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On the non-parametric multivariate control charts in fuzzy environment

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Abstract

Multivariate control chats are generally used in situations where the simultaneous monitoring or control of two or more related quality characteristics is necessary. In most processes in the real world, distribution of the process characteristics are unknown or at least non-normal, so the non-parametric or distribution-free charts are desirable. Most non-parametric statistical process-control techniques depend on ranks. In this survey, we apply the fuzzy set theory to deal with the circumstances that the values of each characteristic are presented in linguistic form, so we propose non-parametric multivariate control charts based on sign and Wilcoxon signed-rank tests. The performance of the proposed charts is investigated in a simulation study. Numerical examples are used to demonstrate the effectiveness and performance of the proposed charts.

Keywords: Multivariate control charts, non-parametric tests, fuzzy logic.

1 Introduction

Control charts are usefull tools that are designed specifically for detecting any out-of-control performance of the production process. Different types of control charts have been developed in the literature, including the Shewhart charts, the cumulative sum (CUSUM) control charts, the exponentially weighted moving average (EWMA) control charts, the control charts based on change-point detection (CPD), the control charts based on likelihood ratio test, and etc. These control charts are based on a parametric models. For more details, see [11], [15].

Statistical models which distributions cannot be indexed by a finite dimensional parameter, i.e., a parametric model is unavailable to describe the process distribution, are called non-parametric models. In these circumstances, non-parametric statistical methods based on the ranking or ordering information of the observed data can be considered for making inferences about the underlying process distribution. Some control charts have been constructed using non-parametric statistical methods for process monitoring, called non-parametric control charts.

In practice, however, the qulity of a product is usually described by multiple quality characteristic variables. That is, most applications require multivariate, instead of univariate methods. Different types of multivariate control charts have been introduced for monitoring multivariate production processes, such as multivariate Shewhart charts, multivariate CUSUM (MCUSUM) charts, multivariate EWMA (MEWMA) charts, multivariate control charts by CPD, multivariate control charts by least absolute shrinkage and selection operator (LASSO), and etc; see [11], [15] for more information. When the normality assumption is violated, the non-parametric multivariate control charts are appropriate.

A thorough review of the literature on non-parametric control charts can be found in [5]. Bakir [1] complied and classified several non-parametric control charts according to the driving non-parametric idea behind each one of them. Qiu and Hawkins [14] introduced a non-parametric multivariate CUSUM control chart. Hamurkaroglu et al. [8] developed non-parametric control charts based on the Mahalanobis depth.

Bakir [2] developed a non-parametric control chart based on sign ranked like statistics when in-control process center is not specified. Zhou et al. [19] worked on non-parametric control charts besed on change point estimate. Das [6] proposed a non-parametric chart for controlling variabilty when location parameter is under control. Das [7] introduced a non-parametric Shewhart-type control chart based on the sign test considered by Puri and Sen [13]. Boone and

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Chakraborti [3] proposed two Shewhart-type based on the multivariate forms of the sign and Wilcoxon signed-rank tests. For more information, see [3], [4], [5], [12].

In this paper, we propose two non-parametric multivariate control charts for the processes with fuzzy data to control location parameter when the variability or scale parameter is under control.

The present paper introduces two non-parametric multivariate control charts for controlling process location parameter based on sign test and Wilcoxon signed-rank test when the data collected from process output are fuzzy. Therefore, the performance of the proposed charts are evaluated with respect to in-control average run length (ARL) and power to detect shift in location parameter.

Here is our paper structure. In the subsequent section, we review fuzzy set theory and some basic definitions. In section 3, we introduce two non-parametric multivariate control charts for the circumstances in which the data collected from the process are fuzzy. The performance of the proposed charts is studied in section 4. Finally, section 5 represents the conclusions.

2 Fuzzy set theory and basic definitions

Crisp consists of two separate parts, that is, yes-or-no form instead of more-or-less form. In traditional dual logic, a statement can be true or false-and nothing in between. Certainty means the structures and parameters of the model are definitely known and there are no doubts about their values or their occasion [20]. But, in the real world there are many situations that we cannot cluster the parameters exactly.

Fuzzy sets were introduced by Zadeh [18] to manipulate data and information possessing nonstatistical uncertainties. Fuzzy logic provides an inference morphology that enables approximate human reasoning capabilities to be applied to knowledge-based systems. Here, some basic definitions are presented. See [10], for more information.

Definition 2.1. ***(Fuzzy set)*** *Suppose X is a nonempty set. A fuzzy set (subset) $\widetilde{A}$ in X is described by its membership function $\mu_{\widetilde{A}} : X \to [0,1]$ and $\mu_{\widetilde{A}}(x)$ is expressed as the degree of membership of element x in fuzzy set $\widetilde{A}$ for each $x \in X$. This function also can be shown by $\widetilde{A}$, that is $\widetilde{A} : X \to [0,1]$. In this paper, we use the second notation.*

Definition 2.2. ***(Support)*** *Let $\widetilde{A}$ be a fuzzy set of X. The support of $\widetilde{A}$, denoted supp$(\widetilde{A})$, is the crisp subset of X whose members all have membership grades greater than zero in $\widetilde{A}$ by, $supp(\widetilde{A}) = \{x \in X | \widetilde{A}(x) > 0\}$.*

Definition 2.3. ***(Normal fuzzy set)*** *A fuzzy set $\widetilde{A}$ of a classical set X is called normal if there exists an $x \in X$ as $\widetilde{A}(x) = 1$. Otherwise, $\widetilde{A}$ is subnormal.*

Definition 2.4. ***(α-cut)*** *An α-level set of a fuzzy set $\widetilde{A}$ of X is a crisp set written by $\widetilde{A}_\alpha$ and is defined by*

$$\widetilde{A}_\alpha = \begin{cases} \{t \in X | \widetilde{A}(t) \geq \alpha\}; & \alpha > 0, \\ cl(supp\widetilde{A}); & \alpha = 0, \end{cases}$$

where, $cl(supp\widetilde{A})$ denotes the closure of the support of $\widetilde{A}$.

Definition 2.5. ***(Convex fuzzy set)*** *A fuzzy set $\widetilde{A}$ of X is called convex if $\widetilde{A}_\alpha$ is a convex subset of X, $\forall \alpha \in [0,1]$.*

Definition 2.6. ***(Fuzzy number)*** *A fuzzy number $\widetilde{A}$ is a fuzzy set of the real line with a normal, (fuzzy) convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by* **F**.

Definition 2.7. ***(LR fuzzy quantity)*** *Any fuzzy quantity $\widetilde{A}$ can be described as;*

$$\widetilde{A}(t) = \begin{cases} L(\frac{a-t}{\alpha}); & t \in [a-\alpha, a], \\ 1; & t \in [a,b], \\ R(\frac{t-b}{\beta}); & t \in [b, b+\beta], \\ 0; & otherwise, \end{cases}$$

where, $L : [0,1] \to [0,1]$ and $R : [0,1] \to [0,1]$, are continuous and non-increasing shape functions with $L(0) = R(0) = 1$ and $L(1) = R(1) = 0$. We call this fuzzy interval of LR-type and refer to as $\widetilde{A} = (a, b, \alpha, \beta)_{LR}$.

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Definition 2.8. ***(Triangular fuzzy number)*** *A fuzzy set $\widetilde{A}$ is called triangular fuzzy number and shown by $T(a,b,c)$ if its membership function is as the following;*

$$\widetilde{A}(t)=\begin{cases}(t-a)/(b-a); & a\le t<b,\\ (c-t)/(c-b); & b\le t<c,\\ 0; & otherwise.\end{cases}$$

In this article, we show $\widetilde{A}=(a,b,c)_T$ as triangular fuzzy number.

Definition 2.9. ***(Trapezoidal fuzzy quantity)*** *A fuzzy set $\widetilde{A}$ is called trapezoidal fuzzy quantity and shown by $T_r(a,b,c,d)$ if its membership function is as the following;*

$$\widetilde{A}(t)=\begin{cases}(t-a)/(b-a); & a\le t<b,\\ 1; & b\le t<c,\\ (d-t)/(d-c); & c\le t<d,\\ 0; & otherwise.\end{cases}$$

Here, we show $\widetilde{A}=(a,b,c,d)_{T_r}$ as trapezoidal fuzzy number.

Definition 2.10. *($D_{p,q}$**-distance**) The $D_{p,q}$-distance indexed by parameters $1\le p\le\infty$ and $0\le q\le 1$, between two fuzzy quantity $\widetilde{A}$ and $\widetilde{B}$ is a nonnegative function as follows;*

$$D_{p,q}(\widetilde{A},\widetilde{B})=\begin{cases}\left[(1-q)\int_0^1 \mid \widetilde{A}_\alpha^- - \widetilde{B}_\alpha^- \mid^p d\alpha + q\int_0^1 \mid \widetilde{A}_\alpha^+ - \widetilde{B}_\alpha^+ \mid^p d\alpha\right]^{\frac{1}{p}}; & p<\infty,\\ \\ (1-q)\sup_{0<\alpha\le 1}(\mid \widetilde{A}_\alpha^- - \widetilde{B}_\alpha^- \mid)+a\inf_{0<\alpha\le 1}(\mid \widetilde{A}_\alpha^+ - \widetilde{B}_\alpha^+ \mid); & p=\infty,\end{cases}$$

where, $\widetilde{A}_\alpha^-$ is the left side of α-cut set of fuzzy quantity $\widetilde{A}$ and $\widetilde{A}_\alpha^+$ is its right side.

The analytical properties of this distance concerns to the parameter p. The parameter q is the weighted index. If there is no reason for distinguishing any side of the fuzzy quantities, $q=\frac{1}{2}$ is recommended. See [16], for more details.

Suppose $\widetilde{A}=(a_1,a_2,a_3,a_4)_{T_r}$ and $\widetilde{B}=(b_1,b_2,b_3,b_4)_{T_r}$ are trapezoidal fuzzy quantities. Then, $D_{p,q}$ distance with $p=2$ and $q=\frac{1}{2}$ is obtained as what follows;

$$D_{2,\frac{1}{2}}(\widetilde{A},\widetilde{B})=\sqrt{\frac{1}{6}\left[\sum_{i=1}^{4}(b_i-a_i)^2+\sum_{i\in\{1,3\}}(b_i-a_i)(b_{i+1}-a_{i+1})\right]}, \tag{1}$$

and for two triangular fuzzy numbers $\widetilde{C}=(c_1,c_2,c_3)_T$ and $\widetilde{D}=(d_1,d_2,d_3)_T$ it is calculated by the followimg formula;

$$D_{2,\frac{1}{2}}(\widetilde{C},\widetilde{D})=\sqrt{\frac{1}{6}\left[(c_1-d_1)^2+2(c_2-d_2)^2+(c_3-d_3)^2+(c_2-d_2)(c_1+c_3-d_1-d_3)\right]}. \tag{2}$$

3 Proposed charts

Suppose the quality of a process products counts on p characteristics $X_1,X_2,...,X_p$ that are not independent. In other words, the quality depends on the vector $\mathbf{X}=(X_1,X_2,...,X_p)'$. Let $\mathbf{X}_1,\mathbf{X}_2,...,\mathbf{X}_n$ be random sample vectors. Therefore, we can define $\mathbf{X}$ as a $p\times n$ sample matrix as the following;

$$\mathbf{X}=\begin{pmatrix}X_{11} & X_{12} & \dots & X_{1n}\\ X_{21} & X_{22} & \dots & X_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ X_{p1} & X_{p2} & \dots & X_{pn}\end{pmatrix},$$

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where, each column in the matrix represents the p measurements recorded from a sample part. Suppose each element of this matrix is fuzzy quantity. Then, the fuzzy sample matrix can be written as

$$\widetilde{\mathbf{X}} = \begin{pmatrix} \widetilde{X}_{11} & \widetilde{X}_{12} & \dots & \widetilde{X}_{1n} \\ \widetilde{X}_{21} & \widetilde{X}_{22} & \dots & \widetilde{X}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{X}_{p1} & \widetilde{X}_{p2} & \dots & \widetilde{X}_{pn} \end{pmatrix}.$$

Assume that we attempt to monitor the in-control medians of underlying variables. Then, the imprecise null hypothesis about the median of each variable is going to be tested. Hence, the null hypothesis is as "the population median is about $\widetilde{\boldsymbol{\theta}}_0$", where $\widetilde{\boldsymbol{\theta}}_0 = (\widetilde{\theta}_{10}, \widetilde{\theta}_{20}, ..., \widetilde{\theta}_{p0})'$ and $\widetilde{\theta}_{i0}$; $i = 1, 2, ..., p$, the median of characteristics, are specified. To provide a procedure for testing this hypothesis, we construct fuzzy non-parametric statistic based on $D_{p,q}$ distance.

3.1 Multivariate sign control chart based on fuzzy data (MSCCF)

Sign statistic vector is obtained as $\mathbf{S} = (S_1, S_2, ..., S_p)'$, that its elements are the univariate sign statistics calculated from fuzzy quantities as what follows;

$$S_i = \sum_{j=1}^{n} sgn(\widetilde{X}_{ij} \ominus \widetilde{\theta}_{i0}), \tag{3}$$

where,

$$sgn(\widetilde{X}_{ij} \ominus \widetilde{\theta}_{i0}) = \begin{cases} 1; & D_{2,\frac{1}{2}}(\widetilde{X}_{ij}, \widetilde{B}_i) > D_{2,\frac{1}{2}}(\widetilde{\theta}_{i0}, \widetilde{B}_i), \\ -1; & \mathrm{D}_{2,\frac{1}{2}}(\widetilde{\mathrm{X}}_{\mathrm{ij}}, \widetilde{\mathrm{B}}_{\mathrm{i}}) < \mathrm{D}_{2,\frac{1}{2}}(\widetilde{\theta}_{\mathrm{i0}}, \widetilde{\mathrm{B}}_{\mathrm{i}}). \end{cases}$$

$\widetilde{B}_i$; $i = 1, 2, ..., p$ are arbitrary values as observation's origin. In fact, $\widetilde{B}_i$s are fuzzy values which are less (greater) than the smallest (largest) observation among sample observations of each characteristic. It should be noted that S_i; $i = 1, 2, ..., p$ are crisp values. Hence, we can apply the asymptotic distribution of sign statistic. See [9], for more information. It is known that $n^{-\frac{1}{2}}\mathbf{S} \longrightarrow^{d} N_p(\mathbf{0}, \mathbf{V})$, where,

$$\widehat{\mathbf{V}} = \begin{pmatrix} v_{11} & v_{12} & \dots & v_{1p} \\ v_{21} & v_{22} & \dots & v_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ v_{p1} & v_{p2} & \dots & v_{pp} \end{pmatrix}, \quad v_{ii} = n, \quad v_{ij} = \sum_{k=1}^{n} sgn(\widetilde{X}_{ik} \ominus \widetilde{\theta}_{i0}) sgn(\widetilde{X}_{jk} \ominus \widetilde{\theta}_{j0}).$$

Therefore, we apply the chart statistic as $SN^2 = \mathbf{S}'\widehat{\mathbf{V}}^{-1}\mathbf{S}$. For large sample sizes, the limiting distribution of the statistic SN^2 is chi-square with p degrees of freedom, i.e., χ^2_p. Hence, the MSCCF's limits are as $LCL = 0$, $UCL = \chi^2_{p,\alpha}$. For every sample or subgroup, the value of statistic SN^2 is plotted on the chart. The process is declared as out-of-control if and only if $SN^2 > \chi^2_{p,\alpha}$.

3.2 Multivariate Wilcoxon signed-rank control chart based on fuzzy data (MWSCCF)

For every sample, we calculate the Wilcoxon signed-rank statistic vector as $\mathbf{W} = (W_1, W_2, ..., W_p)'$, where,

$$W_i = \sum_{j=1}^{n} R(D_{p,q}(\widetilde{X}_{ij}, \widetilde{\theta}_{i0})) sgn(\widetilde{X}_{ij} \ominus \widetilde{\theta}_{i0}),$$

and $R(D_{p,q}(\widetilde{X}_{ij}, \widetilde{\theta}_{i0}))$; $j = 1, 2, ..., n$ are the rank values of $D_{p,q}$ distances. The statistic $n^{-\frac{3}{2}}\mathbf{W}$ is asymptotically distributed as multivariate normal with $N_p(\mathbf{0}, \mathbf{L})$, that $\mathbf{L}$ can be consistently estimated by $n^{-3}\widehat{\mathbf{L}}$. For more detials, see [9]. Therefore, $n^{-\frac{3}{2}}\mathbf{W} \longrightarrow^{d} N_p(\mathbf{0}, n^{-3}\widehat{\mathbf{L}})$, where,

$$\widehat{\mathbf{L}} = \begin{pmatrix} l_{11} & l_{12} & \dots & l_{1p} \\ l_{21} & l_{22} & \dots & l_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ l_{p1} & l_{p2} & \dots & l_{pp} \end{pmatrix}, \quad l_{ii} = \frac{n(n+1)(2n+1)}{6},$$

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and $l_{ij} = \sum_{k=1}^{n} R(D_{p,q}(\widetilde{X}_{ik}, \widetilde{\theta}_{i0}))R(D_{p,q}(\widetilde{X}_{jk}, \widetilde{\theta}_{j0}))sgn(\widetilde{X}_{ik} \ominus \widetilde{\theta}_{i0})sgn(\widetilde{X}_{jk} \ominus \widetilde{\theta}_{j0})$.

For large sample sizes, the MWSCCF statistic is given by $SR^2 = \mathbf{W}'\widehat{\mathbf{L}}^{-1}\mathbf{W}$ which approximately distributed as chi-square with p degrees of freedom. Hence, the control chart's limits are $LCL = 0$, $UCL = \chi^2_{p,\alpha}$. If SR^2 falls above the upper control limit, the system is declared out-of-control.

3.3 Numerical examples

In order to ensure the performance and effectiveness of the proposed control charts, the data from the literature, real world case and simulated data are provided in two examples.

Example 3.1. *In this example, we utilize the proposed charts statistics on a data-set by Sultan* [17]*, which is commonly used when presenting multivariate process capability indices. It is known that, for obtaining process capability, it is needed the data to be collected from in-control process. Therefore, to demonstrate the performance of the proposed procedures, we apply these data set.*

The data consist of 25 observations on Brinell hardness (BH) and Tensile strength (TS) which are two quality characteristics measured on a raw material. Here, we consider all data as triangular fuzzy numbers, and for this purpose, we consider the real data as core of fuzzy data and 5 units smaller than it as the left side and 5 units greater than it as the right side of these fuzzy numbers' support. The data are found in Table 1.

Set $\widetilde{\theta}_{10} = (175, 180, 185)_T$ *and* $\widetilde{\theta}_{20} = (47, 52, 57)_T$ *as the fuzzy median of two characteristics BH and TS, respectively. Computed values are given in Tables 2 and 3. To do the calculations, we consider* $\widetilde{B}_1 = (130, 135, 140)_T$*,* $\widetilde{B}_2 = (19, 24, 29)_T$*. Hence,* $D_{2,\frac{1}{2}}(\widetilde{\theta}_{10}, \widetilde{B}_1) = 45$*,* $D_{2,\frac{1}{2}}(\widetilde{\theta}_{20}, \widetilde{B}_2) = 28$*. Furthermore, we obtain* $\mathbf{S} = \begin{pmatrix} -7 \\ 1 \end{pmatrix}, \mathbf{V} = \begin{pmatrix} 25 & 17 \\ 17 & 25 \end{pmatrix}$*, and then, the MSCCF statistic is gained as* $SN^2 = 4.429$*, which is less than the upper control limit* $\chi^2_{2,0.005} = 10.596$*, and it is concluded that the process is in-control.*

Moreover, we gain $\mathbf{W} = \begin{pmatrix} -106 \\ 64 \end{pmatrix}, \widehat{\mathbf{L}} = \begin{pmatrix} 5525 & 3415.5 \\ 3415.5 & 5525 \end{pmatrix}$*, and the MWSCCF statistic as* $SR^2 = 6.949$ *that is less than* $UCL = \chi^2_{2,0.005} = 10.596$*. Then, we conclude that the process is in-control.*

Based on these data, It is seen that both proposed charts statistics show that the process is in-control.

Table 1: The fuzzy data and $D_{2,\frac{1}{2}}(\widetilde{X}_{ij}, \widetilde{\theta}_{i0})$ for Example 3.1.

j	$\widetilde{X}_{1j}$ (BH)	$\widetilde{X}_{2j}$ (TS)	$D_{2,\frac{1}{2}}(\widetilde{X}_{1j}, \widetilde{\theta}_{10})$	$D_{2,\frac{1}{2}}(\widetilde{X}_{2j}, \widetilde{\theta}_{20})$
1	$(138, 143, 148)_T$	$(29.2, 34.2, 39.2)_T$	37	17.8
2	$(195, 200, 205)_T$	$(52.0, 57.0, 62.0)_T$	20	5
3	$(155, 160, 165)_T$	$(42.5, 47.5, 52.5)_T$	20	4.5
4	$(176, 181, 186)_T$	$(48.4, 53.4, 58.4)_T$	1	1.4
5	$(143, 148, 153)_T$	$(42.8, 47.8, 52.8)_T$	32	4.2
6	$(173, 178, 183)_T$	$(46.5, 51.5, 56.5)_T$	2	0.5
7	$(157, 162, 167)_T$	$(40.9, 45.9, 50.9)_T$	18	6.1
8	$(210, 215, 220)_T$	$(54.1, 59.1, 64.1)_T$	35	7.1
9	$(156, 161, 166)_T$	$(43.4, 48.4, 53.4)_T$	19	3.6
10	$(136, 141, 146)_T$	$(42.3, 47.3, 52.3)_T$	39	4.7
11	$(170, 175, 180)_T$	$(52.3, 57.3, 62.3)_T$	5	5.3
12	$(172, 177, 182)_T$	$(53.5, 58.5, 63.5)_T$	3	6.5
13	$(172, 177, 182)_T$	$(53.2, 58.2, 63.2)_T$	3	6.2
14	$(171, 176, 181)_T$	$(52.0, 57.0, 62.0)_T$	4	5
15	$(167, 172, 177)_T$	$(44.4, 49.4, 54.4)_T$	8	2.6
16	$(177, 182, 187)_T$	$(52.2, 57.2, 62.2)_T$	2	5.2
17	$(172, 177, 182)_T$	$(45.6, 50.6, 55.6)_T$	3	1.4
18	$(199, 204, 209)_T$	$(50.1, 55.1, 60.1)_T$	24	3.1
19	$(173, 178, 183)_T$	$(45.9, 50.9, 55.9)_T$	2	1.1
20	$(191, 196, 201)_T$	$(52.9, 57.9, 62.9)_T$	16	5.9
21	$(155, 160, 165)_T$	$(40.5, 45.5, 50.5)_T$	20	6.5
22	$(178, 183, 188)_T$	$(48.9, 53.9, 58.9)_T$	3	1.9
23	$(174, 179, 184)_T$	$(46.2, 51.2, 56.2)_T$	1	0.8
24	$(189, 194, 199)_T$	$(52.5, 57.5, 62.5)_T$	14	5.5
25	$(176, 181, 186)_T$	$(50.6, 55.6, 60.6)_T$	1	3.6

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Table 2: $D_{2,\frac{1}{2}}(\widetilde{X}_{ij},\widetilde{B}_i)$ and signed for the data of Example 3.1.

j	$D_{2,\frac{1}{2}}(\widetilde{X}_{1j},\widetilde{B}_1)$	$D_{2,\frac{1}{2}}(\widetilde{X}_{2j},\widetilde{B}_2)$	$sgn(\widetilde{X}_{1j}\ominus\widetilde{\theta}_{10})$	$sgn(\widetilde{X}_{2j}\ominus\widetilde{\theta}_{20})$	$sgn(\widetilde{X}_{1j}\ominus\widetilde{\theta}_{10})sgn(\widetilde{X}_{2j}\ominus\widetilde{\theta}_{20})$
1	8	10.2	-1	-1	1
2	65	33	1	1	1
3	25	23.5	-1	-1	1
4	46	29.4	1	1	1
5	13	23.8	-1	-1	1
6	43	27.5	-1	-1	1
7	27	21.9	-1	-1	1
8	80	35.1	1	1	1
9	26	24.4	-1	-1	1
10	6	23.3	-1	-1	1
11	40	33.3	-1	1	-1
12	42	34.5	-1	1	-1
13	42	34.2	-1	1	-1
14	41	33	-1	1	-1
15	37	25.4	-1	-1	1
16	47	33.2	1	1	1
17	42	26.6	-1	-1	1
18	69	31.1	1	1	1
19	43	26.9	-1	-1	1
20	61	33.9	1	1	1
21	25	21.5	-1	-1	1
22	48	29.9	1	1	1
23	44	27.2	-1	-1	1
24	57.77	33.5	1	1	1
25	46	31.6	1	1	1

Table 3: Ranks and signed-rank for the data of Example 3.1.

j	$R(D_{2,\frac{1}{2}}(\widetilde{X}_{1j},\widetilde{\theta}_{10}))$	$R(D_{2,\frac{1}{2}}(\widetilde{X}_{2j},\widetilde{\theta}_{20}))$	$R(D_{2,\frac{1}{2}}(\widetilde{X}_{1j},\widetilde{\theta}_{10}))sgn(\widetilde{X}_{1j}\ominus\widetilde{\theta}_{10})$	$R(D_{2,\frac{1}{2}}(\widetilde{X}_{2j},\widetilde{\theta}_{20}))sgn(\widetilde{X}_{2j}\ominus\widetilde{\theta}_{20})$
1	24.0	25.0	-24.0	-25.0
2	19.0	14.5	19.0	14.5
3	19.0	12.0	-19.0	-12.0
4	2.0	4.5	2.0	4.5
5	22.0	11.0	-22.0	-11.0
6	5.0	1.0	-5.0	-1.0
7	16.0	20.0	-16.0	-20.0
8	23.0	24.0	23.0	24.0
9	17.0	9.5	-17.0	-9.5
10	25.0	13.0	-25.0	-13.0
11	12.0	17.0	-12.0	17.0
12	8.5	22.5	-8.5	22.5
13	8.5	21.0	-8.5	21.0
14	11.0	14.5	-11.0	14.5
15	13.0	7.0	-13.0	-7.0
16	5.0	16.0	5.0	16.0
17	8.5	4.5	-8.5	-4.5
18	21.0	8.0	21.0	8.0
19	5.0	3.0	-5.0	-3.0
20	15.0	19.0	15.0	19.0
21	19.0	22.5	-19.0	-22.5
22	8.5	6.0	8.5	6.0
23	2.0	2.0	-2.0	-2.0
24	14.0	18.0	14.0	18.0
25	2.0	9.5	2.0	9.5

Example 3.2. *We simulated 5 samples of size 20 from a multivarite normal distribution with 3 quality characteristics,*

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fuzzy mean vector $\widetilde{\boldsymbol{\mu}}_0$ *and covariance matrix* $\boldsymbol{\Sigma}$ *as*

$$\widetilde{\boldsymbol{\mu}}_0 = \begin{pmatrix} \widetilde{\mu}_{01} \\ \widetilde{\mu}_{02} \\ \widetilde{\mu}_{03} \end{pmatrix} = \begin{pmatrix} (-0.33, 0, 0.93)_T \\ (-0.49, 0, 0.81)_T \\ (-0.81, 0, 0.96)_T \end{pmatrix}, \quad \boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0.4 & 0.6 \\ 0.5 & 1 & 0.5 \\ 0.6 & 0.5 & 1 \end{pmatrix}.$$

The upper control limits for both control charts MSCCF and MWSCCF are as $\chi^2_{0.995,3} = 12.84$. *Now, we consider shifs in three characteristics mean and then, simulated another 5 samples of size 20 according to the fuzzy mean vector* $\widetilde{\boldsymbol{\mu}}_1$ *as the following;*

$$\widetilde{\boldsymbol{\mu}}_1 = \begin{pmatrix} \widetilde{\mu}_{11} \\ \widetilde{\mu}_{12} \\ \widetilde{\mu}_{13} \end{pmatrix} = \begin{pmatrix} (0.89, 1, 1.74)_T \\ (0.21, 0.5, 1.23)_T \\ (-0.31, 0, 0.01)_T \end{pmatrix}.$$

To do the calculations, we set the observation's origin vector as

$$\widetilde{\boldsymbol{B}} = \begin{pmatrix} \widetilde{B}_1 \\ \widetilde{B}_2 \\ \widetilde{B}_3 \end{pmatrix} = \begin{pmatrix} (-10.19, -10, -9.02)_T \\ (-10.29, -10, -9.18)_T \\ (-10.28, -10, -9.62)_T \end{pmatrix}$$

The computed values of two control charts statistics are presented in Table 4. Figure 1 shows the control charts.

Table 4: Values of MSCCF and MWSCCF statistics for the Example 3.2.

In-control state			Out-of-control state		
sample number	SN^2	SR^2	sample number	SN^2	SR^2
1	7.40	10.56	6	10.07	14.32
2	3.66	4.11	7	11.82	14.96
3	2.02	2.70	8	7.70	10.45
4	0.52	1.13	9	11.71	15.11
5	3.50	3.32	10	16.52	14.68

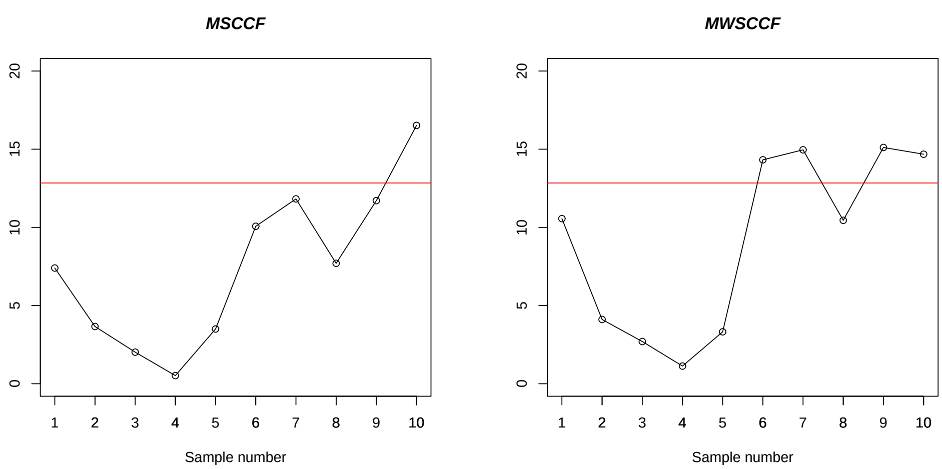

Figure 1: MSCCF and MWSCCF for the Example 3.2.

It is seen that for the data generated from the in-control process, neither MSCCF nor MWSCCF signals and when a shift in the fuzzy mean vector occured, MWSCCF control chart declares signal earlier than MSCCF.

4 Performance study

A popular measure of chart performance is the expected value of the run length (the number of samples or subgroups needed to be collected, before the first out-of-control signal is given by a chart) distribution, called the average run

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length (ARL). It is desirable that when the process is in-control, the ARL value of a chart be large and when it is out-of-control, the amount of ARL be small [7]. The smaller the value of the out-of-control ARL, the better the chart performance.

The main task of a control chart is to detect the change in the process as quikly as possible and give an out-of-control signal. In this section, a comparision between the performace of two proposed control charts is made for the cases that UCLs are detected asymptotically and in which detected based on simulated procedure from empirical distribution.

Here, the performance of the proposed control charts is investigated by the fuzzy data generated by multivariate normal and two multivariate t distributions with 5 and 10 degrees of freedom using a simulation based on 10000 runs for sample sizes of $n = 15, 30$ and 50. Computer programs written in R 3.2.3 are used to study.

The basic parameters of distributions considered in the study are as follows;

- **Bivariate normal**:

$$\boldsymbol{\mu}_0 = \begin{pmatrix}\mu_{01}\\ \mu_{02}\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}, \qquad \boldsymbol{\Sigma} = \begin{pmatrix}1 & 0.5\\ 0.5 & 1\end{pmatrix}.$$

- **Bivariate** $t(5)$:

$$\boldsymbol{\mu}_0 = \begin{pmatrix}\mu_{01}\\ \mu_{02}\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}, \qquad \boldsymbol{\Sigma}_{t(5)} = \begin{pmatrix}0.6 & 0.3\\ 0.3 & 0.6\end{pmatrix}.$$

- **Bivariate** $t(10)$:

$$\boldsymbol{\mu}_0 = \begin{pmatrix}\mu_{01}\\ \mu_{02}\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}, \qquad \boldsymbol{\Sigma}_{t(5)} = \begin{pmatrix}0.8 & 0.4\\ 0.4 & 0.8\end{pmatrix}.$$

To generate fuzzy data, first of all, we simulated data from the detected multivariate distribution, named $\mathbf{x}_i = (x_{i1}, x_{i2})'$. For each datum, we simulate four data from uniform distribution in interval $(0, 1)$, i.e., $U(0, 1)$, called u_{i11}, u_{i12}, u_{i21} and u_{i22}. Then, the triangular fuzzy data is obtained as:

$$\widetilde{\mathbf{x}}_i = \begin{pmatrix}(x_i - u_{i11}, x_i, x_i + u_{i12})_T\\ (x_i - u_{i21}, x_i, x_i + u_{i22})_T\end{pmatrix}.$$

The fuzzy triangular mean vector is as:

$$\widetilde{\boldsymbol{\mu}}_0 = \begin{pmatrix}(\mu_{01} - \frac{\sum_{i=1}^{10000} u_{i11}}{10000},\ \mu_{01},\ \mu_{01} + \frac{\sum_{i=1}^{10000} u_{i12}}{10000})_T\\ \\ (\mu_{02} - \frac{\sum_{i=1}^{10000} u_{i21}}{10000},\ \mu_{02},\ \mu_{02} + \frac{\sum_{i=1}^{10000} u_{i22}}{10000})_T\end{pmatrix}.$$

It is noted that to ensure the covariance matrix of all above distributions is the same, the parameter $\boldsymbol{\Sigma}_{t(v)}$ for each multivariate t distribution is chosen. For investigating the performance, we set the shift δ_1 for μ_{01} and the shift δ_2 for μ_{02}, i.e., $\mu_{11} = \mu_{01} + \delta_1$ and $\mu_{12} = \mu_{02} + \delta_2$, in all above cases, and the fuzzy shifted mean vector obtained from the similar procedure for $\widetilde{\boldsymbol{\mu}}_0$. In this study, we only consider upward shifts to show the ideas of new approaches, but it is not difficult to do so for downward shifts.

Table 5 represents the upper control limits for two proposed control charts based on asymptotic distribution and simulation scheme corresponding to $ARL_0 = 200$, or in other words, $\alpha = 0.005$. It is shown that for all cases, UCLs based on asymptotic distribution are greater than simulated UCLs. Also, UCLs based on asymptotic distribution for both control charts are the same, while simulated UCLs of MWSCCF are lower than UCLs of MSCCF.

Table 5: Upper control limits (UCLs) for two proposed control charts.

Distribution	Sample size	Asymptotic distribution $\chi^2_{2,0.005}$	Simulated UCL ($ARL_0 = 200,\ FAP = 0.005$) MSCCF	MWSCCF
Bivariate normal	15	10.60	8.78	8.67
	30	10.60	9.73	9.65
	50	10.60	9.92	9.71
Bivariate $t(5)$	15	10.60	8.67	8.53
	30	10.60	9.80	9.68
	50	10.60	10.47	10.07
Bivariate $t(10)$	15	10.60	8.60	8.43
	30	10.60	10.33	9.86
	50	10.60	10.38	10.20

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Tables 6 and 7 display run length characteristics of two proposed control charts corresponding to simulated UCLs and UCLs based on asymptotic distribution. The results of these tables for ARL_0 are depicted in Figure 2 for three discused distributions. In this figure, the black solid and black dashed lines correspond to MSCCF and MWSCCF with UCLs based on asymptotic distribution, respectively, and the red solid and red dashed lines concerned to MSCCF and MWSCCF control charts with simulated UCLs, respectively.

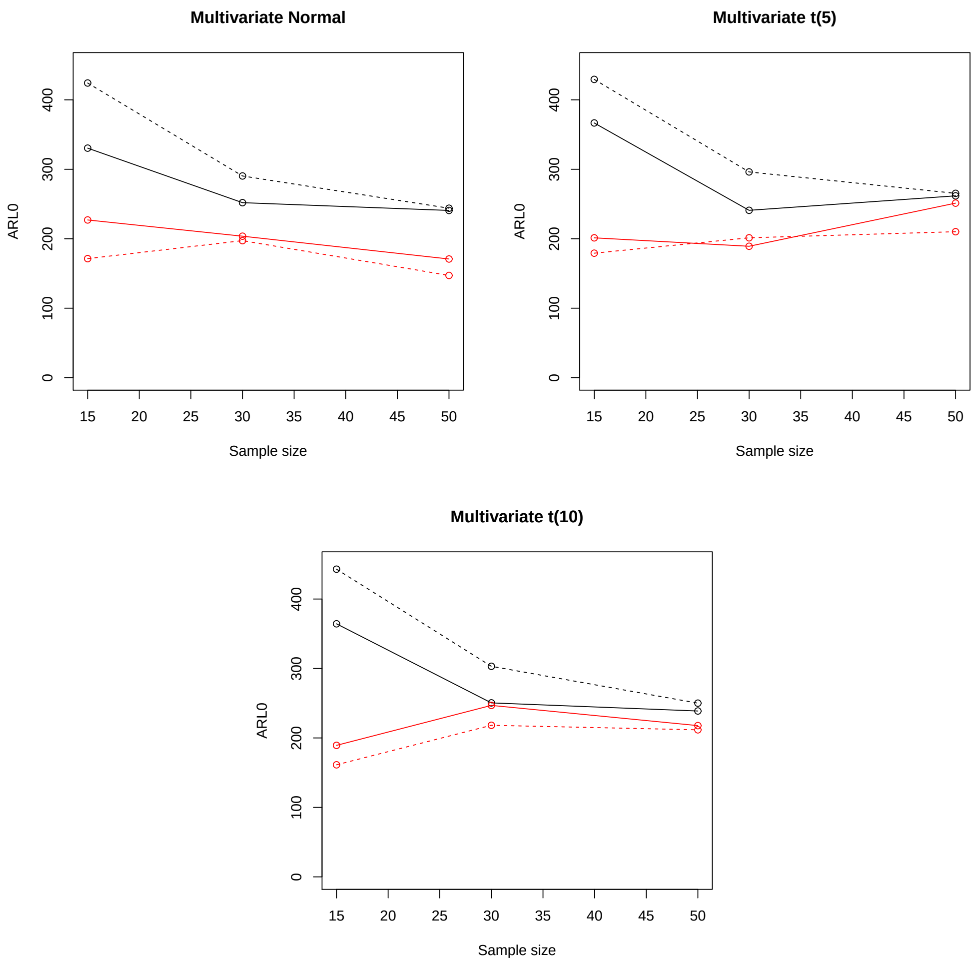

Figure 2: ARL_0 plots based on the data simulated from multivariate normal, multivariate $t(5)$ and multivariate $t(10)$ distributions and for n=15, 30 and 50.

Figure 2 indicates that by utilizing UCL based on asymptotic distribution, ARL_0 for the MWSCCF is greater than the MSCCF, while for the simulated UCLs, it is commonly vice versa. This matter is logical, because MWSCCF has simulated UCL less than MSCCF control chart, in all cases. Furthermore, in all discused distributions, control charts with simulated UCLs achive ARL_0 near to what is nominally expected, while for the charts with USLs based on asymptotic distribution the ARL_0s are more greater than the expected one.

Out-of-control performance in terms of ARL is reported in Tables 8-16 for these control charts based on simulated UCLs and UCLs based on asymptotic distribution. Moreover, the results of these tables are drawn versus δ_1 for some values of δ_2 in Figure 3. It should be noted that the black solid and black dashed lines correspond to MSCCF and MWSCCF with UCLs based on asymptotic distribution, respectively, and the red solid and red dashed lines concerned to MSCCF and MWSCCF with simulated UCLs, respectively.

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Table 6: Run length characteristics of the proposed control charts using the asymptotic distribution.

Distribution	Sample size	Chart statistic	ARL_0	SDRL	5^{th} percentile	MRL	95^{th} percentile
Bivariate normal	15	SN^2	330.47	259.08	16.30	269.00	838.7
		SR^2	424.23	269.16	33.00	394.00	884.75
	30	SN^2	251.94	222.78	13.20	188.00	742.40
		SR^2	290.38	248.68	20.45	197.00	793.05
	50	SN^2	240.75	221.35	11.00	169.00	715.00
		SR^2	243.95	232.01	12.00	159.00	767.2
Bivariate $t(5)$	15	SN^2	366.73	272.29	18.00	315.00	896.20
		SR^2	429.42	304.87	16.35	382.50	935.30
	30	SN^2	240.92	215.71	15.25	170.00	672.75
		SR^2	296.22	241.66	21.00	242.00	762.00
	50	SN^2	261.77	224.11	17.00	202.00	720.70
		SR^2	265.37	223.75	17.00	209.00	762.00
Bivariate $t(10)$	15	SN^2	364.33	285.89	26.75	325.00	872.75
		SR^2	442.98	272.24	37.00	423.00	905.00
	30	SN^2	250.47	215.68	14.00	186.50	694.00
		SR^2	303.14	248.20	17.00	237.00	832.00
	50	SN^2	238.79	203.05	16.45	189.50	645.65
		SR^2	250.10	217.60	16.00	182.00	717.00

Table 7: Run length characteristics of the proposed control charts using the simulated upper control limit.

Distribution	Sample size	Chart statistic	ARL_0	SDRL	5^{th} percentile	MRL	95^{th} percentile
Bivariate normal	15	SN^2	227.05	204.11	13.15	172.00	649.55
		SR^2	171.35	157.40	11.00	123.00	494.60
	30	SN^2	203.67	190.87	10.00	149.50	625.35
		SR^2	197.29	188.08	11.00	138.50	630.25
	50	SN^2	170.85	160.23	8.00	123.50	521.40
		SR^2	147.17	147.81	8.90	102.00	445.60
Bivariate $t(5)$	15	SN^2	201.31	196.11	9.00	143.00	597.00
		SR^2	179.23	166.03	8.00	137.00	534.20
	30	SN^2	189.17	179.55	12.00	133.00	585.00
		SR^2	201.36	191.67	11.7	139.00	586.80
	50	SN^2	251.26	217.37	16.25	191.50	688.25
		SR^2	210.13	183.19	13.00	166.00	565.80
Bivariate $t(10)$	15	SN^2	189.32	183.51	10.00	134.00	552.40
		SR^2	161.30	156.31	8.00	111.00	467.10
	30	SN^2	246.87	213.49	14.00	183.00	686.10
		SR^2	218.20	202.43	12.00	155.50	665.90
	50	SN^2	217.75	187.79	15.60	168.00	586.80
		SR^2	211.68	196.98	14.00	151.00	662.90

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Table 8: ARL values of the proposed control charts for different shifts in location parameter when data simulated from bivariate normal with $n = 15$.

δ_2	UCL	Chart statistic	δ_1						
			0.0	0.2	0.4	0.5	0.6	0.8	1.0
0.0	Asymptotic	SN^2	330.47	223.54	65.21	35.36	18.71	6.80	3.58
		SR^2	424.23	313.88	86.88	42.19	19.70	6.61	3.01
	Simulated	SN^2	227.05	123.36	39.24	21.46	12.52	4.97	2.83
		SR^2	171.35	71.79	18.24	9.44	5.59	2.55	1.60
0.2	Asymptotic	SN^2	225.17	148.36	54.64	31.87	17.68	6.71	3.58
		SR^2	309.21	272.33	95.69	51.03	26.54	8.67	3.73
	Simulated	SN^2	128.06	82.68	35.78	20.63	12.60	5.51	3.08
		SR^2	69.19	62.89	21.71	12.78	7.14	3.08	1.81
0.4	Asymptotic	SN^2	60.78	52.02	31.94	22.24	14.52	6.23	3.45
		SR^2	76.87	84.59	55.79	37.14	24.47	9.05	4.10
	Simulated	SN^2	35.94	32.88	20.91	14.96	10.13	5.04	3.01
		SR^2	16.99	20.46	14.84	10.28	6.68	3.23	1.96
0.5	Asymptotic	SN^2	30.45	27.39	21.01	16.81	11.66	5.83	2.32
		SR^2	37.98	47.97	36.77	29.33	19.82	8.63	4.08
	Simulated	SN^2	19.45	20.45	13.99	10.67	8.04	4.65	2.93
		SR^2	9.02	11.51	10.03	7.61	5.61	3.16	1.95
0.6	Asymptotic	SN^2	17.23	16.39	14.13	12.10	9.06	5.17	3.17
		SR^2	20.30	25.82	23.35	19.95	14.63	7.82	4.02
	Simulated	SN^2	12.21	12.50	9.73	8.15	6.37	4.05	2.70
		SR^2	5.50	6.73	6.85	5.93	5.00	2.91	1.92
0.8	Asymptotic	SN^2	6.87	6.71	6.42	6.06	5.34	3.88	2.70
		SR^2	6.59	8.18	9.26	8.90	7.80	5.40	3.47
	Simulated	SN^2	5.13	5.65	5.36	4.77	4.05	2.88	2.23
		SR^2	2.55	3.01	3.17	3.08	2.97	2.23	1.72
1.0	Asymptotic	SN^2	3.53	3.52	3.44	3.34	3.13	2.61	2.20
		SR^2	2.95	3.64	4.24	4.28	4.14	3.51	2.66
	Simulated	SN^2	2.79	3.08	3.04	2.90	2.62	2.17	1.85
		SR^2	1.54	1.73	1.88	1.91	1.82	1.64	1.44

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Table 9: ARL values of the proposed control charts for different shifts in location parameter when data simulated from bivariate normal with $n = 30$.

δ_2	UCL	Chart statistic	δ_1						
			0.0	0.2	0.4	0.5	0.6	0.8	1.0
0.0	Asymptotic	SN^2	251.94	70.24	12.86	6.55	3.65	1.74	1.20
		SR^2	290.38	53.12	7.37	3.39	1.95	1.16	1.01
	Simulated	SN^2	203.67	52.64	11.23	5.74	3.29	1.64	1.18
		SR^2	197.29	34.71	5.33	2.65	1.67	1.12	1.00
0.2	Asymptotic	SN^2	76.41	43.98	12.08	6.67	3.98	1.83	1.24
		SR^2	53.42	45.47	10.42	5.02	2.72	1.03	1.04
	Simulated	SN^2	57.69	35.1	10.25	6.02	3.65	1.76	1.23
		SR^2	32.77	29.11	7.41	3.90	2.13	1.21	1.02
0.4	Asymptotic	SN^2	12.37	11.91	6.98	4.93	3.46	1.76	1.23
		SR^2	9.65	9.47	6.45	4.39	2.67	1.39	1.07
	Simulated	SN^2	10.29	10.14	6.06	4.39	3.17	1.71	1.20
		SR^2	6.97	6.90	4.92	3.33	2.18	1.28	1.03
0.5	Asymptotic	SN^2	6.90	6.48	4.95	3.93	2.98	1.69	1.21
		SR^2	4.84	4.64	4.13	3.04	2.30	1.37	1.07
	Simulated	SN^2	5.38	5.18	4.51	3.51	2.73	1.62	1.18
		SR^2	3.70	3.55	3.25	2.50	1.92	1.27	1.04
0.6	Asymptotic	SN^2	3.50	3.81	3.35	2.83	2.36	1.56	1.17
		SR^2	1.84	2.56	2.61	2.30	1.93	1.31	1.07
	Simulated	SN^2	3.03	3.58	3.12	2.61	2.20	1.48	1.15
		SR^2	2.20	2.13	2.16	1.95	1.64	1.20	1.03
0.8	Asymptotic	SN^2	1.58	1.77	1.74	1.68	1.57	1.29	1.10
		SR^2	1.14	1.27	1.36	1.36	1.29	1.13	1.03
	Simulated	SN^2	1.49	1.68	1.69	1.59	1.50	1.25	1.08
		SR^2	1.09	1.18	1.26	1.24	1.19	1.07	1.02
1.0	Asymptotic	SN^2	1.16	1.21	1.21	1.20	1.17	1.10	1.04
		SR^2	1.08	1.07	1.06	1.05	1.05	1.03	1.01
	Simulated	SN^2	1.12	1.18	1.20	1.17	1.14	1.08	1.03
		SR^2	1.01	1.02	1.03	1.03	1.03	1.02	1.01

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Table 10: ARL values of the proposed control charts for different shifts in location parameter when data simulated from bivariate normal with $n = 50$.

δ_2	UCL	Chart statistic	δ_1						
			0.0	0.2	0.4	0.5	0.6	0.8	1.0
0.0	Asymptotic	SN^2	240.75	39.21	5.20	2.73	1.71	1.10	1.01
		SR^2	243.95	21.34	2.47	1.43	1.11	1.01	1.00
	Simulated	SN^2	170.85	28.22	4.28	2.45	1.58	1.09	1.01
		SR^2	147.17	15.19	2.11	1.35	1.08	1.00	1.00
0.2	Asymptotic	SN^2	38.60	25.56	5.58	3.05	1.86	1.15	1.02
		SR^2	21.63	16.31	3.37	1.81	1.27	1.01	1.00
	Simulated	SN^2	29.23	17.54	4.73	2.62	1.72	1.11	1.02
		SR^2	15.29	12.12	2.80	1.59	1.18	1.01	1.00
0.4	Asymptotic	SN^2	5.04	5.41	3.14	2.23	1.69	1.14	1.02
		SR^2	2.29	3.27	2.15	1.59	1.26	1.03	1.00
	Simulated	SN^2	4.23	4.53	2.78	1.99	1.52	1.10	1.02
		SR^2	2.04	2.74	1.91	1.42	1.20	1.02	1.00
0.5	Asymptotic	SN^2	2.62	2.44	2.23	1.79	1.46	1.12	1.02
		SR^2	1.82	1.79	1.59	1.35	1.19	1.02	1.00
	Simulated	SN^2	2.27	2.14	1.99	1.63	1.37	1.09	1.02
		SR^2	1.75	1.63	1.46	1.26	1.12	1.01	1.00
0.6	Asymptotic	SN^2	1.70	1.89	1.69	1.49	1.28	1.09	1.01
		SR^2	1.11	1.27	1.25	1.19	1.10	1.01	1.00
	Simulated	SN^2	1.56	1.73	1.55	1.38	1.23	1.06	1.01
		SR^2	1.08	1.19	1.18	1.12	1.07	1.006	1.00
0.8	Asymptotic	SN^2	1.13	1.16	1.15	1.12	1.07	1.03	1.00
		SR^2	1.00	1.03	1.03	1.03	1.02	1.00	1.00
	Simulated	SN^2	1.09	1.13	1.12	1.10	1.06	1.02	1.00
		SR^2	1.00	1.02	1.03	1.02	1.01	1.00	1.00
1.0	Asymptotic	SN^2	1.01	1.03	1.03	1.03	1.02	1.01	1.00
		SR^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Simulated	SN^2	1.00	1.02	1.02	1.02	1.01	1.00	1.00
		SR^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00

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Table 11: ARL values of the proposed control charts for different shifts in location parameter when data simulated from bivariate $t(5)$ with $n = 15$.

δ_2	UCL	Chart statistic	δ_1 0.0	0.2	0.4	0.5	0.6	0.8	1.0
0.0	Asymptotic	SN^2	366.73	171.56	35.82	18.49	10.53	4.29	2.46
		SR^2	429.42	273.33	47.54	23.18	11.38	4.29	2.44
	Simulated	SN^2	201.31	79.29	19.61	10.89	6.54	3.11	1.90
		SR^2	179.23	55.28	11.32	6.14	3.69	2.00	1.39
0.2	Asymptotic	SN^2	179.83	104.31	31.36	17.27	10.17	4.24	2.45
		SR^2	283.77	236.20	59.57	29.41	15.90	5.33	2.79
	Simulated	SN^2	82.75	58.78	19.88	11.41	7.51	3.47	2.11
		SR^2	54.00	49.11	13.88	7.51	4.66	2.41	1.53
0.4	Asymptotic	SN^2	37.67	32.31	19.17	12.50	8.48	3.96	2.39
		SR^2	43.38	61.60	36.65	23.72	14.02	5.72	3.05
	Simulated	SN^2	19.04	19.57	12.03	8.56	6.35	3.34	2.14
		SR^2	11.45	15.07	9.65	6.30	4.32	2.48	1.65
0.5	Asymptotic	SN^2	18.73	17.67	13.20	9.89	7.35	3.66	2.34
		SR^2	22.85	30.22	24.21	17.49	11.93	5.31	3.07
	Simulated	SN^2	10.87	11.90	8.62	6.63	5.15	3.10	2.08
		SR^2	5.99	7.54	6.22	5.09	3.74	2.41	1.65
0.6	Asymptotic	SN^2	10.93	10.54	8.75	7.23	5.74	3.39	2.23
		SR^2	11.31	15.52	14.92	12.46	9.43	4.80	2.91
	Simulated	SN^2	6.73	7.37	5.93	4.84	4.06	2.70	1.97
		SR^2	3.78	4.77	4.60	3.90	3.18	2.26	1.60
0.8	Asymptotic	SN^2	4.26	4.16	3.89	3.70	3.36	2.64	2.00
		SR^2	4.23	5.46	5.75	5.44	4.89	3.71	2.58
	Simulated	SN^2	3.04	3.46	3.26	3.00	2.75	2.18	1.72
		SR^2	2.12	2.42	2.54	2.42	2.24	1.84	1.48
1.0	Asymptotic	SN^2	2.56	2.53	2.44	2.38	2.28	2.02	1.67
		SR^2	2.39	2.80	3.02	3.02	2.95	2.62	2.08
	Simulated	SN^2	2.02	2.27	2.14	2.02	1.93	1.70	1.48
		SR^2	1.39	1.54	1.63	1.64	1.59	1.45	1.31

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Table 12: ARL values of the proposed control charts for different shifts in location parameter when data simulated from bivariate $t(5)$ with $n = 30$.

δ_2	UCL	Chart statistic	δ_1 0.0	0.2	0.4	0.5	0.6	0.8	1.0
0.0	Asymptotic	SN^2	240.92	46.30	6.76	3.42	2.21	1.30	1.07
		SR^2	296.22	39.12	4.58	2.53	1.64	1.12	1.02
	Simulated	SN^2	189.17	37.54	5.85	3.05	2.05	1.26	1.06
		SR^2	201.36	23.91	3.58	2.14	1.47	1.08	1.01
0.2	Asymptotic	SN^2	47.11	27.90	6.81	3.62	2.32	1.35	1.09
		SR^2	37.35	31.68	6.77	3.34	2.11	1.22	1.03
	Simulated	SN^2	31.19	23.35	6.32	3.46	2.22	1.33	1.08
		SR^2	23.86	21.10	4.95	2.69	1.75	1.16	1.02
0.4	Asymptotic	SN^2	7.48	7.04	4.20	2.81	1.99	1.30	1.08
		SR^2	6.96	6.60	3.94	2.67	1.91	1.23	1.06
	Simulated	SN^2	6.55	6.50	3.79	2.54	1.87	1.28	1.07
		SR^2	4.98	4.52	3.11	2.20	1.65	1.16	1.02
0.5	Asymptotic	SN^2	3.58	3.67	2.80	2.11	1.68	1.25	1.07
		SR^2	2.34	3.12	2.59	2.07	1.71	1.21	1.04
	Simulated	SN^2	3.25	3.45	2.57	1.98	1.61	1.22	1.07
		SR^2	1.92	2.54	2.17	1.83	1.48	1.13	1.03
0.6	Asymptotic	SN^2	2.19	2.29	2.04	1.75	1.50	1.17	1.06
		SR^2	1.57	1.87	1.84	1.62	1.44	1.15	1.04
	Simulated	SN^2	2.05	2.17	1.92	1.67	1.43	1.14	1.05
		SR^2	1.39	1.64	1.61	1.45	1.30	1.11	1.02
0.8	Asymptotic	SN^2	1.27	1.30	1.28	1.24	1.18	1.09	1.04
		SR^2	1.08	1.17	1.19	1.18	1.14	1.07	1.03
	Simulated	SN^2	1.24	1.29	1.26	1.21	1.15	1.07	1.03
		SR^2	1.06	1.10	1.13	1.11	1.09	1.05	1.02
1.0	Asymptotic	SN^2	1.07	1.09	1.08	1.07	1.06	1.03	1.01
		SR^2	1.05	1.04	1.04	1.03	1.03	1.03	1.01
	Simulated	SN^2	1.07	1.08	1.08	1.06	1.05	1.02	1.01
		SR^2	1.03	1.02	1.02	1.02	1.02	1.01	1.00

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Table 13: ARL values of the proposed control charts for different shifts in location parameter when data simulated from bivariate $t(5)$ with $n = 50$.

δ_2	UCL	Chart statistic	δ_1						
			0.0	0.2	0.4	0.5	0.6	0.8	1.0
0.0	Asymptotic	SN^2	261.77	23.00	3.07	1.68	1.28	1.02	1.00
		SR^2	265.37	14.85	1.77	1.25	1.07	1.00	1.00
	Simulated	SN^2	251.26	22.09	2.95	1.66	1.27	1.02	1.00
		SR^2	210.13	12.23	1.69	1.21	1.06	1.00	1.00
0.2	Asymptotic	SN^2	21.91	15.03	3.27	1.93	1.38	1.04	1.00
		SR^2	13.40	12.08	2.54	1.51	1.18	1.00	1.00
	Simulated	SN^2	20.96	14.53	3.18	1.90	1.36	1.04	1.00
		SR^2	10.70	10.02	2.39	1.44	1.16	1.01	1.00
0.4	Asymptotic	SN^2	2.85	3.27	2.02	1.58	1.29	1.05	1.01
		SR^2	1.88	2.48	1.82	1.38	1.16	1.01	1.00
	Simulated	SN^2	2.80	3.17	1.99	1.56	1.27	1.05	1.01
		SR^2	1.76	2.32	1.70	1.33	1.14	1.01	1.00
0.5	Asymptotic	SN^2	1.99	1.96	1.57	1.33	1.18	1.04	1.01
		SR^2	1.53	1.44	1.35	1.20	1.11	1.01	1.00
	Simulated	SN^2	1.71	1.63	1.54	1.32	1.18	1.04	1.01
		SR^2	1.42	1.37	1.30	1.17	1.08	1.01	1.00
0.6	Asymptotic	SN^2	1.27	1.36	1.26	1.18	1.12	1.02	1.01
		SR^2	1.06	1.13	1.14	1.10	1.06	1.01	1.00
	Simulated	SN^2	1.25	1.35	1.26	1.18	1.11	1.02	1.00
		SR^2	1.05	1.12	1.11	1.08	1.05	1.01	1.00
0.8	Asymptotic	SN^2	1.03	1.06	1.05	1.03	1.02	1.01	1.00
		SR^2	1.01	1.01	1.01	1.00	1.00	1.00	1.00
	Simulated	SN^2	1.03	1.06	1.05	1.03	1.02	1.01	1.01
		SR^2	1.01	1.01	1.00	1.00	1.00	1.00	1.00
1.0	Asymptotic	SN^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SR^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Simulated	SN^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SR^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00

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Table 14: ARL values of the proposed control charts for different shifts in location parameter when data simulated from bivariate $t(10)$ with $n = 15$.

δ_2	UCL	Chart statistic	δ_1 0.0	0.2	0.4	0.5	0.6	0.8	1.0
0.0	Asymptotic	SN^2	364.33	199.23	47.56	24.52	13.65	5.86	3.01
		SR^2	442.98	321.89	68.57	30.24	15.05	5.79	2.69
	Simulated	SN^2	189.32	93.50	24.05	13.49	8.55	4.16	2.25
		SR^2	161.30	56.48	13.39	7.13	4.40	2.10	1.41
0.2	Asymptotic	SN^2	214.81	126.34	41.03	22.76	13.28	5.87	3.02
		SR^2	293.48	257.30	57.65	37.96	20.21	7.26	3.36
	Simulated	SN^2	80.60	56.24	21.96	13.70	8.77	4.37	2.53
		SR^2	57.38	44.75	16.30	9.11	5.58	2.53	1.53
0.4	Asymptotic	SN^2	48.12	42.49	24.41	16.64	11.10	5.29	2.94
		SR^2	64.92	76.23	49.99	28.65	18.33	7.57	3.70
	Simulated	SN^2	23.22	22.01	12.99	9.59	6.87	3.89	2.41
		SR^2	13.69	15.37	10.20	7.58	5.31	2.70	1.65
0.5	Asymptotic	SN^2	25.64	23.95	17.32	12.79	9.39	5.06	2.86
		SR^2	31.37	36.03	29.18	21.15	14.59	7.14	3.71
	Simulated	SN^2	13.44	13.23	9.32	7.09	5.49	3.40	2.26
		SR^2	7.43	9.56	7.37	6.16	4.74	2.50	1.65
0.6	Asymptotic	SN^2	14.10	13.73	11.12	9.28	7.25	4.41	2.72
		SR^2	15.54	18.55	17.73	15.05	11.36	6.54	3.61
	Simulated	SN^2	8.30	8.55	6.73	5.57	4.46	2.99	2.08
		SR^2	4.71	5.75	5.47	4.71	3.96	2.38	1.64
0.8	Asymptotic	SN^2	5.88	5.82	5.33	5.00	4.51	3.36	2.41
		SR^2	5.69	7.36	7.88	7.25	6.47	4.52	2.99
	Simulated	SN^2	3.88	4.23	3.94	3.46	3.05	2.21	1.75
		SR^2	2.17	2.69	2.85	2.84	2.57	1.93	1.54
1.0	Asymptotic	SN^2	3.12	3.11	3.04	2.99	2.87	2.43	1.98
		SR^2	2.85	3.33	3.59	3.62	3.53	3.06	2.34
	Simulated	SN^2	2.38	2.55	2.44	2.28	2.07	1.73	1.47
		SR^2	1.41	1.55	1.64	1.65	1.59	1.50	1.32

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Table 15: ARL values of the proposed control charts for different shifts in location parameter when data simulated from bivariate $t(10)$ with $n = 30$.

δ_2	UCL	Chart statistic	δ_1 0.0	0.2	0.4	0.5	0.6	0.8	1.0
0.0	Asymptotic	SN^2	250.47	62.87	10.10	5.33	3.04	1.51	1.13
		SR^2	303.14	50.39	6.05	3.18	1.79	1.15	1.01
	Simulated	SN^2	246.87	59.49	9.59	5.12	2.91	1.48	1.12
		SR^2	218.20	36.07	4.81	2.63	1.63	1.10	1.01
0.2	Asymptotic	SN^2	66.99	40.21	10.13	5.67	3.27	1.60	1.14
		SR^2	47.71	44.08	8.98	4.26	2.48	1.26	1.03
	Simulated	SN^2	62.24	39.81	10.00	5.62	3.23	1.59	1.14
		SR^2	32.27	30.87	7.21	3.45	2.12	1.19	1.02
0.4	Asymptotic	SN^2	10.53	10.22	5.85	4.05	2.28	1.54	1.15
		SR^2	8.97	8.24	5.58	3.62	2.44	1.30	1.05
	Simulated	SN^2	10.30	10.07	5.85	4.04	2.77	1.54	1.14
		SR^2	7.90	6.92	4.37	2.90	2.13	1.21	1.04
0.5	Asymptotic	SN^2	5.51	5.71	4.19	3.17	2.40	1.47	1.13
		SR^2	4.13	4.22	3.43	2.68	2.09	1.28	1.06
	Simulated	SN^2	5.33	5.67	4.19	3.17	2.37	1.47	1.13
		SR^2	3.65	3.51	2.87	2.33	1.90	1.20	1.05
0.6	Asymptotic	SN^2	3.35	3.33	2.78	2.46	1.97	1.39	1.11
		SR^2	2.89	2.38	2.37	2.08	1.78	1.23	1.06
	Simulated	SN^2	3.07	3.31	2.78	2.46	1.97	1.39	1.11
		SR^2	2.71	2.08	2.07	1.86	1.59	1.17	1.03
0.8	Asymptotic	SN^2	1.64	1.59	1.53	1.47	1.35	1.18	1.07
		SR^2	1.35	1.33	1.32	1.32	1.26	1.12	1.03
	Simulated	SN^2	1.62	1.59	1.52	1.47	1.35	1.18	1.07
		SR^2	1.28	1.26	1.24	1.23	1.18	1.08	1.02
1.0	Asymptotic	SN^2	1.23	1.16	1.15	1.15	1.12	1.07	1.02
		SR^2	1.09	1.08	1.07	1.07	1.06	1.04	1.01
	Simulated	SN^2	1.18	1.16	1.15	1.15	1.12	1.07	1.02
		SR^2	1.07	1.06	1.05	1.05	1.04	1.02	1.01

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Table 16: ARL values of the proposed control charts for different shifts in location parameter when data simulated from bivariate $t(10)$ with $n = 50$.

δ_2	UCL	Chart statistic	δ_1						
			0.0	0.2	0.4	0.5	0.6	0.8	1.0
0.0	Asymptotic	SN^2	238.79	29.85	3.79	2.17	1.44	1.06	1.00
		SR^2	250.10	17.43	2.11	1.31	1.07	1.00	1.00
	Simulated	SN^2	217.75	27.93	3.62	2.08	1.41	1.06	1.00
		SR^2	211.68	14.87	1.98	1.27	1.05	1.00	1.00
0.2	Asymptotic	SN^2	28.81	19.83	4.26	2.40	1.58	1.09	1.00
		SR^2	18.83	14.91	2.94	1.58	1.15	1.01	1.00
	Simulated	SN^2	26.95	18.04	4.08	2.33	1.54	1.09	1.00
		SR^2	15.95	13.11	2.72	1.52	1.14	1.00	1.00
0.4	Asymptotic	SN^2	3.99	4.44	2.58	1.82	1.41	1.07	1.00
		SR^2	2.19	2.90	1.95	1.42	1.15	1.01	1.00
	Simulated	SN^2	3.74	4.20	2.45	1.72	1.36	1.06	1.00
		SR^2	2.00	2.69	1.83	1.35	1.13	1.01	1.00
0.5	Asymptotic	SN^2	2.97	2.60	1.86	1.51	1.26	1.05	1.00
		SR^2	1.74	1.66	1.45	1.25	1.11	1.00	1.00
	Simulated	SN^2	2.44	2.30	1.78	1.46	1.24	1.04	1.00
		SR^2	1.64	1.60	1.38	1.21	1.09	1.00	1.00
0.6	Asymptotic	SN^2	1.45	1.60	1.46	1.27	1.15	1.04	1.00
		SR^2	1.07	1.22	1.19	1.12	1.06	1.00	1.00
	Simulated	SN^2	1.42	1.57	1.43	1.24	1.12	1.03	1.00
		SR^2	1.07	1.20	1.16	1.09	1.04	1.00	1.00
0.8	Asymptotic	SN^2	1.05	1.08	1.05	1.04	1.02	1.01	1.00
		SR^2	1.01	1.01	1.01	1.01	1.00	1.00	1.00
	Simulated	SN^2	1.05	1.07	1.05	1.04	1.02	1.00	1.00
		SR^2	1.01	1.01	1.01	1.01	1.00	1.00	1.00
1.0	Asymptotic	SN^2	1.01	1.00	1.00	1.00	1.00	1.00	1.00
		SR^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Simulated	SN^2	1.01	1.00	1.00	1.00	1.00	1.00	1.00
		SR^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00

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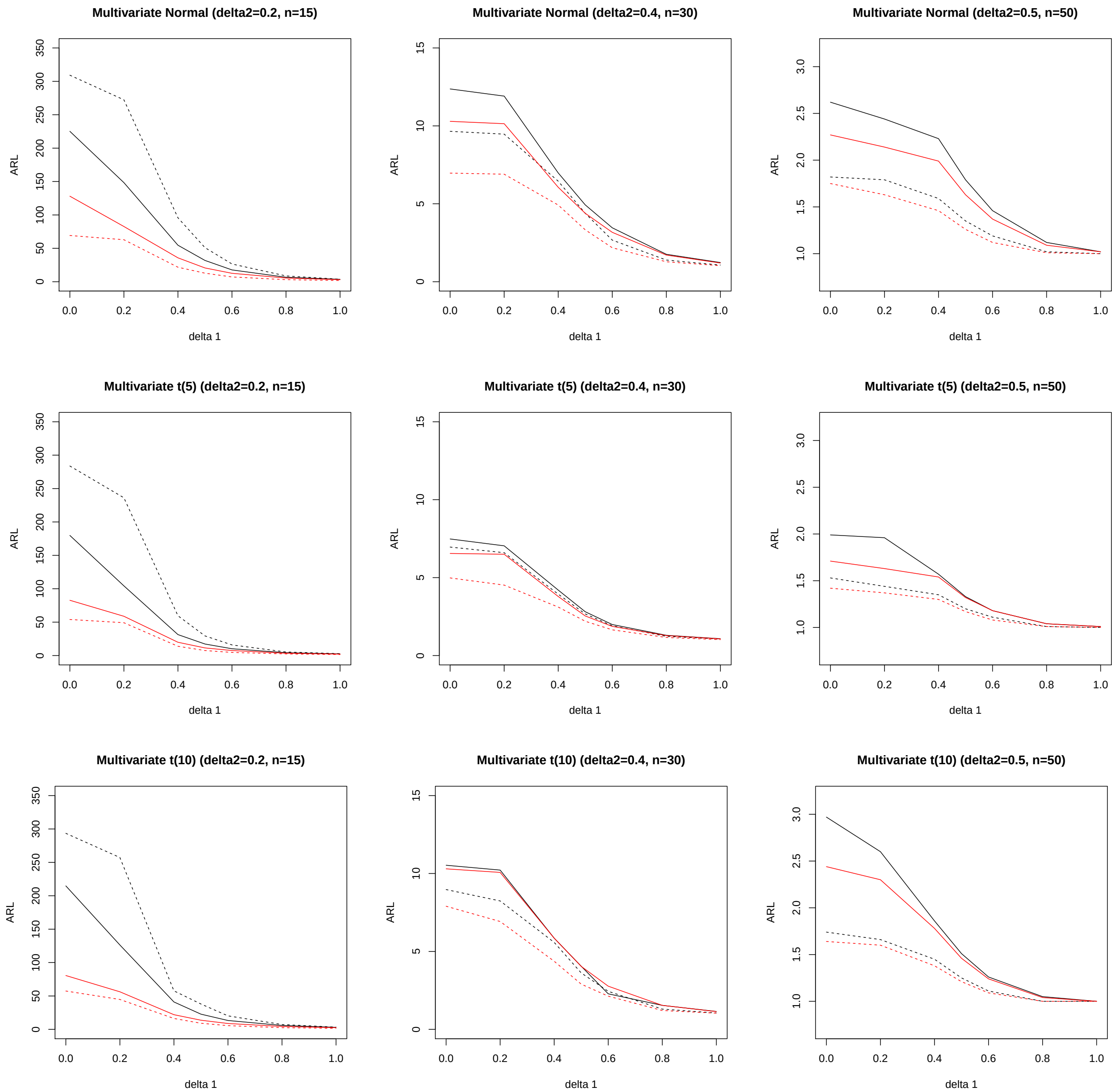

Figure 3: ARL curves of MSCCF and MWSCCF based on multivariate normal, multivariate $t(5)$ and multivariate $t(10)$ simulated data and for n=15, 30 and 50 and some values of δ_2.

It is seen that for small sample size, ARL of MSCCF with simulated UCL is placed above its counterpart in MSCCF with UCL based on asymptotic distribution and then, MWSCCF with UCL based on asymptotic distribution and simulated UCL, respectively. For medium sample size, ALR of MSCCF with UCL based on asymptotic distribution is greater than ARL of MWSCCF with UCL based on asymptotic distribution and MSCCF with simulated UCL, and ARL of MWSCCF with simulated UCL is placed below all of these. For large sample size, ARL of MSCCF with UCL based on asymptotic distribution is placed above its counterparts in MWSCCF with UCL based on asymptotic distribution, MSCCF with simulated UCL, MWSCCF with simulated UCL, respectively.

As a whole, ARL in MWSCCF with simulated UCL is placed below its counterparts in other control charts which means that when the process is out-of-control, MWSCCF with simulated UCL signals earlier than its competitors. Therefore, it is concluded that MWSCCF with simulated UCL has the better performane to detect the shifts in parameters.

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5 Conclusion

In this article, two non-parametric multivariate control charts for fuzzy data, MSCCF and MWSCCF, were developed for monitoring the changes in process location parameter. The performance of the proposed charts was assessesd for both in-control and out-of-control state by simulating data from multivariate normal and multivariate t distribution with two different degrees of freedom. We found that MWSCCF with simulated UCL has better in-control and out-of-control performance than its competitors for any size of shift when the false alarm rate is held at a specified value.

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On the non-parametric multivariate control charts in fuzzy environment

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مباحثی بر نمودارهای کنترل چند متغیره ناپارامتری در محیط فازی

چکیده. نمودارهای کنترل چند متغیره برای کنترل یا پایش هم‌زمان دو یا چند مشخصه کیفی وابسته مورد استفاده قرار می‌گیرند. در دنیای واقعی، توزیع مشخصه کیفی بسیاری از فرآیندها، غیرنرمال یا نامشخص است که در این‌گونه موارد، نمودارهای کنترل ناپارامتری به کار گرفته می‌شود. اکثر روش‌های کنترل فرآیند آمار ناپارامتری بر پایه رتبه‌ها استوار هستند. در این تحقیق، برای فرآیندهایی که مقادیر مشخصه‌ها به صورت مبهم (فازی/مشکک) در دست است، با استفاده از نظریه مجموعه‌های فازی، نمودارهای کنترل چند متغیره ناپارامتری بر اساس آزمون‌های علامت و رتبه‌ای-علامت ویلکاکسون معرفی می‌شود. سپس، با مطالعه شبیه‌سازی عملکرد نمودارهای کنترل ارائه‌شده بررسی و در پایان، برای نشان‌دادن طریقه استفاده از نمودارهای پیشنهادشده، مثال‌های عددی ارائه می‌شود.

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