since $f^{-1}(1_R - \lambda) = 1_Q - (f^{-1}(\lambda))$. This implies $FAcl(f^{-1}(1_R - \lambda)) = FAcl(1_Q - f^{-1}(\lambda)) = 1_Q$. Hence $FAcl(1_Q - f^{-1}(\lambda)) = 1_Q$ in $(Q, \tau(Q))$. □

Proposition 4.7. *Let $(Q_1, \tau(Q_1))$, $(Q_2, \tau(Q_2))$ and $(Q_3, \tau(Q_3))$ be any three fuzzy automata topological spaces. If $f : (Q_1, \tau(Q_1)) \longrightarrow (Q_2, \tau(Q_2))$ is a fuzzy automata open function and $g : (Q_2, \tau(Q_2)) \longrightarrow (Q_3, \tau(Q_3))$ is a somewhat fuzzy automata open function, then $g \circ f : (Q_1, \tau(Q_1)) \longrightarrow (Q_3, \tau(Q_3))$ is a somewhat fuzzy automata open function.*

Proof. Let $0_{Q_1} \neq \lambda \in I^{Q_1}$ be a fuzzy automata open subsystem in $(Q_1, \tau(Q_1))$. Since f is a fuzzy automata open function, $f(\lambda) \in I^{Q_2}$ is a fuzzy automata open subsystem in $(Q_2, \tau(Q_2))$. Since g is a somewhat fuzzy automata open function and $f(\lambda)$ is a fuzzy automata open subsystem in $(Q_2, \tau(Q_2))$ and $f(\lambda) \neq 0_{Q_2}$, there exists a fuzzy automata open subsystem $\mu \in I^{Q_3}$ in $(Q_3, \tau(Q_3))$ such that $\mu \leq g(f(\lambda))$. That is $\mu \leq (g \circ f)(\lambda)$. Hence $FAcl(1_{Q_3} - (g \circ f)(\lambda)) \neq 1_{Q_3}$. Therefore, $g \circ f$ is a somewhat fuzzy automata open function. □

Proposition 4.8. *Let $(Q_1, \tau(Q_1))$, $(Q_2, \tau(Q_2))$ and $(Q_3, \tau(Q_3))$ be any three fuzzy automata topological spaces. If $f : (Q_1, \tau(Q_1)) \longrightarrow (Q_2, \tau(Q_2))$ and $g : (Q_2, \tau(Q_2)) \longrightarrow (Q_3, \tau(Q_3))$ are somewhat fuzzy automata open functions, then $g \circ f : (Q_1, \tau(Q_1)) \longrightarrow (Q_3, \tau(Q_3))$ is a somewhat fuzzy automata open function.*

Proof. Let $0_{Q_1} \neq \mu \in I^{Q_1}$ be a fuzzy automata open subsystem in $(Q_1, \tau(Q_1))$. Since f is a somewhat fuzzy automata open function, there exists a fuzzy automata open subsystem $0_{Q_2} \neq \lambda \in I^{Q_2}$ in $(Q_2, \tau(Q_2))$ such that $\lambda \leq f(\mu)$. Then $g(\lambda) \leq g(f(\mu))$. That is, $g(\lambda) \leq (g \circ f)(\mu)$. Since g is a somewhat fuzzy automata open function and λ is a fuzzy automata open subsystem in $(Q_2, \tau(Q_2))$ and $g(\lambda) \neq 0_{Q_3}$, there exists a fuzzy automata open subsystem $\gamma \in I^{Q_3}$ in $(Q_3, \tau(Q_3))$ such that $\gamma \leq g(\lambda)$. This implies that $\gamma \leq g(\lambda) \leq (g \circ f)(\lambda)$. Hence $FAcl(1_{Q_3} - (g \circ f)(\mu)) \neq 1_{Q_3}$. Therefore, $g \circ f$ is a somewhat fuzzy automata open function. □

Proposition 4.9. *Let* $(Q, \tau(Q))$ *and* $(R, \tau(R))$ *be any two fuzzy automata topological spaces and* $Q = A \cup B$ *where* A *and* B *are subsets of* Q *and* $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ *is a function such that* $f/A : (A, \tau(Q)/A) \longrightarrow (R, \tau(R))$ *and* $f/B : (B, \tau(Q)/B) \longrightarrow (R, \tau(R))$ *are somewhat fuzzy automata open, then* f *is somewhat fuzzy automata open.*

Proof. Let $\lambda \in I^Q$ be a fuzzy automata open subsystem in $(Q, \tau(Q))$ such that $f(\lambda) \neq 0_R$. Now, consider $(f/A)(\lambda)$ and $(f/B)(\lambda)$. Since $f(\lambda) \neq 0_R$, at least $(f/A)(\lambda) \neq 0_A$ or $(f/B)(\lambda) \neq 0_B$. In particular $(f/A)(\lambda) \neq 0_A$. Therefore by assumption, there exists a fuzzy automata open subsystem $0_R \neq \gamma \in I^R$ in $(R, \tau(R))$ such that $\gamma/A \leq (f/A)(\lambda)$. That is, $\gamma \leq f(\lambda)$. Hence $FAcl(1_R - f(\lambda)) \neq 1_R$. This proves that f is somewhat fuzzy automata open. □

5. Functions and Fuzzy Automata Resolvable Spaces, Fuzzy Automata Irresolvable Spaces

In this section, the concepts of fuzzy automata resolvable and fuzzy automata irresolvable spaces are introduced. Some interesting properties are discussed.

Definition 5.1. A fuzzy automata topological space $(Q, \tau(Q))$ is called a fuzzy automata resolvable space if there exists a generating fuzzy automata subsystem $0_Q \neq \lambda \in I^Q$ in $(Q, \tau(Q))$ such that $FAint(\lambda) = 0_Q$. Otherwise $(Q, \tau(Q))$ is called a fuzzy automata irresolvable space.

Example 5.2. Let $M = (Q, X, \delta)$ be a fuzzy automaton where $Q = X = \{0, 1, 2, ...\}$ and $\delta : Q \times X \times Q \to [0, 1]$ is given by

$$\delta(q, 0, p) = \begin{cases} 1, & \text{if } q = p \\ 0, & \text{if } q \neq p \end{cases}$$

with $\delta(q, x_0, p) = 0.7$, $\delta(q, x_0, q) = 0.65$, $\delta(p, x_0, p) = 0.6$, $\delta(p, x_0, q) = 0.75$ for fixed $x_0 \in X (x_0 \neq 0)$ and for fixed $p, q \in Q$. For other $p, q \in Q$ and $x \in X$, $\delta(p, x, q) = 0$. Let $\lambda \in I^Q$ be defined as follows : $\lambda(p) = 0.6$, $\lambda(q) = 0.7$ and for other $r \in Q$, $\lambda(r) = 0$. The Kuratowski saturated fuzzy closure operator $c : I^Q \to I^Q$ on Q is defined as

$$c(\lambda)(q) = \bigvee \Big\{ \bigvee \{\lambda(p) \wedge \delta(p, x, q) : x \in X\} : p \in Q \Big\}, \text{ for all } \lambda \in I^Q \text{ and } q \in Q.$$

It is clear that $c(\lambda) = \lambda$, $c(0_Q) = 0_Q$ and $c(1_Q) = 1_Q$. Then, $\tau(Q) = \{ 0_Q, 1_Q, \lambda \}$ is a fuzzy automata topology on Q and hence the ordered pair $(Q, \tau(Q))$ is a fuzzy automata topological space. Let the fuzzy automata subsystem $\mu \in I^Q$ be defined as follows: $\mu(p) = 0.5$, $\mu(q) = 0.5$ and for other $r \in Q$, $\mu(r) = 0$. Then $FAcl(\mu) = 1_Q$. Therefore, μ is a generating fuzzy automata subsystem in $(Q, \tau(Q))$ and $FAint(\mu) = 0_Q$. Hence $(Q, \tau(Q))$ is a fuzzy automata resolvable space.

Example 5.3. Let $M = (Q, X, \delta)$ be a fuzzy automaton where $Q = X = \{0, 1, 2, ...\}$ and $\delta : Q \times X \times Q \to [0, 1]$ is given by

$$\delta(q, 0, p) = \begin{cases} 1, & \text{if } q = p \\ 0, & \text{if } q \neq p \end{cases}$$

with $\delta(q, x_0, p) = 0.7$, $\delta(q, x_0, q) = 0.65$, $\delta(p, x_0, p) = 0.6$, $\delta(p, x_0, q) = 0.55$ for fixed $x_0 \in X(x_0 \neq 0)$ and for fixed $p, q \in Q$. For other $p, q \in Q$ and $x \in X$, $\delta(p, x, q) = 0$. Let $\lambda \in I^Q$ be defined as follows : $\lambda(p) = 0.3$, $\lambda(q) = 0.4$ and for other $r \in Q$, $\lambda(r) = 0$. The Kuratowski saturated fuzzy closure operator $c : I^Q \to I^Q$ on Q is defined as

$$c(\lambda)(q) = \bigvee \Big\{ \bigvee \{\lambda(p) \wedge \delta(p, x, q) : x \in X\} : p \in Q \Big\}, \text{ for all } \lambda \in I^Q \text{ and } q \in Q.$$

It is clear that $c(\lambda) = \lambda$, $c(0_Q) = 0_Q$ and $c(1_Q) = 1_Q$. Then, $\tau(Q) = \{ 0_Q, 1_Q, \lambda \}$ is a fuzzy automata topology on Q and hence the ordered pair $(Q, \tau(Q))$ is a fuzzy automata topological space. For every generating fuzzy automata subsystem $\mu \in I^Q$ in $(Q, \tau(Q))$, $FAint(\mu) \neq 0_Q$. Therefore, $(Q, \tau(Q))$ is a fuzzy automata irresolvable space.

Proposition 5.4. *Let $(Q, \tau(Q))$ be any fuzzy automata topological space. If a fuzzy automata topological space $(Q, \tau(Q))$ has a pair of generating fuzzy automata subsystems $\mu_1 \in I^Q$ and $\mu_2 \in I^Q$ such that $\mu_1 \leq (1_Q - \mu_2)$, then $(Q, \tau(Q))$ is a fuzzy automata resolvable space.*

Proof. Let the fuzzy automata topological space $(Q, \tau(Q))$ has a pair of generating fuzzy automata subsystems $\mu_1 \in I^Q$ and $\mu_2 \in I^Q$ such that $\mu_1 \leq (1_Q - \mu_2)$. Then to prove $(Q, \tau(Q))$ is a fuzzy automata resolvable space. Assume the contrary. Then, for all generating fuzzy automata subsystems $\mu_i \in I^Q$ where $i \in \mathfrak{J}$ in $(Q, \tau(Q))$ we have $FAcl(1_Q - \mu_i) \neq 1_Q$. In particular, $FAcl(1_Q - \mu_2) \neq 1_Q$ implies that there exists a fuzzy automata closed subsystem $0_Q \neq \lambda \in I^Q$ in $(Q, \tau(Q))$ such that $1_Q - \mu_2 < \lambda < 1_Q$. Then, $\mu_1 \leq (1_Q - \mu_2)$, implies that $\mu_1 \leq (1_Q - \mu_2) < \lambda < 1_Q$ and hence $\mu_1 < \lambda < 1_Q$. This implies $FAcl(1 - \mu_1) \neq 1_Q$.This is a contradiction. Hence $FAcl(1_Q - \mu_i) = 1_Q$ implies $FAint(\mu_i) = 0_Q$ Therefore, $(Q, \tau(Q))$ is a fuzzy automata resolvable space. □

Proposition 5.5. *A fuzzy automata topological space $(Q, \tau(Q))$ is a fuzzy automata irresolvable space if and only if $FAcl(1_Q - \mu) \neq 1_Q$, for each generating fuzzy automata subsystem $\mu \in I^Q$ in $(Q, \tau(Q))$.*

Proof. Let $(Q, \tau(Q))$ be a fuzzy automata irresolvable space. Then, for each generating fuzzy automata subsystem $\mu \in I^Q$ in $(Q, \tau(Q))$, we have $FAint(\mu) \neq 0_Q$ and hence $1_Q - FAcl(1_Q - \mu) = FAint(\mu) \neq 0_Q$. Thus $FAcl(1_Q - \mu) \neq 1_Q$, for each generating fuzzy automata subsystem μ in $(Q, \tau(Q))$.

Conversely, let $FAcl(1_Q - \mu) \neq 1_Q$, for each generating fuzzy automata subsystem $\mu \in I^Q$ in $(Q, \tau(Q))$. Suppose that $(Q, \tau(Q))$ is a fuzzy automata resolvable space. Then, there exists generating fuzzy automata subsystem $0_Q \neq \mu \in I^Q$ in $(Q, \tau(Q))$ such that $FAint(\mu) = 0_Q$ and hence $1_Q - FAcl(1_Q - \mu) = FAint(\mu) = 0_Q$ implies $FAcl(1_Q - \mu) = 1_Q$, a contradiction to the hypothesis. Therefore, $(Q, \tau(Q))$ is a fuzzy automata irresolvable space. □

Proposition 5.6. Let $(Q, \tau(Q))$ and $(R, \tau(R))$ be any two fuzzy automata topological spaces. If the function $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ is a somewhat fuzzy automata open function and if $(Q, \tau(Q))$ is a fuzzy automata irresolvable space, then $(R, \tau(R))$ is a fuzzy automata irresolvable space.

Proof. Let $0_R \neq \lambda \in I^R$ be an arbitrary fuzzy automata subsystem in $(R, \tau(R))$ such that $FAcl(\lambda) = 1_R$. It is enough to prove that $FAcl(1_R - \lambda) \neq 1_R$. Assume the contrary. That is, $FAcl(1_R - \lambda) = 1_R$. Since f is a somewhat fuzzy automata open function, by Proposition 4.6, $FAcl(1_Q - f^{-1}(\lambda)) = 1_Q$ in $(Q, \tau(Q))$ and by Proposition 4.5, $f^{-1}(\lambda)$ is a generating fuzzy automata subsystem in $(Q, \tau(Q))$, for a generating fuzzy automata subsystem λ in $(R, \tau(R))$. Thus $FAcl(1_Q - f^{-1}(\lambda)) = 1_Q$ for a generating fuzzy automata subsystem $f^{-1}(\lambda)$ in $(Q, \tau(Q))$. But this is a contradiction to $(Q, \tau(Q))$, being a fuzzy automata irresolvable space in which $FAcl(1_Q - \mu) \neq 1_Q$, for each generating fuzzy set μ in $(Q, \tau(Q))$ (by Proposition 5.5). Hence our assumption that $FAcl(1_R - \lambda) = 1_R$, for a generating fuzzy automata subsystem λ in $(R, \tau(R))$, does not hold. Hence $FAcl(1_R - \lambda) \neq 1_R$, for a generating fuzzy automata subsystem λ in $(R, \tau(R))$. Therefore, $(R, \tau(R))$ is a fuzzy automata irresolvable space. □

Proposition 5.7. *Let $(Q, \tau(Q))$ and $(R, \tau(R))$ be any two fuzzy automata topological spaces. If the function $f : (Q, \tau(Q)) \longrightarrow (R, \tau(R))$ is a somewhat fuzzy automata continuous and one-to-one function and if $(R, \tau(R))$ is a fuzzy automata irresolvable space, then $(Q, \tau(Q))$ is a fuzzy automata irresolvable space.*

Proof. Let $0_Q \neq \lambda \in I^Q$ be an arbitrary fuzzy automata subsystem in $(Q, \tau(Q))$ such that $FAcl(\lambda) = 1_Q$. It is enough to prove that $FAcl(1_Q - \lambda) \neq 1_Q$. Assume the contrary. That is, $FAcl(1_Q - \lambda) = 1_Q$. Since f is a somewhat fuzzy automata continuous and one-to-one function, by Proposition 3.11, $FAcl(1_R - f(\lambda)) = 1_R$ in $(R, \tau(R))$ and by Proposition 3.10, $f(\lambda)$ is a generating fuzzy automata subsystem in $(R, \tau(R))$, for a generating fuzzy automata subsystem λ in $(Q, \tau(Q))$. Thus, $FAcl(1_R - f(\lambda)) = 1_R$ for a generating fuzzy automata subsystem $f(\lambda)$ in $(R, \tau(R))$. But this is a contradiction to $(R, \tau(R))$, being a fuzzy automata irresolvable space in which $FAcl(1_Q - \mu) \neq 1_Q$, for each generating fuzzy automata subsystem μ in $(Q, \tau(Q))$ (by Proposition 5.5). Hence our assumption that $FAcl(1_Q - \lambda) = 1_Q$, for a generating fuzzy automata subsystem λ in $(Q, \tau(Q))$, does not hold. Hence $FAcl(1_Q - \lambda) \neq 1_Q$, for each generating fuzzy automata subsystem λ in $(Q, \tau(Q))$. Therefore, $(Q, \tau(Q))$ is a fuzzy automata irresolvable space. □

6. **Conclusion**

The concepts of somewhat fuzzy automata continuous functions, somewhat fuzzy automata open functions between fuzzy automata topological spaces are introduced and studied. Also fuzzy automata resolvable spaces and fuzzy automata irresolvable spaces are introduced and studied. Some results concerning fuzzy automata functions that preserve the fuzzy automata resolvable spaces and fuzzy automata irresolvable spaces in the context of images and pre-images are obtained in this paper.

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ON SOMEWHAT FUZZY AUTOMATA CONTINUOUS FUNCTIONS IN FUZZY AUTOMATA TOPOLOGICAL SPACES

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توابع پیوسته خودکار حدوداً فازی در فضاهای توپولوژیکی خودکار فازی

چکیده. در این مقاله ، مفاهیمی چون توابع پیوسته خودکار حدوداً فازی و توابع باز خودکار حدوداً فازی در فضاهای توپولوژیکی خودکار فازی معرفی شده اند و برخی از خواص جالب آنها مورد بررسی قرار گرفته اند . در این ارتباط ، مفاهیم فضاهای حل پذیرخودکار فازی و فضاهای حل ناپذیر خودکار فازی نیز معرفی شده اند و خواص آنها مورد مطالعه قرار گرفته اند.