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An efficient approach for availability analysis through fuzzy differential equations and particle swarm optimization

N. Singhal[1] and S. P. Sharma[2]

[1,2] *Department of Mathematics, Indian Institute of Technology, Roorkee, India*

nehasinghal.iitr@gmail.com, sspprfma@iitr.ac.in

Abstract

This article formulates a new technique for behavior analysis of systems through fuzzy Kolmogorov's differential equations and Particle Swarm Optimization. For handling the uncertainty in data, differential equations have been formulated by Markov modeling of system in fuzzy environment. First solution of these derived fuzzy Kolmogorov's differential equations has been found by Runge-Kutta fourth order method and thereafter the solution has been improved by Particle Swarm Optimization. Fuzzy availability is estimated in its transient as well as steady states. Sensitivity analysis has also been done to find the relative importance of a particular component of the system. Butter oil processing plant as an industrial system has been studied as a case for application of the proposed approach.

Keywords: Availability, Markov Model, Fuzzy numbers, Optimization, Particle Swarm Optimization (PSO), Kolmogorov's differential equations.

1 Introduction

Reliability of a system is ability to execute a required function under operational and environmental conditions in stipulated period of time. Reliability is considered as one of the most important quality features of technical products and used to improve the productivity of system. Availability is also considered as a critical measure of behavior of a system, as mostly the systems are repairable ones. The main objective of reliability/availability study is to provide information as a basis for making decisions. A system normally comprises a number of subsystems that are interconnected in such a manner that the system is able to execute a set of required functions. One of the importance of system reliability/availability is to discover the weakness in the system and quantify the influence of subsystems' failures. Reliability measures are used to estimate and order the impact of a particular subsystem within a system design. In realistic situations, the analyst has to derive stochastic models of the system. A mathematical model is essential in order to handle data and use statistical and mathematical methods to evaluate reliability/availability or risk parameters. For this, number of techniques are available in literature. Some of them are fault tree, petri nets, bayesian and Markovian approach [21, 32, 38, 40, 45] etc.

To estimate and improve reliability/availability of a system from its mathematical model, we need input data. Conventional reliability theory is based on probabilistic approach, but results obtained from probabilistic approach do not always provide helpful information. Data, we collect, are either from past history or as observed by experts. Usually these data are incomplete, vague or uncertain. These types of data usually do not provide certain information. Thus probabilistic approach to the traditional reliability is insufficient to tackle uncertainties in data. To handle these types of difficulties, methodologies based on fuzzy set theory, proposed by Zadeh [47] came into existence. A lot of work has been done in fuzzy set theory [13, 48]. Applications of fuzzy set theory in different fields have been found by many researchers [1, 34, 36, 43].

As far as field of differential equations is concerned, importance of differential equations is well known. Fuzzy set theory has been successfully implemented in differential equations. In 1982, Dubois and Parade [14] have discussed

Corresponding Author: N. Singhal

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differentiation in fuzzy environment. Kaleva [26, 27] has discussed existence and uniqueness of a solution of fuzzy differential equations. Buckley and Feuring [8, 9] have discussed approaches for the solution of fuzzy initial value problem for n^{th} order differential equations. Bede et al. [7] have discussed first order linear fuzzy differential equations under generalized differentiability. The topic of fuzzy differential equations has been growing rapidly in recent years and a lot of work [3, 4, 5, 6, 11, 12, 19] has been done by several authors.

The concept of fuzzy set theory has been implemented in the estimation of reliability/availability of the system in different approaches by several researchers [17, 18, 22, 29, 42]. For instance, Sharma et al. [41] have discussed fuzzy modeling of system behavior for risk and reliability analysis in 2008. Park et al. [35] have discussed the probability of failure in rock slopes through fuzzy set theory in 2012. Garg et al. [23] have presented a technique for examining the reliability using soft computing based techniques. Yazdi et al. [46] have discussed failure probability analysis by employing fuzzy fault tree analysis. In 2018, Garg [20] has discussed analysis of industrial systems using different fuzzy membership functions. Fuzzy set theory has been applied in different approaches for reliability evaluation. Some relevant articles [2, 10, 17, 20, 22, 23, 29, 30, 31, 42, 46] have been depicted in Table 1.

Out of the many discussed mathematical techniques for evaluating reliability, Markov model is a commonly used and widespread technique. In 2005, Gupta et al. [24] has studied the availability of butter oil processing plant through crisp approach. Kumar and Lata [31] in 2012 have discussed reliability evaluation of condensate system using fuzzy Markov model. In 2013, Lata and Kumar [33] have discussed the reliability through solving the fuzzy differential equations. In 2015, Garg [16] has discussed the approach for reliability availability by solving the fuzzy differential equations through numerical techniques. Results provided by existing approaches [16, 31] deal with uncertainties but do not optimize availability through Markov process. This article extends the idea of finding availability by solving fuzzy differential equations through the optimization model. Numerical solution of availability has been optimized. Out of many metaheuristic techniques available in literature, authors have used particle swarm optimization. Among all, PSO is powerful and faster for many benchmark functions [25, 44]. Along with PSO is simple and easy to implement since there is not many parameters to be adjusted. In this article, a novel approach has been discussed for reliability/availability evaluation containing data uncertainty through Markov model and particle swarm optimization.

In this approach, first a system has been mathematically modelled through Markov process. A set of differential equations with fuzzy parameters and fuzzy initial conditions has been composed to handle uncertainty. Here solution of above obtained fuzzy differential equations has been found by α-cut method with the help of fourth order Runge-Kutta method and thereafter solution of the equations has been improved with the help of particle swarm optimization. Obtained solution has been compared with the existing Runge-Kutta method. Apart from that, impact of components on the system have been analyzed by varying its repair and failure rates individually and simultaneously. Based on this analysis, system analyst may analyze system performance and can plan suitable maintenance.

This present article is divided into six sections. Section-2 offers a brief overview of basic concepts used in this article. Proposed approach for reliability/availability evaluation has been described in Section-3. In Section-4, a case study of an industrial system as an application of the proposed approach has been taken. Results for performance analysis of the industrial system have been given in Section-5. Conclusion of the article is given in Section-6.

Author(s)	Year	Techniques discussed by author(s)
Singer	1990	A fuzzy set approach to fault tree and reliability analysis.
Chen	1994	Fuzzy system reliability analysis using fuzzy number arithmetic operations.
Knezevic and Odoom	2001	Reliability modeling of repairable systems using petri nets and fuzzy lambda tau methodology.
Kumar and Lata	2012	Reliability evaluation of condensate system using fuzzy Markov model.
Garg et al.	2014	An approach for analyzing the reliability of industrial systems using soft-computing based technique.
Garg	2014	Reliability, availability and maintainability analysis of industrial systems using pso and fuzzy methodology.
Komal et al.	2015	Fuzzy reliability analysis of dual-fuel steam turbine propulsion system in lng carriers considering data uncertainty.
Garg	2015	An Approach for Analyzing the Reliability of Industrial System Using Fuzzy Kolmogorov's Differential Equations.
Aggarwal et al.	2017	Mathematical modeling and fuzzy availability analysis for serial processes in the crystallization system of a sugar plant.
Yazdi et al.	2017	Failure probability analysis by employing fuzzy fault tree analysis
Garg	2018	Analysis of an industrial system under uncertain environment by using different types of fuzzy numbers

Table 1: Some available techniques for reliability evaluation using fuzzy set theory

2 Preliminaries

In this section, some basic concepts related to proposed approach have been discussed.

2.1 Basic definitions

In classical set theory, an element either belongs to a set or not. A set is defined by characteristic function $\chi_A : U \rightarrow \{0,1\}$ where $\chi_A(u) = 1$, if $u \in A$ and $\chi_A(u) = 0$, if $u \notin A$. But in fuzzy set theory, being an element of A may acquire some uncertainty, so membership is illustrated by a degree of belonging to a set.

Fuzzy Set [13, 47]: A fuzzy set $\tilde{A}$ is completely characterized by the set $\{(u, \mu_{\tilde{A}}(u)) \mid u \in U\}$ where $\mu : U \rightarrow [0,1]$. $\mu_{\tilde{A}}(u)$ determines the degree of belonging of element u in set A.

α-cut[13, 47]: α-cut of a fuzzy set $\tilde{A}$ is the crisp set $\tilde{A}^\alpha$, defined as: $\tilde{A}^\alpha = \{u \in U \mid \mu_{\tilde{A}}(u) \geq \alpha\}$. For this article, we assume $\tilde{A}^0$ as the closure of the union of all $\tilde{A}^\alpha$'s for $\alpha \in (0,1]$.

Fuzzy number [48]: A fuzzy set $\tilde{A} = \{(u, \mu_{\tilde{A}}(u)) \mid u \in R\}$ defined on the real line R is said to be a fuzzy number if

(i) $\tilde{A}$ is normal, i.e. $\exists u_0 \in R$ such that $\mu_{\tilde{A}}(u_0) = 1$.

(ii) $\tilde{A}$ is fuzzy convex set, i.e. for any elements $u_1, u_2, u_3 \in R$ such that $u_1 < u_2 < u_3$, $\mu_{\tilde{A}}(u_2) \geq \min[\mu_{\tilde{A}}(u_1), \mu_{\tilde{A}}(u_3)]$ is satisfied.

(iii) μ is upper piecewise continuous.

(iv) Support of $\tilde{A}$ i.e. $\{u \in R \mid \mu_{\tilde{A}}(u) > 0\}$ is bounded.

Triangular Fuzzy Number[48]: A fuzzy number $\tilde{A}$ with given parameters a_1, a_2, a_3 satisfying $a_1 \leq a_2 \leq a_3$ is called a Triangular Fuzzy Number, denoted by (a_1, a_2, a_3), if its membership function is defined by

$$\mu_{\tilde{A}}(u) = \begin{cases} \frac{u-a_1}{a_2-a_1}, & a_1 \leq u < a_2 \\ \frac{a_3-u}{a_3-a_2}, & a_2 \leq u < a_3 \\ 0, & \text{otherwise} \end{cases}$$

2.2 Assumptions

The following basic assumptions are made in the proposed approach.

- Repair and failure rates are independent to each other and constant.
- Probability of two or more components failed or repaired at the same time is zero.
- Repaired unit is assumed as good as new and repair is done according to first in, first out strategy.
- At any time, system is either in working or in failed state.

3 Proposed Approach

Availability Av is the probability that the system is operating satisfactorily at time t. In order to evaluate the availability of the system by Markov process, having uncertainty in parameters, following steps have been taken:

Step 1: ***Derivation of Kolmogorov differential equations through Markov process*** [39]: Consider the markov process $\{Y(t); t \geq 0\}$ with state space $\Psi = \{0, 1, 2, ...r\}$ and transition probabilities $P_{ij}(t)$. The transition probabilities of Markov process are, $P_{ij}(t) = Pr\{Y(t) = j | Y(0) = i\}$ for all $i, j \in \Psi$. Then by first considering a transition from state i to k in $(0, t)$ and then a transition from k to j in $(t, t + \Delta t)$, Chapman-Kolmogorov equations give $P_{ij}(t + \Delta t)$ as

$$P_{ij}(t + \Delta t) = \sum_{k}^{r} P_{ik}(t).P_{kj}(\Delta t) = \sum_{\substack{k=0, \\ k \neq j}}^{r} P_{ik}(t).P_{kj}(\Delta t) + P_{jj}(\Delta t)P_{ij}(t).$$

or

$$P_{ij}(t + \Delta t) - P_{ij}(t) = \sum_{k=0, k \neq j}^{r} P_{ik}(t).P_{kj}(\Delta t) - [1 - P_{jj}(\Delta t)]P_{ij}(t).$$

Dividing by Δt and taking limit as $\Delta t \rightarrow 0$, one gets

$$\lim_{\Delta t \rightarrow 0} \frac{P_{ij}(t + \Delta t) - P_{ij}(t)}{\Delta t} = \lim_{\Delta t \rightarrow 0} \sum_{k=0, k \neq j}^{r} P_{ik}(t).\frac{P_{kj}(\Delta t)}{\Delta t} - \frac{[1 - P_{jj}(\Delta t)]}{\Delta t} P_{ij}(t).$$

As the summation is finite, it leads to $\frac{dP_{ij}(t)}{dt} = \sum_{k=0,k\neq j}^{r} a_{kj}P_{ik}(t) - \alpha_j P_{ij}(t)$, where α_j is the rate at which process leaves state j and a_{kj} is the transition rate from state k to state j.
This gives rise to a set of first order linear differential equations through Markov process. Here, availability of a system is sum of the probabilities of working states.

Step 2: [***Formulation of Fuzzy differential equations (FDEs)***]: In practical situations, when equation represents a physical situation, the values of coefficients may depend on the various sources, and cannot be obtained accurately. In most of the cases, the information collected from various sources are based on the past behavior of the system and consequently do not necessarily identify the performance of the system. To deal with such type of uncertainties in the coefficients and initial values, the corresponding differential equation becomes fuzzy differential equation. For solution of such differential equations, consider a general set of linear first order fuzzy differential equation in fuzzy function $\tilde{Z}(t) = (\tilde{z}_1(t), \tilde{z}_2(t), ..., \tilde{z}_n(t))^T$, as

$$\frac{d\tilde{Z}(t)}{dt} = \tilde{C}\tilde{Z}(t) + h(t), \text{ with initial conditions } \tilde{Z}(0) = (\tilde{\phi}_1, \tilde{\phi}_2, ..., \tilde{\phi}_n)^T, \tag{1}$$

where
(i) $\tilde{\phi}_i$'s are fuzzy numbers.
(ii) $\tilde{C} = [\tilde{c}_{ij}]$ is an $n \times n$ matrix of fuzzy numbers.
(iii) $(h(t))^T = (h_1(t), h_2(t), ..., h_n(t))$, with all the $h_i(t)$'s for $i = 1, 2..., n$ as continuous functions on the interval I.
(iv) all the $\tilde{z}_i(t)$'s for $i = 1, 2, ..., n$ are fuzzy subsets of real numbers for $t \in I$.

Step 3: [***Computation of α-cuts***]: Let $(\tilde{z}_i(t))^\alpha$ be closed and bounded intervals for all t and i, defined as $(\tilde{z}_i(t))^\alpha = [(\tilde{z}_i(t))^\alpha_{(L)}, (\tilde{z}_i(t)^\alpha_{(R)}]$, where $(\tilde{z}_i(t))^\alpha_{(L)}$ and $(\tilde{z}_i(t)^\alpha_{(R)}$ are functions of t and α. Assume that all $(\tilde{z}_i)^\alpha_{(L)}$ and $(\tilde{z}_i)^\alpha_{(R)}$ are continuously differentiable functions of t for all $\alpha \in (0, 1)$, $1 \le i \le n$.

Step 4: [***Substitution of α-cuts in FDEs***]: Now substitute the α-cuts of $\tilde{Z}(t)$ into Equation (1). Then using the concepts of interval arithmetic, system of differential equations (1) reduces to following differential equations:

$$(\tilde{z}'_i(t))^\alpha_{(L)} = \sum_{j=1}^{n} b_{ij}x_j + h_i(t), \tag{2}$$

where $b_{ij}x_j = \min\{(\tilde{c}_{ij})^\alpha_{(L)}(\tilde{z}_j)^\alpha_{(L)}, (\tilde{c}_{ij})^\alpha_{(L)}(\tilde{z}_j)^\alpha_{(R)}, (\tilde{c}_{ij})^\alpha_{(R)}(\tilde{z}_j)^\alpha_{(L)}, (\tilde{c}_{ij})^\alpha_{(R)}(\tilde{z}_j)^\alpha_{(R)}\}$.

$$(\tilde{z}'_i(t))^\alpha_{(R)} = \sum_{j=1}^{n} d_{ij}x_j + h_i(t), \tag{3}$$

where $d_{ij}x_j = \max\{(\tilde{c}_{ij})^\alpha_{(L)}(\tilde{z}_j)^\alpha_{(L)}, (\tilde{c}_{ij})^\alpha_{(L)}(\tilde{z}_j)^\alpha_{(R)}, (\tilde{c}_{ij})^\alpha_{(R)}(\tilde{z}_j)^\alpha_{(L)}, (\tilde{c}_{ij})^\alpha_{(R)}(\tilde{z}_j)^\alpha_{(R)}\}$. with the initial conditions $(\tilde{z}_i(0))^\alpha_{(L)} = (\tilde{\phi})^\alpha_{(L)}$, $(\tilde{z}_i(0))^\alpha_{(R)} = (\tilde{\phi})^\alpha_{(R)}$ for $1 \le i \le n$.

Step 5: [***Formulation of Optimization Problem***]: With the help of above equations, an optimization problem is developed by using $\tilde{c}_{ij}$'s and $\tilde{z}_j$'s for α-cut level. In the form of bounded interval, input data at α-cut level is substituted in the expression. Lower and upper boundary values of these equations are obtained at α-cut level by solving the following optimization problem. $\min / \max g(\tilde{c}_{ij}, \tilde{z}_j)$, subject to $\mu_{\tilde{c}_{ij}} \ge \alpha$, $\mu_{\tilde{z}_j} \ge \alpha$, $0 \le \alpha \le 1$, where g is the fitness function, obtained by solving the differential equations (1) by using fourth order Runge-Kutta method. The obtained maximum and minimum value of g denoted by $g_{\max}$ and $g_{\min}$ respectively corresponding to α-cut level satisfy $\mu_{\tilde{g}}(g_{min}) = \mu_{\tilde{g}}(g_{max}) = \alpha$. There are many efficient techniques available for finding the global or near global solution. Out which PSO is one of the most popular evolutionary algorithm, briefly described in the next section.

Step 6: [***Solution of FDEs***]: We say $\tilde{Z}(t)$ is a fuzzy solution of Equation (1) for all t if the obtained values of $(\tilde{z}_i(t))^\alpha_{(L)}$ and $(\tilde{z}_i(t))^\alpha_{(R)}$ define the α-cuts $[(\tilde{z}_i(t))^\alpha_{(L)}, (\tilde{z}_i(t))^\alpha_{(R)}]$ of fuzzy numbers. Thus we can say that $\tilde{Z}(t)$ is a fuzzy solution of Equation (2) if the following conditions are met out.
(i) $\frac{\partial(\tilde{z}_i)^\alpha_{(L)}}{\partial\alpha} \ge 0$ and $\frac{\partial(\tilde{z}_i)^\alpha_{(R)}}{\partial\alpha} \le 0$, i.e. $(\tilde{z}_i)^\alpha_{(L)}$ increases while $(\tilde{z}_i)^\alpha_{(R)}$ decreases as α increases.
(ii) $(\tilde{z}_i)^\alpha_{(L)} \le (\tilde{z}_i)^\alpha_{(R)}$ for $\alpha = 1$.
for all $\alpha \in [0, 1]$, $t \in I$ and $1 \le i \le n$.

3.1 Particle Swarm Optimization(PSO)

PSO [28, 37] is a population based stochastic algorithm motivated by the social behavior of bird flocking and fish schooling in nature. PSO was introduced by Kennedy and Eberhart[15, 28] in 1995. As in GA, PSO exploits a population of potential solutions to explore the search space. Different from GA, in PSO, no operators motivated by

natural evolution are applied to extract a new generation of solutions. PSO relies on the exchange of information between individuals called particles of population. Starting from a randomly distributed set of particles, algorithm tries to improve the solutions according to fitness function. In PSO, global sharing of information takes place and previous experience of all other companions during the search for promising regions of environment is taken into account. The improvisation is performed through moving the particles around the search space by means of a set of simple mathematical expressions. These mathematical expressions, in the most basic form, suggest the movement of each particle towards its own best experienced position and swarm's best position so far along with some random disturbance.
This algorithm starts by initializing a flock randomly over a search space where every bird is called a particle. These particles have certain position and fly with a certain velocity. At each iteration, each particle adjusts its velocity based on its momentum and impact of its best position (Pbest) as well as the best position of its neighbors (Gbest) and then evaluate new position that the particle is fly to.

Suppose the searching space dimension is D, total number of particles are N. Position of i^{th} particle X_i can be expressed by the vector $X_i = [x_{i1}, x_{i2}, ..., x_{iD}]$. Best position of i^{th} particle is represented by $Pbest_i = [Pbest_{i1}, Pbest_{i2}, ..., Pbest_{iD}]$ and best position of the total particle swarm is denoted by the vector $Gbest = [Gbest_1, Gbest_2, ..., Gbest_D]$. Velocity of the i_{th} particle is denoted by vector $V_i = [v_{i1}, v_{i2}, ..., v_{iD}]$. Numerically, let $X_i(k)$ and $V_i(k)$ be the position and velocity of i_{th} particle in search space at k^{th} iteration respectively then its position and velocity at $(k+1)^{th}$ iteration is updated through the following equations:

$$V_i(k+1) = w.V_i(k) + c_1.r_1.(Pbest_i(k) - X_i(k)) + c_2.r_2.(Gbest(k) - X_i(k)) \tag{4}$$

$$X_i(k+1) = X_i(k) + V_i(k+1) \tag{5}$$

where c_1 and c_2 are constants and have influence in the movement of particles, r_1 and r_2 are random variables with uniform distribution between 0 and 1. w is inertia weight which shows the effect of previous velocity vector on the new velocity vector. The parameter c_1 (cognitive factor) characterizes the level of importance given by the particle to its previous positions whereas parameter c_2 (social factor) characterizes the level of importance that particle gives to the overall position. Initial position $X_i(0)$ of i^{th} particle is taken randomly from uniform distribution in the range from lower to upper bounds of i^{th} particle.

The pseudo code for PSO is given below.

```
1:  For each particle
      Initialize particle position and velocity
    end
2:  Do
3:    For each particle:
        (a) Evaluate fitness value.
        (b) If the fitness value is better than the best fitness value (Pbest) in history.
        (c) Set current value as the new Pbest.
      end
4:    Select the particle with the best fitness value of all the particles as the Gbest.
5:    For each particle:
        (a) Evaluate particle velocity according Equation (4).
        (b) Update particle position according to Equation (5).
      end
6:  While minimum error criteria or maximum iterations is not attained.
```

Table 2: Pseudo Code of Particle Swarm Optimization

4 Case Study

To illustrate the suggested approach, Butter Oil manufacturing plant [24], as a repairable industrial system has been considered. Concise description of the system is given here.

4.1 System Description

The considered industrial system is composed of six subsystems having following outlines:

(i) **Subsystem A (Separator):** This subsystem of plant consists of motors, bearings and high speed gearbox in series. It works on the principle of centrifugal force. Fats from milk are taken away in the form of cream and the retained skimmed milk is used for preparation of milk powder.

(ii) **Subsystem B (Pasteuriser):** Pasteuriser includes a motor and bearings connected in series. This subsystem is used to destroy pathogenic organisms, to desirable organisms and to inactivate the enzymes present, and

to remove volatile flavours by heating the cream up to 80^oC. Tanning substance present in cream is also removed by this subsystem. It fails through reduced state B^1 only.
(iii) **Subsystem C (Continuous Butter Making):** The subsystem includes gearbox, motor and bearings in series. First butter granules are obtained with the help of this machine. The homogenous butter is taken out from machine into butter trolleys and shifted to melting vats.
(iv) **Subsystem D (Melting Vats):** This subsystem consists of monoblock pumps, motors and bearing in series. In this section, butter is melted at about 107^oC very gently so that water evaporates from butter.
(v) **Subsystem E (Butter-Oil Clarifier):** This subsystem consists of gearbox and motor in series. In this subsystem, fine particles of butter-oil are separated from butter-oil by settling it for few hours. For storage of butter-oil, it is cooled to a temperature of $28^o - 30^oC$.
(vi) **Subsystem F (Packaging):** In this Subsystem (F), packets of processed butter-oil are produced using pouch filling machine. This subsystem is composed of circuit board and pneumatic cylinder.

Component	Separator (A)	Pasteuriser (B)	Continuous Butter Making (C)	Melting Vats (D)	Butter oil clarifier (E)	Packaging (F)
Failure rate	0.008	0.01111*, 0.0055**	0.0054	0.0027	0.0009	0.0027
Repair rate	0.41	6.00	0.40	0.70	0.30	0.65

Tablenotes 4.1.

**corresponding to failure rate of Pasteuriser (B) and **corresponding to failure rate of Pasteuriser (B) in reduced state.*

Table 3: Input data for the system

4.2 Notations

In this section, notations that are used for performance analysis of the system are given below.

○	Represents working state of the system.
▭	Represents failed state of the system.
A, B, C, D, E, F	Working states of the subsystem.
a, b, c, d, e, f	Failed states of the subsystem.
B^1	Represents the reduced state of subsystem B.
$P_1(t)$	Indicates the probability of the system working in full capacity at time $'t'$.
$P_2(t)$	Indicates the probability of the system in reduced state at time $'t'$.
$P_3(t)$ to $P_{13}(t)$	Indicates the probability of the system in failed state at time $'t'$.
$\lambda_i, i = 1, 2..., 7$	Represent failure rates of the subsystems A, C, D, E, F, B^1 and B respectively.
$\mu_i, i = 1, 2..., 6$	Represents repair rates of the subsystems A, C, D, E, F and B respectively.
Av	Availability of system

The transition diagram of this subsystem is given here as Figure 1.

4.3 Mathematical Formulation

Applying the concepts of Markov modeling and probability theory as described in Step-1 and Step-2 of proposed approach, the transition diagram (Figure 1) of this system leads to the formulation of following fuzzy differential equations:

$$\frac{d\tilde{P}_1(t)}{dt} \oplus \tilde{\delta}_1 \tilde{P}_1(t) = \sum_{j=1}^{5} \tilde{\mu}_j \tilde{P}_{j+2}(t) \oplus \tilde{\mu}_6 \tilde{P}_{13}(t) \tag{6}$$

$$\frac{d\tilde{P}_2(t)}{dt} \oplus \tilde{\delta}_2 \tilde{P}_2(t) = \sum_{j=1}^{5} \tilde{\mu}_j \tilde{P}_{j+7}(t) \oplus \tilde{\lambda}_6 \tilde{P}_1(t) \tag{7}$$

$$\frac{d\tilde{P}_{i+2}(t)}{dt} \oplus \tilde{\mu}_i \tilde{P}_{i+2}(t) = \tilde{\lambda}_i \tilde{P}_1(t), \qquad i = 1, 2, ...5 \tag{8}$$

$$\frac{d\tilde{P}_{i+7}(t)}{dt} \oplus \tilde{\mu}_i \tilde{P}_{i+7}(t) = \tilde{\lambda}_i \tilde{P}_2(t), \qquad i = 1, 2, ...5$$

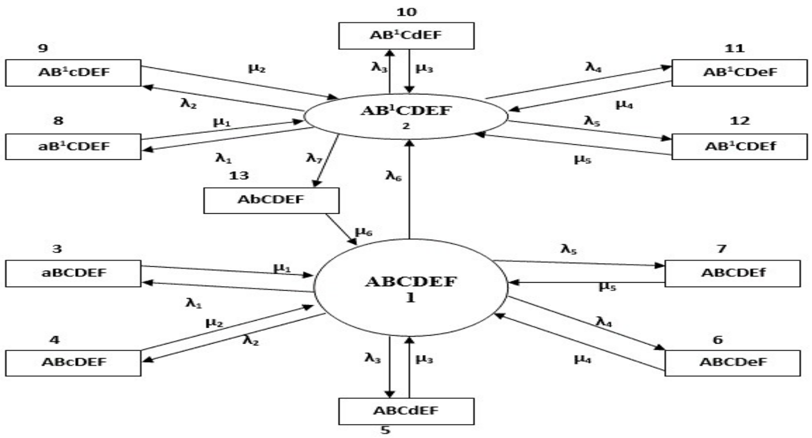


Figure 1: Transition diagram of Butter oil processing plant

(9)

$$\frac{d\tilde{P}_{13}(t)}{dt} \oplus \tilde{\mu}_6 \tilde{P}_{13}(t) = \tilde{\lambda}_7 \tilde{P}_2(t) \tag{10}$$

with $\tilde{\delta}_1 = \sum_{j=1}^{6} \tilde{\lambda}_j$ and $\tilde{\delta}_2 = \sum_{j=1}^{5} \tilde{\lambda}_j \oplus \tilde{\lambda}_7$ and initial conditions are given as, $\tilde{P}_1(0) = (1,1,1)$ and $\tilde{P}_j(0) = (0,0,0)$ for j=2 to 13.

Availability function $\tilde{Av}(t)$ of the system in terms of $\tilde{P}_1(t)$ and $\tilde{P}_2(t)$ can be obtained by $\tilde{Av}(t) = \tilde{P}_1(t) \oplus \tilde{P}_2(t)$.

4.4 Steady State Analysis

For long term availability of the system, steady state probabilities of the system are obtained by applying following limitations on probabilities: $\frac{d}{dt} \to 0$ as $t \to \infty$. In this case study, following system of equations are obtained by imposing the above restrictions.

$$P_2 = \frac{\lambda_6}{\lambda_7}P_1; \quad P_3 = \frac{\lambda_1}{\mu_1}P_1; \quad P_4 = \frac{\lambda_2}{\mu_2}P_1; \quad P_5 = \frac{\lambda_3}{\mu_3}P_1; \quad P_6 = \frac{\lambda_4}{\mu_4}P_1; \quad P_7 = \frac{\lambda_5}{\mu_5}P_1;$$

$$P_8 = (\frac{\lambda_1}{\mu_1})(\frac{\lambda_6}{\lambda_7})P_1; \quad P_9 = (\frac{\lambda_2}{\mu_2})(\frac{\lambda_6}{\lambda_7})P_1; \quad P_{10} = (\frac{\lambda_3}{\mu_3})(\frac{\lambda_6}{\lambda_7})P_1;$$

$$P_{11} = (\frac{\lambda_4}{\mu_4})(\frac{\lambda_6}{\lambda_7})P_1; \quad P_{12} = (\frac{\lambda_5}{\mu_5})(\frac{\lambda_6}{\lambda_7})P_1; \quad P_{13} = (\frac{\lambda_6}{\mu_6})P_1;$$

Substituting these values of P_1 to P_{13} in the normalizing condition $\sum_{i=1}^{13} P_i = 1$, Steady state availability becomes:

$$Av = P_1 + P_2 = \Big[\Big(1 + \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \frac{\lambda_3}{\mu_3} + \frac{\lambda_4}{\mu_4} + \frac{\lambda_5}{\mu_5}\Big) + \Big\{\mu_6\Big(\frac{1}{\lambda_6} + \frac{1}{\lambda_7}\Big)\Big\}^{-1}\Big]^{-1}.$$

5 Results and Discussion

System availability for Butter oil processing plant obtained through proposed approach in terms of transient and steady state has been discussed in this section.

5.1 Transient State

The proposed approach has been performed in MATLAB (Mathworks). In the present analysis, 20 independent runs have been made that imply 20 different random initial solutions with swarm size is 25×(no. of variables). In this case, acceleration coefficient parameters c_1 and c_2 are taken as c_1=c_2=2 with inertia weight w is explained as $w = w_{max} - (w_{max} - w_{min}) * iter/iter_{max}$, here $w_{max} = 0.9$ and $w_{min} = 0.4$ are maximum and minimum values of inertial weight respectively and $iter_{max}$ indicates the maximum generation number(=100). The termination criterion has been set either to relative error equal to 10^{-6} or maximum number of generations, whichever is obtained first.

Fuzzy system availability has been evaluated by solving the fuzzy differential equations (6)-(10) for mission time t=100 days for $\alpha = 0, 0.1, 0.2, ..., 1$ with ±15% uncertainty. It has been observed that fuzzy system availability lies in the internal $[0.9573611, 0.9574162]$ and $[0.9431705, 0.9681699]$ by existing and proposed approach respectively. Results for system availability for different α-cuts have been summarized in tabular form (Table 4) obtained by proposed and existing approaches. Comparison has also been shown through graphs. Following results are concluded by using the proposed method.

- Results provided by the existing approach [24] do not deal with uncertainty and imprecise information. Proposed approach deals with uncertainty and provides more realistic results. System availability for mission time $t = 100$ days is 0.9573854 by existing approach while keeping uncertainty in view in the light of proposed approach it is $(0.9431705, 0.9573854, 0.9681699)$. For instance, it can be seen (from Table 4) for level of uncertainty $\alpha = 0.5$, availability lies in the interval [0.9508474, 0.9631240] which shows the possibility to optimize the availability from 0.9573854 to 0.9631240.
- Results obtained by existing approach [16] deal with uncertainty and are obtained by solving fuzzy differential equations by Runge-Kutta method. Fuzzy system availability is $(0.9574162, 0.9573854, 0.9573611)$ and $(0.94317\text{-}05, 0.9573854, 0.9681699)$ by existing and proposed approach. It can be seen (from Table 4) that proposed approach has an improvement on the availability by the improvement on the solutions through particle swarm optimization. For instance, for $\alpha = 0.7$ availability lies in the interval $[0.9573775, 0.9573939]$ and $[0.9535481, 0.9609193]$ by existing and proposed approach, whose spread increases by 0.40% and 0.37%in left and right cut respectively, which shows the possibility to optimize the availability. For $\alpha = 0.5$, it is observed that right cut of the availability increases 0.60% from 0.9573999 to 0.9631240 by using proposed approach instead of existing approach. Based on these obtained results, system analyst may improve their target goals rather from traditional analysis. In case, if system analyst wants to optimize availability of system, then new target would be greater than 0.9574162 rather it will be 0.9681699 which comes from proposed approach.

	Existed Approach [24]		Existed Approach [16]		Proposed Approach	
$\alpha \downarrow$	$(\tilde{Av}(t))^{\alpha}_{(L)}$	$(\tilde{Av}(t))^{\alpha}_{(R)}$	$(\tilde{Av}(t))^{\alpha}_{(L)}$	$(\tilde{Av}(t))^{\alpha}_{(R)}$	$(\tilde{Av}(t))^{\alpha}_{(L)}$	$(\tilde{Av}(t))^{\alpha}_{(R)}$
0	0.9573854	0.9573854	0.9573611	0.9574162	0.9431705	0.9681699
0.1	0.9573854	0.9573854	0.9573633	0.9574128	0.9447922	0.9672100
0.2	0.9573854	0.9573854	0.9573655	0.9574095	0.9463640	0.9662264
0.3	0.9573854	0.9573854	0.9573679	0.9574062	0.9478880	0.9651708
0.4	0.9573854	0.9573854	0.9573702	0.9574030	0.9493665	0.9641844
0.5	0.9573854	0.9573854	0.9573725	0.9573999	0.9508474	0.9631240
0.6	0.9573854	0.9573854	0.9573750	0.9573969	0.9521947	0.9620068
0.7	0.9573854	0.9573854	0.9573775	0.9573939	0.9535481	0.9609193
0.8	0.9573854	0.9573854	0.9573801	0.9573910	0.9548803	0.9597728
0.9	0.9573854	0.9573854	0.9573827	0.9573881	0.9561419	0.9585872
1.0	0.9573854	0.9573854	0.9573854	0.9573854	0.9573854	0.9573854

Table 4: Availability of the System at t = 100 days

5.2 Steady state

To find the long term availability of the system, fuzzy steady state availability has been found at uncertainty levels ±15% and ±20%. Corresponding to different α-cuts ($\alpha = 0, 0.1, 0.2, ..., 1$), fuzzy steady state availability has been given in Table 5 at uncertainty levels ±15% and ±20%. From these results, it has been seen that steady state availability lies in the intervals $[0.9430492, 0.9680618]$ and $[0.9372473, 0.9711025]$ for ±15% and ±20% uncertainty levels respectively.

As the goal of system analyst is to maximize profit and failure free performance of system, it is important to find the impact of individual parameters of the components on the system. For this analysis, individual effects of failure and repair have been done to find the impact of that particular subsystem on the system performance. Effect of each parameter on the system availability has been shown through subplots given in Figure 2. For instance, failure rate of subsystem Separator varies from 0.005 to 0.015, keeping other parameters fixed, shows that availability varies from 0.9418774 to 0.9640235,

	Existed Approach [24]		Proposed Approach (±15% uncertainty)		Proposed Approach (±20% uncertainty)	
$\alpha\downarrow$	$(\tilde{Av})^{\alpha}_{(L)}$	$(\tilde{Av})^{\alpha}_{(R)}$	$(\tilde{Av})^{\alpha}_{(L)}$	$(\tilde{Av})^{\alpha}_{(R)}$	$(\tilde{Av})^{\alpha}_{(L)}$	$(\tilde{Av})^{\alpha}_{(R)}$
0	0.9572711	0.9572711	0.9430492	0.9680618	0.9372473	0.9711025
0.1	0.9572711	0.9572711	0.9446717	0.9671013	0.9396443	0.9699149
0.2	0.9572711	0.9572711	0.9462442	0.9661171	0.9419386	0.9686893
0.3	0.9572711	0.9572711	0.9477690	0.9651082	0.9441365	0.9674241
0.4	0.9572711	0.9572711	0.9492482	0.9640738	0.9462442	0.9661171
0.5	0.9572711	0.9572711	0.9506838	0.9630128	0.9482670	0.9647663
0.6	0.9572711	0.9572711	0.9520778	0.9619242	0.9502100	0.9633694
0.7	0.9572711	0.9572711	0.9534318	0.9608069	0.9520778	0.9619242
0.8	0.9572711	0.9572711	0.9547477	0.9596598	0.9538746	0.9604279
0.9	0.9572711	0.9572711	0.9560270	0.9584816	0.9556045	0.9588779
1.0	0.9572711	0.9572711	0.9572711	0.9572711	0.9572711	0.9572711

Table 5: Steady state availability of the system

Component	Failure rate (λ)	Availability (Min,Max)	Repair rate (μ)	Availability (Min,Max)
Separator (A)	0.005-0.015	(0.9418774,0.9640235)	0.30-0.50	(0.9507596, 0.9605004)
2*Pasteuriser (B)	0.0025-0.025*	(0.9571443,0.9575704)	2*5.00-7.00	2*(0.4900507,0.4901012)
	0.0045-0.0065**	(0.4900633,0.4900992)		
Continuous Butter Making (C)	0.002-0.008	(0.9513516,0.9651241)	0.30-0.50	(0.9531651, 0.9597518)
Melting Vats (D)	0.001-0.005	(0.9542696,0.9595018)	0.60-0.70	(0.9566824, 0.9572711)
Butter-Oil Clarifier (E)	0.0005-0.0012	(0.9563556,0.9584945)	0.20-0.40	(0.9558985, 0.9579589)
Packaging (F)	0.001-0.005	(0.3457415,0.7087912)	0.55-0.75	(0.9565795,0.95777891)

Tablenotes 5.1.

**corresponding to failure rate of Pasteuriser (B) and **corresponding to failure rate of Pasteuriser (B) in reduced state.*

Table 6: Individual effects of failure and repair rates on system availability

which gives rise an increment of 2.35%. Minimum and maximum value of the availability by variation in each parameter in each component is outlined in the Table 6.

In order to find the crucial components in the system, in preferential order, so that suitable maintenance may be taken by system analyst, the effects on availability have been studied by varying failure and repair rates of a component simultaneously. Effect of each component on the system behavior has been illustrated through the subplots given in Figure 3. From this investigation, it may be observed that, variation in the failure and repair rates of Separator (Subsystem A) from 0.005 to 0.015 and 0.30 to 0.50 respectively shows an increment of 3.86% on system availability. The complete ranges of impacts on availability by varying failure and repair rates are depicted in Table 7.

From this analysis, it has been seen that for long term availability, more attention should be followed as per the preferential order; Pasteuriser, Separator, Continuous Butter Making, Packaging, Melting Vats, Butter-Oil Clarifier. Thus system analyst may plan suitable maintenance policy to attain target goals.

6 Conclusions

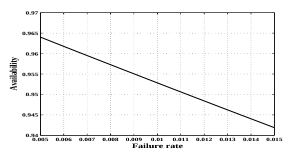
In this article, authors have discussed an efficient approach based on Runge-Kutta and Particle swarm optimization for examining the availability indices of an industrial system. Performance analysis of Butter-oil processing plant as a case study of repairable industrial system has also been discussed. In this paper, an organized framework has been developed to handle uncertain, vague information related to system behavior. This approach has been discussed for analyzing the availability through Markov process and handles uncertainty through fuzzy set theory. It has been noticed that the solution found by Runge-Kutta fourth order method is improved by Particle swarm optimization. This methodology optimizes the spread of quantitative availability which may be useful to system analyst to draw more relevant conclusions. It has been observed that improvement of solution also provides the possibility to increase the availability, which may be useful for the system analyst. Results provided by existing approach [16] deal with uncertainties but do not optimize availability through Markov process. Sensitivity analysis as well as individual performance analysis in order to study the behavior effect has been carried out for various combinations of availability parameters. On the basis of these results, system analyst may analyze the behavior of system and plan the suitable maintenance to enhance the performance of system and therefore reduce maintenance and operational cost.

Component	Failure rate (λ)	Repair rate (μ)	Availability (Min,Max)
Separator (A)	0.005-0.015	0.30-0.50	(0.9301253, 0.9660679)
2*Pasteuriser (B)	0.0025-0.025*	2* 5.00-7.00	(0.4900109,0.4901685)
	0.0045-0.0065**		(0.4900305, 0.4901176)
Continuous Butter Making (C)	0.002-0.008	0.30-0.50	(0.9453558, 0.9660565)
Melting Vats (D)	0.001-0.005	0.60-0.70	(0.9531868, 0.9595018)
Butter-Oil Clarifier (E)	0.0005-0.0012	0.20-0.40	(0.9545299, 0.9588775)
Packaging (F)	0.001-0.005	0.55-0.75	(0.9527682,0.9598627)

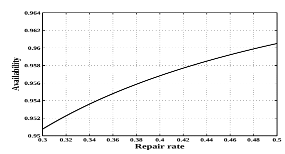
Tablenotes 5.2.

**corresponding to failure rate of Pasteuriser (B) and **corresponding to failure rate of Pasteuriser (B) in reduced state.*

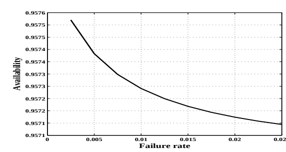
Table 7: Simultaneous effect of failure and repair rate on system availability



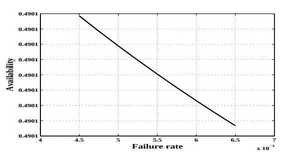
(a) Separator



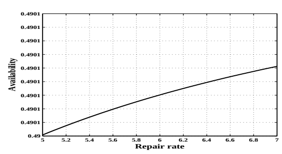
(b) Separator



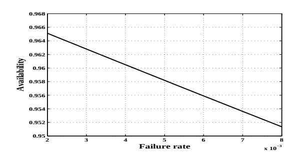
(c) Pasteuriser



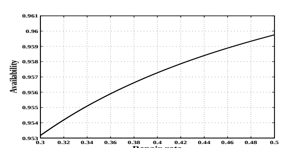
(d) Pasteuriser (reduced state)



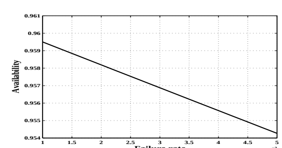
(e) Pasteuriser



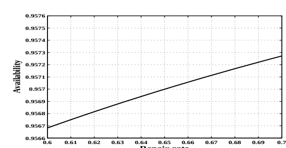
(f) Continuous Butter Making



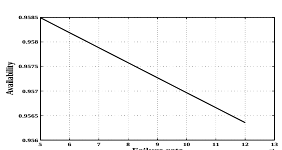
(g) Continuous Butter Making



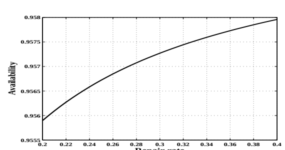
(h) Melting Vats



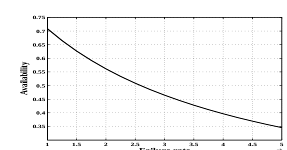
(i) Melting Vats



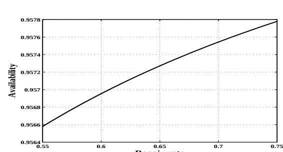
(j) Butter-Oil Clarifier



(k) Butter-Oil Clarifier



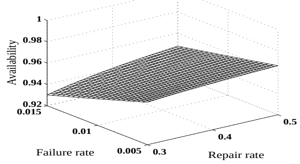
(l) Packaging



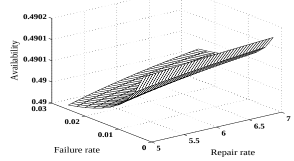
(m) Packaging

Figure 2: Individual effect of repair and failure rates on availability of system

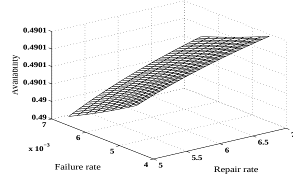
In future, other important factors like maintainability and risk analysis involving uncertainty may be studied. Further more, general types of fuzzy membership functions, proposed approach may be examined.



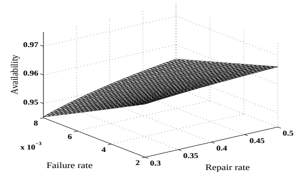
(a) Seperator



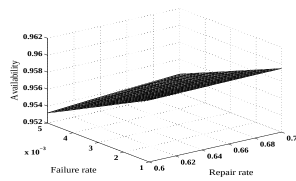
(b) Pasteuriser



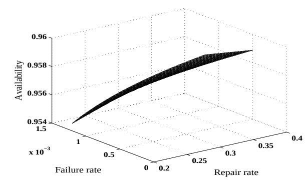
(c) Pasteuriser (reduced state)



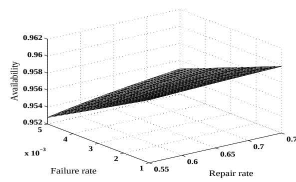
(d) Continuous Butter Making



(e) Melting Vats



(f) Butter-Oil Clarifier



(g) Packaging

Figure 3: Simultaneous effect of repair and failure rates on availability

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An efficient approach for availability analysis through fuzzy differential equations and particle swarm optimization

N. Singhal, and S. P. Sharma

یک روش کارا برای بررسی قابل دسترس بودن از طریق معادلات دیفرانسیل فازی و بهینه سازی ذره ذره

چکیده. این مقاله یک تکنیک جدید برای بررسی رفتار سیستم‌ها از طریق معادلات دیفرانسیل Kolmogorov و بهینه سازی ذره ذره را تنظیم می‌کند. برای بررسی عدم قطعیت در داده، معادلات دیفرانسیل توسط مدلسازی مارکف سیستم در محیط فازی فرمولبندی شده‌اند. اولین جواب این معادلات دیفرانسیل Kolmogorov فازی مشتق شده توسط روش مرتبه چهارم Runge-Kutta بدست آمده و بعد از آن جواب بهینه‌سازی ذره ذره بهبود یافته است. قابل دسترسی بودن فازی در چگونگی ناپایداری و ثبات خودش برآورد شده‌است. بررسی حساس بودن نیز جهت یافتن اهمیت نسبی یک مؤلفه خاص از سیستم انجام شده‌است. دستگاه پردازش روغن کره به عنوان سیستم صنعتی به عنوان موردی از کاربرد روش پیشنهاد شده مورد مطالعه قرار گرفته‌است.