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An extended hesitant group decision-making technique based on the prospect theory for emergency situations

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Abstract

Throughout the present manuscript, we are going to introduce a novel group emergency decision-making technique in which the application of prospect theory explains the psychological behaviour of the decision maker who is affected by the hesitancy and uncertainty of cognition in decision making problems.
Instead of usual aggregation procedure, we implement here a new fusion technique that is based on modified version of extended hesitant fuzzy set (EHFS), and it definitely keeps possible amount of expert's information more than the existing fusion technique of hesitant fuzzy set (HFS). The main motivation of re-visiting the concept of EHFS comes from its potential role in increasing the richness of numerical appearance in the form of value-groups, and of course its ability in identifying various decision makers in decision making situations. Such a definition further expands the practical applications of HFSs.
Finally, we employ the barrier lake problem to illustrate the feasibility and the validity of the presented technique.

Keywords: Extended hesitant fuzzy set, multiple criteria group decision making, prospect theory, emergency event.

1 Introduction

Usually, a group decision-making technique is understood as a process in which a number of individuals interact simultaneously. Then, they evaluate all possible alternatives by the help of multiple conflicting criteria, and consequently they choose a suitable alternative solution to the problem [1]-[3]. However, these days, the human beings play an important role in the social problems, specially in the technological development of society and also has been further studied in economy problems.

This kind of attention has greatly been given to the role of emergency events that affect negatively the social development, and of course the life of a society. In simple words, an emergency event describes a situation which is taken place suddenly, increases the rate of death, injury, ecological damage, property loss, and also social hazards (for instance, air crash, earthquakes, terrorist attacks, and so on).

The key point in such happenings is the use of appropriate and effective way of controlling the probable undesired situation. Needless to say that selecting a proper strategy in such situations is more effective, and prevents more ineffective treatments in the first steps of an emergency event. Such a strategy selection is named emergency decision making and it has became a very active research field in recent years [5, 16, 17, 15, 30]. Other recent works in this field include those by Sun et al. [27] where the prediction of emergency material demanded by adopting the fuzzy rough set theory is proposed; Fan et al.'s [12] analysing technique which is presented for the emergency response by taking the prospect theory into account;

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Wang and Wang's [32] technique of emergency decision making in which the prospect theory roles as the main part of specifying a reference point; Wang and Sun's [31] PROMETEE approaches that implement the trapezoidal concept of fuzzy numbers in order for investigating a variety of uncertain decision making problems.

In addition to the works mentioned above, Liu et al. [18] extended the procedure of dealing with multiple criteria decision making (MCDM) problems involving prospect theory by using the concept of probability. Moreover, it may be referred to TOPSIS-based methods that are investigated by Fan et al. [13] and Wang et al. [33] in order to handle MCDM problems that are equipped with the prospect theory. Recently, Ren et al. [23] have worked on a method to handle emergency decision making by taking the advantages of the hesitant fuzzy element for representing the fuzziness of objects and moreover the hesitant thoughts of each expert. Zhang et al. [39] presented a technique on the basis of prospect theory in which emergency situations are taken into consideration. Two factors including the psychological behaviour of decision makers and various emergency situations in emergency decision making procedures play the main role in Zhang et al.'s [39] technique.

Liu and Li [19] studied an emergency decision-making technique mainly focusing on the following three aspects: (1) quantification of impact of psychological behaviours, (2) consideration of group decision-making instead of single-person technique, and (3) introduction of an algorithm for integrating the different individual assessments.

Generally, judging about emergency cases is not comprehensive, and this comes back to the lack of information and time restrictions that a decision maker may have. This is the main reason that a group of experts should help the decision maker to make a proper decision by taking various backgrounds into account, and in fact this leads to the form of group decision making in such a case. Here, what should be paid attention is that the process of a group decision making is usually based on (i) aggregation and (ii) selection steps. In the first step, it needs to aggregate all individual information [35, 37], and in the second step, the aggregated information should be considered for selecting the best alternative(s) [4].

In a recent contribution, Zhang et al. [38] attempted to address and resolve these two key issues by suggesting a hesitant group decision making technique that is formed on the basis of prospect theory. But there remains a problem with the concept of hesitant fuzzy set (HFS) which roles the main part of the latter context. There exist a large number of contributions that have been devoted to the HFS which was first introduced by Torra [28]. Here, we are not intended to discuss the details concerned with HFS theory, and the interested reader is referred to numerous recent works in this regard [6]-[10].

However, by a simple investigation in the most-recent manuscripts, it can be found that the HFS has some shortages, specially, in the case where the opinion of decision makers with different preferences should be taken into account in a decision-making process. More specifically, consider a decision situation in which two decision makers should return their evaluation results corresponding to a criterion. The first one may assign 0.2, and the other decision maker may assign 0.2 or 0.5. In this regard, if we *ignore* the different importance and different social importance of decision makers, then it results in the collection of the assessment in the form of HFE $\{0.2, 0.5\}$. While, the logical collection of decision makers' assessment should be in the form of two value-groups as $(0.2, 0.2)$ and $(0.2, 0.5)$. This reason clearly shows that the necessity of considering concept of extended hesitant fuzzy set (EHFS) being first introduced by Zhu and Xu [40]. By the way, Zhu and Xu's [40] definition of EHFS is not completely satisfactory, and it needs to be modified due to its limited capability. The modification of EHFS definition given by Zhu and Xu [40] has been recently suggested by Farhadinia and Herrera-Viedma [11], where an element of EHFE does not generally play a role as the element included in Cartesian product of HFEs like that considered by Zhu and Xu [40].

In this contribution, the main intention is to deal with a new aspect of emergency event in which instead of usual aggregation procedure, we implement a new fusion technique that is based on modified version of EHFS, and it definitely keeps all possible amount of expert's information more than the existing fusion technique of HFS. As we will demonstrate in the last part of this contribution, the two concepts of *pessimistic/optimistic* HFS and EHFS are quite structurally different. The main difference is related to the implication of sorted values in representation forms of *pessimistic/optimistic* HFS or EHFSs. Therefore, in accordance with the HFS form, we cannot exactly judge which value refers to which expert, meanwhile, such a connection is quite evident in the EHFS form. This difference is so noticeable, specially in the real-world applications where we may confront a problem involved a group of n-fixed experts, and therefore, by implementing EHFS concept in modelling the problem gives rise to more reliable decision than the case where it just takes the HFS concept into account.

The remainder of this paper is set out as follows: In Section 2, it is given some basic knowledge on EHFSs together with the introduction of prospect theory in brief. Section 3 describes completely the structure of a group emergency decision making algorithm. Section 4 employs the barrier lake problem to show the practicality and effectiveness of the proposed method. Moreover, this section is ended by making comparison analyses between the proposed method for EHFSs and that for HFSs. The paper ends with some conclusions remarks in Section 5.

Nomenclature

A	Hesitant fuzzy set (HFS);
h_A	Hesitant fuzzy element (HFE);
$h_A^{\delta(i)}$	The i-th element of h_A;
$\mathbf{H}$	Extended hesitant fuzzy set (EHFS);
$\mathbf{h}$	Extended hesitant fuzzy element (EHFE);
$\mathbf{h}_A^{\delta(i)} = (\gamma_A^{1,\delta(i)}, ..., \gamma_m^{A,\delta(i)})$	The i-th element of $\mathbf{h}_A$;
$C_j^k(A_i)$	The opinion of decision maker e_k provided for the alternative A_i regarding to the criterion C_j;
$\overline{C}_j^k(A_i)$	The normalization form of $C_j^k(A_i)$;
$I_{ij}^{l_h} = [I_{ij}^{L,1}, I_{ij}^{U,1}] \times ... \times [I_{ij}^{L,l_h}, I_{ij}^{U,l_h}]$	Effective control scope over the alternative A_i regarding the criterion C_j;
$J_j^{l_h} = [J_j^{L,1}, J_j^{U,1}] \times ... \times [J_j^{L,l_h}, J_j^{U,l_h}]$	Reference point given by the decision maker for the criterion C_j;
$\mathbf{G}$	Gain matrix;
$\mathbf{L}$	Loss matrix;
$\mathbf{V}$	Prospect value matrix;
$\mathbf{OPV}_i$	Overall prospect value of the alternative A_i;
$\mathbf{S}_{AM}$	Arithmetic-mean score function.

2 Preliminaries

This section mainly deals with the revised version of extended hesitant fuzzy set (EHFS), and it provides some requirements, such as, describing the prospect theory and the value function that are needed in the subsequent sections.

In this portion, the concept of hesitant fuzzy set (HFS) which seems to be as a seminal concept of next discussions is firstly introduced.

Definition 2.1. [28] *Suppose that X is the reference set. Then, a hesitant fuzzy set (HFS) on X is in terms of a function that when it is applied to X, it returns a subset of* $[0,1]$.

For a better understanding, Xia and Xu [36] represented subsequently the HFS in the following form of mathematical symbol, $A = \{\langle x, h_A(x)\rangle : x \in X\}$, where $h_A(x)$ stands for all possible membership degrees of $x \in X$ belonging to the set A, and it is afterward named as the hesitant fuzzy element (HFE) of A.

In order to have a better insight of the HFE, the following operations of HFEs are to be presented.

Let $h_1 = \{h_1^{\delta(i)} \mid i = 1, ..., l_{h_1}\}$ and $h_2 = \{h_2^{\delta(i)} \mid i = 1, ..., l_{h_2}\}$ be two HFEs, then the following operations are defined as [34]:

- Addition: $h_1 \oplus h_2 = \bigcup_{h_1^{\delta(i)} \in h_1, h_2^{\delta(j)} \in h_2} \{h_1^{\delta(i)} + h_1^{\delta(j)} - h_1^{\delta(i)} h_1^{\delta(j)}\}$;
- Multiplication: $h_1 \otimes h_2 = \bigcup_{h_1^{\delta(i)} \in h_1, h_2^{\delta(j)} \in h_2} \{h_1^{\delta(i)} h_2^{\delta(j)}\}$;
- Multiplication by scalar: $\lambda h_1 = \bigcup_{h_1^{\delta(i)} \in h_1} \{1 - (1 - h_1^{\delta(i)})^\lambda\},\ \lambda > 0$;
- Power: $h_1^\lambda = \bigcup_{h_1^{\delta(i)} \in h_1} \{(h_1^{\delta(i)})^\lambda\},\ \lambda > 0$.

What is essential to be considered in computation procedures is the unification of length of all taken HFEs. In most situations, for any two HFEs h_1 and h_2, it is observed that $l_{h_1} \neq l_{h_2}$, and in order to compare h_1 and h_2 correctly, it needs to extend shorter HFE until the length of both HFEs are the same [7, 36]. This unification is conducted by three manners: (see, for example, [9, 10]) if $l = \max\{l_{h_1}, l_{h_2}\}$, then,

- it is repeated the shortest value (in the *pessimistic* case);

- the largest value is repeated (in the *optimistic* case);
- it can be taken the convex combination of maximum and minimum values of a HFE (in the case that depends on the decision makers' risk preference).

In a recent study given by Zhu and Xu [40], an extension of a HFS, called extended HFS (EHFS), is defined in terms of a function which returns a finite set of membership value-groups.

Definition 2.2. [40] *Suppose that X is the reference set, and $h_k(x) = \{h_k^{\delta(i)}(x) \mid i = 1, ..., l_{h_k}\}$ for $k = 1, .., m_x$ denote a family of m_x hesitant fuzzy elements defined for a given $x \in X$. Then, the version of Zhu and Xu's [40] extended hesitant fuzzy set (Z-EHFS) is defined as:*

$$\begin{aligned}\mathbf{H} &= \{\langle x, h_1(x) \times h_2(x) \times ... \times h_{m_x}(x)\rangle \mid x \in X\} \\ &= \{\langle x, \bigcup_{(\gamma_1(x),...,\gamma_m(x)) \in h_1(x) \times h_2(x) \times ... \times h_{m_x}(x)} \{(\gamma_1(x), ..., \gamma_m(x))\}\rangle \mid x \in X\}.\end{aligned} \tag{1}$$

Farhadinia and Herrera-Viedma [11] indicated that each element of a EHFS, called the extended hesitant fuzzy element (EHFE), is indeed a set of n-tuples demonstrating the opinion of n number of decision makers. If there exist m criteria to be evaluated, then the corresponding concept of EHFS proposed by Zhu and Xu [40] is to be in form of m-tuples elements which describes really Cartesian product of m HFEs. This asserts that an element of EHFE does not generally play the role of element included in Cartesian product of HFEs similar to that considered in Zhu and Xu [40].

Now, we are in a position to recall once again the re-visited concept of EHFS as the following form:

Definition 2.3. [11] *Suppose that X is the reference set. Then, an extended hesitant fuzzy set (EHFS) is characterized by the following mathematical symbol:*

$$\mathbf{H} = \{\langle x, \mathbf{h}(x)\rangle \mid x \in X\} = \{\langle x, \bigcup_{(\gamma_1(x),...,\gamma_m(x)) \in \mathbf{h}(x)} \{(\gamma_1(x), ..., \gamma_m(x))\}\rangle \mid x \in X\}, \tag{2}$$

where

$$\mathbf{h} = \bigcup_{(\gamma_1,...,\gamma_m) \in \mathbf{h}} \{(\gamma_1, ..., \gamma_m)\}, \tag{3}$$

stands for an extended HFE (EHFE).

Example 2.4. *Suppose that $X = \{x_1, x_2\}$ is the reference set, and $\mathbf{h}_1(x) = \{(0.6, 0.3, 0.3), (0.5, 0.2, 0.2)\}$ and $\mathbf{h}_2(x) = \{(0.3, 0.2, 0.1)\}$ are two EHFEs on X. Then, the EHFS $\mathbf{H}$ is characterized by*

$$\mathbf{H} = \{\langle x_1, \mathbf{h}_1(x)\rangle, \langle x_2, \mathbf{h}_2(x)\rangle\} = \{\langle x_1, \{(0.6, 0.3, 0.3), (0.5, 0.2, 0.2)\}\rangle, \langle x_2, \{(0.3, 0.2, 0.1)\}\rangle\}. \tag{4}$$

Farhadinia and Herrera-Viedma [11] described the following operational laws on EHFEs for aggregating the extended hesitant fuzzy information:

For two EHFEs $\mathbf{h}_1 = \{\mathbf{h}_1^{\delta(t)} := (\gamma_1^{1,\delta(t)}, ..., \gamma_m^{1,\delta(t)}) \mid t = 1, ..., l_{h_1}\}$ and $\mathbf{h}_2 = \{\mathbf{h}_2^{\delta(t)} := (\gamma_1^{2,\delta(t)}, ..., \gamma_m^{2,\delta(t)}) \mid t = 1, ..., l_{h_2}\}$, the arithmetic operations are defined as:

- Addition:

$$\begin{aligned}\mathbf{h}_1 \oplus \mathbf{h}_2 &= \bigcup_{\mathbf{h}_1^{\delta(t)} \in \mathbf{h}_1, \mathbf{h}_2^{\delta(r)} \in \mathbf{h}_2} \{\mathbf{h}_1^{\delta(t)} + \mathbf{h}_2^{\delta(r)} - \mathbf{h}_1^{\delta(t)}\mathbf{h}_2^{\delta(r)}\} \\ &= \bigcup_{(\gamma_1^{1,\delta(t)},...,\gamma_m^{1,\delta(t)}) \in \mathbf{h}_1, (\gamma_1^{2,\delta(r)},...,\gamma_m^{2,\delta(r)}) \in \mathbf{h}_2} \{(\gamma_1^{1,\delta(t)} + \gamma_1^{2,\delta(r)} - \gamma_1^{1,\delta(t)}\gamma_1^{2,\delta(r)}, ..., \gamma_m^{1,\delta(t)} + \gamma_m^{2,\delta(r)} - \gamma_m^{1,\delta(t)}\gamma_m^{2,\delta(r)})\};\end{aligned} \tag{5}$$

- Multiplication:

$$\begin{aligned}\mathbf{h}_1 \otimes \mathbf{h}_2 &= \bigcup_{\mathbf{h}_1^{\delta(t)} \in \mathbf{h}_1, \mathbf{h}_2^{\delta(r)} \in \mathbf{h}_2} \{\mathbf{h}_1^{\delta(t)}\mathbf{h}_2^{\delta(r)}\} \\ &= \bigcup_{(\gamma_1^{1,\delta(t)},...,\gamma_m^{1,\delta(t)}) \in \mathbf{h}_1, (\gamma_1^{2,\delta(r)},...,\gamma_m^{2,\delta(r)}) \in \mathbf{h}_2} \{(\gamma_1^{1,\delta(t)}\gamma_1^{2,\delta(r)}, ..., \gamma_m^{1,\delta(t)}\gamma_m^{2,\delta(r)})\};\end{aligned} \tag{6}$$

- Multiplication by scalar:

$$\begin{aligned}\lambda\mathbf{h}_1 &= \bigcup_{\mathbf{h}_1^{\delta(t)}\in\mathbf{h}_1}\{1-(1-\mathbf{h}_1^{\delta(t)})^{\lambda}\}\\ &= \bigcup_{(\gamma_1^{1,\delta(t)},...,\gamma_m^{1,\delta(t)})\in\mathbf{h}_1}\{(1-(1-\gamma_1^{1,\delta(t)},...,1-\gamma_m^{1,\delta(t)})^{\lambda})\}, \qquad \lambda>0;\end{aligned} \tag{7}$$

- Power:

$$\begin{aligned}\mathbf{h}_1^{\lambda} &= \bigcup_{\mathbf{h}_1^{\delta(t)}\in\mathbf{h}_1}\{(\mathbf{h}_1^{\delta(t)})^{\lambda}\}\\ &= \bigcup_{(\gamma_1^{1,\delta(t)},...,\gamma_m^{1,\delta(t)})\in\mathbf{h}_1}\{([\gamma_1^{1,\delta(t)}]^{\lambda},...,[\gamma_m^{1,\delta(t)}]^{\lambda})\}, \qquad \lambda>0.\end{aligned} \tag{8}$$

By the help of following theorem, we will guarantee the well-defined property of above-mentioned operations.

Theorem 2.5. *For any two EHFEs* $\mathbf{h}_1=\{\mathbf{h}_1^{\delta(t)}:=(\gamma_1^{1,\delta(t)},...,\gamma_m^{1,\delta(t)})\mid t=1,...,l_{h_1}\}$ *and* $\mathbf{h}_2=\{\mathbf{h}_2^{\delta(t)}:=(\gamma_1^{2,\delta(t)},...,\gamma_m^{2,\delta(t)})\mid t=1,...,l_{h_2}\}$*, the arithmetic operations introduced above, that is,* $\mathbf{h}_1\oplus\mathbf{h}_2$*,* $\mathbf{h}_1\otimes\mathbf{h}_2$*,* $\lambda\mathbf{h}_1$ *and* $\mathbf{h}_1^{\lambda}$ *are all well-defined.*

Proof. We only prove the first aim, and the other cases are proven similarly. From definition of two EHFEs $\mathbf{h}_1$ and $\mathbf{h}_2$, we find that $\mathbf{h}_1\in\mathcal{U}([0,1]^m,l_{h_1})$, and $\mathbf{h}_2\in\mathcal{U}([0,1]^m,l_{h_2})$, in which $\mathcal{U}([0,1]^m,l_{h_i})$ denotes the union of all l_{h_i} vectors in $[0,1]^m$ for $i=1,2$.

On the other hand, from the fact that $\mathbf{h}_1^{\delta(t)}:=(\gamma_1^{1,\delta(t)},...,\gamma_m^{1,\delta(t)})\in[0,1]^m$ for any $t=1,...,l_{h_1}$, and moreover, $\mathbf{h}_2^{\delta(t)}:=(\gamma_1^{2,\delta(t)},...,\gamma_m^{2,\delta(t)})\in[0,1]^m$ for any $t=1,...,l_{h_2}$, we result in

$$0\le\gamma_j^{1,\delta(t)}+\gamma_j^{2,\delta(r)}-\gamma_j^{1,\delta(t)}\gamma_j^{2,\delta(r)}\le 1, \quad j=1,2,...,m,$$

for any $t=1,...,l_{h_1}$ and $r=1,...,l_{h_2}$. Thus, we conclude that

$$(\gamma_1^{1,\delta(t)}+\gamma_1^{2,\delta(r)}-\gamma_1^{1,\delta(t)}\gamma_1^{2,\delta(r)},...,\gamma_m^{1,\delta(t)}+\gamma_m^{2,\delta(r)}-\gamma_m^{1,\delta(t)}\gamma_m^{2,\delta(r)})\in[0,1]^m$$

for any $(\gamma_1^{1,\delta(t)},...,\gamma_m^{1,\delta(t)})\in\mathbf{h}_1$ and $(\gamma_1^{2,\delta(r)},...,\gamma_m^{2,\delta(r)})\in\mathbf{h}_2$.

Therefore,

$$\bigcup_{(\gamma_1^{1,\delta(t)},...,\gamma_m^{1,\delta(t)})\in\mathbf{h}_1,(\gamma_1^{2,\delta(r)},...,\gamma_m^{2,\delta(r)})\in\mathbf{h}_2}\{(\gamma_1^{1,\delta(t)}+\gamma_1^{2,\delta(r)}-\gamma_1^{1,\delta(t)}\gamma_1^{2,\delta(r)},...,\gamma_m^{1,\delta(t)}+\gamma_m^{2,\delta(r)}-\gamma_m^{1,\delta(t)}\gamma_m^{2,\delta(r)})\}$$
$$\in\mathcal{U}([0,1]^m,l_h:=l_{h_1}\times l_{h_2}).$$

The above result concludes that $\mathbf{h}_1\oplus\mathbf{h}_2\in\mathcal{U}([0,1]^m,l_h)$, implying that the operation $\oplus$ on EHFEs is well-defined.

Now, we are in a position to explain how two EHFEs may be unified according to their number of elements. Let $\mathbf{h}_1$ and $\mathbf{h}_2$ be two EHFEs. In the case where $l_{h_1}\ne l_{h_2}$, the shorter EHFE is extended until the length of both EHFEs are the same. To do this, assume that $l=\max\{l_{h_1},l_{h_2}\}$. Then, the shorter EHFE is extended by appending the same value repeatedly. The appended value depends on the risk preferences in a decision making case. In the *pessimistic* case, it is repeated the shortest value, and in the *optimistic* case, the largest value is repeated. Although, another technique is to append the convex combination of maximum and minimum values of an EHFE according to risk preferences in a decision making case, but this technique considers the virtual values which are not those to be added.

Besides that and in the general case, whenever there exist K number of EHFEs $\mathbf{h}_1$, $\mathbf{h}_2$,..., $\mathbf{h}_K$ which are to be compared, it is considered the extension of each shorter EHFE until the length of all EHFEs are the same as $l=\max\{l_{h_1},...,l_{h_K}\}$.

Let us here describe a concept that will be used hereafter, and plays an important role in the next discussions.

Prospect theory describes generally a decision making process which takes the conditions of risk into consideration. In fact, the prospect theory is based on two phases including edition and evaluation [29]. Through the edition phase, each outcome is stated by using gain or loss parameters from a reference point. Then, by the help of a value function, the edited prospect is evaluated in the evaluation phase, and moreover, the prospect of highest value is selected. Needless to say that the value function depends on deviation distance from the reference point, and it is described by the following definition (see [29])

$$V(x)=\begin{cases} x^{\alpha}, & x\geq 0;\\ -\lambda(-x)^{\beta}, & x<0, \end{cases} \tag{9}$$

in which the first rule comes to the gain value, and the second one indicates the loss value. Moreover, $0\leq\alpha,\beta\leq 1$,

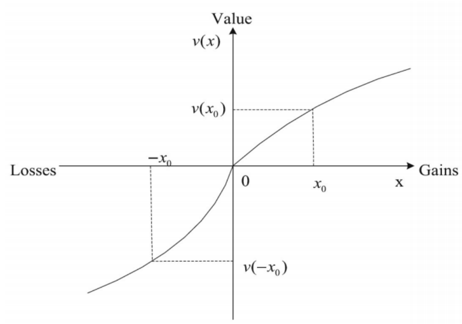


Figure 1: The S-shaped value function of prospect theory.

and $\lambda>1$. This can be clearly seen from Figure 1.

3 Group emergency decision making technique based on EHFSs

In the current section, it is going mainly to propose the extended hesitant group emergency decision making technique being based on the prospect theory, and depicted graphically in Figure 2.

3.1 The problem structure

Suppose that $A=\{A_1,...,A_{m_a}\}$ denotes a collection of alternatives, and the set $C=\{C_1,...,C_{n_c}\}$ indicates the collection of criteria with corresponding weight vector $w=(w_1,...,w_{n_c})$ where $w_j\in[0,1]$ for $j=1,...,n_c$ and $\sum_{j=1}^{n_c}w_j=1$. Moreover, let $E=\{e_1,...,e_m\}$ be the set of fixed number of m decision makers.

If $C_j^k(A_i)$ stands for the opinion of the decision maker e_k providing for the alternative A_i with respect to the criterion C_j, then the notation $\overline{C}_j^k(A_i)$ is used to denote the normalization form of $C_j^k(A_i)$. Moreover, in the next discussions, the set $\mathbf{h}_{ij}=\{\mathbf{h}_{ij}^{\delta(t)}=(\gamma_1^{ij,\delta(t)},...,\gamma_m^{ij,\delta(t)})\mid t=1,...,l_h\}$ is used to indicate the EHFE which is formed by taking the normalized terms $\overline{C}_j^k(A_i)$ for $k=1,...,m$, $i=1,...,m_a$ and $j=1,...,n_c$ into account.

It is assumed that Cartesian product of intervals $I_{ij}^{l_h}=[I_{ij}^{L,1},I_{ij}^{U,1}]\times...\times[I_{ij}^{L,l_h},I_{ij}^{U,l_h}]$ describes the effective control scope [33] over the alternative A_i regarding the criterion C_j, and also, Cartesian product of intervals $J_j^{l_h}=[J_j^{L,1},J_j^{U,1}]\times...\times[J_j^{L,l_h},J_j^{U,l_h}]$ stands for that reference point given by the decision maker corresponding to the criterion C_j.

As explained before, considering the concept of HFS may lead to loss of information that is important to identify the opinion of decision makers, and such a consideration causes probably wrong decisions without respect to which expert evaluates which opinion.

In order to remove such a limitation, it is assumed that the experts' preferences to be described by the EHFSs in a group decision making problem for keeping information as much as possible.

Now, it is time to specify the effective control scope of all alternatives being described by hesitation vectors in form of EHFSs. Keeping this in mind, the decision making algorithm is now described by using the following steps:

Step 1. Information of the emergency alternative
Using the analysis of emergency alternatives, the decision maker e_k returns the information $C_j^k(A_i)$ which describes the emergency alternative A_i with respect to the criterion C_j.

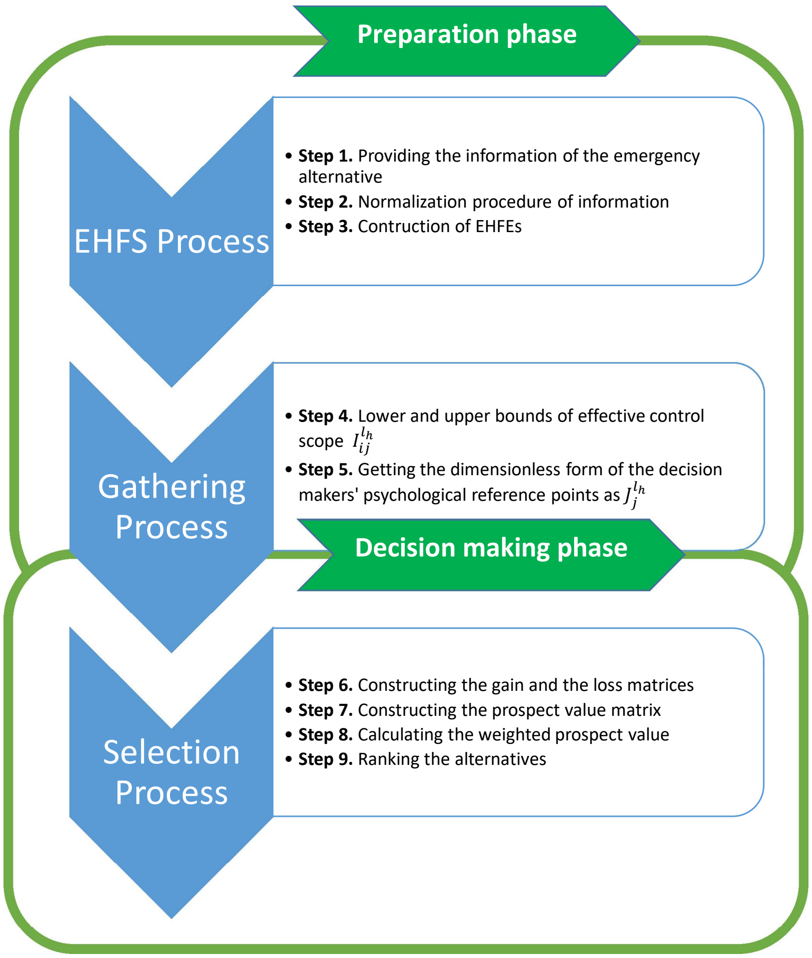


Figure 2: General scheme of the proposed technique.

Step 2. Normalization process
Using the following rule

$$\overline{C}_j^k(A_i) = \frac{C_j^k(A_i)}{\max_{i=1,\dots,m_a}\{\max_{k=1,\dots,m}\{C_j^k(A_i)\}\}}, \quad j = 1, ..., n_c, \tag{10}$$

the information $C_j^k(A_i)$ is normalized to $\overline{C}_j^k(A_i)$ for $k = 1, ..., m$, $i = 1, ..., m_a$ and $j = 1, ..., n_c$.

Step 3. Construction of the EHFEs
Being $\overline{C}_j^k(A_i)$ for $k = 1, ..., m$, $i = 1, ..., m_a$ and $j = 1, ..., n_c$ at hand, it is constructed the EHFEs

$$\mathbf{h}_{ij} = \{\mathbf{h}_{ij}^{\delta(t)} = (\overline{C}_j^{1,\delta(t)}(A_i), ..., \overline{C}_j^{m,\delta(t)}(A_i)) \mid t = 1, ..., l_h\}, \quad i = 1, ..., m_a, \; j = 1, ..., n_c. \tag{11}$$

Step 4. Lower and upper bounds of effective control scope
The lower and upper bounds of effective control scope $I_{ij}^{l_h} = [I_{ij}^{L,1}, I_{ij}^{U,1}] \times ... \times [I_{ij}^{L,l_h}, I_{ij}^{U,l_h}]$ are specified as follows:

$$I_{ij}^{L,\delta(t)} = \min\{\overline{C}_j^{1,\delta(t)}(A_i), ..., \overline{C}_j^{m,\delta(t)}(A_i)\}; \tag{12}$$

$$I_{ij}^{U,\delta(t)} = \max\{\overline{C}_j^{1,\delta(t)}(A_i), ..., \overline{C}_j^{m,\delta(t)}(A_i)\}, \tag{13}$$

where $t = 1, ..., l_h$, $i = 1, ..., m_a$ and $j = 1, ..., n_c$.

Here, it should be mentioned that the Cartesian product of intervals $I_{ij}^{l_h}$ enables maintaining the decision maker' opinions to the highest level possible.

Step 5. Dimensionlessing of the decision makers' psychological reference points
For making the decision makers' psychological reference points $J_j^{l_h} = [J_j^{L,1}, J_j^{U,1}] \times ... \times [J_j^{L,l_h}, J_j^{U,l_h}]$ to be dimensionless, they are defined as:

$$\overline{J}_j^{L,\delta(t)} = \frac{J_j^{L,\delta(t)}}{\max_{i=1,...,m_a}\{\max_{k=1,...,m}\{C_j^k(A_i)\}\}}, \tag{14}$$

$$\overline{J}_j^{U,\delta(t)} = \frac{J_j^{U,\delta(t)}}{\max_{i=1,...,m_a}\{\max_{k=1,...,m}\{C_j^k(A_i)\}\}}, \tag{15}$$

where $t = 1, ..., l_h$ and $j = 1, ..., n_c$.

Remark 3.1. *As explained above, in the case where the EHFEs are not with the same length, the extension of the shorter EHFEs are constructed in such a way that the lengths of all EHFEs are the same.*

What needs to be explained here is the calculation of reference points that is mainly based on the decision maker's psychology state being different from a psychology state to another one, even for the same commodity. Therefore, as expected, the reference point may be varied from case to case, and simply it is assumed that the reference point is given by the decision maker [37].

Step 6. Gain and loss matrices
Taking the Cartesian product of intervals $I_{ij}^{l_h}$ and the decision makers' psychological reference points $J_j^{l_h}$ into account, it is time to construct the gain and loss matrices.

It is pointed out that the interested reader refers to [13, 20, 33, 38] for the deep study of calculating the gain and loss matrices with respect to all the possible cases in form of different extensions of fuzzy sets. Here, it is not intended to present the same process of calculating, and therefore that process for EHFSs is compacted as follows:

Before presenting the calculation formulae of gain and loss matrices, it is here highlighted that the criteria are generally classified into the benefit and cost ones [21].

Firstly, let the EHFSs be degenerated to the HFSs, that is, it is thought that $l_h = 1$. Then, the gain and loss formulaes for the benefit and cost criteria with respect to all possible cases are given below. Here, note that the positional relationships of the Cartesian product of intervals $I_{ij}^{l_h=1}$ and the decision makers' psychological reference points $J_j^{l_h=1}$ are shown as in Figure 3.

In order to specify the gain and loss definitions regarding to each alternative, it is taken the following probability density function into consideration for the effective control scope I_{ij}^1 of alternatives. Let

$$f^{(1)}(x) = \begin{cases} \frac{1}{I_{ij}^{U,1} - I_{ij}^{L,1}}, & I_{ij}^{L,1} \le x \le I_{ij}^{U,1}; \\ 0, & \text{otherwise}, \end{cases} \tag{16}$$

such that $\int_{I_{ij}^{L,1}}^{I_{ij}^{U,1}} f^{(1)}(x)dx = 1$, and moreover, $f^{(1)}(x) \ge 0$ for any $x \in [I_{ij}^{L,1}, I_{ij}^{U,1}]$.

Now, it is time to characterize the general forms of gain and loss by keeping Table 1 in the mind:

Benefit case:
(Loss:)

$$-[sgn(\frac{\overline{J}_j^{L,1} - I_{ij}^{L,1}}{2})\frac{\overline{J}_j^{L,1} - I_{ij}^{L,1}}{2} + sgn(\frac{\overline{J}_j^{L,1} - I_{ij}^{U,1}}{2})\frac{\overline{J}_j^{L,1} - I_{ij}^{U,1}}{2}], \tag{17}$$

(Gain:)

$$-[[1 - sgn(\frac{\overline{J}_j^{U,1} - I_{ij}^{L,1}}{2})]\frac{\overline{J}_j^{U,1} - I_{ij}^{L,1}}{2} + [1 - sgn(\frac{\overline{J}_j^{U,1} - I_{ij}^{U,1}}{2})]\frac{\overline{J}_j^{U,1} - I_{ij}^{U,1}}{2}], \tag{18}$$

Cost case:
(Loss:)

$$sgn(\frac{\overline{J}_j^{L,1} - I_{ij}^{L,1}}{2})\frac{\overline{J}_j^{L,1} - I_{ij}^{L,1}}{2} + sgn(\frac{\overline{J}_j^{L,1} - I_{ij}^{U,1}}{2})\frac{\overline{J}_j^{L,1} - I_{ij}^{U,1}}{2}, \tag{19}$$

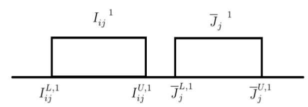
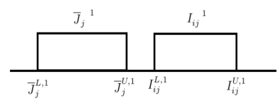
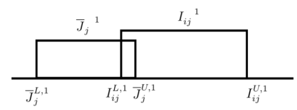
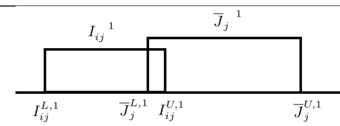
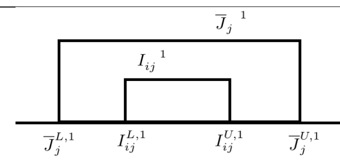
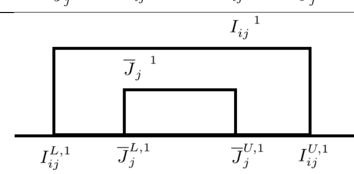
Cases		Comparison relationship between I_{ij}^1 and $\bar{J}_j^1$
Case 1	$I_{ij}^{U,1} < \bar{J}_j^{L,1}$	
Case 2	$\bar{J}_j^{U,1} < I_{ij}^{L,1}$	
Case 3	$\bar{J}_j^{L,1} < I_{ij}^{L,1} \leq \bar{J}_j^{U,1} < I_{ij}^{U,1}$	
Case 4	$I_{ij}^{L,1} < \bar{J}_j^{L,1} \leq I_{ij}^{U,1} < \bar{J}_j^{U,1}$	
Case 5	$\bar{J}_j^{L,1} < I_{ij}^{L,1} < I_{ij}^{U,1} < \bar{J}_j^{U,1}$	
Case 6	$I_{ij}^{L,1} < \bar{J}_j^{L,1} < \bar{J}_j^{U,1} < I_{ij}^{U,1}$	

Figure 3: Possible comparison of the interval $I_{ij}^1 = [I_{ij}^{L,1}, I_{ij}^{U,1}]$ and decision makers' psychological reference point $\overline{J}_j^1 = [J_j^{L,1}, J_j^{U,1}]$.

(Gain:)

$$[1 - sgn(\frac{\overline{J}_j^{U,1} - I_{ij}^{L,1}}{2})]\frac{\overline{J}_j^{U,1} - I_{ij}^{L,1}}{2} + [1 - sgn(\frac{\overline{J}_j^{U,1} - I_{ij}^{U,1}}{2})]\frac{\overline{J}_j^{U,1} - I_{ij}^{U,1}}{2}. \tag{20}$$

In all the above formulaes, the notation $sgn(.)$ refers to the sign function:

$$sgn(x) = \begin{cases} 1, & x \geq 0; \\ 0, & x < 0. \end{cases} \tag{21}$$

According to the benefit criterion, it can be deduced that the higher value indicates the better one, while the higher value of cost criterion shows the worse one. Based on this issue, and in the case of the cost criterion, it is found that the relation $I_{ij}^{U,1} < \overline{J}_j^{L,1}$ implies that the decision maker feels gains, and the relation $I_{ij}^{L,1} > \overline{J}_j^{U,1}$ shows that the decision maker feels losses.

Remark 3.2. *Once again, it is emphasized here that all the above-mentioned relations are considered only for the special case of the positional relationship of the Cartesian product of interval $I_{ij}^{l_h} := I_{ij}^1$ and the decision makers' psychological reference point $J_j^{l_h} := J_j^1$, and they can be easily extended to the general cases*

$$I_{ij}^{l_h} := I_{ij}^1 \times ... \times I_{ij}^{l_h} = [I_{ij}^{L,1}, I_{ij}^{U,1}] \times ... \times [I_{ij}^{L,l_h}, I_{ij}^{U,l_h}], \tag{22}$$

and

$$J_j^{l_h} := J_j^1 \times ... \times J_j^{l_h} = [J_j^{L,1}, J_j^{U,1}] \times ... \times [J_j^{L,l_h}, J_j^{U,l_h}]. \tag{23}$$

Taking the benefit and cost cases into account, the gain matrix is constructed as:

$$\mathbf{G} = G^{(1)} \times ... \times G^{(l_h)} \tag{24}$$

and the loss matrix is constructed as:

$$\mathbf{L} = L^{(1)} \times ... \times L^{(l_h)}, \tag{25}$$

where $G^{(t)}$ (respectively, $L^{(t)}$) stands for the gain matrice (respectively, the loss matrice) of t-th Cartesian product of the interval I_{ij}^t and the decision makers' psychological reference point J_j^t for $t = 1, ..., l_h$.

Step 7. Prospect value matrix

Now, similar to that work done in [13] and by the use of the gain and loss matrices **G** and **L**, it is constructed the prospect value matrix:

$$\mathbf{V} = V^{(1)} \times ... \times V^{(l_h)} \tag{26}$$

which is given by

$$\begin{aligned}&(V^{(1)} \times ... \times V^{(l_h)})_{ij} = \\ &[(G^{(1)})_{ij}]^{\alpha}\chi_{(G^{(1)}{}_{ij})} + (-\lambda(-[(L^{(1)})_{ij}]^{\beta}\chi_{(L^{(1)}{}_{ij})})) \times ... \times [(G^{(l_h)})_{ij}]^{\alpha}\chi_{(G^{(l_h)}{}_{ij})} + (-\lambda(-[(L^{(l_h)})_{ij}]^{\beta}\chi_{(L^{(l_h)}{}_{ij})})),\end{aligned} \tag{27}$$

where $(V^{(1)} \times ... \times V^{(l_h)})_{ij}$ is calculated according to the i-th alternative and the j-th criterion, and

$$\chi_{(G^{(t)}{}_{ij})} = \begin{cases} 1, & G^{(t)}ij > 0; \\ 0, & G^{(t)}ij \leq 0, \end{cases} \qquad \chi_{(L^{(t)}{}_{ij})} = \begin{cases} 1, & L^{(t)}ij > 0; \\ 0, & L^{(t)}ij \leq 0, \end{cases} \tag{28}$$

for $t = 1, ..., l_h$. Moreover, as considered in [29], it is hereafter set $\alpha = 0.89$, $\beta = 0.92$ and $\lambda = 2.25$.

Step 8. Weighted prospect value of alternatives

In this stage, it is time to deal with the computation of weighted *overall prospect value* of alternatives by the help of the simple additive weighting method [14]:

$$\mathbf{OPV}_i = (OPV^{(1)} \times ... \times OPV^{(l_h)})_i = \sum_{j=1}^{n_c} w_j (V^{(1)} \times ... \times V^{(l_h)})_{ij}, \tag{29}$$

where w_j indicates the relative weight of j-th criterion, and it satisfies $\sum_{j=1}^{n_c} w_j = 1$.

Step 9. Ranking of the alternatives

Before giving any more details on the achievement of values $\mathbf{OPV}_i$ $(i = 1, ..., m_a)$, here it is referred to Farhadinia [6] where a collection of aggregation operations are presented to map a collection of sets to a combined set in a desirable way. Following that procedure, the best alternative can be selected regarding to the biggest value of aggregation operation on overall prospect values. Note that the aggregated values will be ranked in a descending order.

Up till now, we have addressed an extended group emergency decision-making technique in which by the use of the prospect theory we explained the psychological behaviour of that decision makers being effected by the hesitancy and uncertainty features. We presented here a fusion technique being based on the modified version of EHFS instead of implementing the typical aggregation procedure. Such an implementation obviously keeps the possible amount of expert's information compared to the employment of HFSs. This clearly comes from EHFS potential role in increasing the richness of numerical appearance in the form of value-groups, and in expanding the practical applications of HFSs in decision making situations.

4 Application of extended hesitant group emergency decision making

In the following, it will be dealt with the barrier lake problem which is caused by a huge earthquake in the southwestern area of China, and it was investigated by Zhang et al. [38] based on the concept of HFS.

4.1 The experimental problem

As it is known, a barrier lake causes more or less damage to the lives and of course the properties of a considerable number of people, and in such a situation a decision maker should do an urgent response for controlling adequately that situation, and more importantly does not permit to increase the rate of deterioration.

Zhang et al. [38] considered the three criteria including C_1: Alternative cost; C_2: Casualty number; and C_3: The loss of property. Keeping such a description into the mind, Zhang et al. [38] described the emergency alternatives as the following: A_1: Conducting people from the most critical places to the safe ones; A_2: Increasing the accessibility to the basic requirements for reducing the barrier lake pressure; A_3: Reducing the parameters of risk that come from the break of dam; A_4: Increasing the accessibility to the basic requirements for reducing the barrier lake pressure, meanwhile, the parameters of risk of dam breaking are reduced. What should be considered in such a modelling is that the above emergency alternatives are happened consequently.

According to the knowledge of certified experts, there exist four emergency cases during the three first days which may be considered as the following rates for the barrier lake: r_1: The rate of breaking of dam body is zero; r_2: The rate of breaking of dam body is $\frac{1}{3}$; r_3: The rate of breaking of dam body is $\frac{1}{2}$; and r_4: The rate of breaking of dam body is 1.

In order to make an appropriate decision, the decision maker is supported by the help of three experts (i.e., $k = 3$) such that they represent their idea about four emergency alternatives with respect to the effective control scopes in the form of preferences $C_j^k(A_i)$. In fact, the preferences $C_j^k(A_i)$ which are given in Table 1 describe the emergency alternatives A_i $(i = 1, 2, 3, m_a = 4)$ with respect to the criteria C_j $(j = 1, 2, n_c = 3)$. Moreover, all these preferences are to be normalized by the help of the rule (10) in the form of $\overline{C}_j^k(A_i)$.

Alternative	Expert	Criterion [weight]					
		C_1 [0.3]		C_2 [0.4]		C_3 [0.3]	
		$C_1^k(A_i)$	$\overline{C}_1^k(A_i)$	$C_2^k(A_i)$	$\overline{C}_2^k(A_i)$	$C_3^k(A_i)$	$\overline{C}_3^k(A_i)$
A_1	e_1	$250, 280$	$0.42, 0.47$	5000	0.59	3500	0.64
	e_2	280	0.47	5000	0.59	$3000, 3500$	$0.55, 0.64$
	e_3	300	0.50	4000	0.47	4000	0.73
A_2	e_1	300	0.50	6500	0.76	$4000, 4500$	$0.73, 0.82$
	e_2	$300, 350$	$0.50, 0.58$	5500	0.65	4000	0.73
	e_3	350	0.58	5000	0.59	4500	0.82
A_3	e_1	400	0.67	7000	0.82	$4500, 5000$	$0.82, 0.91$
	e_2	350	0.58	7500	0.88	4500	0.82
	e_3	$350, 400$	$0.58, 0.67$	6500	0.76	5000	0.91
A_4	e_1	600	1.00	8000	0.94	5000	0.91
	e_2	$500, 550$	$0.83, 0.92$	8500	1.00	5300	0.96
	e_3	550	0.92	7500	0.88	$5300, 5500$	$0.96, 1.00$

Table 1. The preferences $C_j^k(A_i)$ together with their normalized forms $\overline{C}_j^k(A_i)$.

Alternative	Expert	Criterion [weight]					
		C_1 [0.3]		C_2 [0.4]		C_3 [0.3]	
		$C_1^k(A_i)$	$\overline{C}_1^k(A_i)$	$C_2^k(A_i)$	$\overline{C}_2^k(A_i)$	$C_3^k(A_i)$	$\overline{C}_3^k(A_i)$
A_1	$(e_1^{(1)}, e_2^{(1)}, e_3^{(1)})$	$(250, 280, 300)$	$(0.42, 0.47, 0.50)$	$(5000, 5000, 4000)$	$(0.59, 0.59, 0.47)$	$(3500, 3000, 4000)$	$(0.64, 0.55, 0.73)$
	$(e_1^{(2)}, e_2^{(2)}, e_3^{(2)})$	$(280, 280, 300)$	$(0.47, 0.47, 0.50)$			$(3500, 3500, 4000)$	$(0.64, 0.64, 0.73)$
A_2	$(e_1^{(1)}, e_2^{(1)}, e_3^{(1)})$	$(300, 300, 350)$	$(0.50, 0.50, 0.58)$	$(6500, 5500, 5000)$	$(0.76, 0.65, 0.59)$	$(4000, 4000, 4500)$	$(0.73, 0.73, 0.82)$
	$(e_1^{(2)}, e_2^{(2)}, e_3^{(2)})$	$(300, 350, 350)$	$(0.50, 0.58, 0.58)$			$(4500, 4000, 4500)$	$(0.82, 0.73, 0.82)$
A_3	$(e_1^{(1)}, e_2^{(1)}, e_3^{(1)})$	$(400, 350, 350)$	$(0.67, 0.58, 0.58)$	$(7000, 7500, 6500)$	$(0.82, 0.88, 0.76)$	$(4500, 4500, 5000)$	$(0.82, 0.82, 0.91)$
	$(e_1^{(2)}, e_2^{(2)}, e_3^{(2)})$	$(400, 350, 400)$	$(0.67, 0.58, 0.67)$			$(5000, 4500, 5000)$	$(0.91, 0.82, 0.91)$
A_4	$(e_1^{(1)}, e_2^{(1)}, e_3^{(1)})$	$(600, 500, 550)$	$(1.00, 0.83, 0.92)$	$(8000, 8500, 7500)$	$(0.94, 1.00, 0.88)$	$(5000, 5300, 5300)$	$(0.91, 0.96, 0.96)$
	$(e_1^{(2)}, e_2^{(2)}, e_3^{(2)})$	$(600, 550, 550)$	$(1.00, 0.92, 0.92)$			$(5000, 5300, 5500)$	$(0.91, 0.96, 1.00)$

Table 1. Continued.

By the help of all the vectors of $\overline{C}_1^k(A_i)$ which are represented in Table 1, it can be constructed the corresponding HFEs as given by (11). Moreover, it can be calculated the interval forms of the effective control scope $I_{ij}^{l_h} = [I_{ij}^{L,1}, I_{ij}^{U,1}] \times \ldots \times [I_{ij}^{L,l_h}, I_{ij}^{U,l_h}]$ by the use of (12)-(13). These results are all summarized in Table 2.

	Criterion [weight]		
Alternative	C_1 [0.3]	C_2 [0.4]	C_3 [0.3]
A_1	$\mathbf{h}_{11} = \{(0.42, 0.47, 0.50), (0.47, 0.47, 0.50)\}$ $I_{11}^2 = [0.42, 0.50] \times [0.47, 0.50]$	$\mathbf{h}_{12} = \{(0.59, 0.59, 0.47)\}$ $I_{12}^1 = [0.47, 0.59]$	$\mathbf{h}_{13} = \{(0.64, 0.55, 0.73), (0.64, 0.64, 0.73)\}$ $I_{13}^2 = [0.55, 0.73] \times [0.64, 0.73]$
A_2	$\mathbf{h}_{21} = \{(0.50, 0.50, 0.58), (0.50, 0.58, 0.58)\}$ $I_{21}^2 = [0.50, 0.58] \times [0.50, 0.58]$	$\mathbf{h}_{22} = \{(0.76, 0.65, 0.59)\}$ $I_{22}^1 = [0.59, 0.76]$	$\mathbf{h}_{23} = \{(0.73, 0.73, 0.82), (0.82, 0.73, 0.82)\}$ $I_{23}^2 = [0.73, 0.82] \times [0.73, 0.82]$
A_3	$\mathbf{h}_{31} = \{(0.67, 0.58, 0.58), (0.67, 0.58, 0.67)\}$ $I_{31}^2 = [0.58, 0.67] \times [0.58, 0.67]$	$\mathbf{h}_{32} = \{(0.82, 0.88, 0.76)\}$ $I_{32}^1 = [0.76, 0.88]$	$\mathbf{h}_{33} = \{(0.82, 0.82, 0.91), (0.91, 0.82, 0.91)\}$ $I_{33}^2 = [0.82, 0.91] \times [0.82, 0.91]$
A_4	$\mathbf{h}_{41} = \{(1.00, 0.83, 0.92), (1.00, 0.92, 0.92)\}$ $I_{41}^2 = [0.83, 1.00] \times [0.92, 1.00]$	$\mathbf{h}_{42} = \{(0.94, 1.00, 0.88)\}$ $I_{42}^1 = [0.88, 1.00]$	$\mathbf{h}_{43} = \{(0.91, 0.96, 0.96), (0.91, 0.96, 1.00)\}$ $I_{43}^2 = [0.91, 0.96] \times [0.91, 1.00]$

Table 2. The EHFE $\mathbf{h}_{ij}$ and the interval form of the effective control scope $I_{ij}^{l_h}$ for each alternative.

In the above-considered problem, the decision makers' psychological reference points have been provided on the basis of four mentioned emergency cases regarding to the barrier lake. In this regard and according to the equations (14) and (15), the $J_j^{l_h} = [J_j^{L,1}, J_j^{U,1}] \times ... \times [J_j^{L,l_h}, J_j^{U,l_h}]$ results are that provided in the form of the interval values given in Table 3.

	Criterion		
Reference point	C_1	C_2	C_3
$J_j^{l_h}$	$[300, 350] \times [300, 400]$	$[6000, 7000]$	$[2000, 3000] \times [3000, 4000]$
$\overline{J}_j^{l_h}$	$[0.5, 0.58] \times [0.5, 0.67]$	$[0.71, 0.82]$	$[0.36, 0.55] \times [0.55, 0.73]$

Table 3. Decision makers' psychological reference points $J_j^{l_h}$ and the dimensionless form of $\overline{J}_j^{l_h}$.

In order to have a comprehensive comparison, it is extended all the normalized decision makers' psychological reference points $\overline{J}_j^{l_h}$ to that with the same dimension (see the results given in Table 4).

	Criterion		
Reference point	C_1	C_2	C_3
$\overline{J}_j^{l_h}$	$[0.5, 0.67] \times [0.5, 0.58]$	$[0.71, 0.82] \times [0.71, 0.82]$	$[0.36, 0.55] \times [0.55, 0.73]$

Table 4. Normalized decision makers' psychological reference points $\overline{J}_j^{l_h(=2)} = [\overline{J}_j^{L,1}, \overline{J}_j^{U,1}] \times [\overline{J}_j^{L,2}, \overline{J}_j^{U,2}]$.

Moreover, by extending the interval form of the effective control scope $I_{ij}^{l_h}$, the results of Table 5 are concluded.

	Criterion [weight]		
Alternative	C_1 [0.3]	C_2 [0.4]	C_3 [0.3]
A_1	$I_{11}^2 = [0.42, 0.50] \times [0.47, 0.50]$	$I_{12}^1 = [0.47, 0.59] \times [0.47, 0.59]$	$I_{13}^2 = [0.55, 0.73] \times [0.64, 0.73]$
A_2	$I_{21}^2 = [0.50, 0.58] \times [0.50, 0.58]$	$I_{22}^1 = [0.59, 0.76] \times [0.59, 0.76]$	$I_{23}^2 = [0.73, 0.82] \times [0.73, 0.82]$
A_3	$I_{31}^2 = [0.58, 0.67] \times [0.58, 0.67]$	$I_{32}^1 = [0.76, 0.88] \times [0.76, 0.88]$	$I_{33}^2 = [0.82, 0.91] \times [0.82, 0.91]$
A_4	$I_{41}^2 = [0.83, 1.00] \times [0.92, 1.00]$	$I_{42}^1 = [0.88, 1.00] \times [0.88, 1.00]$	$I_{43}^2 = [0.91, 0.96] \times [0.91, 1.00]$

Table 5. The interval form of the effective control scope $I_{ij}^{l_h(=2)} = [I_{ij}^{L,1}, I_{ij}^{U,1}] \times [I_{ij}^{L,2}, I_{ij}^{U,2}]$ for each alternative.

In what follows, by taking equations (17)-(18) and (19)-(20) into account, the gain matrix and also the loss matrix are respectively constructed in according to the rules given by (24) and (25):

$$\mathbf{G} = G^{(1)} \times G^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.0300 & 0 \\ 0.2450 & 0.1200 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.0450 & 0.0300 & 0 \\ 0.3800 & 0.1200 & 0 \end{pmatrix}, \tag{30}$$

and

$$\mathbf{L} = L^{(1)} \times L^{(2)} = \begin{pmatrix} -0.0400 & -0.1800 & -0.0900 \\ 0 & -0.0600 & -0.2550 \\ 0 & 0 & -0.3150 \\ 0 & 0 & -0.3850 \end{pmatrix} \times \begin{pmatrix} -0.0150 & -0.1800 & 0 \\ 0 & -0.0600 & -0.0450 \\ 0 & 0 & -0.3150 \\ 0 & 0 & -0.2250 \end{pmatrix}. \quad (31)$$

Since both the interval form of the effective control scope $I_{ij}^{l_h(=2)}$ and the normalized decision makers' psychological reference point $\overline{J}_j^{l_h(=2)}$ are now dimensionless, the prospect value matrix $\mathbf{V}$ is now constructed by the help of equation (26):

$$\mathbf{V} = V^{(1)} \times V^{(2)} = \begin{pmatrix} -0.1164 & -0.4646 & -0.2455 \\ 0 & -0.1691 & -0.6400 \\ 0 & 0.0441 & -0.7774 \\ 0.2860 & 0.1515 & -0.9350 \end{pmatrix} \times \begin{pmatrix} -0.0472 & -0.4646 & 0 \\ 0 & -0.1691 & -0.1298 \\ 0.0633 & 0.0441 & -0.7774 \\ 0.4227 & 0.1515 & -0.5704 \end{pmatrix}. \quad (32)$$

Now, it is time to construct the weighted overall prospect value of alternatives by the help of equation (29):

$$\begin{aligned} \mathbf{OPV} &= OPV^{(1)} \times OPV^{(2)} \\ &= w_1(V^{(1)} \times V^{(2)}) + w_2(V^{(1)} \times V^{(2)}) + w_3(V^{(1)} \times V^{(2)}) = \begin{pmatrix} -0.2944 \\ -0.2596 \\ -0.2156 \\ -0.1341 \end{pmatrix} \times \begin{pmatrix} -0.2000 \\ -0.1066 \\ -0.1966 \\ 0.0163 \end{pmatrix}. \end{aligned} \quad (33)$$

In this part of the contribution, it can be adopted any kind of Farhadinia [6]'s score functions that fulfill the two properties, known as the monotone non-decreasing and the boundary conditions properties, to achieve the ranking order of alternatives. For this purpose, it is selected here only the arithmetic-mean score function, and thus it is obtained that:

$$\mathbf{S}_{AM}(\mathbf{OPV}) = \mathbf{S}_{AM}(\begin{pmatrix} -0.2944 \\ -0.2596 \\ -0.2156 \\ -0.1341 \end{pmatrix} \times \begin{pmatrix} -0.2000 \\ -0.1066 \\ -0.1966 \\ 0.0163 \end{pmatrix}) = \begin{pmatrix} \mathbf{S}_{AM}(-0.2944, -0.2000) \\ \mathbf{S}_{AM}(-0.2596, -0.1066) \\ \mathbf{S}_{AM}(-0.2156, -0.1966) \\ \mathbf{S}_{AM}(-0.1341, \ 0.0163) \end{pmatrix} = \begin{pmatrix} -0.2472 \\ -0.1831 \\ -0.2061 \\ -0.0589 \end{pmatrix}. \quad (34)$$

Consequently, the ranking of alternatives is as

$$A_4 \succ A_2 \succ A_3 \succ A_1. \quad (35)$$

4.2 Comparative study and discussion

In this part of the article, we are going to verify the accuracy of the proposed EHFE-based technique by means of comparing the numerical results obtained from the proposed EHFE-based technique and the HFS-based techniques including the information fusion process [38] and the aggregation process constructed by the help of weighted average technique [24].

Following the same assumption as that made in [38] and [24], we suppose here that all the three experts' opinions are equally important. Therefore, the effective control scope $I_{ij}^{l_h}$ is now stated as the following aggregation operator:

$$I_{ij}^{l_h} = \frac{1}{3}\sum_{k=1}^{3} \overline{C}_j^k(A_i), \quad (36)$$

which is associated with the alternatives A_i $(i = 1, 2, 3, m_a = 4)$ and the criteria C_j $(j = 1, 2, n_c = 3)$.

Before going more into the details, let us take a look at the difference between modelling a problem using the concepts of EHFS and HFS. As it was pointed out in Introduction, EHFS concept is employed to describe a situation in which the number of experts is fixed, and the resulted outcomes must be vectors with length equal to the number of experts. Meanwhile, the outcomes resulted from HFS concept are not necessarily in the form of vectors with equal length. Definitely, such a deficiency of HFS implementation may reduce the efficiency of modelling accurately a group-values-based optimization problem.

By the way, let us here recall the real case study illustrated by Zhang et al. [38] about a barrier lake emergency which is caused by a huge earthquake occurring in the south western area of China.
What is interesting in Zhang et al.'s description of case study in [38] is the fact that they employed indeed the concept of EHFS instead of HFS without knowing and defining it priorly. This is because, in the modelling of that case study, the number of experts are considered to be fix, and therefore, the next decision making process is based on three-expert-vectors.

Now, we re-state the preferences of effective control scopes for alternatives like that supposed by Zhang et al. [38] in the form of HFEs (see Table 6).

	Criterion [weight]		
Alternative	C_1 [0.3]	C_2 [0.4]	C_3 [0.3]
A_1	$\mathbf{h}_{11} = \{(0.42, 0.47, 0.50)\}$	$\mathbf{h}_{12} = \{(0.59, 0.59, 0.47)\}$	$\mathbf{h}_{13} = \{(0.64, 0.55, 0.73)\}$
A_2	$\mathbf{h}_{21} = \{(0.50, 0.50, 0.58)\}$	$\mathbf{h}_{22} = \{(0.76, 0.65, 0.59)\}$	$\mathbf{h}_{23} = \{(0.73, 0.73, 0.82)\}$
A_3	$\mathbf{h}_{31} = \{(0.67, 0.58, 0.67)\}$	$\mathbf{h}_{32} = \{(0.82, 0.88, 0.76)\}$	$\mathbf{h}_{33} = \{(0.82, 0.82, 0.91)\}$
A_4	$\mathbf{h}_{41} = \{(1.00, 0.83, 0.92)\}$	$\mathbf{h}_{42} = \{(0.94, 1.00, 0.88)\}$	$\mathbf{h}_{43} = \{(0.91, 0.96, 1.00)\}$

Table 6. The preferences of the effective control scopes for alternatives in the form of HFEs $\mathbf{h}_{ij}$.

If we do not think about the above experts' opinions as the group of three-fixed experts, then, the above data will be in the form of HFEs as represented in Table 7.

	Criterion [weight]		
Alternative	C_1 [0.3]	C_2 [0.4]	C_3 [0.3]
A_1	$\mathbf{h}_{11} = \{0.42, 0.47, 0.50\}$	$\mathbf{h}_{12} = \{0.59, 0.47\}$	$\mathbf{h}_{13} = \{0.64, 0.55, 0.73\}$
A_2	$\mathbf{h}_{21} = \{0.50, 0.58\}$	$\mathbf{h}_{22} = \{0.76, 0.65, 0.59\}$	$\mathbf{h}_{23} = \{0.73, 0.82\}$
A_3	$\mathbf{h}_{31} = \{0.58, 0.67\}$	$\mathbf{h}_{32} = \{0.82, 0.88, 0.76\}$	$\mathbf{h}_{33} = \{0.82, 0.91\}$
A_4	$\mathbf{h}_{41} = \{1.00, 0.83, 0.92\}$	$\mathbf{h}_{42} = \{0.94, 1.00, 0.88\}$	$\mathbf{h}_{43} = \{0.91, 0.96, 1.00\}$

Table 7. The preferences of the effective control scopes for alternatives in the form of original HFEs $\mathbf{h}_{ij}$.

In this step, the length unification process of HFEs is recommended for further works [6, 7], and such a unification process may be *pessimistic* or *optimistic*. Both forms of unification, that is, the pessimistic and optimistic unification processes are given in Table 8.

		Criterion [weight]		
Alternative		C_1 [0.3]	C_2 [0.4]	C_3 [0.3]
A_1	*Pessimistic*	$\mathbf{h}_{11} = \{(0.42, 0.47, 0.50)\}$	$\mathbf{h}_{12} = \{(0.47, 0.47, 0.59)\}$	$\mathbf{h}_{13} = \{(0.55, 0.64, 0.73)\}$
	Optimistic	$\mathbf{h}_{11} = \{(0.42, 0.47, 0.50)\}$	$\mathbf{h}_{12} = \{(0.47, 0.59, 0.59)\}$	$\mathbf{h}_{13} = \{(0.55, 0.64, 0.73)\}$
A_2	*Pessimistic*	$\mathbf{h}_{21} = \{(0.50, 0.50, 0.58)\}$	$\mathbf{h}_{22} = \{(0.59, 0.65, 0.76)\}$	$\mathbf{h}_{23} = \{(0.73, 0.73, 0.82)\}$
	Optimistic	$\mathbf{h}_{21} = \{(0.50, 0.58, 0.58)\}$	$\mathbf{h}_{22} = \{(0.59, 0.65, 0.76)\}$	$\mathbf{h}_{23} = \{(0.73, 0.82, 0.82)\}$
A_3	*Pessimistic*	$\mathbf{h}_{31} = \{(0.58, 0.58, 0.67)\}$	$\mathbf{h}_{32} = \{(0.76, 0.82, 0.88)\}$	$\mathbf{h}_{33} = \{(0.82, 0.82, 0.91)\}$
	Optimistic	$\mathbf{h}_{31} = \{(0.58, 0.67, 0.67)\}$	$\mathbf{h}_{32} = \{(0.76, 0.82, 0.88)\}$	$\mathbf{h}_{33} = \{(0.82, 0.91, 0.91)\}$
A_4	*Pessimistic*	$\mathbf{h}_{41} = \{(0.83, 0.92, 1.00)\}$	$\mathbf{h}_{42} = \{(0.88, 0.94, 1.00)\}$	$\mathbf{h}_{43} = \{(0.91, 0.96, 1.00)\}$
	Optimistic	$\mathbf{h}_{41} = \{(0.83, 0.92, 1.00)\}$	$\mathbf{h}_{42} = \{(0.88, 0.94, 1.00)\}$	$\mathbf{h}_{43} = \{(0.91, 0.96, 1.00)\}$

Table 8. The preferences of the effective control scopes for alternatives in the form of *pessimistic/optimistic* HFEs $\mathbf{h}_{ij}$.

On the other hand, let us consider once again the experts' opinions as the group of three-fixed experts in the form of EHFEs (see Table 9).

	Criterion [weight]		
Alternative	C_1 [0.3]	C_2 [0.4]	C_3 [0.3]
A_1	$\mathbf{h}_{11} = \{(0.42, 0.47, 0.50)\}$	$\mathbf{h}_{12} = \{(0.59, 0.59, 0.47)\}$	$\mathbf{h}_{13} = \{(0.64, 0.55, 0.73)\}$
A_2	$\mathbf{h}_{21} = \{(0.50, 0.50, 0.58)\}$	$\mathbf{h}_{22} = \{(0.76, 0.65, 0.59)\}$	$\mathbf{h}_{23} = \{(0.73, 0.73, 0.82)\}$
A_3	$\mathbf{h}_{31} = \{(0.67, 0.58, 0.67)\}$	$\mathbf{h}_{32} = \{(0.82, 0.88, 0.76)\}$	$\mathbf{h}_{33} = \{(0.82, 0.82, 0.91)\}$
A_4	$\mathbf{h}_{41} = \{(1.00, 0.83, 0.92)\}$	$\mathbf{h}_{42} = \{(0.94, 1.00, 0.88)\}$	$\mathbf{h}_{43} = \{(0.91, 0.96, 1.00)\}$

Table 9. The preferences of the effective control scopes for alternatives in the form of EHFE $\mathbf{h}_{ij}$.

As can be seen from Tables 8 and 9, the preference matrix of the effective control scopes in the forms of *pessimistic/optimistic* HFEs $\mathbf{h}_{ij}$ and EHFEs $\mathbf{h}_{ij}$ are quite structurally different. The main difference is related to the implication of sorted values in the *pessimistic/optimistic* HFEs $\mathbf{h}_{ij}$ or EHFEs $\mathbf{h}_{ij}$. That is, from the HFE forms, we cannot exactly judge which value refers to which expert, meanwhile, such a connection is quite evident in the EHFE form.
By the way, in the present work, such an important difference is not effective, and this is because of the fact that in all subsequent steps of optimization, the essential parameter is the interval of the lower and upper bounds of effective control scope which is constructed respectively by the use of *minimum* and *maximum* elements. Therefore, from the relations (12) and (13), we easily find that $I_{ij}^{l_h} := I_{ij}^1 = [I_{ij}^{L,1}, I_{ij}^{U,1}]$ is constructed by the followings:

$$I_{ij}^{L,1} = \min\{\overline{C}_j^{1,1}(A_i), ..., \overline{C}_j^{m,1}(A_i)\};$$
$$I_{ij}^{U,1} = \max\{\overline{C}_j^{1,1}(A_i), ..., \overline{C}_j^{m,1}(A_i)\},$$

where $i = 1, ..., m_a$ and $j = 1, ..., n_c$.

However, regardless of $\mathbf{h}_{ij}$ being in the forms of *pessimistic/optimistic* HFEs or EHFEs, the effective control scope I_{ij}^1 is stated in the form of intervals $I_{ij}^1 = [I_{ij}^{L,1}, I_{ij}^{U,1}]$ which are summarized in Table 10.

	Criterion [weight]		
Alternative	C_1 [0.3]	C_2 [0.4]	C_3 [0.3]
A_1	$\mathbf{h}_{11}$ $I_{11}^2 = [0.42, 0.50]$	$\mathbf{h}_{12}$ $I_{12}^1 = [0.47, 0.59]$	$\mathbf{h}_{13}$ $I_{13}^2 = [0.55, 0.73]$
A_2	$\mathbf{h}_{21}$ $I_{21}^2 = [0.50, 0.58]$	$\mathbf{h}_{22}$ $I_{22}^1 = [0.59, 0.76]$	$\mathbf{h}_{23}$ $I_{23}^2 = [0.73, 0.82]$
A_3	$\mathbf{h}_{31}$ $I_{31}^2 = [0.58, 0.67]$	$\mathbf{h}_{32}$ $I_{32}^1 = [0.76, 0.88]$	$\mathbf{h}_{33}$ $I_{33}^2 = [0.82, 0.91]$
A_4	$\mathbf{h}_{41}$ $I_{41}^2 = [0.83, 1.00]$	$\mathbf{h}_{42}$ $I_{42}^1 = [0.88, 1.00]$	$\mathbf{h}_{43}$ $I_{43}^2 = [0.91, 1.00]$

Table 10. The general form of $\mathbf{h}_{ij}$ and the interval form of the effective control scope $I_{ij}^1 = [I_{ij}^{L,1}, I_{ij}^{L,1}]$ for each alternative.

If the normalized decision makers' psychological reference points $J_j^{l_h=1}$ is considered as before (like that in Table 4), that is,

	Criterion		
Reference point	C_1	C_2	C_3
$\overline{J}_j^1$	$[0.5, 0.58]$	$[0.71, 0.82]$	$[0.36, 0.55]$

Table 12. Normalized decision makers' psychological reference points $\overline{J}_j^{l_h(=1)} = [\overline{J}_j^{L,1}, \overline{J}_j^{U,1}]$.

Then, the rankings of alternatives determined by Zhang et al's HFE-based techniques [38] and the EHFE-based proposed technique will be obtained as those given in Table 13.

Alternative	A_1	A_2	A_3	A_4	Ranking
The HFE-based GEDM technique	-0.1278	0.0150	0.0912	-0.04823	$A_3 \succ A_2 \succ A_4 \succ A_1$
The HFE-based aggregation technique	-0.1096	0.0319	0.0573	-0.0480	$A_3 \succ A_2 \succ A_4 \succ A_1$
The EHFE-based proposed technique	-0.1278	0.0150	0.0912	-0.04823	$A_3 \succ A_2 \succ A_4 \succ A_1$

Table 13. The overall prospect values and the ranking of alternatives.

Anyway, in the real-world applications, we may confront a problem involved a group of n-fixed experts, hence, implementing EHFS concept in modelling the problem definitely gives rise to more reliable decision than the case where it just takes the HFS concept into account.

5 Conclusions and future directions

In this paper, it was interesting to resolve the problem of information losing in emergencies or selecting the ideal emergency alternative(s) by encountering the modified form of EHFS instead of the HFS form because with the latter form, we cannot exactly judge which value refers to which expert, while, such a connection is quite evident from the former form. Here, we dealt with a new aspect of emergency event in which instead of usual aggregation procedure, we implemented a new fusion technique being based on the modified version of EHFS, and such a technique definitely keeps all possible amount of expert's information more than the existing fusion technique of HFS. Eventually, the comparison analysis between the EHFS-based group emergency decision-making technique with the existing HFS-based one showed that the EHFS-based approach has some prominent advantages over the other approach based on HFEs. Among features derived from this contribution, we find that the implementation of EHFS concept in modelling those problems involved a group of n-fixed experts gives rise to more reliable decision than the case where it just takes the HFS concept into account. In terms of future research, a promising research direction is the attention being given to the risk decision making problem with extended hesitant fuzzy information, where the attribute values are random variables. Further, the introduced technique would be applied to solve those problems of extended hesitant fuzzy risk decision making.

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An extended hesitant group decision-making technique based on the prospect theory for emergency situations

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یک روش تصمیم‌گیری گروهی مردد توسیع‌یافته براساس نظریه احتمال برای شرایط اضطراری

چکیده. در سراسر مقاله حاضر تصمیم داریم یک روش تصمیم‌گیری اضطراری گروهی جدید را معرفی کنیم که در آن کاربرد نظریه احتمال، رفتار مربوط به روانشناسی تصمیم گیرنده را که تحت تأثیر تردید و عدم اطمینان شناخت در مسائل تصمیم‌گیری قرار گرفته، بیان می‌کند. بجای روند انباشتگی معمول، یک تکنیک ادغام جدید را که بر اساس نسخه تعدیل یافته مجموعه‌ی فازی مردد توسیع یافته (EHFS) می‌باشد، و بطور اطمینان مقادیر ممکن اطلاعات متخصصین را بیشتر از روش ادغام مجموعه ی فازی مردد (HFS) موجود حفظ می‌کند، به کار می‌بریم. انگیزه اصلی بازنگری مفهوم EHFS ، از نقش کارآمدی آن در افزایش توانمندی ظهور عددی به شکل گروه‌های ارزشیابی، ودوره توانایی آن در شناسایی تصمیم گیرنده‌های مختلف در شرایط تصمیم‌گیری نشئت می‌گیرد. یک چنین تعریفی بیشتر کاربرد عملی HFSs را گسترش می‌دهد. در آخر، جهت نشان دادن امکان و اعتبار تکنیک ارائه شده مسئله دریاچه مخزنی را بخدمت می‌گیریم.