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The 2-additive fuzzy Choquet integral-based TODIM method with improved score function under hesitant fuzzy environment

C. Wu[1] and D. Zhang[2]

[1,2] *School of Management, Harbin Institute of Technology, Harbin 150001, Peoples Republic of China*

wuchong@hit.edu.cn, ponder_action@163.com

Abstract

Recently, the TODIM[1](an acronym in Portuguese of interactive and multi-criteria decision making) method has attracted increasing attention and many researchers have extended it to deal with multiple attribute decision making (MADM) problems under different situations. However, none of them can be used to handle MADM problems with positive, independent, and negative interactions among attributes, which restricts the applicability of TODIM method. Therefore, in this paper, we propose the 2-additive fuzzy Choquet integral-based hesitant fuzzy TODIM method to deal with this situation. To begin with, we propose the novel measured function to compare the magnitude of hesitant fuzzy elements, which has been proved to be more rational and efficient than existing approaches. Then we use nonlinear programming to obtain 2-additive fuzzy measures and then put forward novel Choquet integral based-dominance degree to calculate the dominance degree of one alternative over another under all attributes. Consequently, we then calculate the global value of each alternative whereby we can rank all the alternatives. Finally, an illustrate example is used to demonstrate the efficiency and applicability of the proposed approach with sensitivity analysis.

Keywords: Multiple attribute decision making (MADM), TODIM, interactive attributes, 2-additive fuzzy measure, Choquet integral based-dominance degree, hesitant fuzzy set, measured function.

1 Introduction

The multiple attribute decision making (MADM) has been extensively studied in recent years, which refers to the problem of selecting better alternatives under several attributes. In order to handle MADM problems, abundant methods have been proposed such as analytic hierarchy process (AHP) approach [3, 7],Technique for order Preference by Similarity to ideal solution (TOPSIS) method [4, 25],VlseKriterij- umska Optimizacija I Kompromisno Resenje (VIKOR) method [9, 31], etc. In addition, more and more studies have focus on approaches for MADM problems under uncertain environment with the advent of fuzzy set theory and its extensions which have been proved to the better resolution for MADM problems concerning uncertain information [1, 2, 32].

Useful as these methods, they do not take psychological behavior of decision makers (DMs) into consideration. Indeed, DMs are bounded rational in the decision process and the psychological of DMs should be taken into consideration so as to deal with MADM problems [5, 6, 8, 10]. As a result, this highlights the need to consider decision makers (DMs) psychological behavior in MADM.

Based on prospect theory proposed by [28], the TODIM (an acronym in Portuguese of interactive and multi-criteria decision making) method has been proved to be a useful way to handle MADM problems concerning DMs psychological behavior [13].

Based on the value function [28], the dominance degree of each alternative over others can be calculated in TODIM. Consequently, the global dominance of each alternative can be obtained, which determines the ranking of all available alternatives. Owing to the efficiency of the TODIM, it has been extensively studied in the literature. [19] combined

[1]The full name of TODIM in Portuguese is TOmada de Decisão Interativa e Multicritério.

*Corresponding Author: D. Zhang

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fuzzy set theory with TODIM method so as to deal with challenging MADM problems under uncertainty. [12] proposed the novel hybrid TODIM method to deal with attribute values that are expressed in terms of crisp numbers, interval numbers and fuzzy numbers. Moreover, the application of the TODIM method in the real world has motivated researchers to verify its rationality and applicability. [21]studied the function of distributor in the CFPR process of the supply management and evaluate the available distributors with the help of intuitionistic fuzzy TODIM method. [20] studied the mechanism of house pricing and use the fuzzy TODIM method to aid the real estate broker.

However, these above mentioned methods are based on the assumption that criteria are mutually independent, which means that there is no interaction among the criteria. Indeed, this restricts development of the TODIM method because attributes are interactive in MADM [18, 24]. Consequently, it would be interesting to improve the TODIM method so that it can deal with MADM problems concerning interactive criteria. Up to now, very few research has paid attention to this problem. [14] attempted to study the TODIM method with criteria interactions. However, this attempt was less successful. For one thing, the proposed method is difficult for calculation, which means that $2^n - 2$ parameters have to be determined if the number of attributes is n. For another, the proposed approach can only deal with crisp values, which restricts the method to certain areas. Also, [26] proposed the extended TODIM method based λ-measure function to deal with MADM problems with interactive attributes. However, λ-measure function can only handle positive interaction or negative interaction among attributes, which limited the application of TODIM method. For this reason, it is necessary to propose an approach that can describe positive, independent and negative interactions among attributes, which can be further integrated with TODIM method to deal with MADM problems. And this is our first motivation to write this paper.

Meantime, hesitant fuzzy set (HFS) has attracted increasing attention in recent years. According to HFS [27], DMs can use serval possible values to express their attitudes towards the alternative. Therefore, HFS is more useful in MADM problems under uncertain environment, especially when DMs have different views on alternatives. However, the score functions proposed in previous studies are not valid and effective in distinguishing different HFSs. Furthermore, the existing methods did not take the non-membership degree of HFS into account when constructing the measured function. Therefore, in this paper, we propose novel measured function to better differentiate HFEs. And this is the second motivation to write this paper.

The aim of the present paper is to study the criteria interactions for TODIM method under hesitant fuzzy environment. To begin with, we combine TODIM approach with the theory of HFS. Then, we proposed the novel measured function for comparing the magnitude of hesitant fuzzy elements (HFEs), which performs better than the existing score functions. Further, we can obtain all the possible relationships among available criteria based on Shapley value and 2-additive fuzzy measure and Choquet integral (2-additive fuzzy Choquet integral).

The reminder of the paper is organized as follows. In section 2, we describe the basic knowledge of TODIM method, HFS, and introduce the theory of 2-additive fuzzy Choquet integral which can be used to describe attributes with interactions. In section 3, we aim to put forward the novel measured function to better compare the magnitude of HFEs. Then in section 4, we put forward the proposed 2-additive fuzzy Choquet integral-based TODIM method with hesitant fuzzy information, where detailed steps of calculation for the proposed method are present. Further, we use a numerable example to illustrate the rationality and efficiency of the proposed method in section 5. Finally, some conclusions are made in section 6.

2 Preliminaries

In this section, we present the basic concepts of TODIM method, hesitant fuzzy set, and the theory of 2-additive fuzzy Choquet integral, which will be used in next sections.

2.1 The description of TODIM method

Based on prospect theory proposed by [28], the TODIM approach aims to select appropriate alternatives under several attributes as well as to capture the psychological behavior of DMs. Indeed, the core part of the TODIM method is the adaptation of value function whereby the dominance degree of each alternative over others can be acquired.

Let $A = \{ A_i | i \in M\}$ be a finite alternative set and $C = \{ C_j | j \in N\}$ be a finite attribute set, where $M = \{1, 2, 3, \ldots, m\}$ and $N = \{1, 2, 3, \ldots, n\}$. Moreover, the weight vector of attributes is $w = (w_1, w_2, \cdots, w_n)^T$ and $\sum_{i=1}^{n} w_i = 1$. Further, we can obtain the decision matrix R=$[r_{ij}]_{m\times n}$, which is described as below.

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}_{m\times n}$$

Where r_{ij} denotes the rating of alternative A_i under attribute C_j by DMs. The process of TODIM method is described as follows.

Step 1: Normalize the original decision matrix $R = [r_{ij}]_{m\times n}$ into $X = [x_{ij}]_{m\times n}$, where $M = \{1, 2, ..., m\}$, $N = \{1, 2, ..., n\}$ and $i \in M, j \in N$.

Step 2: Figure out the relative weight w_{jr} of attribute C_j to the highest reference point C_r.

$$w_{jr} = C_j/C_r \tag{1}$$

where $w_r = \max\{ w_j | \, j \in N\}$

Step 3: Calculate the dominance degree of A_i over A_k under attribute C_j using the formulas as follows:

$$\Phi_j(A_i, A_k) = \begin{cases} \sqrt{(x_{ij} - x_{kj})\, w_{jr} \Big/ \left(\sum\limits_{j=1}^{n} w_{jr}\right)}\ , & x_{ij} - x_{kj} > 0 \\ 0, & x_{ij} - x_{kj} = 0 \\ -\frac{1}{\theta}\sqrt{(x_{kj} - x_{ij})\left(\sum\limits_{j=1}^{n} w_{jr}\right) \Big/ w_{jr}}, & x_{ij} - x_{kj} < 0 \end{cases} \tag{2}$$

Where θ indicates the attenuation factor of losses. There are two cases: a) if the $(x_{ij} - x_{kj})$ is positive, it donates a gain; b) if the $(x_{ij} - x_{kj})$ is negative or nil, it donates a loss.

Step 4: Calculate the overall dominance degree of A_i over A_k under all attributes.

$$\delta(A_i, A_k) = \sum_{j=1}^{n} \Phi(A_i, A_k) \quad i, k \in M \tag{3}$$

Step 5: Calculate the global value of alternative A_i with the final matrix of dominance by using the following equation:

$$\xi(A_i) = \frac{\sum\limits_{k=1}^{m} \delta(A_i, A_k) - \min\left\{\sum\limits_{k=1}^{m} \delta(A_i, A_k)\right\}}{\max\limits_{i\in M}\left\{\sum\limits_{k=1}^{m} \delta(A_i, A_k)\right\} - \min\limits_{i\in M}\left\{\sum\limits_{k=1}^{m} \delta(A_i, A_k)\right\}},\ \ i \in M \tag{4}$$

2.2 Concepts of theory of hesitant fuzzy set (HFS)

In this section, we present the definition of HFS and the Euclidean distance formula for calculating the difference between two hesitant fuzzy sets.

Definition 2.1. *Let $M = \{\mu_1, \mu_2, \ldots, \mu_n\}$ be a set of membership functions. Then the hesitant fuzzy set (HFS) associated with M, h_M, is defined as below.*

$$h_M(x) = \cup_{\mu\in M}\{\mu(x)\}$$

Definition 2.2. [30] *The hesitant Euclidean distance for HFEs is defined as below:*

$$d(\alpha, \beta) = \sqrt{\frac{1}{l}\sum_{i=1}^{l}\left|\alpha_{\sigma(i)} - \beta_{\sigma(i)}\right|^2} \tag{5}$$

Where α and β are two HFEs.$\alpha_{\sigma(i)}$ and $\beta_{\sigma(i)}$ are the i-th smallest values for α and β, respectively.

2.3 Theory of 2-additive fuzzy measure and Choquet integral

In this section, basic concepts of 2-additive fuzzy measure, Shapley index and Choquet integral are presented, which will be further used in section 4. In the sequel,$|A|$ denotes the cardinality of a set A, while $A\backslash B$ is the set difference between A and B.

Definition 2.3. [16] *Let* $X=\{x_1, x_2, \ldots, x_n\}$ *be a finite set and P(X) be the power set of X. Let* μ *be a fuzzy measure on X. Then* μ *is called 2-additive fuzzy measure if it satisfies the following conditions: (1) If* $|A|>2$*, then* m_A*=0 for* $\forall A \in P(X)$*. (2)* $\exists B \in X, |B|=2$*, and* $m_B \neq 0$*.*
Since the Mobius transform was just an abstract conception, no calculating algorithm had been put forward to figure the value of m. Consequently, [17]proposed the interaction index of a fuzzy measure so as to calculate the value of m, which is defined as below:

Definition 2.4. [17] *Let* μ *be a set measure on X. The Shapley index for every* $i \in X$ *is defined as*

$$I_i = \sum_{k=0}^{n-1} \gamma_k \sum_{K \subset X \backslash i, |K|=k} (\mu_{K \cup i} - \mu_K) \tag{6}$$

Where $\gamma_k = \frac{(n-|K|-1)!|K|!}{n!}$, $0 \leq I_i \leq 1$ *and* $\sum_{i=1}^{n} I_i = 1$.

Definition 2.5. [17] *Let* μ *be a fuzzy measure on X. The interaction index of element i, j is defined as*

$$I_{ij} = \sum_{k=0}^{n-2} \zeta \sum_{K \subset X \backslash \{i,j\}, |K|=k} (\mu_{ijK} - \mu_{iK} - \mu_{jK} + \mu_K) \tag{7}$$

Where $\zeta = \frac{(n-k-2)!k!}{(n-1)!}$.

The I_{ij} interaction index can be used as a kind of average value of the added value given by putting element i and j together, where all coalitions have been considered. For example, when the value of I_{ij} is negative, then the interaction is negative. Further, the interaction index for any subset is defined as below. Based on the definitions mentioned preciously, [17]proposed the relationships between Mobius transform and the interaction index, which is defined as follows:

Definition 2.6. [17] *For a given Mobius representation m, the corresponding interaction I is given as below:*

$$I_T = \sum_{k=0}^{n-|T|} \frac{1}{k+1} \sum_{K \subset X \backslash T, |K|=k} m_{(T \cup K)} \forall T \subset P(X) \tag{8}$$

Being one efficient way to model interactive characteristics among criteria, Choquet integral has been used as an innovative tool for criteria aggregation in MADM, which can be defined as below.

Definition 2.7. [17] *Let f be a real-valued function on X and* μ *be a fuzzy measure on X. The discrete Choquet integral of with respective to* μ *is defined by*

$$CI_\mu(f(x_1), f(x_2), ..., f(x_m)) = \sum_{j=1}^{m} f(x_{\sigma(j)}) \left[\mu(A_{\sigma(j)}) - \mu(A_{\sigma(j+1)})\right] \tag{9}$$

Where $\sigma(\cdot)$ indicates a permutation on X such that $0 \leq f(x_{\sigma(1)}) \leq f(x_{\sigma(1)}) \leq ... \leq f(x_{\sigma(m)})$. Meanwhile, $A_{\sigma(j)} = \{x_{\sigma(j)}, x_{\sigma(j+1)}, ..., x_{\sigma(m)}\}$, $A_{\sigma(m+1)}$=$\emptyset$, and $f(x_{\sigma(0)})$=0. It is apparent that the discrete Choquet integral is a linear function that reorders the elements. Furthermore, it equals to weighted mean on condition that the fuzzy measure is additive.

3 The proposed measured function under hesitant fuzzy environment

The measured function plays an important role in calculating the dominance degree of one alternative over another and thus influence the ranking of available alternatives. Accordingly, improving the measured function to make the dominance degree of each alternative over others more accurate and rational is necessary. In consideration of this, we proposed the measured function under hesitant fuzzy environment, which performs better than the existing measured functions in term of comparing the magnitude of HFEs. To begin with, we review the previous measured functions for HFEs.

Definition 3.1. [29] *For a HFE h,* $s(h) = \frac{1}{\#h}\sum_{\gamma\in h}\gamma$ *is called the score function of h, where* $\#h$ *is the number of the elements in h. For two HFEs,* h_1 *and* h_2*, if* $h_1 \succ h_2$*, then* $s(h_1) > s(h_2)$*; if* $h_1 = h_2$*, then* $s(h_1) = s(h_2)$*.*

It is worth mentioning that the score function cannot distinguish the difference of HFEs in certain cases. In order to demonstrate this, we give an example as below.

Example 3.2. *Let* $h_1 = \{0.2, 0.6\}$*,* $h_2 = \{0.4\}$ *and* $h_3 = \{0.1, 0.7\}$ *be three HFEs, respectively. Based on the score function mention above, we can calculated the final results as follows:*
$s(h_1) = 0.4$*,* $s(h_2) = 0.4$*,* $s(h_3) = 0.4$*.*

With the calculated results, we can draw the conclusion that there is no difference among these three HFEs. It is apparent that the results are not rationale for they agaist the rule of intuition, which means the score function cease to be effective and thus cause unreasonable outcome. Consequently, [11]put forward the novel ranking method for hesitant fuzzy values, which is defined as follows:

Definition 3.3. [11] *Given the following assumption: (1) The arrangement of elements in a HFE is in an increasing order; (2) For any two HFEs* h_1 *and* h_2*,* $l(h_1) \neq l(h_2)$*, where* $l(h_1)$ *and* $l(h_2)$ *denote the number of values in* h_1 *and* h_2*, respectively. In order to compute, we extend the short one by adding the maximum element until both of HFEs have the same length. And the measured function can be described as below.*

Let $h = \cup_{\gamma\in h}\{h\} = \{\gamma_j\}_{j=1}^{l(h)}$ be a HFE. The measured function of a HFE is defined by

$$\bar{S}(h) = \frac{\sum_{j=1}^{l(h)} \delta(j)\gamma_j}{\sum_{j=1}^{l(h)} \delta(j)} \tag{10}$$

Where $\{\delta(j)\}_{j=1}^{l(h)}$ is a positive-valued monotonic increasing sequence of index j. If $\bar{S}(h_1) > \bar{S}(h_2)$, then $h_1 \succ h_2$; if $\bar{S}(h_1) = \bar{S}(h_2)$, then $h_1 = h_2$. We use example 3.4 to illustrate the measured function.

Example 3.4. *Let* $h_1 = \{0.2, 0.6\}$*,* $h_2 = \{0.4\}$ *and* $h_3 = \{0.1, 0.7\}$ *be three HFEs, respectively. To begin with, we need to extend the shorter HFEs so that* h_1*,* h_2 *and* h_3 *have the same length. In other words, we can obtain* $h_1 = \{0.2, 0.6\}$*,* $h_2 = \{0.4, 0.4\}$ *and* $h_3 = \{0.1, 0.7\}$*. As a result, we can calculated the final results as follows:* $s(h_1) = 0.47$*,* $s(h_2) = 0.4$*,* $s(h_3) = 0.5$*.*

Based on example 3.2 and example 3.4, we can see that the improved measured function can distinguish the difference among h_1, h_2 and h_3, namely, $h_3 \succ h_1 \succ h_2$. It seems to be more rational and valid. However, the ranking method proposed by [11] did not obey the basic rule of HFE. To begin with, the ranking method assumed that the arrangements of elements in a HFE is in an increasing order. In other words, the method assigned different weight for different elements in a HFE, which is not in accord with the fact that the importance of each element is the same. Further, the ranking method extended the short one by adding the maximum element until both of HFEs have the same length. In that case, the risk preference of the ranking method is risk seeking. Accordingly, the ranking results would be different with the changing preference of DMs. Therefore, it is not a useful tool for ranking HFEs. On the other hand, [22]put forward the novel measured function based on the score function proposed by [29]to overcome the shortcoming described after definition 3.3, which is defined as below.

Definition 3.5. [22] *For a HFE h,* $v(h) = \frac{1}{l_h}\sqrt{\sum_{\gamma_i,\gamma_j\in h}(\gamma_i - \gamma_j)^2}$ *is called the variance function of h, where* l_h *is the number of values in h. And* $v(h)$ *is called the variance degree of h. For two HFEs* h_1 *and* h_2*, if* $v(h_1) > v(h_2)$*, then* $h_1 \prec h_2$*; if* $v(h_1) < v(h_2)$*, then* $h_1 \succ h_2$*; if* $v(h_1) = v(h_2)$*, then*$h_1 \sim h_2$*.*

Based on the above definition, [22] developed a scheme below to compare any two HFEs. Let $s(h), v(h)$ be defined as above. For any two HFEs h_1 and h_2, if $s(h_1) < s(h_2)$, then $h_1 \prec h_2$; if $s(h_1) = s(h_2)$, then (1) if $v(h_1) < v(h_2)$, then $h_1 \succ h_2$; (2) if $v(h_1) = v(h_2)$, then$h_1 \sim h_2$. We use example 3.6 to demonstrate the advantages of the measured function.

Example 3.6. *Let $h_1 = \{0.2, 0.6\}$, $h_2 = \{0.4\}$ and $h_3 = \{0.1, 0.7\}$ be three HFEs, respectively. It is easy to compute the $s(h), v(h)$, for each HFE, of which the results are as follows: $s(h_1)$=0.4, $s(h_2)$=0.4, $s(h_2)$=0.4; $v(h_1)$=0.08, $v(h_2)$=0, $v(h_3)$=0.18. It is obvious that the ranking is $h_2 \succ h_1 \succ h_3$.*

Based on the above analysis and examples, we can easily draw the conclusion that the ranking method proposed by [22] performs better than the previous two. However, the method is not effective sometimes. We use example 3.7 to demonstrate this problem:

Example 3.7. *Let $h_1 = \{0.2, 0.3, 0.7\}$, $h_2 = \{0.15, 0.35, 0.7\}$,$h_3 = \{0.16, 0.34, 0.7\}$ and $h_4 = \{0.1, 0.5, 0.6\}$ be four HFEs.*

It is easy to compute the $s(h), v(h)$, for each HFE, of which the results are as follows: $s(h_1)$=0.4, $s(h_2)$=0.4, $s(h_3)$=0.4 $s(h_4)$=0.4; $v(h_1)$=0.07, $v(h_2)$=0.0775, $v(h_3)$=0.0756, $v(h_4)$=0.07. It is apparent that the method cannot tell the difference between h_1 and h_4. Furthermore, the method cease to be effective in terms of calculation steps for it has two steps and thus may be more time-consuming compared with methods with only one step. Because measured function of fuzzy information can be classified into two categories: algorithmic approaches and non-algorithmic approaches [33]. Obviously, the method put forward by [22] belongs to algorithmic approach for it has two steps. And the method proposed by [11] belongs to non-algorithmic approach. Therefore, we aim to propose the non-algorithmic method for reducing the complexity of comparing the magnitude of HFEs. Besides, all of the existing score functions did not take the non-membership degree of HFS into consideration. Based on the two aspects mentioned above, we propose the novel score function, which is defined as below:

Definition 3.8. *For a HFE $h = \{\gamma_1, \gamma_2, \ldots, \gamma_n\}$, a new measured function $S_{new}(h)$ can be defined as below:*

$$S_{new}(h) = \frac{\prod_{i=1}^{n} (\gamma_i + \lambda)}{\prod_{i=1}^{n} (\gamma_i + \lambda) + \prod_{i=1}^{n} ((1 - \gamma_i) + \lambda)} \tag{11}$$

Where n denotes the number of values in h, $0 < \lambda < 1$. For two HFEs h_1 and h_2, if $S_{new}(h_1) > S_{new}(h_2)$, then $h_1 \succ h_2$; if $S_{new}(h_1) < S_{new}(h_2)$, then $h_1 \prec h_2$; if $S_{new}(h_1) = S_{new}(h_2)$, then $h_1 \sim h_2$. Note that $(1 - \gamma_i)$ denotes the non-membership degree of HFS. Also we add the parameter λ to avoid such cases that the HFS contains element 0. For example, if parameter λ equals to 0, the value of proposed score function for $h = \{0.5, 0, 0.6, 0.7\}$ is 0. Besides, λ is a parameter determined by the DM, which can be tuned according to the problem at hand. Based on the Definition 14, we put forward some propositions of the novel measured function, which are as follows:

Proposition 3.9. *For any HFE $h = \{\gamma_1, \gamma_2, \ldots, \gamma_n\}$, the proposed score function $S_{new}(h) \in (0, 1)$.*

Proof.

$$S_{new}(h) = \frac{\prod_{i=1}^{n} (\gamma_i+\lambda)}{\prod_{i=1}^{n} (\gamma_i+\lambda) + \prod_{i=1}^{n} ((1-\gamma_i)+\lambda)} = \frac{1}{1+\prod_{i=1}^{n} \frac{((1-\gamma_i)+\lambda)}{(\gamma_i+\lambda)}} = \frac{1}{1+\prod_{i=1}^{n} \frac{(1+2\lambda-\gamma_i-\lambda)}{(\gamma_i+\lambda)}}$$
$$= \frac{1}{1+\prod_{i=1}^{n} \left(\frac{1+2\lambda}{\gamma_i+\lambda}-1\right)}$$

since $0 \le \gamma_i \le 1$,then $\frac{\lambda}{1+\lambda} \le \frac{1+2\lambda}{\gamma_i+\lambda} - 1 \le \frac{1}{\lambda} + 1$, then $\frac{1}{1+\left(\frac{1}{\lambda}+1\right)^n} \le S_{new}(h) \le \frac{1}{1+\left(\frac{\lambda}{1+\lambda}\right)^n}$. Since $0 < \lambda < 1$, obviously, $0 < \lim_{n\to\infty} S_{new}(h) < 1$, which completes the proof. □

Proposition 3.10. *For a single-valued HFE $h = \{\gamma\}$, the proposed score function $S_{new}(h) = \frac{\gamma+\lambda}{2\lambda+1}$. Specifically, if $h = \{0\}$, then $S_{new}(h) = \frac{1}{2+\frac{1}{\lambda}}$; if $h = \{1\}$,then $S_{new}(h) = 1 - \frac{1}{2+\frac{1}{\lambda}}$.*

λ	**Ranking order**
0.1	$h_3 \succ h_1 \succ h_4 \succ h_2$
0.5	$h_3 \succ h_1 \succ h_4 \succ h_2$
0.8	$h_3 \succ h_1 \succ h_4 \succ h_2$

Table 1: Ranking orders with different values of parameter λ.

λ	**Ranking order**
Definition 3.1	h_2=h_1=h_3
Definition 3.3	$h_3 \succ h_1 \succ h_2$
Definition 3.5	$h_2 \succ h_1 \succ h_3$
Definition 3.8	$h_2 \succ h_1 \succ h_3$

Table 2: Ranking orders for different approaches.

Proof. Since the single-valued HFE has only one element γ, then the proposed score function

$$S_{new}(h) = \frac{\prod_{i=1}^{n}(\gamma_i+\lambda)}{\prod_{i=1}^{n}(\gamma_i+\lambda)+\prod_{i=1}^{n}((1-\gamma_i)+\lambda)} = \frac{\gamma+\lambda}{\gamma+\lambda+(1-\gamma)+\lambda} = \frac{\gamma+\lambda}{2\lambda+1}$$

If γ=0, then $S_{new}(h) = \frac{0+\lambda}{2\lambda+1} = \frac{\lambda}{2\lambda+1} = \frac{1}{2+\frac{1}{\lambda}}$. If γ=1, then $S_{new}(h) = \frac{1+\lambda}{2\lambda+1} = \frac{1+2\lambda-\lambda}{2\lambda+1} = 1-\frac{\lambda}{2\lambda+1} = 1-\frac{1}{2+\frac{1}{\lambda}}$. □

Proposition 3.11. *For two HFEs, $h_1 = \{\gamma_1,\gamma_2,\ldots,\gamma_m\}$, $h_2 = \{\gamma_1,\gamma_2,\ldots,\gamma_n\}$, that have same length and are arranged in ascending order. If $h_1 \le h_2$, then $S_{new}(h_1) \le S_{new}(h_2)$.*

Proof. Since h_1 and h_2 have the same length, then $l(h_1) = l(h_2)$, namely m=n. Also, h_1 and h_2 are arranged in ascending order, then $\gamma_1^1 \le \gamma_2^1 \le \ldots \le \gamma_{m-1}^1 \le \gamma_m^1$ and $\gamma_1^2 \le \gamma_2^2 \le \ldots \le \gamma_{m-1}^2 \le \gamma_m^2$, where γ_i^j denotes the i-th element in h_j, $i = 1,2,...,n$, $j = 1,2,...,n$. In addition, $h_1 \le h_2$, then $\gamma_i^1 \le \gamma_i^2$. Since $0 < \lambda < 1$ and $0 \le \gamma_i^j \le 1$, then $\frac{1+2\lambda}{\gamma_i^j+\lambda} > 1$. Thus, we can obtain $\prod_{i=1}^{n}\left(\frac{1+2\lambda}{\gamma_i^1+\lambda}-1\right) \ge \prod_{i=1}^{n}\left(\frac{1+2\lambda}{\gamma_i^2+\lambda}-1\right)$. Consequently, we can get $\frac{1}{1+\prod_{i=1}^{n}\left(\frac{2\lambda+1}{\gamma_i^1+\lambda}-1\right)} \le \frac{1}{1+\prod_{i=1}^{n}\left(\frac{2\lambda+1}{\gamma_i^2+\lambda}-1\right)}$. According to Proposition 1, $S_{new}(h) = \frac{1}{1+\prod_{i=1}^{n}\left(\frac{1+2\lambda}{\gamma_i+\lambda}-1\right)}$. As a result, we can obtain that $S_{new}(h_1) \le S_{new}(h_2)$, which completes the proof. □

We use the following to illustrate the calculation process of the proposed method.

Example 3.12. *Let $h_1 = \{0.2, 0.3, 0.7\}$, $h_2 = \{0.1, 0.4, 0.5\}$, $h_3 = \{0.3, 0.7\}$, $h_4 = \{0.1, 0.5, 0.6\}$ be four HFEs, respectively. Let λ be 0.5, based on the score function mention above, we can calculated the final results as follows: $S_{new}(h_1)$=0.35; $S_{new}(h_2)$=0.2596; $S_{new}(h_3)$=0.5, $S_{new}(h_4)$=0.3438. Obviously, we can obtain the ranking order of these three HFEs, namely, $h_3 \succ h_1 \succ h_4 \succ h_2$.*

In order to examine the influence of parameter on the ranking result, we obtain the ranking orders with different values of parameter, which can be seen in Table 1. From Table 1, it is clear that the values of parameter do not affect the ranking orders.

In order to present the difference among the four methods mentioned above, we put the ranking orders of data in above examples using different methods in Table 2. As can be seen in Table 2, the ranking order of the three HFEs calculated by the proposed method is in accord with the ranking order computed by Definition 3.5 and different with the results obtained by Definition 3.1, Definition 3.3. The reason for the difference among these results is that the method in Definition 3.1 only consider the mean value of elements in a HFE, which can result in the same result even when the HFEs are different and thus cease to be effective. Besides, the approach mentioned in Definition 3.3 does not obey the fact that all elements in a HFE have the same important and it also extent the shorter HFEs by adding the maximum element so that they can reach the same length based on the assumption that the DMs preference is risk seeking. As a result, the method is not rational compared with the existing methods. Further, as mention above, the approach mentioned in Definition 3.5 belongs to the algorithmic method, which performs less efficient compared with the proposed method belonging to non-algorithmic approach. Because the proposed can reduce the calculation complexity and thus save the time. As for the accuracy of the proposed score function, we use 10 examples to valid its efficiency (see more details in appendix). The results show that performance of the proposed score function is excellent.

4 The 2-additive fuzzy Choquet integral-based TODIM method under hesitant fuzzy environment

In this section, we present the 2-additive fuzzy Choquet integral-based TODIM for handing MADM concerning interactive characteristics among attributes under hesitant fuzzy environment. To begin with, we describe how to use nonlinear programming to obtain criteria interactions and construct Choquet integral based-dominance degree. Then, we present the 2-additive fuzzy Choquet integral-based hesitant fuzzy TODIM method.

4.1 Nonlinear programming based on maximum-entropy principal and Choquet integral based-dominance degree

This section aims to explain how to obtain the criteria interactions with nonlinear programming and integrate Choquet integral with dominance degree of TODIM method. In the following, we first put forward the nonlinear programming to get the interactions among attributes and then propose the Choquet integral-based dominance degree. In the beginning, we get the deductions based on definition 5 and 6, which can be used in the nonlinear programming. Since $I_T = \sum_{k=0}^{n-|T|} \frac{1}{k+1} \sum_{K\subset X\setminus T, |K|=k} m_{(T\cup K)}$, we can obtain the following deductions:

$$\begin{cases} I_\emptyset = m_\emptyset + \frac{1}{2}m_i + \frac{1}{3}\sum_{\{i,j\}\subset X} m_{ij} \\ I_i = m_i + \frac{1}{2}\sum_{j\in X\setminus i} m_{ij} \\ I_{ij} = m_{ij} \\ I_S = 0 \quad if\,|S| \geq 3 \end{cases} \tag{12}$$

The equation (12) has been further simplified, of which the results can be seen as below.

$$\begin{cases} m_\emptyset = I_\emptyset - \frac{1}{2}\sum_{i\in X} I_i + \frac{1}{6}\sum_{\{i,j\}\subset X} I_{ij} \\ m_i = I_i - \frac{1}{2}\sum_{j\in X\setminus i} I_{ij} \\ m_{ij} = I_{ij} \\ m_S = 0 \quad if\,|S| \geq 3 \end{cases} \tag{13}$$

Moreover, in order to determine the unique 2-additive fuzzy measure, [17] defined monotonicity constraints, $-2I_i/(n-1) \leq I_{ij} \leq 2I_i/(n-1)$ and $-2I_j/(n-1) \leq I_{ij} \leq 2I_j/(n-1)$. Based on monotonicity constraint, we can define the range of I_{ij}, where different intervals describe different interactions among attributes. The above constraints can be rewritten as $|I_{ij}| \leq \min\{2I_i/(n-1), 2I_j/(n-1)\}$. Let $g_{ij} = \min\{2I_i/(n-1), 2I_j/(n-1)\}$. Obviously, $|I_{ij}| \leq g_{ij}$, namely, $I_{ij} \in (-g_{ij}, g_{ij})$.Based on the theory of correlation coefficient in statistics, we divide the into three intervals, $[-g_{ij}, -0.3g_{ij}]$, $[-0.3g_{ij}, 0.3g_{ij}]$ and $[0.3g_{ij}, g_{ij}]$, to describe the negative interaction, mutual independent, positive interaction among attributes, respectively. And the selected interval can be defined as $\hat{g}_{ij}$. Then, the next step is to obtain the values of I_{ij}. With the constraints mentioned above, we need to find out proper objective function. Shannon entropy can be used to measure the amount of information in fuzzy measure [23], which can be taken as the objective function. As a result, the more information in fuzzy measure, the better the objective function. For 2-additive fuzzy measure, the objective function is defined as follows:

$$\sum_{i=1}^{n} \sum_{T\subset X\setminus i} \frac{(n-[T]-1)!\,[T]!}{n!} \times h\left[\mu_{T\cup c_i} - \mu_T\right]$$

Where $h(x) = -x\ln(x)$ We can further simplify the objective function. Since

$$\begin{aligned} \mu_{T\cup i} - \mu_T &= I_i - \tfrac{1}{2}\sum_{j\in X/i} I_{ij} + \sum_{j\in T} I_{ij} = I_i - \tfrac{1}{2}\left(\sum_{j\in (T\cup X\setminus(T))/i} I_{ij}\right) + \sum_{j\in T} I_{ij} \\ &= I_i - \tfrac{1}{2}\left(\sum_{j\in X\setminus(T\cup i)} I_{ij} + \sum_{j\in T} I_{ij}\right) + \sum_{j\in T} I_{ij} \\ &= I_i - \tfrac{1}{2}\sum_{j\in X\setminus(T\cup i)} I_{ij} + \tfrac{1}{2}\sum_{j\in T} I_{ij} \end{aligned}$$

Then we can obtain the nonlinear programming, which is defined as below:

$$\max z = \sum_{i=1}^{n} \sum_{T\subset X\backslash i} \frac{(n-[T]-1)!\,[T]!}{n!} \times h\left[I_i - \frac{1}{2}\sum_{j\in X\backslash(T\cup i)} I_{ij} + \frac{1}{2}\sum_{j\in T} I_{ij}\right]$$

$$s.t. \begin{cases} I_{ij} \in \hat{g}_{ij} \\ I_i = w_i \\ \{i,j\} \in X, i \neq j \end{cases} \tag{14}$$

Where w_i is the weight of attribute i. With the nonlinear programming, we can obtain the values of I_{ij} whereby we can calculate the values of Mobius and the corresponding 2-additive fuzzy measure μ_{ij}. Then, we can use the μ_{ij} value and Choquet integral to obtain Choquet integral based-dominance degree of one alternative over another.

In order to describe the methods, we assume that the alternative set is $A = \{A_1, A_2, \ldots, A_m\}$ and the attribute set is $C = \{c_1, c_2, \ldots, c_n\}$. In the following, indicates the gain value or loss value of alternative i over alternative k under attribute c_j. We can calculate the $z_{ik}^{\sigma(j)}$ value based on our proposed score function and equation (5). Further, based on the theory of Choquet integral, the values of $z_{ik}^{\sigma(j)}$ are rearranged in ascending order. In other words, ,where j denotes the j-th smallest value. According to the Choquet integral, we can obtain the weight of c_j, namely $\mu\left(U_{\sigma(j)} - U_{\sigma(j+1)}\right)$, where $U_{\sigma(j)}$ denotes $c_{(j)}, c_{(j+1)}, \ldots, c_{(n)}$.

Based on TODIM, we can obtain the Choquet integral based-dominance degree of A_i over A_k under attribute c_j, which can be defined as below:

$$\Phi^-\left(A_i, A_k\right) = -\frac{1}{\theta}\sum_{j=1}^{d}\sqrt{-z_{ik}^{\sigma(j)} \Big/ \left(\mu\left(U_{\sigma(j)}\right) - \mu\left(U_{\sigma(j+1)}\right)\right)}, i,k = 1,2,\ldots,m \tag{15}$$

$$\Phi^+\left(A_i, A_k\right) = \sum_{d+1}^{n}\sqrt{z_{ik}^{\sigma(j)}\left(\mu\left(U_{\sigma(j)}\right) - \mu\left(U_{\sigma(j+1)}\right)\right)}, i,k = 1,2,\ldots,m \tag{16}$$

Where $\Phi^-\left(A_i, A_k\right)$ and $\Phi^+\left(A_i, A_k\right)$ are the negative and positive dominance degree of A_i over A_k under attribute c_j. Besides, θ indicates the attenuation factor of losses. Consequently, we can obtain the dominance degree of alternative i and alternative k by the following expression.

$$\Phi(A_i, A_k) = \Phi^-(A_i, A_k) + \Phi^+(A_i, A_k) \tag{17}$$

With the above nonlinear programming and Choquet integral based-dominance degree, we can further rank all the alternatives. We talk about the decision steps in the following section 4.2.

4.2 The 2-additive fuzzy Choquet integral based- hesitant fuzzy TODIM method

This section describes how to combine 2-additive fuzzy Choquet integral and TODIM method to deal with MADM problems concerning criteria interactions under hesitant fuzzy environment. To demonstrate the proposed method, we the above mentioned alternative set and the attribute set in section 4.1. Further, we can obtain original decision matrix with hesitant fuzzy information, which is described as below:

$$D = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mn} \end{bmatrix}$$

Where h_{ij} denotes the rating of alternative A_i under attribute c_j, $A_i \in A$, $C_j \in C$. The steps of proposed method are explained as follows:

Step 1:

Normalize the original decision matrix D into standardized decision matrix X. For benefit attribute c_j, $x_{ij} = h_{ij}$ and for cost attribute c_j, $x_{ij} = (h_{ij})^C$. Where $(h_{ij})^C$ is the complement of h_{ij} and $(h_{ij})^C = \cup_{\gamma_{ij}\in h_{ij}}\{1-\gamma_{ij}\}$. In addition, x_{ij} denotes the standardized rating, whereby we can obtain the standardized matrix $X = [x_{ij}]_{m\times n}$.

Step 2:

Calculating the Shapley value. It can be determined by decision makers with AHP method. Moreover, Shapley vector can be expressed as $I = (I_{C_1}, I_{C_2}, \ldots, I_{C_n})^T$. Then, we can calculate interaction index I_{ij} for any criteria i, j with the following nonlinear programming.

$$\max z = \sum_{i=1}^{n} \sum_{T \subset X \backslash i} \frac{(n - [T] - 1)!\,[T]!}{n!} \times h \left[I_i - \frac{1}{2} \sum_{j \in X \backslash (T \cup i)} I_{ij} + \frac{1}{2} \sum_{j \in T} I_{ij} \right]$$

$$s.t. \begin{cases} I_{ij} \in \hat{g}_{ij} \\ I_i = w_i \\ \{i, j\} \in X, i \neq j \end{cases} \tag{18}$$

Note that decision makers can tell the possible interactions between any two attributes, with which different intervals can be used in the constraints. With the above nonlinear programming, we can compute the I_{ij}, based on which we can obtain the 2-additive fuzzy measure values for all subsets from C.

Step 3:

Calculate the dominance degree of one alternative over another under all attributes based on the Choquet integral, which are described using the following expressions:

$$\Phi^-(A_i, A_k) = -\frac{1}{\theta} \sum_{j=1}^{d} \sqrt{-z_{ik}^{\sigma(j)} \Big/ \left(\mu\left(U_{\sigma(j)}\right) - \mu\left(U_{\sigma(j+1)}\right)\right)}, i, k = 1, 2, \ldots, m \tag{19}$$

$$\Phi^+(A_i, A_k) = \sum_{d+1}^{n} \sqrt{z_{ik}^{\sigma(j)} \left(\mu\left(U_{\sigma(j)}\right) - \mu\left(U_{\sigma(j+1)}\right)\right)}, i, k = 1, 2, \ldots, m \tag{20}$$

Where $\Phi^-(A_i, A_k)$ and $\Phi^+(A_i, A_k)$ are the negative and positive dominance degree of A_i over A_k under attribute c_j. Besides, θ indicates the attenuation factor of losses. Consequently, we can obtain the dominance degree of alternative i and alternative k by the following expression.

$$\Phi(A_i, A_k) = \Phi^-(A_i, A_k) + \Phi^+(A_i, A_k) \tag{21}$$

Step 4:

Calculate the overall dominance of alternative i over other alternatives with the following equation.

$$\delta(A_i) = \sum_{k=1}^{m} \Phi(A_i, A_k) \quad i, k \in \{1, 2, \ldots, m\}, i \neq k \tag{22}$$

Step 5:

Compute the global value of alternative A_i with the final matrix of dominance by using the following equation:

$$\xi(A_i) = \frac{\delta(A_i) - \min_{i \in M}\{\delta(A_i)\}}{\max_{i \in M}\{\delta(A_i)\} - \min_{i \in M}\{\delta(A_i)\}}, \quad i \in \{1, 2, \ldots, m\} \tag{23}$$

According to Equation (4), we can get the value $\xi(A_i)$ for each alternative. Consequently, we can rank all the alternatives.

5 Illustration example

In this section, we use a numerable example of selecting the better project under four attributes to demonstrate the valid and applicability of the proposed approach.

	c_1	c_2	c_3	c_4
A_1	0.2,0.4,0.7	0.2,0.6,0.8	0.2,0.3,0.6,0.7,0.9	0.3,0.4,0.5,0.7,0.8
A_2	0.2,0.4,0.7,0.9	0.1,0.2,0.4,0.5	0.3,0.4,0.6,0.9	0.5,0.6,0.8,0.9
A_3	0.3,0.5,0.6,0.7	0.2,0.4,0.6	0.3,0.5,0.7,0.8	0.2,0.5,0.6,0.7
A_4	0.3,0.5,0.6	0.2,0.4	0.5,0.6,0.7	0.8,0.9

Table 3: Original decision information presented with hesitant fuzzy information

Subset	m	μ	Subset	m	μ
c_1	0.4015	0.4015	c_2, c_4	0.016	0.3515
c_2	0.1060	0.1060	c_3, c_4	-0.05	0.3835
c_3	0.2040	0.2040	c_1, c_2, c_3	0	0.7695
c_4	0.2295	0.2295	c_1, c_2, c_4	0	0.810
c_1, c_2	0.016	0.5235	c_1, c_3, c_4	0	0.8040
c_1, c_3	0.026	0.6315	c_2, c_3, c_4	0	0.5215
c_1, c_4	0.035	0.666	c_1, c_2, c_3, c_4	0	1
c_2, c_3	0.016	0.3260			

Table 4: The Mobius value and fuzzy measure for subset of attribute set

5.1 Decision steps

One company is to plan the development of large projects (strategy initiatives) for the following five years. There are four available projects A_1, A_2, A_3, A_4. In addition, four attributes are considered in decision-making process, which are c_1(financial perspective), c_2(the customer satisfaction), c_3(internal business process perspective) and c_4(learning and growth perspective), respectively. Moreover, five experts are invited to assess these four alternatives. In the example, we use the proposed method to select the best project. To begin with, we obtain the original decision matrix $D{=}[h_{ij}]_{m\times n}$ (see Table 3), where $h_{ij}1$ is expressed with hesitant fuzzy information. With all the information above, the decision steps for the problem based on the proposed method are as follows.

Step 1:

Normalized the original decision information. Since all the attributes are benefit criteria, the standardized decision matrix equals to the original decision matrix.

Step 2:

Calculate the Shapley values for the four criteria based on AHP method (see details in appendix). The results are as follows.

$$I = (0.44, 0.13, 0.20, 0.23)^T$$

Based on their knowledge, the experts determine that the interaction between c_3 and c_4 is negative. And the interactions among other attributes are positive. In other words, $I_{(c_3,c_4)} \in [-\hat{g}_{ij}, -0.3\hat{g}_{ij}]$ and $*I_{(c_3,c_4)} \in [0.3\hat{g}_{ij}, \hat{g}_{ij}]*$.Then we can compute the interaction index for any two criteria $i, j \in C$ with equation (18), of which results can be seen as below. Note that $g_{ij} = \min\{2I_i/(n-1), 2I_j/(n-1)\}$.Then, we can get $I_{(c_1,c_2)}$=0.016, $I_{(c_1,c_3)}$=0.026, $I_{(c_1,c_4)}$=0.035, $I_{(c_2,c_3)}$=0.016, $I_{(c_2,c_4)}$=0.016, $I_{(c_3,c_4)}$=-0.05. Then we can obtain the Mobius value and fuzzy measure for any subset $T \in C$, where $C{=}\{c_1, c_2, c_3, c_4\}$. The results can be seen in Table 4.

Step 3:

Calculate the Choquet integral-based dominance degree of one alternative over another under all attributes with equations (15)-(17). In the problem, we assume the value of parameter θ(the attenuation factor for the losses) is 1 so as to reduce the calculation complexity. Then we can obtain the dominance matrix $\Phi(A_i, A_k)$, which can be seen as below.

$$\Phi(A_i, A_k){=}\begin{vmatrix} 0 & 0.2147 & 0.1425 & -0.8577 \\ -0.1292 & 0 & -0.0906 & -0.6826 \\ -0.0625 & 0.2048 & 0 & -0.7657 \\ 0.9167 & 0.7097 & 0.8646 & 0 \end{vmatrix}$$

Step 4:

Calculate the overall dominance of alternative i and alternative k with equation (3), of which the results are as follows. $\delta(A_1)$=-0.5006; $\delta(A_2)$=-0.9025; $\delta(A_3)$=-0.6233; $\delta(A_4)$=2.4909.

Step 5:

Compute the global value of alternative A_i with the final matrix of dominance by using the equation (4). $\xi(A_1)$=0.1184; $\xi(A_2)$=0; $\xi(A_3)$=0.0823; $\xi(A_4)$=1. Apparently, we can the ranking order of the four subjects, which is $A_4 \succ A_1 \succ A_3 \succ A_2$.

θ	Ranking order of alternatives
0.1	$A_4 \succ A_3 \succ A_1 \succ A_2$
1	$A_4 \succ A_3 \succ A_1 \succ A_2$
2	$A_4 \succ A_3 \succ A_1 \succ A_2$
3	$A_4 \succ A_3 \succ A_1 \succ A_2$

Table 5: Ranking orders of alternatives with varying values of θ.

Research method	Ranking order of alternatives
Original TODIM [15]	$A_4 \succ A_3 \succ A_1 \succ A_2$
Choquet integral based TODIM [14]	$A_4 \succ A_1 \succ A_2 \succ A_3$
Generalized Choquet integral based TODIM [26]	$A_1 \succ A_4 \succ A_3 \succ A_2$
The proposed method	$A_4 \succ A_1 \succ A_3 \succ A_2$

Table 6: Ranking orders of alternatives based on different research methods

5.2 Sensitivity analysis

The sensitivity analysis is constructed to find out whether parameter θ(the attenuation factor of losses) affects the final ranking of alternatives. As a result, we observe the results of final ranking of alternatives with the value of parameter θ varying from small to large. In the sensitivity analysis, we change the value of θ from 0.1 to 3 and then obtain the final ranking orders of alternatives as below in Table 5. As is shown in table 5, it is apparent that the final ranking orders of alternatives are sensitive to the parameter θ, for the ranking order changes with the varying values of θ. That is to say that the final ranking orders of alternatives are not constant with the changing attenuation factor of losses. This has significance in the real world for that reflect the different attitudes of DMs towards risks. For a mature enterprise which has the strong ability to affect the market, it is more likely that it will select a smaller value of θ for it can afford the losses. For a small enterprise which just starts its business in the market, it is more likely to choose a higher value of θ because it has weaker ability to have influence on the market and cannot stand the losses. On the other hand, the ranking orders of alternatives remain steady when the values of θ higher than 1. That is to say the ranking orders are usual consistent when DMs tend to be risk-seeking rather risk-aversion.

5.3 Comparative analysis

The comparative analysis is performed to compare the proposed method with other related research methods. In this section, only results are presented so as to save space, which are displayed in Table 6. Then, we discuss the results from theoretical and numerical view. From Table 6, it is apparent that the ranking results of these four methods are different. The detailed discussion can be seen as below.

1) Theoretical view

The original TODIM method and Choquet integral based TODIM method can only deal with crisp values. Therefore, we transform fuzzy sets into crisp values so as to carry out the comparative analysis. Moreover, $2^n - 2$ parameters have to be determined, in Choquet integral based TODIM method, if the number of criteria is n. Besides, the original TODIM method assumes that the criteria are independent. Furthermore, in generalized Choquet integral based TODIM method, λ-measure function is used to deal with criteria interactions. However, λ-measure function can only handle positive interaction or negative interaction among criteria, which limited the application of TODIM method. The proposed method can deal with positive, independent and negative criteria interactions at the same time and only C_n^2 parameters have to be computed, which reduce the complexity of calculation process.

2) Numerical view

In this section, we calculate the Spearman coefficient of correlation among these four ranks, which aims to check the relationship between each of the three other methods and the proposed method. The Spearman coefficient of correlation between original TODIM method and the proposed method is 0.4. The Spearman coefficient of correlation between Choquet integral based method and the proposed method is 0.8. The Spearman coefficient of correlation between generalized Choquet integral based method and the proposed method is 0.8. From the above results, it can be seen that the ranking of Choquet integral based TODIM method and generalized Choquet based TODIM method are quite related with the ranking of proposed method, while the ranking of original TODIM method not. The above results indicate the efficiency of the proposed method because it considers all possible criteria interactions including positive, independent and negative interactions.

	h_1	h_2	h_3	h_4
1	0.7,0.8,0.95	0.5,0.6,0.9	0.5,0.6,0.8	0.4,0.4,0.5
2	0.4,0.55,0.79	0.5,0.6,0.8,0.9	0.3,0.4,0.5	0.65,0.8,0.9
3	0.4,0.5,0.6,0.7	0.5,0.8,0.9	0.6,0.8	0.6,0.7,0.95
4	0.7,0.9	0.35,0.4,0.5	0.65, 0.8, 0.9, 0.95	0.4, 0.5
5	0.3, 0.45, 0.6,0.8	0.2, 0.4, 0.6,0.7	0.4, 0.5, 0.9	0.4, 0.6, 0.7
6	0.5, 0.8	0.4, 0.5	0.5, 0.8, 0.9	0.7, 0.7, 0.9
7	0.3, 0.4, 0.5,0.65	0.4, 0.5, 0.6,0.7	0.5, 0.6, 0.8,0.9	0.4, 0.5,0.7, 0.9
8	0.2, 0.3, 0.7	0.15, 0.35, 0.7	0.16, 0.34, 0.7	0.1, 0.5, 0.6
9	0.2, 0.4, 0.7	0.2, 0.6, 0.8	0.2, 0.3, 0.6,0.7,0.9	0.3, 0.4, 0.5,0.7,0.8
10	0.3, 0.5, 0.6,0.7	0.2, 0.4, 0.5,0.6	0.3, 0.5, 0.7,0.8	0.2, 0.5, 0.6,0.7

Table 7: 10 examples for method validation

6 Conclusions

In this paper, we extend the TODIM method to handle interactive characteristics among attributes under hesitant fuzzy environment. Further, the innovations of the paper can be described as follows.

To begin with, we propose the novel measure function of HFS that performs better than the existing measured functions, which is further used in calculating the dominance degree of each alternatives over others. Moreover, we put forward the 2-additive fuzzy Choquet integral-based TODIM method to deal with criteria interactions in MADM problems because 2-additve fuzzy measure can describe positive interaction, negative interaction and independent characteristic among criteria. Furthermore, we extend the interactive TODIM method with hesitant fuzzy information to handle more complicated problems.

In addition, we use the illustration example to demonstrate the rationality of our proposed method. Besides, we make full use of the sensitivity analysis and comparative analysis to valid the efficiency of the proposed approach, of which the results show that the ranking orders of alternative are sensitive to the values of parameter θ and the proposed method is superior to other existing methods. In conclusion, the proposed method is qualified to deal with uncertain MADM problems concerning interactive criteria and can be a more efficient tool for DMs.

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7 Appendix

1. Accuracy of the proposed score function In this paper, we use 10 examples (Table 7) to examine the efficiency of the proposed method. Note that the results are obtained from Liaos method (see definition 15, 16) and our proposed method (see definition 17) because Liaos method is better than the previous two approaches mentioned in section 3. The results are shown as follows. The results in Table 8 show the accuracy of the proposed method.

2. AHP method in calculation process

At first, the decision makers got the judgement matrix, which is as below:

$$R = \begin{bmatrix} 1 & 4 & 2 & 3/2 \\ 1/4 & 1 & 2/3 & 1/2 \\ 1/2 & 3/2 & 1 & 3/4 \\ 2/3 & 2 & 4/3 & 1 \end{bmatrix}$$

Then we got the maximum eigenvalue $\lambda_{\max}$ and the eigenvector $u = (u_1, u_2, u_3, u_4)$.

$\lambda_{\max}$=3.8981, u=(-0.7912, -0.2274, -0.3664, -0.4337). Obviously, we can calculate the weight of four attributes using the following equation:

As a result, the weight vector is w_i =(0.4350, 0.1250, 0.2015, 0.2385). In order the make the weight vector, we carry out the consistency check. We calculate the consistency index and the consistency ratio. The equation for consistency index is $C.I. = \frac{\lambda_{\max}-n}{n-1}$ and the equation for consistency ration is $C.R. = \frac{C.I.}{R.I.}$, where R.I. equals to 0.9 when n is 4. Consequently, C.I.=-0.034, C.R.=-0.0378. It is apparent that C.R. is less than 0.1, which indicates that the calculated weight vector is reliable.

Example	methods	Ranking order
1	Liao's method	$h_1 \succ h_2 \succ h_3 \succ h_4$
	The proposed method	$h_1 \succ h_2 \succ h_3 \succ h_4$
2	Liao's method	$h_4 \succ h_2 \succ h_1 \succ h_3$
	The proposed method	$h_4 \succ h_2 \succ h_1 \succ h_3$
3	Liao's method	$h_4 \succ h_2 \succ h_3 \succ h_1$
	The proposed method	$h_4 \succ h_2 \succ h_3 \succ h_1$
4	Liao's method	$h_3 \succ h_1 \succ h_4 \succ h_2$
	The proposed method	$h_3 \succ h_1 \succ h_4 \succ h_2$
5	Liao's method	$h_3 \succ h_4 \succ h_1 \succ h_2$
	The proposed method	$h_3 \succ h_4 \succ h_1 \succ h_2$
6	Liao's method	$h_4 \succ h_3 \succ h_1 \succ h_2$
	The proposed method	$h_4 \succ h_3 \succ h_1 \succ h_2$
7	Liao's method	$h_3 \succ h_4 \succ h_2 \succ h_1$
	The proposed method	$h_3 \succ h_4 \succ h_2 \succ h_1$
8	Liao's method	$h_1 \sim h_4 \succ h_3 \succ h_2$
	The proposed method	$h_1 \succ h_3 \succ h_2 \succ h_4$
9	Liao's method	$h_4 \succ h_3 \succ h_2 \succ h_1$
	The proposed method	$h_3 \succ h_4 \succ h_2 \succ h_1$
10	Liao's method	$h_3 \succ h_1 \succ h_4 \succ h_2$
	The proposed method	$h_3 \succ h_1 \succ h_4 \succ h_2$

Table 8: Comparative analysis between Liaos method and the proposed method

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The 2-additive fuzzy Choquet integral-based TODIM method with improved score function under hesitant fuzzy environment

C. Wu and D. Zhang

روش TODIM انتگرال – مبنای Choquet فازی 2- جمعی با تابع امتیاز اصلاح شده تحت محیط فازی مردد

چکیده. اخیراً، روش (یک سرنام پرتغالی، در تصمیم‌گیری چند معیاره و فعل و انفعالی) TODIM توجه بسیاری را جلب کرده و محققین بسیاری آن‌را به مسائل تصمیم‌گیری چند معیاره (MADM) تحت شرایط مختلف گسترش داده‌اند. با این وجود، هیچ یک از آن‌ها نمی‌تواند در مسائل MADM با فعل و انفعالات مثبت، مستقل و منفی در بین خواصی که توانایی روش TODIM را محدود می‌سازد بکار برده شود. بنابراین، در این مقاله، ما روش TODIM فازی مردد انتگرال مبنای **Choquet** فازی 2- جمعی را جهت مواجهه با این شرایط پیشنهاد می‌کنیم. برای شروع، تابع اندازه‌دار شده جدید را جهت مقایسه مقدار عناصر فازی مردد، که نشان داده شده است گویاتر و کاراتر از روش‌های موجود می‌باشد، پیشنهاد می‌کنیم بعد از آن برنامه‌ریزی خطی را جهت بدست آوردن اندازه‌های فازی 2-جمعی بکار می‌بریم و سپس درجه برتری انتگرال – مبنای **Choquet** جدید را جهت محاسبه‌ی درجه برتری یک گزینه روی دیگری تحت تمام خواص، نامزد می‌کنیم. نتیجتاً، مقدار جهانی هرگزینه را که به موجب آن می‌توانیم تمام گزینه‌های را رتبه‌بندی کنیم، محاسبه می‌کنیم. نهایتاً، یک مثال جهت نشان دادن کارایی و کاربردی بودن روش پیشنهادی با آنالیز میزان حساسیت ارائه گردیده‌است.