ADAPTIVE FUZZY TRACKING CONTROL FOR A CLASS OF PERTURBED NONLINEARLY PARAMETERIZED SYSTEMS USING MINIMAL LEARNING PARAMETERS ALGORITHM

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Abstract. In this paper, an adaptive fuzzy tracking control approach is proposed for a class of single-input single-output (SISO) nonlinear systems in which the unknown continuous functions may be nonlinearly parameterized. During the controller design procedure, the fuzzy logic systems (FLS) in Mamdani type are applied to approximate the unknown continuous functions, and then, based on the minimal learning parameters (MLP) algorithm and the adaptive backstepping dynamic surface control (DSC) technique, a new adaptive fuzzy backstepping control scheme is developed. The main advantages of our approach include: (i) unlike the existing results which deal with the nonlinearly parameterized functions by using the separation principle, the nonlinearly parameterized functions are lumped into the continuous functions which can be approximated by using the FLS, (ii) only one parameter needs to be adjusted online in controller design procedure, which reduces the online computation burden greatly, and our development is able to eliminate the problem of "explosion of complexity" inherent in the existing backstepping-based methods. It is proven that the proposed design method is able to guarantee that all the signals in the closed-loop system are bounded and the tracking error is smaller than a prescribed error bound. Finally, two examples are used to show the effectiveness of the proposed approach.

1. Introduction

During the past several decades, many scholars have dedicated a lot of effort to handle the control design of nonlinear systems and some interesting control methods have been proposed [3, 7, 12, 13, 16, 21, 29, 30]. Among them, adaptive backstepping control, a recursive design procedure, has become an effective tool for controlling strict-feedback nonlinear systems with uncertain parameters, and many significant developments have been achieved, for example [3, 7, 12, 16] and the references therein. To extend the applications of adaptive backstepping control, fuzzy or neural network adaptive control approaches were developed to deal with the tracking or regulation control problems of nonlinear systems with unknown continuous functions. The main idea of the adaptive fuzzy or neural network control methodology is that fuzzy logic systems (FLS) or neural networks (NN) are applied to approximate the unknown nonlinear continuous functions of the investigated

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systems, and the classical adaptive technique is used to construct controllers based on the backstepping technique. Up to now, the issues of combining the FLS or NN with backstepping technique to approximate unknown nonlinearities have been well investigated in [4, 5, 6, 10, 11, 32, 35] and the references therein.

During the control design process by using the FLS and the backstepping technique, the number of the online adjusted parameters of the controller can affect the closed system running efficiency. Thus, in [1, 2, 11, 17, 25, 32], by using the "minimal learning parameters" (MLP) algorithm, the controllers with much fewer adjusted parameters in the control schemes were proposed, which reduced the online computational burden greatly such that the closed-loop systems running efficiency were improved. In [1, 25], the adaptive fuzzy controllers were designed for multiple-input multiple-output (MIMO) and single-input single-output (SISO) nonlinear systems, respectively, and the authors considered the norm of the ideal weighting vector in fuzzy logic systems as the estimation parameter instead of the elements of weighting vector, therefore, the number of the online adaptive parameters was not more than the order of the original system. In [2], by estimating the maximum value of the norm of weight vector of fuzzy systems, only one adaptive parameter was needed for an n-order nonlinear systems, and the computational burden was alleviated significantly, which might render this control design more suitable for practical applications. Then, this idea was extended to several different cases, such as the SISO systems with time-varying delays [32], the MIMO nonlinear systems [11, 17] in which the number of parameters updated online for each subsystem was reduced dramatically to 1.

In recent years, the adaptive fuzzy backstepping control has become one of the most popular design approaches to handle the control design of a large class of nonlinear systems. However, a major drawback is that the problem of "explosion of complexity" existing in conventional adaptive fuzzy backstepping design, which is caused by the repeated differentiations of certain nonlinear functions as the order of the system increases. The dynamic surface control (DSC) technique has been proposed to avoid this problem by introducing a first order low-pass filter at each step of the conventional backstepping design procedure [23, 31]. Recently, the combination of the DSC approach with the MLP algorithm was extended to the fuzzy adaptive tracking control for a class of SISO nonlinear systems [33] and MIMO nonlinear systems [14], respectively.

Although the adaptive control theory has achieved a great progress, the considered plants in the previous adaptive control approaches are mainly focused on the systems with linear parameterizations, for details see [12, 16, 20, 27] and the references therein. It is well known that nonlinear parameterizations are inevitable in any realistic practical problem, and designing the algorithms for nonlinearly parameterized systems is an interesting and meaningful problem. Therefore, many valuable results have been achieved for the nonlinearly parameterized systems with the unknown continuous functions $f(x, \rho)$ [8, 9, 15, 18, 19, 22, 26, 34]. In [18], in order to design the effective adaptive controllers for nonlinearly parameterized systems, the authors constructed a monotone function, which explicitly depended on some of the estimator tuning parameters. In [15], Lin and Qian introduced the separation principle, in which the nonlinearly parameterized functions were expressed as linear parameterized functions. And then, this design idea was extended to investigate the stability and the controller design problem of different systems with the nonlinearly parameterizations problem, such as the systems with unknown input nonlinearities [26], switched nonlinear system [19, 22], and the stochastic nonlinear systems [8, 9, 34]. However, the disadvantage of the separation principle is that the separated out parameters should be designed the adaptive laws such that the online computation burden increases greatly. Therefore, for the nonlinearly parameterized systems, if not using the separation principle, how to handle with the nonlinearly parameterized functions by combining the DSC method and the MLP algorithm is a challenging problem.

Based on the above observation, in this paper, the tracking control problem is revisited for a class of perturbed nonlinear systems using fuzzy control and in the systems the functions may be nonlinearly parameterized. During the controller design procedure, the nonlinearly parameterized functions will be lumped into the other unknown continuous functions, and then, they will be approximated by using the FLS as a whole. Finally, by utilizing the MLP algorithm, the DSC and the backstepping technique, the controller containing only one adjusted parameter is designed. It should be pointed out that the DSC technique is used to avoid the "explosion of complexity" within the conventional backstepping technique. The two main advantages of the scheme are that:

(1) Unlike the existing results [8, 9, 15, 19, 22, 26, 34] in which the nonlinearly parameterized functions were expressed as the linear parametrization by using the separation principle, in this paper, the nonlinearly parameterized functions are lumped into the continuous functions, then, as a whole they will be approximated by utilizing the FLS.

(2) By using the MLP algorithm and the DSC technique, the controller with only one adjusted parameter is constructed, which reduces the online computation burden greatly, and meanwhile the "explosion of complexity" problem is overcome.

It can be proven that all the signals in the closed loop system are bounded and the tracking error is smaller than a prescribed error bound. Simulation results are provided to show the effectiveness of the proposed approach.

2. Problem Formulation and Preliminaries

2.1. **Problem Formulation.** Consider the following nonlinear system

$$\begin{cases} \dot{x}_i = g_i(\bar{x}_i)x_{i+1} + f_i\left(\bar{x}_i, \rho\right) + \Delta_i\left(x, t\right), \\ \dot{x}_n = g_n(x)u\left(t\right) + f_n\left(x, \rho\right) + \Delta_n\left(x, t\right), \\ y = x_1, \end{cases} \tag{1}$$

where $x = [x_1, \cdots, x_i, \cdots, x_n]^T$ are the system states with $\bar{x}_i = [x_1, \cdots, x_i]^T$. y and u represent the system output and system control input, respectively. Functions $f_i\left(\bar{x}_i, \rho\right)$ and $g_i\left(x_i\right)$ are unknown smooth nonlinear functions, and $f_i\left(\bar{x}_i, \rho\right)$ may be nonlinearly parameterized. $\rho = [\rho_1, \ldots, \rho_m]^T$ is a parameter vector with $\rho_1, \ldots, \rho_m$ being unknown parameters. And for $n = 1, 2, \ldots, n$, $\Delta_i\left(\cdot\right)$ stands for the unknown external disturbance input. In this paper, the target is to design an adaptive fuzzy controller such that the system output can track the reference signal y_d, while all the signals in the closed loop system remain bounded.

Throughout this paper, we make the following assumptions.

Assumption 1. [33] The reference signal $y_d(t)$ is a sufficiently smooth function. $y_d(t)$, $\dot{y}_d(t)$ and $\ddot{y}_d(t)$ are bounded, i.e., there exists a positive constant B_0, such that $\Pi := \left\{(y_d, \dot{y}_d, \ddot{y}_d) : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leq B_0\right\} \subset \mathbf{R}^3$.

Assumption 2. [25] For $1 \leq i \leq n$, the sign of $g_i(\bar{x}_i)$ is known, and there exist unknown constants g_0 and g_1 such that $0 < g_0 \leq |g_i(\bar{x}_i)| \leq g_1, \forall \bar{x}_i \in R^i$. Without loss of generality, it is assumed that $g_1 \geq g_i(\bar{x}_i) \geq g_0 > 0$.

Assumption 3. [25] For $1 \leq i \leq n$, there exist unknown positive smooth functions $\omega_i(\bar{x}_i)$ such that $|\Delta_i(t,x)| \leq \omega_i(\bar{x}_i)$.

2.2. **Fuzzy Logic Systems.** In this paper, the following rules are used to develop the adaptive fuzzy controller

R^l: if x_1 is F_1^l and x_2 is F_2^l and ... and x_n is F_n^l , then y is $G^l, l = 1, 2..., Q$,

where $x = \begin{bmatrix} x_1, & \ldots, & x_n \end{bmatrix}^T$ and y are the FLS input and output, respectively. Fuzzy sets F_i^l and G^l, associated with the membership function $\mu_{F_i^l}(x_i)$ and $\mu_{G^l}(y)$, respectively. Q is the rule number. Through singleton function, center average defuzzification, the FLS can be expressed as follows

$$y(x) = \frac{\sum_{l=1}^{Q} \Phi_l \Pi_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{Q} \Pi_{i=1}^{n} \mu_{F_i^l}(x_i)}, \tag{2}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$, $\mu_{F_i^l}(x_i)$ is the membership function of F_i^l, and $\Phi_l = \arg\sup_{y \in R} \mu_{G^l}(y)$. Define $\phi = [\Phi_1, \ldots, \Phi_Q]^T$ and $\xi(x) = [\xi_1(x) \ldots, \xi_Q(x)]^T$ with the fuzzy basis function ξ_l given by

$$\xi_l(x) = \frac{\Pi_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{Q} \Pi_{i=1}^{n} \mu_{F_i^l}(x_i)}. \tag{3}$$

Then, the fuzzy logic system (2) can be rewritten as

$$y(x) = \phi^T \xi(x). \tag{4}$$

Our first choice for the membership function is the Gaussian function $\mu_{F_i^l}(x_i) = \exp\left(-\frac{1}{2}\left(\left(x_i - a_i^l\right)/\sigma_i^l\right)^2\right)$, where σ_i^l and a_i^l are fixed parameters. It has been proven that when the membership functions are chosen as Gaussian functions, the above fuzzy logic system is capable of uniformly approximating any continuous nonlinear function over a compact set with any degree of accuracy. This property is shown by the following lemma.

Lemma 2.1. [24] *Let $f(x)$ be a continuous function defined on compact set Ω. Then, for any constant $\iota > 0$, there exists an FLS (4) such that*

$$\sup_{x \in \Omega} \left| f(x) - \phi^T \xi(x) \right| \leq \iota. \tag{5}$$

3. Controller Design

The design of adaptive DSC laws is based on the following change of coordinates:

$$z_1 = x_1 - y_d, \ldots, z_i = x_i - \alpha_{i-1}, \ldots, z_n = x_n - \alpha_{n-1}, \tag{6}$$

where α_i, $i = 1, \ldots, n-1$ are the output of a first order filter with $\alpha_{i,d}$ as the input, and $\alpha_{i,d}$ are intermediate controllers which are developed in (7). Finally, the input $u(t)$ is constructed in the step n.

In this section, we will use the recursive backstepping technique to develop the adaptive fuzzy tracking control laws as follows

$$\alpha_{i,d} = -k_i z_i - \frac{\hat{\theta}\xi_i^T(\mathrm{Z}_i)\,\xi_i(\mathrm{Z}_i)\,z_i}{2\eta^2}, \quad 1 \le i \le n-1, \tag{7}$$

$$u(t) = -k_n z_n - \frac{\hat{\theta}\xi_n^T(\mathrm{Z}_n)\,\xi_n(\mathrm{Z}_n)\,z_n}{2\eta^2}, \tag{8}$$

$$\dot{\hat{\theta}} = \sum_{i=1}^{n} \frac{r}{2\eta^2}\xi_i^T(\mathrm{Z}_i)\,\xi_i(\mathrm{Z}_i)\,z_i^2 - \sigma\hat{\theta}, \tag{9}$$

where k_i, σ, r and η are positive design parameters, moreover, $k_i > \frac{1}{g_0\lambda}$ for $i = 1, \ldots, n$ with λ being positive design parameters. The parameter $\hat{\theta}$ is the estimation of the unknown constant θ, which is defined as

$$\theta = \max\left\{\theta_1 = \frac{\|\phi_1\|^2}{g_0}, \ldots, \ldots, \theta_n = \frac{\|\phi_n\|^2}{g_0}\right\}, \tag{10}$$

where ϕ_i is an unknown weight parameter vector and will be specified later. The parameter estimation error is $\tilde{\theta} = \hat{\theta} - \theta$. Furthermore, in order to obtain the filtered virtual controller α_i, we pass $\alpha_{i,d}$ through a first-order filter with time constant $\tau_i > 0$ for $i = 1, \ldots, n-1$

$$\tau_i\dot{\alpha}_i + \alpha_i = \alpha_{i,d}, \quad \alpha_i(0) = \alpha_{i,d}(0). \tag{11}$$

Step 1: From (1) and (6) yields that

$$\dot{z}_1 = \dot{y} - \dot{y}_d = g_1(x_1)x_2 + f_1(x_1, \rho) + \Delta_1(x, t) - \dot{y}_d,$$

choose the Lyapunov function candidate as

$$V_1 = \frac{1}{2}z_1^2, \tag{12}$$

by applying Assumption 3 and the triangular inequality, the following inequality can be obtained

$$z_1\Delta_1(x, t) \le \frac{z_1^2\omega_1^2(x_1)}{2a_{11}^2} + \frac{a_{11}^2}{2},$$

then, we can get

$$\begin{aligned}\dot{V}_1 &\le z_1\left(g_1(x_1)x_2 + f_1(x_1, \rho) + \frac{z_1\omega_1^2(x_1)}{2a_{11}^2} - \dot{y}_d + \nu_1 g_1^2(x_1)z_1\right) + \frac{a_{11}^2}{2} - \nu_1 g_1^2(x_1)z_1^2 \\ &= z_1\left(g_1(x_1)x_2 + \varphi_1(Z_1)\right) - \nu_1 g_1^2(x_1)z_1^2 + \frac{a_{11}^2}{2},\end{aligned} \tag{13}$$

where

$$\varphi_1(Z_1) = f_1(x_1,\rho) - \dot{y}_d + \frac{z_1\omega_1^2\,(x_1)}{2a_{11}^2} + \nu_1 g_1^2(x_1)z_1, \tag{14}$$

with $Z_1 = [x_1, z_1, \dot{y}_d]^T \in \Omega_{Z_1} \subset R^3$ and Ω_{Z_1} being some known compact set in R^3. Notice that the function $\varphi_1(Z_1)$ is continuous, therefore, it can be approximated by the FLS $\phi_1^T\xi_1\,(\mathrm{Z}_1)$ such that

$$\varphi_1\,(\mathrm{Z}_1) = \phi_1^T\xi_1\,(\mathrm{Z}_1) + \delta_1\,(\mathrm{Z}_1)\,, \tag{15}$$

furthermore, using the triangular inequality yields that

$$z_1\left(\phi_1^T\xi_1\,(\mathrm{Z}_1) + \delta_1\,(\mathrm{Z}_1)\right) \le \frac{g_0\theta_1}{2\eta^2}\xi_1^T\xi_1 z_1^2 + \frac{z_1^2}{2\lambda} + \frac{\eta^2}{2} + \frac{\lambda\varepsilon_1^2}{2}, \tag{16}$$

from (6), (10), (13)-(16), we can get that

$$\begin{aligned}\dot{V}_1 &\le z_1\left(g_1(x_1)z_2 + g_1(x_1)\alpha_1 + \frac{g_0\theta_1}{2\eta^2}\xi_1^T\xi_1 z_1 + \frac{z_1}{2\lambda}\right)\\ &\quad + \frac{\eta^2}{2} + \frac{\lambda\varepsilon_1^2}{2} - \nu_1 g_1^2(x_1)z_1^2 + \frac{a_{11}^2}{2}\\ &\le z_1\left(g_1(x_1)z_2 + g_1(x_1)\alpha_1 + \frac{g_0\theta}{2\eta^2}\xi_1^T\xi_1 z_1 + \frac{z_1}{2\lambda}\right)\\ &\quad + \frac{\eta^2}{2} + \frac{\lambda\varepsilon_1^2}{2} - \nu_1 g_1^2(x_1)z_1^2 + \frac{a_{11}^2}{2},\end{aligned} \tag{17}$$

where λ, ν_1, η and a_{11} are the positive design parameters with ε_1 being the upper bound of $\delta\,(Z_1)$, and the term $\nu_1 g_1^2(x_1)z_1^2$ will be canceled later.

Remark 3.1. If the unknown continuous function $f_1(x_1,\rho)$ is nonlinearly parameterized, in many existing results [8, 9, 15, 22, 19, 26, 34], the separation principle was used to deal with the nonlinearly parameterized function $f_1(x_1,\rho)$ such that it can be expressed as linear parameterized function. Nevertheless, in this paper, whether the unknown continuous function $f_1(x_1,\rho)$ is linearly parameterized or not, since ρ is the constant vector rather than the variable vector, that is to say there is only one variable x_1 in function $f_1(x_1,\rho)$, the function $f_1(x_1,\rho)$ can be lumped into the continuous function $\varphi_1(Z_1)$ such that it can be approximated by using the FLS.

***Step i*:** Considering $z_i = x_i - \alpha_{i-1}$, where α_{i-1} defined in (11), then

$$\dot{z}_i = \dot{x}_i - \dot{\alpha}_{i-1} = g_i\,(\bar{x}_i)\,x_{i+1} + f_i\,(\bar{x}_i,\rho) + \Delta_i\,(x,t) - \dot{\alpha}_{i-1}. \tag{18}$$

Choose the Lyapunov function candidate as

$$V_i = \frac{1}{2}z_i^2, \tag{19}$$

using the Assumption 3 yields that

$$z_i\Delta_i\,(x,t) \le \frac{z_i^2\omega_i^2\,(\bar{x}_i)}{2a_{ii}^2} + \frac{a_{ii}^2}{2},$$

where a_{ii} is a positive design constant. Then, we have

$$\dot{V}_i \le z_i\left(g_i\,(\bar{x}_i)\,x_{i+1} + f_i\,(\bar{x}_i,\rho) + \frac{z_i\omega_i^2\,(\bar{x}_i)}{2a_{ii}^2} - \dot{\alpha}_{i-1} + g_{i-1}(\bar{x}_{i-1})z_{i-1} + \nu_i g_i\,(\bar{x}_i)\,z_i\right)$$

$$-g_{i-1}(\bar{x}_{i-1})z_i z_{i-1} - \nu_i g_i(\bar{x}_i) z_i^2 + \frac{a_{ii}^2}{2}$$
$$= z_i\left(g_i(\bar{x}_i)x_{i+1} + \varphi_i(Z_i)\right) - g_{i-1}(\bar{x}_{i-1})z_i z_{i-1} - \nu_i g_i(\bar{x}_i) z_i^2 + \frac{a_{ii}^2}{2}, \quad (20)$$

where

$$\varphi_i(Z_i) = f_i(\bar{x}_i, \rho) - \dot{\alpha}_{i-1} + \frac{z_i \omega_i^2(\bar{x}_i)}{2a_{ii}^2} + g_{i-1}(\bar{x}_{i-1})z_{i-1} + \nu_i g_i(\bar{x}_i) z_i, \quad (21)$$

with ν_i bing a positive design parameter, $Z_i = [x_1, \ldots, x_i, z_{i-1}, z_i, \dot{\alpha}_{i-1}]^T \in \Omega_{Z_i} \subset R^{i+3}$ and Ω_{Z_i} being some known compact set in R^{i+3}. Similar to step 1, the nonlinear function $\varphi(\mathrm{Z_i})$ can be approximated by an FLS $\phi_i^T \xi_i(\mathrm{Z}_i)$ such that

$$\varphi_i(\mathrm{Z}_i) = \phi_i^T \xi_i(\mathrm{Z}_i) + \delta_i(\mathrm{Z}_i), \quad (22)$$

using

$$z_i\left(\phi_i^T \xi_i(\mathrm{Z}_i) + \delta_i(\mathrm{Z}_i)\right) \le \frac{g_0 \theta_i}{2\eta^2} \xi_i^T \xi_i z_i^2 + \frac{z_i^2}{2\lambda} + \frac{\eta^2}{2} + \frac{\lambda \varepsilon_i^2}{2},$$

and similar to (17) yields that

$$\dot{V}_i \le z_i\left(g_i(\bar{x}_i)x_{i+1} + \frac{g_0\theta}{2\eta^2}\xi_i^T\xi_i z_i + \frac{z_i}{2\lambda}\right) + \frac{a_{ii}^2}{2}$$
$$-g_{i-1}(\bar{x}_{i-1})z_i z_{i-1} - \nu_i g_i(\bar{x}_i) z_i^2 + \frac{\eta^2}{2} + \frac{\lambda\varepsilon_i^2}{2},$$

where ε_i is the upper bound of $\delta_i(Z_i)$ with $|\delta_i(Z_i)| \le \varepsilon_i$, and the term $g_{i-1}(\bar{x}_{i-1})z_i z_{i-1}$ can be canceled in the next step.

Step n: Considering $z_n = x_n - \alpha_{n-1}$, similar to step i, we can have

$$\dot{z}_n = \dot{x}_n - \dot{\alpha}_{n-1} = g_n(\bar{x}_n)u(t) + f_n(\bar{x}_n, \rho) + \Delta_n(x,t) - \dot{\alpha}_{n-1}, \quad (23)$$

choose the Lyapunov function candidate as

$$V_n = \frac{1}{2} z_n^2, \quad (24)$$

then, similar to step i, one gets

$$\dot{V}_n \le z_n\left(g_n(\bar{x}_n)u + f_n(\bar{x}_n, \rho) + g_{n-1}(\bar{x}_{n-1})z_{n-1} - \dot{\alpha}_{n-1}\right) - g_{n-1}(\bar{x}_{n-1})z_n z_{n-1} + \frac{a_{ii}^2}{2}$$
$$= z_n\left(g_n(\bar{x}_n)u + \varphi_n(Z_n)\right) - g_{n-1}(\bar{x}_{n-1})z_n z_{n-1} + \frac{a_{ii}^2}{2}, \quad (25)$$

where a_{nn} is a positive design parameter and

$$\varphi_n(Z_n) = g_{n-1}(\bar{x}_{n-1})z_{n-1} + \frac{z_n \omega_n^2(\bar{x}_n)}{2a_{nn}^2} + f_n(\bar{x}_n, \rho_n) - \dot{\alpha}_{n-1},$$

with $Z_n = [x_1, \ldots, x_n, z_{n-1}, z_n, \dot{\alpha}_{n-1}]^T \in \Omega_{Z_n} \subset R^{n+3}$ and Ω_{Z_n} being some known compact set in R^{n+3}. Similar to step 1, the nonlinear function $\varphi(\mathrm{Z_n})$ can be approximated by an FLS $\phi_n^T \xi_n(\mathrm{Z}_n)$ such that

$$\varphi_n(\mathrm{Z}_n) = \phi_n^T \xi_n(\mathrm{Z}_n) + \delta_n(\mathrm{Z}_n), \quad (26)$$

combining

$$z_n\left(\phi_n^T\xi_n\left(\mathrm{Z}_n\right)+\delta_n\left(\mathrm{Z}_n\right)\right)\leq\frac{g_0\theta_n}{2\eta^2}\xi_n^T\xi_n z_n^2+\frac{z_n^2}{2\lambda}+\frac{\eta^2}{2}+\frac{\lambda\varepsilon_n^2}{2},$$

gives that

$$\dot{V}_n\leq z_n\left(g_n\left(\bar{x}_n\right)u(t)+\frac{g_0\theta}{2\eta^2}\xi_n^T\xi_n z_n+\frac{z_n}{2\lambda}\right)-g_{n-1}(\bar{x}_{n-1})z_n z_{n-1}+\frac{\eta^2}{2}+\frac{\lambda\varepsilon_n^2}{2},\quad(27)$$

where ε_n is the upper bound of $\delta_n(Z_n)$ with $|\delta_n(Z_n)|\leq\varepsilon_n$.

In the following section, we will design the adaptive law for $\hat{\theta}$. Choose the Lyapunov function as

$$V=\sum_{i=1}^{n}V_i+\frac{g_0\tilde{\theta}^2}{2r},\quad(28)$$

the derivative of V is given by

$$\begin{aligned}\dot{V}=\sum_{i=1}^{n}\dot{V}_i+\frac{g_0\tilde{\theta}\dot{\hat{\theta}}}{r}&\leq\sum_{i=1}^{n-1}\left(g_i\left(\bar{x}_i\right)\alpha_i z_i-\nu_i g_i\left(\bar{x}_i\right)z_i^2\right)+g_n\left(\bar{x}_n\right)u(t)z_n\\&+\sum_{i=1}^{n}\frac{g_0\theta}{2\eta^2}\xi_i^T\xi_i z_i^2+\sum_{i=1}^{n}\frac{z_i^2}{2\lambda}+\sum_{i=1}^{n}\left(\frac{\eta^2}{2}+\frac{\lambda\varepsilon_i^2}{2}+\frac{a_{ii}^2}{2}\right)+\frac{g_0\tilde{\theta}\dot{\hat{\theta}}}{r}.\end{aligned}\quad(29)$$

Now, define the boundary layer errors as

$$y_i=\alpha_i-\alpha_{i,d},\quad i=1,\ldots,n-1.\quad(30)$$

By using $\alpha_{i,d}=\tau_i\dot{\alpha}_i+\alpha_i,\ i=1,\ldots,n-1$, we obtain that

$$\dot{y}_i=-\frac{y_i}{\tau_i}-\dot{\alpha}_{i,d},\quad i=1,\ldots,n-1,\quad(31)$$

then, similar to step i, combining Assumption 2 and (7)-(10), (31) yields that

$$\begin{aligned}\dot{V}\leq&\sum_{i=1}^{n-1}\left(g_i\left(\bar{x}_i\right)y_i z_i-\nu_i g_i\left(\bar{x}_i\right)z_i^2\right)-\sum_{i=1}^{n}g_0 k_i z_i^2\\&+\sum_{i=1}^{n}\frac{z_i^2}{2\lambda}+\sum_{i=1}^{n}\left(\frac{\eta^2}{2}+\frac{\lambda\varepsilon_i^2}{2}+\frac{a_{ii}^2}{2}\right)-\frac{g_0\sigma\hat{\theta}\tilde{\theta}}{r}.\end{aligned}\quad(32)$$

Furthermore, using $g_i\left(\bar{x}_i\right)y_i z_i\leq\frac{y_i^2}{4\nu_i}+\nu_i g_i^2\left(\bar{x}_i\right)z_i^2$ gives that

$$\dot{V}\leq\sum_{i=1}^{n-1}\frac{y_i^2}{4\nu_i}-\sum_{i=1}^{n}g_0 k_i z_i^2+\sum_{i=1}^{n}\frac{z_i^2}{2\lambda}+\sum_{i=1}^{n}\left(\frac{\eta^2}{2}+\frac{\lambda\varepsilon_i^2}{2}+\frac{a_{ii}^2}{2}\right)-\frac{g_0\sigma\hat{\theta}\tilde{\theta}}{r},\quad(33)$$

where ν_i is a positive design parameter.

4. Stability Analysis

Theorem 4.1. *Consider the closed-loop system consisting of system (1) under Assumptions 1- 3, the control laws (7)-(8) and the parameter adaptive law (9). Suppose that the packaged uncertain functions $\varphi_i(Z_i)$, $1\leq i\leq n$ can be approximated by the FLS in the sense that approximation errors are bounded.*

Then, for any initial condition satisfying $\sum_{i=1}^{n} V_i + \sum_{i=1}^{n-1} \frac{1}{2}y_i^2 + \frac{g_0\tilde{\theta}^2}{2r} \leq p$, *where* p *is any positive constant, there exist* k_i, τ_i, σ, r *and* η *such that the solution of the closed-loop system is semi-globally uniformly ultimately bounded, while the tracking errors may be made arbitrarily small by appropriately adjusting the design parameters.*

Proof. Choose the Lyapunov function candidate as

$$\bar{V} = \sum_{i=1}^{n} V_i + \sum_{i=1}^{n-1} \frac{1}{2}y_i^2 + \frac{g_0\tilde{\theta}^2}{2r}, \tag{34}$$

using (9) and (33) gives that

$$\begin{aligned}\dot{\bar{V}} &= \sum_{i=1}^{n} \dot{V}_i + \sum_{i=1}^{n-1} y_i\dot{y}_i + \frac{g_0\tilde{\theta}\dot{\hat{\theta}}}{r} \\ &\leq \sum_{i=1}^{n-1} \frac{y_i^2}{4\nu_i} + \sum_{i=1}^{n}\left(\frac{\eta^2}{2} + \frac{\lambda\varepsilon_i^2}{2} + \frac{a_{ii}^2}{2}\right) + \sum_{i=1}^{n}\frac{z_i^2}{2\lambda} \\ &\quad - \sum_{i=1}^{n} g_0k_iz_i^2 - \frac{g_0\sigma\hat{\theta}\tilde{\theta}}{r} + \sum_{l=1}^{n-1} y_i\left(-\frac{y_i}{\tau_i} - \dot{\alpha}_{i,d}\right) \\ &= \sum_{i=1}^{n-1}\left(\frac{1}{4\nu_i} - \frac{1}{\tau_i}\right)y_i^2 - \sum_{i=1}^{n}\left(g_0k_i - \frac{1}{2\lambda}\right)z_i^2 \\ &\quad + \sum_{i=1}^{n}\left(\frac{\eta^2}{2} + \frac{\lambda\varepsilon_i^2}{2} + \frac{a_{ii}^2}{2}\right) - \frac{g_0\sigma\hat{\theta}\tilde{\theta}}{r} - \sum_{l=1}^{n-1} y_i\dot{\alpha}_{i,d},\end{aligned} \tag{35}$$

furthermore, according to the definitions of $\tilde{\theta}$ and $\dot{\alpha}_{i,d}$ yields that

$$\begin{aligned}-\frac{g_0\sigma\hat{\theta}\tilde{\theta}}{r} &= -\frac{g_0\sigma\left(\tilde{\theta}+\theta\right)\tilde{\theta}}{r} \leq -\frac{g_0\sigma\tilde{\theta}^2}{2r} + \frac{g_0\sigma\theta^2}{2r}, \\ \dot{\alpha}_{i,d} = -k_i\dot{z}_i &- \frac{\dot{\hat{\theta}}\xi_i^T(\mathrm{Z}_i)\,\xi_i(\mathrm{Z}_i)\,z_i}{2\eta^2} - \frac{\hat{\theta}\dot{\xi}_i^T(\mathrm{Z}_i)\,\xi_i(\mathrm{Z}_i)\,z_i}{2\eta^2} - \frac{\hat{\theta}\xi_i^T(\mathrm{Z}_i)\,\xi_i(\mathrm{Z}_i)\,\dot{z}_i}{2\eta^2} \\ &= \chi_i\left(z_1,\ldots,z_n,y_1,\ldots,y_{n-1},\hat{\theta},y_d,\dot{y}_d,\ddot{y}_d\right),\end{aligned} \tag{36}$$

where χ_i is a continuous function with $1 \leq i \leq n-1$. Then, we have

$$\dot{\bar{V}} \leq \sum_{i=1}^{n-1}\left(\frac{1}{4\nu_i} - \frac{1}{\tau_i}\right)y_i^2 - \sum_{i=1}^{n}\left(g_0k_i - \frac{1}{2\lambda}\right)z_i^2 - \frac{g_0\sigma\tilde{\theta}^2}{2r} + \sum_{l=1}^{n-1}|y_i\dot{\alpha}_{i,d}| + C, \tag{37}$$

where $C = \frac{g_0\sigma\theta^2}{2r} + \sum_{i=1}^{n}\left(\frac{\eta^2}{2} + \frac{\lambda\varepsilon_i^2}{2} + \frac{a_{ii}^2}{2}\right)$. According to $k_i > \frac{1}{g_0\lambda}$ gives that $g_0k_i - \frac{1}{2\lambda} > 0$ with $i = 1, \ldots, n$. Moreover, according to Assumption 1, for any $B_0 > 0$ and $p > 0$, the sets $\Pi = \left\{(y_d, \dot{y}_d, \ddot{y}_d) : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leq B_0\right\}$ and $\bar{\Pi} = \left\{\sum_{i=1}^{n}\frac{1}{2}z_i^2 + \frac{g_0\tilde{\theta}^2}{2r} + \sum_{i=1}^{n-1}\frac{1}{2}y_i^2 \leq p\right\}$ are compact in R^3 and R^{2n}, respectively. Therefore, $\Pi \times \bar{\Pi}$ is also compact in R^{2n+3}. Thus, χ_i has a maximum M_i on $\Pi \times \bar{\Pi}$. Using $|y_i \dot{\alpha}_{i,d}| \le \frac{\tau}{2} + \frac{1}{2\tau} y_i^2 \chi_i^2$ yields

$$\dot{\bar{V}} \le \sum_{i=1}^{n-1} \left(\frac{1}{4\nu_i} - \frac{1}{\tau_i} + \frac{1}{2\tau}\chi_i^2 \right) y_i^2 - \sum_{i=1}^{n} \bar{k}_i z_i^2 - \frac{g_0 \sigma \tilde{\theta}^2}{2r} + \bar{C}, \tag{38}$$

where $\bar{C} = C + \frac{(n-1)\tau}{2}$, $\bar{k}_i = g_0 k_i - \frac{1}{2\lambda}$ and τ is a positive parameter. Now, choose $\bar{k}_i = \ldots = \bar{k}_n = k^*$, $\frac{1}{\tau_i} = \frac{1}{4\nu_i} + \frac{M_i^2}{2\tau} + \tau_i^*$ with k^* and τ_i^* being positive constants, we have

$$\begin{aligned} \dot{\bar{V}} &\le \sum_{i=1}^{n-1} \left(\frac{1}{4\nu_i} - \left(\frac{1}{4\nu_i} + \frac{M_i^2}{2\tau} + \tau_i^* \right) + \frac{1}{2\tau}\chi_i^2 \right) y_i^2 - \sum_{i=1}^{n} k^* z_i^2 - \frac{g_0 \sigma \tilde{\theta}^2}{2r} + \bar{C} \\ &= \sum_{i=1}^{n-1} \left(-\frac{M_i^2}{2\tau} - \tau_i^* + \frac{1}{2\tau}\chi_i^2 \right) y_i^2 - \sum_{i=1}^{n} k^* z_i^2 - \frac{g_0 \sigma \tilde{\theta}^2}{2r} + \bar{C}, \end{aligned} \tag{39}$$

according to $|\chi_i| \le M_i$ yields that

$$\dot{\bar{V}} \le -\sum_{i=1}^{n-1} \tau_i^* y_i^2 - \sum_{i=1}^{n} k^* z_i^2 - \frac{g_0 \sigma \tilde{\theta}^2}{2r} + \bar{C} \le -\alpha_0 \bar{V} + \bar{C}, \tag{40}$$

where $0 < \alpha_0 < \min\left\{k^*, \frac{\sigma}{2}, \tau_i{}^*\right\}$. Let $\alpha_0 > \frac{\bar{C}}{2p}$, then, $\dot{\bar{V}} < 0$ on $\bar{V}(0) = p$. Thus, $\bar{V} \le p$ is an invariant set, i.e. if $\bar{V}(0) \le p$, then $\bar{V}(t) \le p$ for all $t \ge 0$. Thus, (40) holds for all $\bar{V}(0) \le p$ and all $t \ge 0$. Solving (40) gives $0 \le \bar{V}(t) \le \frac{\bar{C}}{2\alpha_0} + \left(\bar{V}(0) - \frac{\bar{C}}{2\alpha_0} \right) e^{-2\alpha_0 t}, \forall t \ge 0$, which means that $\bar{V}(t)$ eventually is bounded by $\frac{\bar{C}}{2\alpha_0}$. Thus, the boundedness of $\tilde{\theta}$, $z_i(t)$ for $i = 1, \ldots, n$ and y_i for $i = 1, \ldots, n-1$ are obtained. Finally, the boundedness of $u(t)$ can be obtained.

Moreover, by increasing the values of k_i, σ and reducing the value of τ_i, η, i.e. increasing the value of α_0, while reducing the value of $\bar{C}$, $\frac{\bar{C}}{2\alpha_0}$ can be made arbitrarily small. Thus, the tracking error z_1 may be made arbitrarily small.

This concludes the proof. □

5. **Simulation**

In this section, two numerical simulation examples are given to demonstrate the effectiveness of the proposed control method. In Example 5.1, the Brusselator model in dimensionless form with disturbance is considered. And, in Example 5.2, the system with unknown nonlinearly parameterized functions is investigated.

Example 5.1. The Brusselator model in dimensionless form is considered in this section:

$$\begin{cases} \dot{x}_1 = \rho_1 - (\rho_2 + 1)x_1 + x_1^2 x_2, \\ \dot{x}_2 = \rho_2 x_1 - x_1^2 x_2, \end{cases} \tag{41}$$

where x_1 and x_2 denote the concentrations of the reaction intermediates: $\rho_1, \rho_2 > 0$ are parameters describing the (constant) supply of "reservoir" chemicals. The Brusselator model is a simplified model describing a certain set of chemical reactions. As a simplified model depicting chemical reactions, the Brusselator model is derived from partial differential equations after a series of approximations.

Thus, there must exist modelling errors and other types of unknown nonlinearities in the practical chemical reactions. The controller Brusselator with disturbance [28] is assumed as

$$\begin{cases} \dot{x}_1 = \rho_1 - (\rho_2 + 1)x_1 + x_1^2 x_2 + \Delta_1 (t, x), \\ \dot{x}_2 = \rho_2 x_1 - x_1^2 x_2 + (2 + \cos(x_1))\, u + \Delta_2 (t, x), \\ y = x_1, \end{cases} \tag{42}$$

where Δ_1 and Δ_2 are the disturbance terms, the nonlinearities $f_1(x_1, \rho) = \rho_1 - (\rho_2 + 1)x_1$, $f_2(\bar{x}_2, \rho) = \rho_2 x_1$, $g_1(x_1) = x_1^2$, $g_2(\bar{x}_2) = 2 + \cos(x_1)$ are assumed unknown functions. In the simulation, we assume that $x_1 \neq 0$, $\rho_1 = 1$, $\rho_2 = 3$, $\Delta_1(t, x) = x_1^3 \cos(1.5t)$ and $\Delta_2(t, x) = x_2^2 \sin x_1 \sin^3 t$.

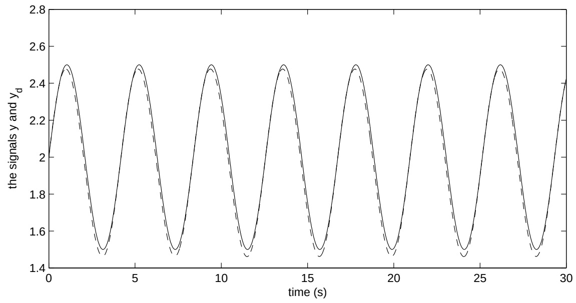

FIGURE 1. The Trajectories of x_1 and y_d (Dashed Line)

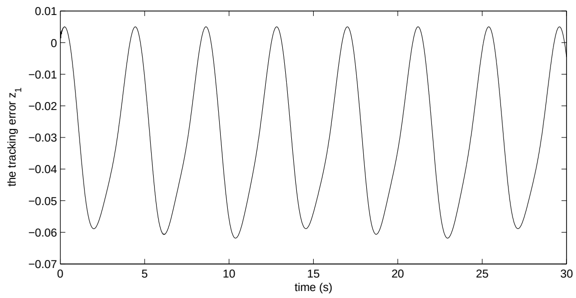

FIGURE 2. The Tracking Error z_1

The control objective is to guarantee (i) all the signals in the closed-loop system remain bounded, and (ii) the output y follows the reference signal $y_d = 2 + 0.5\sin(1.5t)$. The adaptive fuzzy tracking controller is chosen according to Theorem 1 as follows:

$$\alpha_{1,d} = -k_1 z_1 - \frac{\hat{\theta}\xi_1^T(\mathrm{Z}_1)\,\xi_1(\mathrm{Z}_1)\,z_1}{2\eta^2}, \quad u(t) = -k_2 z_2 - \frac{\hat{\theta}\xi_2^T(\mathrm{Z}_2)\,\xi_2(\mathrm{Z}_2)\,z_2}{2\eta^2}, \tag{43}$$

$$\dot{\hat{\theta}} = \sum_{i=1}^{2} \frac{r}{2\eta^2}\xi_i^T(\mathrm{Z}_i)\,\xi_i(\mathrm{Z}_i)\,z_i^2 - \sigma\hat{\theta}. \tag{44}$$

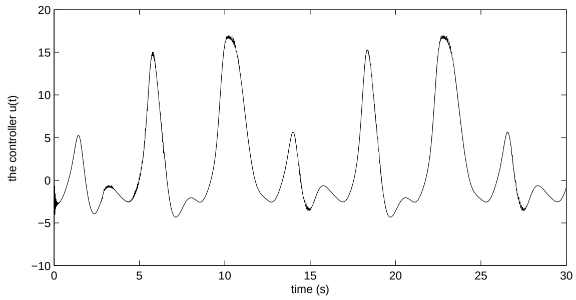

FIGURE 3. The Trajectory of $u(t)$

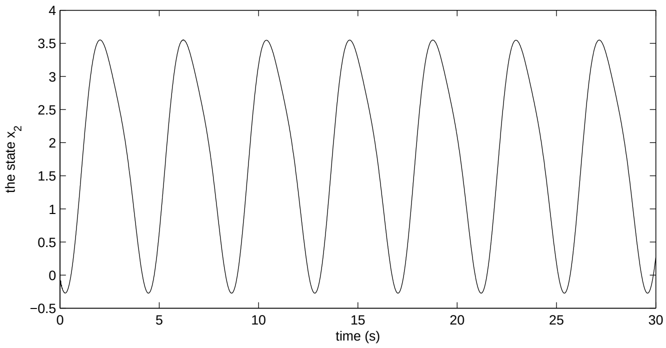

FIGURE 4. The State x_2

The initial states are chosen as $x_1(0) = 2,\ x_2(0) = 0,\ \hat{\theta}(0) = 0$ and $y_1(0) = 0$ and select the design parameters as $\eta = 1,\ \sigma = 0.5,\ \ k_1 = 60,\ k_2 = 100,\ \tau_1 = 1/150$ and $r = 1$. Simulation results in Figures 1-6 show the effectiveness of the developed adaptive fuzzy control schemes. From figure 1 it can be seen that good tracking performance is obtained. The boundedness of the variables z_1 and $u(t)$ are illustrated in Figures 2 and 3, respectively. The state x_2 and the adaptive parameter θ are also bounded by Figures 4 and 5. Finally, we can deduce that the error z_2 is also bounded from Figure 6.

Example 5.2. Consider the following nonlinearly parameterized system:

$$\begin{cases} \dot{x}_1 = x_2 + \rho_1^{x_1} + 0.7x_1\cos(1.5t), \\ \dot{x}_2 = (3+\cos(x_1x_2))u(t) + \ln\left(1+(\rho_2x_2)^2\right) + 2x_2^2\sin^3(t), \\ y = x_1, \end{cases} \tag{45}$$

in the simulation, $\rho_1 = 0.9$, $\rho_2 = 0.5$ and the initial states are chosen as $x_1(0) = x_2(0) = 0$, $\hat{\theta}(0) = 0$ and $y_1(0) = 0$. The simulation objective is to apply the developed adaptive fuzzy controller such that the boundedness of all the signals in the closed-loop system is guaranteed and the system output y follows the reference signal y_d to a small neighborhood of zero with $y_d = \sin t + \sin(0.5t)$.

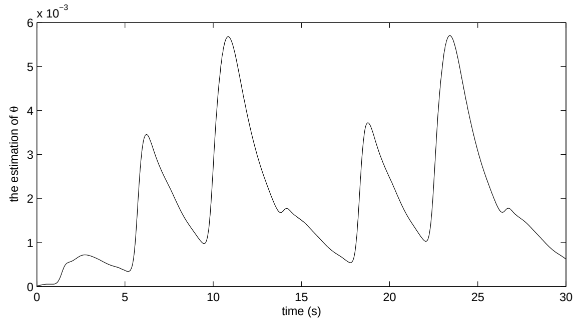

Figure 5. The Trajectory of $\hat{\theta}$

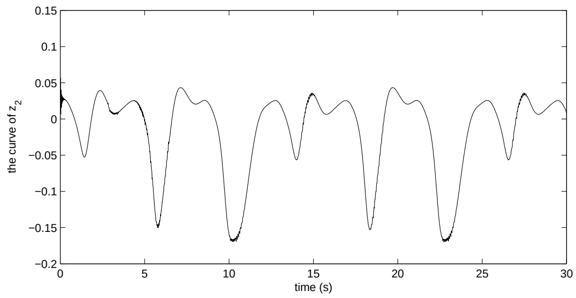

Figure 6. The Curve of z_2

The controller and the parameter adaptive laws are chosen as

$$\alpha_{1,d} = -k_1 z_1 - \frac{\hat{\theta}\xi_1^T\left(\mathrm{Z}_1\right)\xi_1\left(\mathrm{Z}_1\right)z_1}{2\eta^2}, \quad u(t) = -k_2 z_2 - \frac{\hat{\theta}\xi_2^T\left(\mathrm{Z}_2\right)\xi_2\left(\mathrm{Z}_2\right)z_2}{2\eta^2}, \tag{46}$$

$$\dot{\hat{\theta}} = \sum_{i=1}^{2}\frac{r}{2\eta^2}\xi_i^T\left(\mathrm{Z}_i\right)\xi_i\left(\mathrm{Z}_i\right)z_i^2 - \sigma\hat{\theta}, \tag{47}$$

select the design parameters as $\eta = 10,\ \sigma = 0.3,\ \ k_1 = 60,\ k_2 = 80,\ \tau_1 = 0.01$ and $r = 1$.

The simulation results are shown by Figures 7-12. Apparently, the simulation results show that under the action of the suggested controller, a good tracking performance has been achieved.

Remark 5.3. It should be emphasized that in many existing results, for example [22, 26, 34] and the references therein, by using the separation principle, the nonlinearly parameterized functions are separated into two parts, and one part is the unknown parameter which should be designed the adaptive law, therefore, the constructed controller contains much more adjustable parameters, from the view point of the engineering application, the online computation burden is heavy (See Figures 5-6 of the reference [34] and Figure 3 of the reference [22]).

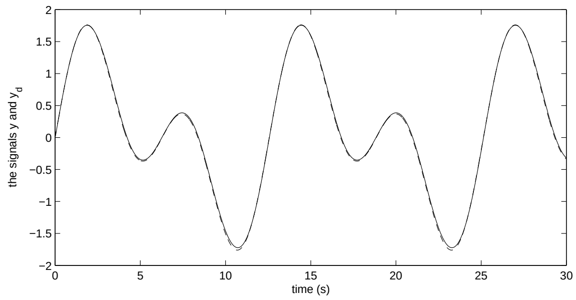

FIGURE 7. The Trajectories of x_1 and y_d (Dashed Line)

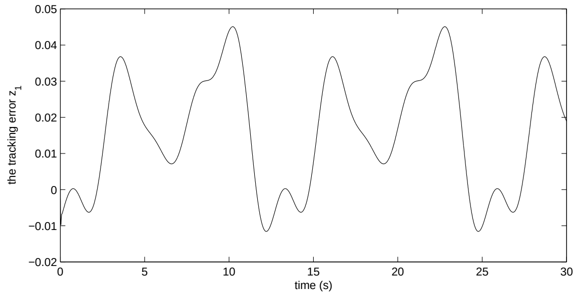

FIGURE 8. The Tracking Error z_1

However, in this manuscript, the nonlinearly parameterized functions are lumped into the nonlinear continuous functions which can be approximated by using the FLS, and then, by employing the MLP algorithm and the DSC technique, the controller contains only one adjusted parameter is constructed in (9) which reduces the online computation burden greatly than that of [22, 26, 34], and in the two examples, the boundedness of $\hat{\theta}$ is shown in Figures 5 and 11.

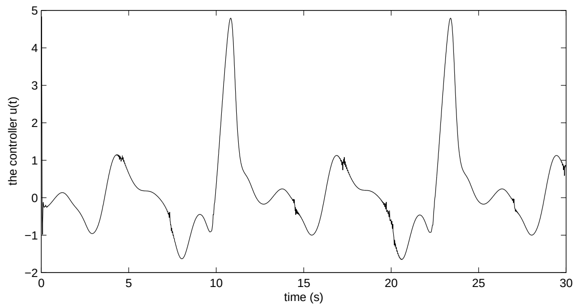

FIGURE 9. The Trajectory of $u(t)$

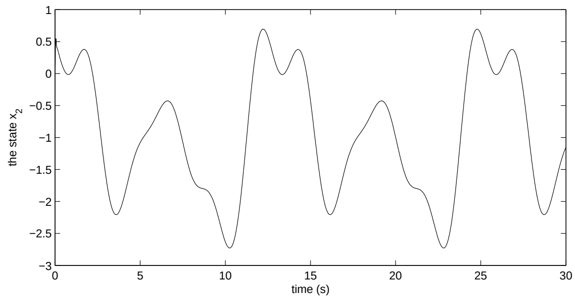

Figure 10. The State x_2

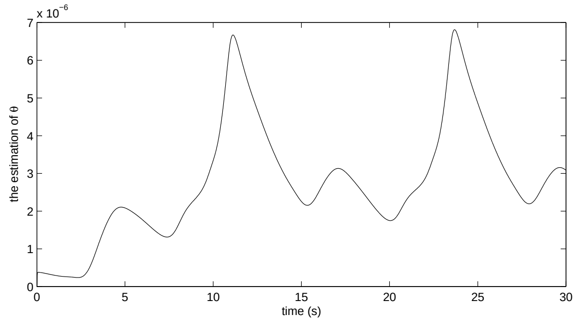

Figure 11. The Trajectory of $\hat{\theta}$

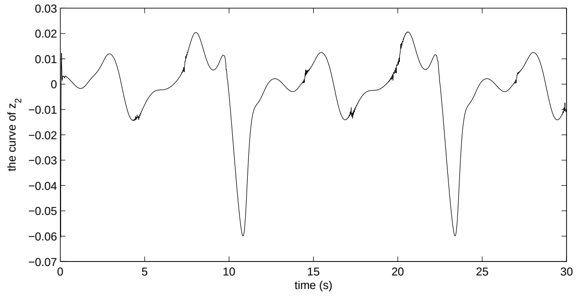

Figure 12. The Curve of z_2

6. **Conclusion**

In this paper, the problem of adaptive fuzzy tracking control has been addressed for a class of systems in which the unknown functions may be not linearly parameterized. When the functions is nonlinearly parameterized, unlike the existing results which deal with the nonlinearly parameterized functions by using the separation principle, they are lumped into the unknown continuous functions which can be approximated by using the fuzzy logic systems. The adaptive fuzzy controller has been constructed by using the backstepping approach, the MLP algorithm and the DSC technique. The proposed controller ensures that all the signals of the resulting closed-loop system are bounded, and the tracking error may be made arbitrarily small by adjusting the parameters. Moreover, it should be emphasized that the proposed controller contains only one adjustable parameter which significantly reduces the online computation burden.

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