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Synchronization criteria for T-S fuzzy singular complex dynamical networks with Markovian jumping parameters and mixed time-varying delays using pinning control

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Abstract

In this paper, we are discuss about the issue of synchronization for singular complex dynamical networks with Markovian jumping parameters and additive time-varying delays through pinning control by Takagi-Sugeno (T-S) fuzzy theory. The complex dynamical systems consist of m nodes and the systems switch from one mode to another, a Markovian chain with glorious transition probability. Based on the control strategies are designed, the singular complex dynamical networks are synchronized. A new class of Lyapunov-Krasovskii functional, which contains integral terms is constructed to derive the stability criteria. Some sufficient conditions for synchronization in the form of linear matrix inequality (LMI) approach. Finally, numerical example is presented to support the main results of this paper.

Keywords: Singular complex networks, additive time-varying delay, T-S fuzzy theory, Markovian jump, pinning control, LMIs.

1 Introduction

In recent years, complex dynamical networks have been acquired an extremely good deal of interest due to the fact they occur widely appearing every where in various fields inside the real-world. For instance, internet, World Wide Web (WWW), food webs, scientific citation webs neural networks, coordinate systems, informal communities, electrical control networks, biology, mathematics and physics, chemical systems, robotic manipulator systems, aircraft control systems, and so on [50], are extensively studied by the researchers. Complex dynamical networks are made out of singular nodes and coupled nodes, every node represents a dynamical device and association among nodes, in which a node is primary unit. Several researchers developed various economical synchronization problem for collective behavior of complex dynamical networks (see [6, 7, 8, 18, 23, 30] and references therein).

Generally representing, the synchronization of complex dynamical networks have been extensively explored in diverse fields of science and engineering, internet, signal synchronization, biological systems such as synchronous fireflies, flocking of birds, geostationary satellite, synchronous motor, because of its many ability realistic packages [45, 59]. If two structures have something in not unusual, then synchronization may also occur among them after they are concerned immediately. In reality, many problems in real world have close courting, with synchronization, which is an critical research difficulty with the growing examination, and there are measures of results [11, 36]. Synchronization of coupled oscillators can give an explanation properly for some of the natural phenomena. Within the past few years widespread interests inside the study of synchronization issues in coupled unique networks have been broadly explored, because of its potential applications in secure communication [4, 26, 40, 41], signal generators configuration, modeling brain activity and sample recognition phenomenon [16, 32, 56].

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It merits pointing out that, the theory of "Fuzzy Sets" was introduced by Zadeh, which plays a fundamental job in the modeling and controlling of complex nonlinear systems. The Takagi–Sugeno (T-S) fuzzy model is a kind of fuzzy system proposed by Takagi and Sugeno [33], which is described by a set of fuzzy **IF-THEN** rules which can give local linear representation of the nonlinear system by decomposing the whole input space into several partial fuzzy spaces and representing each output space with a linear equation. In reality, complex networks may display a special characteristic called fuzzy reasoning. For example, in [35], revealed that small-world networks can be modeled by fuzzy logic. In [24] the fuzzy neural networks, have advantages over pure neural networks since they incorporate the capability of fuzzy reasoning in handling uncertain information. In [2], fuzzy neural networks can be approximate to a wide range of nonlinear functions to any desired degree of accuracy under certain circumstances. Among various fuzzy systems, one of the most famous model is the Takagi-Sugeno(T-S) fuzzy model [54].

Accordingly, most of researchers pay more attention to the singular Markovian jump and singular systems have their intensive programs in control theory, chemical processes, economics, robots, aerospace engineering, mechanical systems and other areas, power systems, magnetic-ball suspension systems, a broad concern in articles [2, 29, 34], that incorporates type of physical systems higher than the regular (nonsingular) one. Control of singular structures has been extensively studied within the past years due to the very fact that singular structures higher describe physical systems than regular (nonsingular) one. Those singular systems are also referred to as descriptor systems, semi-state systems, differential-algebraic systems or semi-state systems [22, 61], implicit systems, generalized state-space systems. A great number of results based on the theory of regular systems (or state-space systems) had been extended to the place of singular systems (see, e.g.,[22, 61]). Singular systems can be acquainted with improve the customary complex networks to clarify the singular unique practices of nodes [58]. Singular systems have some specific complex properties, which need not be considered in typical frameworks. One of the main research topics in control theory for nearly half a century as such systems have broad applications in different areas, for example in the Leontief dynamic model, electrical systems and mechanical systems.

Markovian jump parameters are a collection of structures with transition the various models ruled taking values in a finite set, which changed into initial delivered by Krasovskii and Lidskii [25]. As is outstanding to all or any, Markov jump system, a extraordinary elegance of hybrid and random systems, is explicit created up of two additives. The primary segment refers back to the mode outlined by method of a continuous-time finite-notion inside the method of Markov [12, 13, 42, 55]. Markovian jump systems have received several analysis interest [14, 49, 38, 48, **?**, 9, 21, 28, 63]. The reason of Markovian jump systems have been paid a great deal of attention is that they are usually utilized to model the the abrupt phenomena such as random failures of the components and sharp environmental disturbances, changing subsystems interconnections, and so on. Markovian jump systems play a crucial function in describing several real world applications, inclusive networked control systems, manufacturing systems, conversation systems, monetary systems, flight systems, electricity systems, further as demonstrating generation frameworks somewhat [1, 53, 10].

Hypothetically taking, control is likewise fundamental method utilized for guiding or driving the system to attain favored synchronization, that is shape for circumstance that given system of dynamical structures is not synchronized or the synchronized state is not an predicted one. However, from engineering point of view, it is, sometimes, hard to control a complex network by means of adding the controllers to all or any nodes. A natural manner to decrease the quantity of controllers is to utilize the pinning control technique. The creators, in [52, 31] looked into the pinning problem control for linearly coupled systems by determined, simply can pin the coupled system by method for presenting less domestically locally feedback controllers. To delight this example, pinning control, in which controllers are implemented handiest to little big variety of nodes, has been projected.

Recently, the authors in [17, 15, 27, 43] reported that the signals transmitted, within the network control system from one purpose to different passes through few segments of networks, that may possibly induce successive delays with different properties due to the variable network transmission conditions which may purpose time delay with some various characteristics in realistic applications. Primarily based during this, a brand new version for neural networks with two additive time-varying delays has been planned in [47, 51]. For instance, the time delay within the dynamical model like $\dot{x}(t) = \mathbf{A}x(t) + \mathbf{BK}x(t - \tau_1(t) - \tau_2(t))$ wherein $\tau_1(t)$ is that the time delay brought on from device to controller and $\tau_2(t)$ is the delay caused from controller to the mechanism. The stability analysis for such systems has been completed in [17, 15, 27, 43] by usage two additive time varying delay components, $\tau_1(t) + \tau_2(t) = \tau(t)$. Contrasted with the single-delay frameworks, this model is under a stronger background of application. Consequently, taking the model with additive time-varying delay additives into consideration is significant. However, to the best of our knowledge, there is no work that considers the problem of synchronization for T-S fuzzy singular complex dynamical networks with Markovian jumping parameters and two additive time-varying delays via pinning control available in the present literature.

Motivated by the above discussions, the main object of this paper, we investigate the problem of synchronization criteria for T-S fuzzy singular complex dynamical networks (CDNs) with Marovian jumping parameters and additive time-varying delays using pinning control approach. We introduce a Lyapunov-Krasovskii functional and integral techniques, Jensen's inequality, various globally Lipschitz continuous activation functions and convexity of matrix functions, a new delay-dependent stability criteria for Markovian jump T-S fuzzy singular CDNs with additive time-varying delays are established in terms of LMIs. The proposed LMIs can be effectively solved by utilizing Matlab LMI Toolbox. A numerical model is introduced to show the adequacy of the proposed outcomes.

This paper is organized as follows. In Section 2 the network model is introduced and a few necessary definitions and lemmas are presented. In Section 3, we have tendency to propose our novel linear technique to resolve issue of synchronization for fuzzy singular complex systems with Markovian jump by the pinning control method. In Section 4 we have a tendency to provides numerical model. Finally, the paper is concluded in Section 5.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denotes, severally, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscript "T" denotes matrix operation and also the notation $X\geq Y$ (respectively, $X>Y$) wherever X and Y are symmetric matrices, which $X-Y$ is positive semi-definite (respectively, positive definite), I is the identity matrix with appropriate dimension. The asterisk "$*$" in a matrix is used to represent the term which is induced by symmetry. $\|\cdot\|$ refers to the Euclidean vector norm. If A is the square matrix, denote by $\lambda_{max}(A)$ (respectively, $\lambda_{min}(A)$) means the largest (respectively, smallest) eigenvalue of A. Moreover $(\Omega,\mathcal{F},\{\mathcal{F}_t\},\mathcal{P})$ complete probability space with a filtration, $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., the filtration contains all P-null sets and is right continuous). $\mathbb{E}\{\cdot\}$ represents the mathematical expectation.

2 Model description and preliminaries

Let $\{\sigma(t)(t\geq 0)\}$ be a right-continuous Markovian chain on the probability space $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathcal{P})$ taking values within the finite space $\mathcal{S}=\{1,2,....,m\}$ with generator $\Pi=\{\pi_{ij}\}_{m\times m}$ $(i,j\in S)$ given by

$$Pr\Big\{\sigma(t+\Delta t)=j|\sigma(t)=i\Big\}=\begin{cases}\pi_{ij}\Delta t+o(\Delta t), & \text{if } i\neq j,\\ 1+\pi_{ij}\Delta t+o(\Delta t), & \text{if } i=j,\end{cases}$$

where $\Delta t>0$, $lim_{t\to 0}(o(\Delta t)/\Delta t)=0$ and π_{ij} is the transition rate from mode i to mode j satisfying $\pi_{ij}\geq 0$ for $i\neq j$ with $\pi_{ij}=-\sum_{j=1,j\neq i}\pi_{ij}$ $(i,j\in\mathcal{S})$.

In this section, we consider the problem of Markovian jumping T-S fuzzy singular complex dynamical networks with two additive time-varying delays consisting of N identical nodes coupling which is described by a T-S fuzzy model composed a set of fuzzy implications. In view of T-S fuzzy model concept, a general class of T-S fuzzy in which each node is an n-dimensional dynamical subsystem as follows:

Rule: **IF** $\{\theta_1(t)$ is $F_{l1}\}$ **and** $\{\theta_2(t)$ is $F_{l2}\}$ **and** ..... **and** $\theta_g(t)$ is F_{lg}, **THEN**

$$\begin{cases}\mathbf{E}\dot{x}_k(t) &= \mathbf{A}(\sigma(t))x_k(t)+f(x_k(t),t)+a_1\sum_{w=1}^N g_{kw}\Gamma_1(\sigma(t))x_w(t)\\ &\quad +a_2\sum_{w=1}^N g_{kw}\Gamma_2(\sigma(t))x_w(t-\tau_1(t)-\tau_2(t))+a_3\sum_{w=1}^N g_{kw}\mathbf{D}x_j(t-\tau(t)),\\ x_k(t) &= \phi_k(t),\quad \forall t\in[-\tau,0],\quad k=1,2,....,N,\end{cases}\tag{1}$$

where $\mathbf{E}\in\mathbb{R}^{n\times n}$ is a singular matrix satisfying $rank(\mathbf{E})=r(0<r<n)$, $x_k(t)\in\mathbb{R}^n$ is that the state vector associated with k nodes, $\{\sigma(t)\ (t\geq 0)\}$ is that the continuous-time Markov process that describes the evolution of the mode at time t, $F_{lj}(l=1,2,....,r;j=1,2,...,g)$ are the fuzzy sets; r represents the wide variety of **IF-THEN** regulations. θ_j stands for the premise variables and $\theta(t)=[\theta_1(t),\theta_2(t),....,\theta_g(t)]$. $F_{lj}(\theta_j(t))$ is the grade of the membership of $\theta_j(t)$ in F_{lj}. $\mathbf{A}(\sigma(t))\in\mathbb{R}^{n\times n}$ is a constant matrix, $a_m>0$ $(m=1,2,3)$ are positive constants which coupling strengths, $\Gamma_u(\sigma(t))=diag\{b_{u1}(\sigma(t)),b_{u2}(\sigma(t)),....b_{un}(\sigma(t))\}$ $(u=1,2)$ is an inner-coupling matrices. $\phi_k(t)$ is continuously differential vector valued initial function on $[0,\tau_1+\tau_2]$ of the system. $\mathbf{D}=diag\{\vartheta_1,\vartheta_2,...,\vartheta_n\}\in\mathbb{R}^{n\times n}$ describes a constant diagonal inner-coupling matrix, $G=(g_{kw})_{N\times N}$ $(k=1,2,...,N)$ is the outer-coupling matrix representing the topological structure of the complex networks, in which g_{kw} is defined as follows: if there is a connection among node k node w $(k\neq w)$ then $g_{kw}=g_{wk}=1$; otherwise, $g_{kw}=g_{wk}=0$ $(k\neq w)$. The row sums of zero, i.e., $\sum_{w\neq k}g_{kw}=-g_{kk}$, $k=1,2,...,N$.

Assumption 1: For $k=1,2,.....,N$ the nonlinear function $f(\cdot)$ satisfies the globally Lipschitz condition:

$$\|f(x_k(t),t)-f(y_k(t),t)\|\leq l_k\|x_k(t)-y_k(t)\|,\tag{2}$$

where l_k is a positive constant.

For convenience, each possible value of $\sigma(t)$ is denoted as i, $i\in\mathcal{S}$ in the sequel. Then we get $\mathbf{A}(\sigma(t)=\mathbf{A}_i$, $\Gamma_1(\sigma(t))=\Gamma_{1i}$, $\Gamma_2(\sigma(t))=\Gamma_{2i}$.

Assumption 2: The transmission delays $\tau_1(t)$ and $\tau_2(t)$ represents the two delay components in the state vector and we denote $\tau(t) = \tau_1(t) + \tau_2(t)$ also if satisfies

$$0 \leq \tau_1(t) \leq \tau_1 < \infty, \ \dot{\tau}_1(t) < d_1 < \infty,$$
$$0 \leq \tau_2(t) \leq \tau_2 < \infty, \ \dot{\tau}_2(t) < d_2 < \infty,$$

where τ_1, τ_2, d_1 and d_2 are known constants. Naturally, we denote $\tau = \tau_1 + \tau_2$, $d = d_1 + d_2$.

We denote by $C([0, \tau_1 + \tau_2], \mathbb{R}^n)$ the Banach space of all continuous functions $\phi(t) = (\phi_1(t), \phi_2(t), ..., \phi_n(t))^T$: $[0, \tau_1 + \tau_2] \to \mathbb{R}^n$ with norm $\|\phi\| = sup_{0 \leq \theta \leq (\tau_1+\tau_2)} \|\phi(\theta)\|$.

Using a standard inference method, the system (1) is inferred as follows

$$\begin{cases} \mathbf{E}\dot{x}_k(t) & = \sum_{l=1}^{r} \mu_l(\theta(t)) \Big[\mathbf{A}_i x_k(t) + f(x_k(t), t) + a_1 \sum_{w=1}^{N} g_{kw} \Gamma_{1i} x_w(t) \\ & \quad + a_2 \sum_{w=1}^{N} g_{kw} \Gamma_{2i} x_w(t - \tau_1(t) - \tau_2(t)) + a_3 \sum_{w=1}^{N} g_{kw} \mathbf{D} x_w(t - \tau(t)) \Big], \\ x_k(t) & = \phi_k(t), \quad \forall t \in [-\tau, 0], \quad k = 1, 2, ..., N, \end{cases} \tag{3}$$

where $\mu_l(\theta(t))$ is the normalized membership function of the inferred fuzzy set $\rho_l(\theta(t))$, that is,

$$\mu_l(\theta(t)) = \frac{\rho_l(\theta(t))}{\sum_{l=1}^{r} \rho_1(\theta(t))}, \ \rho_l(\theta(t)) = \prod_{j=1}^{g} F_{lj}(\theta(t)),$$

and $F_{lj}(\cdot)$ is the grade membership function of $\theta_g(t)$ in F_{lj}. We assume

$$\rho_l(\theta(t)) \geq 0, \ l = 1, 2.....r, \ \sum_{l=1}^{r} \rho_l(\theta(t)) > 0, \ \text{for any } \theta(t).$$

Hence $\mu_l(\theta(t))$ satisfies $\mu_l(\theta(t)) \geq 0, \ l = 1, 2, ..., r, \ \ \sum_{l=1}^{r} \mu_l(\theta(t)) = 1, \ \text{for any } \theta(t)$.
Correspondingly the response complex network with the control inputs $u_k(t) \in \mathbb{R}^n$ $(k = 1, 2, ..., N)$, can be written as

$$\begin{cases} \mathbf{E}\dot{y}_k(t) & = \sum_{l=1}^{r} \mu_l(\theta(t)) \Big[\mathbf{A}_i y_k(t) + f(y_k(t), t) + a_1 \sum_{w=1}^{N} g_{kw} \Gamma_{1i} y_w(t) \\ & \quad + a_2 \sum_{w=1}^{N} g_{kw} \Gamma_{2i} y_w(t - \tau_1(t) - \tau_2(t)) + a_3 \sum_{w=1}^{N} g_{kw} \mathbf{D} y_w(t - \tau(t)) + u_k(t) \Big], \\ y_k(t) & = \varphi_k(t), \quad \forall t \in [-\tau, 0], \quad k = 1, 2, ..., N, \end{cases} \tag{4}$$

where $\varphi_k(t)$ is continuously differential vector-valued initial functions on $[0, \tau_1 + \tau_2]$; $u_k(t)$ is defined by

$$u_k(t) = \begin{cases} -a_4 \sigma_k \Gamma_3 (y_k(t) - x_k(t)), \quad k = 1, 2, ..., l, \\ 0, \quad k = l+1, l+2, ..., N. \end{cases} \tag{5}$$

Let $e_k(t) = y_k(t) - x_k(t)$ be the synchronization error of the kth node with CDNs (3) and (4), the following model can be obtained:

$$\begin{cases} \mathbf{E}\dot{e}_k(t) & = \sum_{l=1}^{r} \mu_l(\theta(t)) \Big[\mathbf{A}_i e_k(t) + F_k(e_k(t), t) + a_1 \sum_{w=1}^{N} g_{kw} \Gamma_{1i} e_w(t) \\ & \quad + a_2 \sum_{w=1}^{N} g_{kw} \Gamma_{2i} e_w(t - \tau(t)) + a_3 \sum_{w=1}^{N} g_{kw} \mathbf{D} e_w(t - \tau(t)) - c_4 \sigma_k \Gamma_3 e_k(t) \Big], \\ e_k(t) & = \psi_k(t), \ t \in [-\tau, 0], \ \sigma(0) = \sigma_0, \quad k = 1, 2, ..., N, \end{cases} \tag{6}$$

where $F_k(e_k(t), t) = f(y_k(t), t) - f(x_k(t), t)$ and $\psi_k(t) = \phi_k(t) - \varphi_k(t)$.

Denoting $\sigma_k = 0$ $(k = l+1, l+2, ..., N)$, at that point we may also compose the error system in its compact structure as

$$\mathbf{E}\dot{e}(t) = \sum_{l=1}^{r} \mu_l(\theta(t)) \Big[\mathbf{A}_i e(t) + F(e(t), t) + a_1 \Gamma_{1i} G e(t) + a_2 \Gamma_{2i} G e(t - \tau(t)) + a_3 \mathbf{D} e_w(t - \tau(t)) U^T - a_4 \sigma \Gamma_3 e_k(t) \Big], \tag{7}$$

where $e(t) = (e_1(t), e_2(t), ..., e_N(t))$, $F(e(t), t) = (F_1(e_1(t)), F_2(e_2(t)), ..., F_N(e_N(t)))$ and $\sigma = diag\{\sigma_1, \sigma_2, ..., \sigma_N\}$. By the properties of the outer-coupling matrix G, there exists a unitary matrix $U = [U_1, U_2, ..., U_N] \in \mathbb{R}^{N \times N}$ to such an extent that $U^TG = \Lambda U^T$ with $\Lambda = diag\{\lambda_1, \lambda_2, ..., \lambda_N\}$ and $UU^T = I$. Utilizing the nonsingular transform $e(t)U = z(t) = [z_1(t), z_2(t), ..., z_N(t)] \in \mathbb{R}^{N\times N}$, from condition (7), it follows the network condition

$$\mathbf{E}\dot{z}(t) = \sum_{l=1}^{r} \mu_l(\theta(t))\Big[\mathbf{A}_i z(t) + F(e(t), t)U + a_1\Gamma_{1i}\Lambda z(t) + a_2\Gamma_{2i}\Lambda z(t-\tau(t)) + a_3\mathbf{D}z(t-\tau(t))\Lambda - a_4\sigma\Gamma_3 z(t)\Big]. \quad (8)$$

In a similar manner, display (8) can be composed as

$$\mathbf{E}\dot{z}_k(t) = \sum_{l=1}^{r} \mu_l(\theta(t))\Big[(\mathbf{A}_i + a_1\Gamma_{1i}\lambda_k - a_4\sigma_k\Gamma_3)z_k(t) + h_k(t) + a_2\Gamma_{2i}\lambda_k z_k(t-\tau(t)) + a_3\mathbf{D}\lambda_k z_k(t-\tau(t))\Big],\ k = 1, 2, ..., N, \quad (9)$$

where $h_k(t) = F(e(t), t)U_k$.

Definition 2.1. [46] *Complex dynamical network (1) is said to be global (asymptotically) synchronized by pinning control, if*

$$lim_{t\to\infty} = \|x_k(t) - y_k(t)\| = 0, \quad k = 1, 2, ..., N. \quad (10)$$

Definition 2.2. *The pair $(\boldsymbol{E}, \boldsymbol{A}_i + a_1\Gamma_{1i}\lambda_k - a_4\sigma_k\Gamma_3)$ is said to be regular, if there exist a scalar a such that $det(a\boldsymbol{E} - (\boldsymbol{A}_i + a_1\Gamma_{1i}\lambda_k - a_4\sigma_k\Gamma_3))$ is not identically zero.*

Definition 2.3. *The pair $(\boldsymbol{E}, \boldsymbol{A}_i + a_1\Gamma_{1i}\lambda_k - a_4\sigma_k\Gamma_3)$ is said to be impulsive free, if it is regular and satisfies $deg(det(a\boldsymbol{E} - (\boldsymbol{A}_i + a_1\Gamma_{1i}\lambda_k - a_4\sigma_k\Gamma_3))) = rank(\boldsymbol{E})$.*

Lemma 2.4. [16] *The eigenvalues of an irreducible matrix $G = (g_{kw}) \in \mathbb{R}^{N\times N}$ with $\sum_{w\neq k}^{N} g_{kw} = -g_{kk}$, $k = 1, 2, ..., N$ satisfies the properties properties:*
(i) Real parts of all eigenvalues of G are not exactly or equivalent to 0 with multiplicity 1,
(ii) G has a right eigenvector $(1, 1, ..., 1)^T$ corresponding to the eigenvalue 0.

Lemma 2.5. [16] *In the event that for any constant matrix $\boldsymbol{R} \in \mathbb{R}^{m\times m}$, $\boldsymbol{R} = \boldsymbol{R}^T > 0$, scalar $\gamma > 0$ and a vector function $\Phi : [0, \gamma] \to \mathbb{R}^m$ such that the integrations concerned is well defined, then the following inequality holds:*

$$-\gamma \int_{t-\gamma}^{t} \dot{\chi}^T(s)\dot{\chi}(s)ds \leq \begin{pmatrix} \chi(t) \\ \chi(t-\gamma) \end{pmatrix}^T \begin{pmatrix} -\boldsymbol{R} & \boldsymbol{R} \\ * & -\boldsymbol{R} \end{pmatrix} \begin{pmatrix} \chi(t) \\ \chi(t-\gamma) \end{pmatrix}.$$

Lemma 2.6. [39] *The pair $(\boldsymbol{E}, \boldsymbol{A}_i + a_1\Gamma_{1i}\lambda_k - a_4\sigma_k\Gamma_3)$ is said to regular and impulsive free if and only if there exist matrices $\boldsymbol{P}_{ki}$ such that the following inequalities hold for $k = 2, 3, ..., N$:*
(1) $\boldsymbol{E}^T\boldsymbol{P}_{ki} = \boldsymbol{P}_{ki}\boldsymbol{E} \geq 0$ and
(2) $(\boldsymbol{A}_i + a_1\Gamma_{1i}\lambda_k - a_4\sigma_k\Gamma_3)^T\boldsymbol{P}_{ki} + \boldsymbol{P}_{ki}^T(\boldsymbol{A}_i + a_1\Gamma_{1i}\lambda_k - a_4\sigma_k\Gamma_3) < 0$.

Lemma 2.7. [20] *(Jensen's inequality) For a positive matrix $\boldsymbol{M} \in \mathbb{R}^{m\times m}$, $\boldsymbol{M} = \boldsymbol{M}^T > 0$, two scalars $h_U > h_L > 0$, such that the following concerned integrations are well defined, then*

$$-(h_U - h_L)\int_{t-h_U}^{t-h_L} x^T(s)\boldsymbol{M}x(x)ds \leq -\Big(\int_{t-h_U}^{t-h_L} x^T(s)ds\Big)\boldsymbol{M}\Big(\int_{t-h_U}^{t-h_L} x(s)ds\Big),$$
$$-\frac{h_U^2 - h_L^2}{2}\int_{t-h_U}^{t-h_L}\int_{s}^{t} x^T(u)\boldsymbol{M}x(u)duds \leq -\Big(\int_{t-h_U}^{t-h_L}\int_{s}^{t} x^T(u)duds\Big)\boldsymbol{M}\Big(\int_{t-h_U}^{t-h_L}\int_{s}^{t} x(u)duds\Big).$$

Lemma 2.8. *(Schur complement)* [5] *Given constant matrices Ω_1, Ω_2 and Ω_3 with appropriate dimensions, where $\Omega_1^T = \Omega_1$ and $\Omega_2^T = \Omega_2 > 0$, then $\Omega_1 + \Omega_3^T\Omega_2^{-1}\Omega_3 < 0$, if and only if*

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} < 0 \ \ or \ \ \begin{bmatrix} -\Omega_2 & \Omega_3 \\ * & \Omega_3 \end{bmatrix} < 0.$$

To date, we converted the synchronization issue of the T-S fuzzy singular complex networks (1) into the synchronization issue of the N bits of the relating error dynamical network (9). We will see from Lemma 2.4 that $\lambda_1 = 0$ and $z_1(t) = e(t)U_1 = 0$. Consequently, if the following $(N-1)$ bits of the relating error dynamical system

$$\mathbf{E}\dot{z}_k(t) = \sum_{l=1}^{r} \mu_l(\theta(t))\Big[(\mathbf{A}_i + a_1\Gamma_{1i}\lambda_k - a_4\sigma_k\Gamma_3)z_k(t) + h_k(t) + a_2\Gamma_{2i}\lambda_k z_k(t-\tau(t)) + a_3\mathbf{D}\lambda_k z_k(t-\tau(t))\Big],\ k = 2, 3, ..., N \quad (11)$$

Remark 2.9. *The main contributions of this paper are as follows: (i). Since singular systems give a more general description of physical systems than the normal one, there are many extended concepts and results are studies from the regular systems theory into singular systems (ii). Master-slave Markovian jumping T-S fuzzy singular complex dynamical networks. By using pinning control approach, can be equivalently expressed as the error dynamical system with all globally Lipschitz continuous activation function and additive time-varying delay function. (iii). Singular systems are introduced to improve the ordinary complex networks based on T-S fuzzy theory. (iv). And suitable Lyapunov-Krasovskii functional and utilize integral inequality technique, piecewise analysis method to derive in terms of linear matrix inequalities (LMIs). The proposed LMIs can be effectively understood by utilizing Matlab LMI Toolbox. (v). At last, a numerical example is presented to illustrate the effectiveness of the proposed results.*

Remark 2.10. *Network (1) is a Markovian jumping singular complex network model with additive time-varying delays. It means that [62] the communication of information at each node with other nodes takes place at time t as well as at time $(t-\tau_1-\tau_2)$. Truly, it happens in the real world as we can see in stock market, the decision making of trade-off is impacted by the information at time t and at $(t-\tau_1-\tau_2)$. It is quite clear that the error occurrence with communication of information with both nodes at time t and at $(t-\tau_1-\tau_2)$ is much smaller than that occurred with only one of them in [57].*

3 Synchronization of singular complex dynamical networks with additive time-varying delays

In this section, the delay-dependent stability condition for the synchronization problem for singular complex dynamical network system (11) is investigated by utilizing the Lyapunov functional method combining with the LMI techniques. Let us define

$$\eta_k^T(t) = \Big[z_k^T(t) \;\; z_k^T(t-\tau(t)) \;\; z_k^T(t-\tau_1(t)) \;\; z_k^T(t-\tau_1) \;\; z_k^T(t-\tau_1-\tau_2) \int_{t-\tau_1-\tau_2}^{t-\tau_1} z_k^T(s)ds \;\; \int_{t-\tau_1}^{t} z_k^T(s)ds \;\; h_k^T(t)\Big], \quad (12)$$

$$\delta_k = \Big[(\mathbf{A}_i + a_1\lambda_k\Gamma_{1i} - a_4\sigma_k\Gamma_3) \;\; a_2\Gamma_{2i}\lambda_k + a_3\mathbf{D}\lambda_k \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; I\Big]. \quad (13)$$

Then,

$$\mathbf{E}\dot{z}_k(t) = \delta_k\eta_k(t). \quad (14)$$

The inequality (2) and the Lipschitz continuity of $h_k(t)$ can be utilized to make $h_k(t)$ to satisfies

$$\begin{aligned}\Big\|h_k(t)\Big\| &= \Big\|\sum_{w=1}^{N}[f(x_w(t),t)-f(y_w(t),t)]u_{kw}\Big\| \le \sum_{w=1}^{N}\Big\|[f(x_w(t),t)-f(y_w(t),t)]\Big\||u_{kw}| \\ &\le \sum_{w=1}^{N} l_k\Big\|x_w(t)-y_w(t)\Big\| = \sum_{w=1}^{N} l_k\Big\|e_w(t)\Big\| \le \sum_{w=1}^{N}\bar{l}\Big\|z_w(t)\Big\| = \sum_{w=2}^{N}\bar{l}\Big\|z_w(t)\Big\|,\end{aligned} \quad (15)$$

where u_{kw} is the w-th element of U_k and $\bar{l} = max\ l_k$. Therefore the following inequalities holds:

$$\sum_{k=2}^{N}\Big(\Big\|h_k(t)\Big\| - \bar{l}\sum_{w=2}^{N}\Big\|z_w(t)\Big\|\Big) = \sum_{k=2}^{N}\Big\|h_k(t)\Big\| - \bar{l}\sum_{k=2}^{N}\sum_{w=2}^{N}\Big\|z_w(t)\Big\| = \sum_{k=2}^{N}\Big(\Big\|h_k(t)\Big\| - (N-1)\bar{l}\Big\|z_k(t)\Big\|\Big) \le 0, \quad (16)$$

if the following inequality is satisfied

$$\Big\|h_k(t)\Big\| - (N-1)\bar{l}\Big\|z_k(t)\Big\| \le 0, \quad k = 2,3,...,N, \quad (17)$$

where $\bar{l} = max\ l_k$. From equation (10) and inequality (15)-(17), there exist positive diagonal matrices $\mathbf{S}_k$ such that

$$\eta_k^T(t)diag\Big\{-(N-1)\bar{l}\mathbf{S}_k \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \mathbf{S}_k\Big\}\eta_k(t) = \eta_k^T(t)\Phi_k\eta_k(t) \le 0. \quad (18)$$

Theorem 3.1. *Assume that (1) and (2) holds. Then the T-S fuzzy singular Markovian jumping error dynamical network (11) is asymptotically stable. For a given scalars $\tau_1 > 0$, $\tau_2 > 0$, $d_1 > 0$, and $d_2 > 0$, if there exist nonnegative* *constants* α_k*, nonnegative definite matrices* $\boldsymbol{P}_{ki} > 0$*,* $\boldsymbol{Q}_{kq} > 0$*,* $\boldsymbol{M}_{kq} > 0$ $(q = 1,2,3,4)$*,* $\boldsymbol{N}_{k1} > 0$*,* $\boldsymbol{N}_{k2} > 0$ *and positive diagonal matrices* $\boldsymbol{S}_k$ *such that the following LMIs hold* $\forall\ i \in S$*:*

$$\boldsymbol{E}^T\boldsymbol{P}_{ki} = \boldsymbol{P}_{ki}\boldsymbol{E} \geq 0, \quad \Psi_i < 0, \quad i = 1,2,....,N, \;\; k = 2,3,....N, \tag{19}$$

$$\Psi_i = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & 0 & \Upsilon_{14} & 0 & \Upsilon_{16} & \frac{2}{\tau_1}\boldsymbol{E}^T\boldsymbol{N}_{k1}\boldsymbol{E} & P_{ki} & \Upsilon_{19} \\ * & -(1-d)\boldsymbol{Q}_{k2} & 0 & 0 & 0 & 0 & 0 & 0 & \Upsilon_{29} \\ * & * & \Upsilon_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Upsilon_{44} & \frac{1}{\tau_2}\boldsymbol{E}^T\boldsymbol{M}_{k2}\boldsymbol{E} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Upsilon_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Upsilon_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Upsilon_{77} & 0 & 0 \\ * & * & * & * & * & * & * & -\alpha_k\boldsymbol{S}_k & I \\ * & * & * & * & * & * & * & * & -R^{-1} \end{bmatrix}, \tag{20}$$

where

$$\begin{aligned} \Upsilon_{11} &= \boldsymbol{P}_{ki}(\boldsymbol{A}_i + a_1\lambda_k\Gamma_{1i} - a_4\sigma_k\Gamma_3) + (\boldsymbol{A}_i + a_1\lambda_k\Gamma_{1i} - a_4\sigma_k\Gamma_3)^T\boldsymbol{P}_{ki} + \sum_{j=1}^{m}\pi_{ij}\boldsymbol{P}_{kj} + \boldsymbol{Q}_{k1} + \boldsymbol{Q}_{k3} \\ &\quad + \tau_2^2\boldsymbol{M}_{k3} + \tau_1^2\boldsymbol{M}_{k4} - \frac{1}{\tau_1}\boldsymbol{E}^T\boldsymbol{M}_{k1}\boldsymbol{E} - 2\boldsymbol{E}^T\boldsymbol{N}_{k2}\boldsymbol{E} + \alpha_k(N-1)\bar{l}\boldsymbol{S}_k, \;\; \Upsilon_{12} = \boldsymbol{P}_{ki}a_2\Gamma_{2i}\lambda_k + \boldsymbol{P}_{ki}a_3\boldsymbol{D}\lambda_k \\ \Upsilon_{14} &= \frac{1}{\tau_1}\boldsymbol{E}^T\boldsymbol{M}_{k1}\boldsymbol{E}, \;\; \Upsilon_{16} = \frac{2}{\tau_2}\boldsymbol{E}^T\boldsymbol{N}_{k2}\boldsymbol{E}, \;\; \Upsilon_{19} = \boldsymbol{A}_i + a_1\Gamma_{1i}\lambda_k - a_4\sigma_k\Gamma_3, \;\; \Upsilon_{29} = a_2\Gamma_{2i}\lambda_k + a_3\boldsymbol{D}\lambda_k, \\ \Upsilon_{33} &= -(1-d_1)(\boldsymbol{Q}_{k1} - \boldsymbol{Q}_{k2}), \;\; \Upsilon_{44} = \boldsymbol{Q}_{k4} - \boldsymbol{Q}_{k3} - \frac{1}{\tau_1}\boldsymbol{E}^T\boldsymbol{M}_{k1}\boldsymbol{E} - \frac{1}{\tau_2}\boldsymbol{E}^T\boldsymbol{M}_{k2}\boldsymbol{E}, \;\; \Upsilon_{55} = -\boldsymbol{Q}_{k4} \\ &\quad - \frac{1}{\tau_2}\boldsymbol{E}^T\boldsymbol{M}_{k2}\boldsymbol{E}, \;\; \Upsilon_{66} = -\boldsymbol{M}_{k3} - \frac{2}{\tau_2^2}\boldsymbol{E}^T\boldsymbol{N}_{k2}\boldsymbol{E}, \;\; \Upsilon_{77} = -\boldsymbol{M}_{k4} - \frac{2}{\tau_1^2}\boldsymbol{E}^T\boldsymbol{N}_{k1}\boldsymbol{E}, \\ R &= \boldsymbol{E}^T(\tau_1\boldsymbol{M}_{k1} + \tau_2\boldsymbol{M}_{k2})\boldsymbol{E} + \frac{\tau_1^2}{2}\boldsymbol{E}^T\boldsymbol{N}_{k1}\boldsymbol{E} + \frac{\tau_2^2}{2}{}^T\boldsymbol{N}_{k2}\boldsymbol{E}. \end{aligned} \tag{21}$$

Proof. Let us consider the following Lyapunov-Krasovskii functional candidate to be

$$\mathbb{V}_k(z_k(t),i,t) = \mathbb{V}_{k1}(z_k(t),i,t) + \mathbb{V}_{k2}(z_k(t),i,t) + \mathbb{V}_{k3}(z_k(t),i,t) + \mathbb{V}_{k4}(z_k(t),i,t), \tag{22}$$

where

$$\mathbb{V}_{k1}(z_k(t),i,t) = z_k^T(t)\mathbf{P}_{ki}Ez_k(t), \tag{23}$$

$$\begin{aligned} \mathbb{V}_{k2}(z_k(t),i,t) &= \int_{t-\tau_1(t)}^{t} z_k^T(s)\mathbf{Q}_{k1}z_k(s)ds + \int_{t-\tau(t)}^{t-\tau_1(t)} z_k^T(s)\mathbf{Q}_{k2}z_k(s)ds \\ &\quad + \int_{t-\tau_1}^{t} z_k^T(s)\mathbf{Q}_{k3}z_k(s)ds + \int_{t-\tau_1-\tau_2}^{t-\tau_1} z_k^t(s)\mathbf{Q}_{k4}z_k(s)ds, \end{aligned} \tag{24}$$

$$\begin{aligned} \mathbb{V}_{k3}(z_k(t),i,t) &= \int_{-\tau_1}^{0}\int_{t+\rho}^{t} \dot{z}_k^T(s)\mathbf{E}^T\mathbf{M}_{k1}\mathbf{E}\dot{z}_k(s)dsd\rho + \int_{-\tau_1-\tau_2}^{-\tau_1}\int_{t+\rho}^{t} \dot{z}_k^T(s)\mathbf{E}^T\mathbf{M}_{k2}\mathbf{E}\dot{z}_k(s)dsd\rho \\ &\quad + \tau_2\int_{-\tau_1-\tau_2}^{-\tau_1}\int_{t+\rho}^{t} z_k^T(s)\mathbf{M}_{k3}z_k(s)dsd\rho + \tau_1\int_{-\tau_1}^{0}\int_{t+\rho}^{t} z_k^T(s)\mathbf{M}_{k4}z_k(s)dsd\rho, \end{aligned} \tag{25}$$

$$\mathbb{V}_{k4}(z_k(t),i,t) = \int_{-\tau_1}^{0}\int_{\theta}^{0}\int_{t+\rho}^{t} \dot{z}_k^T(s)\mathbf{E}^T\mathbf{N}_{k1}\mathbf{E}\dot{z}_k(s)dsd\rho d\theta + \int_{-\tau_1-\tau_2}^{-\tau_1}\int_{\theta}^{0}\int_{t+\rho}^{t} \dot{z}_k^T(s)\mathbf{E}^T\mathbf{N}_{k2}\mathbf{E}\dot{z}_k(s)dsd\rho d\theta. \tag{26}$$

Then

$$\begin{aligned} \dot{\mathbb{V}}_{k1}(z_k(t),i,t) &= 2z_k^T(t)\mathbf{P}_{ki}E\dot{z}_k(t), \\ &= \sum_{l=1}^{r}\mu_l(\theta(t))\Big[z_k^T(t)\Big(\mathbf{P}_{ki}(\mathbf{A}_i + a_1\lambda_k\Gamma_{1i} - a_4\sigma_k\Gamma_3) + (\mathbf{A}_i + a_1\lambda_k\Gamma_{1i} - a_4\sigma_k\Gamma_3)^T\mathbf{P}_{ki}\Big)z_k(t) \\ &\quad + z_k^T(t)\Big(2\mathbf{P}_{ki}(a_2\Gamma_{2i}\lambda_k + a_3\mathbf{D}\lambda_k)\Big)z_k(t-\tau(t)) + z_k^T(t)(2\mathbf{P}_{ki})h_k(t) \end{aligned}$$

$$+ \sum_{j=1}^{m} \pi_{ij} z_k^T(t) \mathbf{P}_{kj} z_k(t) \Big], \tag{27}$$

$$\begin{aligned} \dot{\mathbb{V}}_{k2}(z_k(t), i, t) &= z_k^T(t)\Big[\mathbf{Q}_{k1} + \mathbf{Q}_{k3}\Big] z_k(t) - z_k^T(t-\tau_1(t))\Big[(1-\dot{\tau}_1(t))\Big(\mathbf{Q}_{k1} - \mathbf{Q}_{k2}\Big)\Big] z_k(t-\tau_1(t)) \\ &+ z_k^T(t-\tau_1)\Big[\mathbf{Q}_{k4} - \mathbf{Q}_{k3}\Big] z_k(t-\tau_1) - z_k^T(t-\tau_1-\tau_2)\mathbf{Q}_{k4} z_k(t-\tau_1-\tau_2) \\ &+ z_k^T(t-\tau(t))\Big[-(1-\dot{\tau}_1(t)-\dot{\tau}_2(t))\mathbf{Q}_{k2}\Big] z_k(t-\tau(t)), \end{aligned} \tag{28}$$

$$\begin{aligned} \dot{\mathbb{V}}_{k3}(z_k(t), i, t) &= \dot{z}_k^T(t)\mathbf{E}^T\Big[\tau_1\mathbf{M}_{k1} + \tau_2\mathbf{M}_{k2}\Big]\mathbf{E}\dot{z}_k(t) + z_k^T(t)\Big[\tau_2^2\mathbf{M}_{k3} + \tau_1^2\mathbf{M}_{k4}\Big] z_k(t) \\ &- \int_{t-\tau_1}^{t} \dot{z}_k^T(s)\mathbf{E}^T\mathbf{M}_{k1}\mathbf{E}\dot{z}_k(s)ds - \int_{t-\tau_1-\tau_2}^{t-\tau_1} \dot{z}_k^T(s)\mathbf{E}^T\mathbf{M}_{k2}\mathbf{E}\dot{z}_k(s)ds \\ &- \tau_2\int_{t-\tau_1-\tau_2}^{t-\tau_1} z_k^T(s)\mathbf{M}_{k3} z_k(s)ds - \tau_1\int_{t-\tau_1}^{t} z_k^T(s)\mathbf{M}_{k4} z_k(s)ds, \end{aligned} \tag{29}$$

$$\begin{aligned} \dot{\mathbb{V}}_{k4}(z_k(t), i, t) &= \frac{\tau_1^2}{2}\dot{z}_k^T(t)\mathbf{E}^T\mathbf{N}_{k1}\mathbf{E}\dot{z}_k(t) - \int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{z}_k^T(s)\mathbf{E}^T\mathbf{N}_{k1}\mathbf{E}\dot{z}_k(s)dsd\theta + \frac{\tau_2^2}{2}\dot{z}_k^T(t)\mathbf{E}^T\mathbf{N}_{k2}\mathbf{E}\dot{z}_k(t) \\ &- \int_{-\tau_1-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \dot{z}_k^T(s)\mathbf{E}^T\mathbf{N}_{k2}\mathbf{E}\dot{z}_k(s)dsd\theta. \end{aligned} \tag{30}$$

By Lemma 2.5 and 2.7, it can be seen that

$$-\int_{t-\tau_1}^{t} \dot{z}_k^T(s)E^T M_{k1} E\dot{z}_k(s)ds \le \frac{1}{\tau_1}\begin{pmatrix} z_k(t) \\ z_k(t-\tau_1) \end{pmatrix}^T \begin{pmatrix} -\mathbf{E}^T\mathbf{M}_{k1}\mathbf{E} & \mathbf{E}^T\mathbf{M}_{k1}\mathbf{E} \\ * & -\mathbf{E}^T\mathbf{M}_{k1}\mathbf{E} \end{pmatrix} \times \begin{pmatrix} z_k(t) \\ z_k(t-\tau_1) \end{pmatrix}, \tag{31}$$

$$-\int_{t-\tau_1-\tau_2}^{t-\tau_1} \dot{z}_k^T(s)E^T M_{k2} E\dot{z}_k(s)ds \le \frac{1}{\tau_2}\begin{pmatrix} z_k(t-\tau_1) \\ z_k(t-\tau_1-\tau_2) \end{pmatrix}^T \begin{pmatrix} -\mathbf{E}^T\mathbf{M}_{k2}\mathbf{E} & \mathbf{E}^T\mathbf{M}_{k2}\mathbf{E} \\ * & -\mathbf{E}^T\mathbf{M}_{k2}\mathbf{E} \end{pmatrix} \times \begin{pmatrix} z_k(t-\tau_1) \\ z_k(t-\tau_1-\tau_2) \end{pmatrix}, \tag{32}$$

$$-\tau_2\int_{t-\tau_1-\tau_2}^{t-\tau_1} z_k^T(s)\mathbf{M}_{k3} z_k(s)ds \le -\int_{t-\tau_1-\tau_2}^{t-\tau_1} z_k^T(s)ds\mathbf{M}_{k3}\int_{t-\tau_1-\tau_2}^{t-\tau_1} z_k(s)ds, \tag{33}$$

$$-\tau_1\int_{t-\tau_1}^{t} z_k^T(s)\mathbf{M}_{k4} z_k(s)ds \le -\int_{t-\tau_1}^{t} z_k^T(s)ds\mathbf{M}_{k4}\int_{t-\tau_1}^{t} z_k(s)ds. \tag{34}$$

Similarly,

$$-\int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{z}_k^T(s)\mathbf{E}^T\mathbf{N}_{k1}\mathbf{E}\dot{z}_k(s)dsd\theta \le -\frac{2}{\tau_1^2}\int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{z}_k^T(s)dsd\theta\mathbf{E}^T\mathbf{N}_{k1}\mathbf{E} \quad \times \int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{z}_k(s)dsd\theta, \tag{35}$$

$$-\int_{-\tau_1-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \dot{z}_k^T(s)\mathbf{E}^T\mathbf{N}_{k2}\mathbf{E}\dot{z}_k(s)dsd\theta \le -\frac{2}{\tau_2^2}\int_{-\tau_1-\tau_2}^{-\tau_1}\int_{t+\theta}^{t-\tau_1} \dot{z}_k^T(s)dsd\theta\mathbf{E}^T\mathbf{N}_{k2}\mathbf{E} \quad \times \int_{-\tau_1-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \dot{z}_k(s)dsd\theta. \tag{36}$$

From equations (27)-(36) we get,

$$\begin{aligned} \dot{\mathbb{V}}_k(z_k(t), i, t) &= \sum_{l=1}^{r}\mu_l(\theta(t))\Big\{z_k^T(t)\Big[\mathbf{P}_{ki}(\mathbf{A}_i + a_1\lambda_k\Gamma_{1i} - a_4\sigma_k\Gamma_3) + (\mathbf{A}_i + a_1\lambda_k\Gamma_{1i} - a_4\sigma_k\Gamma_3)^T\mathbf{P}_{ki}\Big] z_k(t) \\ &+ z_k^T(t)[2\mathbf{P}_{ki}a_2\lambda_k\Gamma_{2i} + a_3\mathbf{D}\lambda_k] z_k(t-\tau(t)) + z_k^T(t)[2\mathbf{P}_{ki}]h_k(t) + \sum_{j=1}^{m}\pi_{ij} z_k^T(t)\mathbf{P}_{kj} z_k(t) \\ &+ z_k^T(t)\Big[\mathbf{Q}_{k1} + \mathbf{Q}_{k3}\Big] z_k(t) - z_k^T(t-\tau_1(t))\Big[(1-\dot{\tau}_1(t))(\mathbf{Q}_{k1} - \mathbf{Q}_{k2})\Big] z_k(t-\tau_1(t)) \\ &+ z_k^T(t-\tau_1)\Big[\mathbf{Q}_{k4} - \mathbf{Q}_{k3}\Big] z_k(t-\tau_1) - z_k^T(t-\tau_1-\tau_2)\mathbf{Q}_{k4} z_k(t-\tau_1-\tau_2) \\ &+ z_k^T(t-\tau(t))\Big[-(1-\dot{\tau}_1(t)-\dot{\tau}_2(t))\mathbf{Q}_{k2}\Big] z_k(t-\tau(t)) + \dot{z}_k^T(t)\mathbf{E}^T\Big[\tau_1\mathbf{M}_{k1} + \tau_2\mathbf{M}_{k2}\Big]\mathbf{E}\dot{z}_k(t) \end{aligned}$$

$$+ z_k^T(t)\Big[\tau_2^2\mathbf{M}_{k3} + \tau_1^2\mathbf{M}_{k4}\Big]z_k(t) + \frac{1}{\tau_1}\begin{pmatrix} z_k(t) \\ z_k(t-\tau_1) \end{pmatrix}^T \begin{pmatrix} -\mathbf{E}^T\mathbf{M}_{k1}\mathbf{E} & \mathbf{E}^T\mathbf{M}_{k1}\mathbf{E} \\ * & -\mathbf{E}^T\mathbf{M}_{k1}\mathbf{E} \end{pmatrix}$$
$$\times \begin{pmatrix} z_k(t) \\ z_k(t-\tau_1) \end{pmatrix} + \frac{1}{\tau_2}\begin{pmatrix} z_k(t-\tau_1) \\ z_k(t-\tau_1-\tau_2) \end{pmatrix}^T \begin{pmatrix} -\mathbf{E}^T\mathbf{M}_{k2}\mathbf{E} & \mathbf{E}^T\mathbf{M}_{k2}\mathbf{E} \\ * & -\mathbf{E}^T\mathbf{M}_{k2}\mathbf{E} \end{pmatrix}$$

$$\times \begin{pmatrix} z_k(t-\tau_1) \\ z_k(t-\tau_1-\tau_2) \end{pmatrix} - \int_{t-\tau_1-\tau_2}^{t-\tau_1} z_k^T(s)ds\mathbf{M}_{k3}\int_{t-\tau_1-\tau_2}^{t-\tau_1} z_k(s)ds - \int_{t-\tau_1}^{t} z_k^T(s)ds\mathbf{M}_{k4}\int_{t-\tau_1}^{t} z_k(s)ds$$
$$+ \frac{\tau_1^2}{2}\dot{z}_k^T(t)\mathbf{E}^T\mathbf{N}_{k1}\mathbf{E}\dot{z}_k(t) - \frac{2}{\tau_1^2}\Big[\tau_1 z_k(t) - \int_{t-\tau_1}^{t} z_k(s)ds\Big]^T\mathbf{E}^T\mathbf{N}_{k1}\mathbf{E}\Big[\tau_1 z_k(t) - \int_{t-\tau_1}^{t} z_k(s)ds\Big]$$
$$- \frac{2}{\tau_2^2}\Big[\tau_2 z_k(t) - \int_{t-\tau_1-\tau_2}^{t-\tau_1} z_k(s)ds\Big]^T\mathbf{E}^T\mathbf{N}_{k2}\mathbf{E}\Big[\tau_2 z_k(t) - \int_{t-\tau_1-\tau_2}^{t-\tau_1} z_k(s)ds\Big]\Big\}. \tag{37}$$

It can be obtained from (13),(18) and (37) that

$$\dot{\mathbb{V}}_k(z_k(t),i,t) = \sum_{l=1}^{r}\mu_l(\theta(t))\eta_k(t)^T(\Delta + \delta_k^T R\delta_k)\eta_k(t), \tag{38}$$

where

$$\Delta = \begin{bmatrix} \overline{\Upsilon}_{11} & \Upsilon_{12} & 0 & \frac{1}{\tau_1}\mathbf{E}^T\mathbf{M}_{k1}\mathbf{E} & 0 & \frac{2}{\tau_2}\mathbf{E}^T\mathbf{N}_{k2}\mathbf{E} & \frac{2}{\tau_1}\mathbf{E}^T\mathbf{N}_{k1}\mathbf{E} & \mathbf{P}_{ki} \\ * & -(1-d)\mathbf{Q}_{k2} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Upsilon_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Upsilon_{44} & \frac{1}{\tau_2}\mathbf{E}^T\mathbf{M}_{k2}\mathbf{E} & 0 & 0 & 0 \\ * & * & * & * & \Upsilon_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \Upsilon_{66} & 0 & 0 \\ * & * & * & * & * & * & \Upsilon_{77} & 0 \\ * & * & * & * & * & * & * & -\alpha_k\mathbf{S}_k \end{bmatrix},$$

$$\overline{\Upsilon}_{11} = \mathbf{P}_{ki}(\mathbf{A}_i + a_1\lambda_k\Gamma_{1i} - a_4\sigma_k\Gamma_3) + (\mathbf{A}_i + a_1\lambda_k\Gamma_{1i} - a_4\sigma_k\Gamma_3)^T\mathbf{P}_{ki} + \sum_{j=1}^{m}\pi_{ij}\mathbf{P}_{kj}$$
$$+ \mathbf{Q}_{k1} + \mathbf{Q}_{k3} + \tau_2^2\mathbf{M}_{k3} + \tau_1^2\mathbf{M}_{k4} - \frac{1}{\tau_1}\mathbf{E}^T\mathbf{M}_{k1}\mathbf{E} - 2\mathbf{E}^T\mathbf{N}_{k2}\mathbf{E} + \alpha_k(N-1)\bar{l}\mathbf{S}_k,$$
$$\delta_k = \Big[(\mathbf{A}_i + a_1\lambda_k\Gamma_{1i} - a_4\sigma_k\Gamma_3) \;\; a_2\Gamma_{2i}\lambda_k + a_3\mathbf{D}\lambda_k \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; I\Big],$$

and Υ_{12}, Υ_{33}, Υ_{44}, Υ_{55}, Υ_{66}, Υ_{77} and R defined in (21). It follows readily from (38) that

$$\dot{\mathbb{V}}_k(z_k(t),i,t) \le \sum_{l=1}^{r}\mu_l(\theta(t))\eta_k^T(t)(\Delta + \delta_k^T R\delta_k)\eta_k(t), \tag{39}$$

By using the Schur complement Lemma 2.8, it can be seen that the inequality (20) is equivalent to $\Delta + \delta_k^T R\delta_k$, which implies $\dot{\mathbb{V}}(z_k(t),i,t) \le -\rho\|\eta_k^T(t)\|^2 < 0$, we get $\dot{\mathbb{V}}(z_k(t),i,t) < 0$, where $\rho = -\lambda_{max}(\Delta + \delta_k^T R\delta_k)$. Considering that $\mathbf{E}^T\mathbf{P}_{ki} = \mathbf{P}_{ki}\mathbf{E} \ge 0$, the stable result cannot be received via Lyapunov stability theory because the rank of $\mathbf{E}^T\mathbf{P}_{ki}$ within the Lyapunov function $\mathbb{V}_{k1}(z_k(t),i,t)$ is $r < n$. In view of Lemma 2.6 it is clear that the pair $(\mathbf{E}, \mathbf{A}_i + a_1\lambda_k\Gamma_{1i} - a_4\sigma_k\Gamma_3)$ is regular and impulse free on every occasion the inequalities (19)-(22) keep. Then there exist matrices $U_{k1} \in \mathbb{R}^{r\times n}$,

$U_{k2} \in \mathbb{R}^{n-r}$, $W_{k1} \in \mathbb{R}^{n\times r}$, $W_{k2} \in \mathbb{R}^{n-r}$, such that $U_k = [U_{k1}^T \ U_{k2}^T]$ and $W_k = [W_{k1}^T \ W_{k2}^T]$ are two nonsingular matrices and the following standard decomposition holds;

$$U_k\mathbf{E}W_k = diag\{I_r, 0\}, \tag{40}$$
$$U_k(\mathbf{A}_i + a_1\lambda_k\Gamma_{1i} - a_4\sigma_k\Gamma_3)W_k = diag\{\bar{\mathbf{A}}_{ki}, I_{n-r}\}, \tag{41}$$

where $\bar{\mathbf{A}}_{ki} \in \mathbb{R}^{r\times r}$, $k = 2, 3, ..., N$. The network system (11) is equivalent to

$$\begin{cases} \dot{z}_k^{(1)} &= \bar{\mathbf{A}}_{ki}z_k^{(1)} + U_{k1}h_k + \Big(a_2\lambda_kU_{k1}\Gamma_{2ri}W_{k1} + a_3\mathbf{D}U_{k1}\lambda_kW_{k1}\Big)z_k^{(1)}(t-\tau(t)), \\ 0 &= z_k^{(2)} + U_{k2}h_k + \Big(a_2\lambda_kU_{k2}\Gamma_{2(n-r)i}W_{k2} + a_3\mathbf{D}\lambda_kU_{k2}W_{k2}\Big)z_k^{(2)}(t-\tau(t)), \\ k &= 2, 3, ..., N, \end{cases} \tag{42}$$

where $W_k^{-1}z_k(t) = \begin{pmatrix} z_k^{(1)} \\ z_k^{(2)} \end{pmatrix}$, $\Gamma_{2ri} = diag\{d_1(i), d_2(i), ..., d_r(i)\}$ and $\Gamma_{2(n-r)i} = diag\{d_{r+1(i)}, d_{r+2(i)}, ..., d_n(i)\}$.

Let $U_k^{-T}\mathbf{P}_{ki}W_k = \begin{pmatrix} \mathbf{P}_{ki}^{(1)} & \mathbf{P}_{ki}^{(2)} \\ \mathbf{P}_{ki}^{(3)} & \mathbf{P}_{ki}^{(4)} \end{pmatrix}$. t that point as indicated by equations (19), (40) and (41), anything but difficult to see that $\mathbf{P}_{ki}^{(1)} = \mathbf{P}_{ki}^{(1)T}$ and $\mathbf{P}_{ki}^{(2)} = 0$. Subsequently,

$$\mathbb{V}_{k1}(z_k(t), i, t) = z_k^{(1)T}(t)\mathbf{P}_{ki}^{(1)}\mathbf{E}z_k^{(1)}(t). \tag{43}$$

From $\dot{\mathbb{V}}_k(z_k(t), i, t) < 0$, $z_k^{(1)}(t)$ of system (11) is asymptotically stable, i.e., $lim_{t\to\infty}\|z_k^{(1)}(t)\| = 0$, $k = 2, 3, ..., N$. In the following, we display that $z_k^{(2)}(t)$ additionally are asymptotically stable. From equation (42) and comparing with [37], deciding U_{k2} such that $U_{k2}U_{k2}^T = I_{n-r}$ which means that $\|U_{k2}\| = 1$ and the use of Lemma 2.4, we have

$$\begin{aligned} \Big\|z_k^{(2)}(t)\Big\| &= \Big\|U_{k2}h_k + (a_2\lambda_kU_{k2}\Gamma_{2(n-r)i}W_{k2} + a_3\mathbf{D}\lambda_kU_{k2}W_{k2})z_k^{(2)}(t-\tau(t))\Big\| \\ &\le \Big\|U_{k2}\Big\|\Big\|h_k\Big\| + \Big(a_2max(\lambda_k)\Big\|U_{k2}\Big\|\Big\|\Gamma_{2(n-r)i}\Big\|\Big\|W_{k2}\Big\| + a_3max(\lambda_k)\Big\|\mathbf{D}\Big\|\Big\|W_{k2}\Big\|U_{k2}\Big\|\Big\|\Big) \end{aligned}$$
$$\Big\|z_k^{(2)}(t-\tau(t))\Big\| \le \Big\|h_k(t)\Big\| = \sum_{k=2}^{N}\bar{l}\Big\|z_k(t)\Big\|$$
$$\Big(1 - \sum_{k=2}^{N}\bar{l}\Big\|W_k\Big\|\Big)\Big\|z_k^{(2)}(t)\Big\| \le \sum_{k=2}^{N}\bar{l}\Big\|W_k\Big\|\Big\|z_k^{(1)}(t)\Big\|.$$

If we choose W_k such that, $\Big(1 - \sum_{k=2}^{N}\bar{l}\|W_k\|\Big) > 0$ which leads $lim_{t\to\infty}\|z_k^{(2)}\| = 0$, $k = 2, 3, ..., N$. This completes the proof. □

Remark 3.2. *Numerous scientific models for real-world phenomena are characteristically nonlinear, and the stability investigation and synthesis issues for nonlinear systems are typically difficult. In the previous, the fuzzy logic theory has been shown to be successful in managing with a variety of complex nonlinear systems, which has subsequently received a set of consideration in the literature (see [3],[44]). It has turned out to be one of the most valuable approaches for using the qualitative knowledge of a framework to the structure a controller for complex networks [60]. The local dynamics of each fuzzy rule could be expressed by nonlinear systems with discrete delay. The overall fuzzy model can be achieved by fuzzy "blending" of these nonlinear systems.*

Remark 3.3. *Then $\Gamma_1 = 0$, the system (11) reduces to*

$$E\dot{z}_k(t) = \sum_{l=1}^{r}\mu_l(\theta(t))\Big[A_i - a_4\sigma_k\Gamma_3)z_k(t) + h_k(t) + (a_2\Gamma_{2i}\lambda_k + a_3\boldsymbol{D}\lambda_k)z_k(t-\tau(t))\Big], \quad k = 2, 3, ..., N. \tag{44}$$

Remark 3.4. *In this paper, we constructed a suitable Lyapunov Krasovskii functionals containing useful integral terms. Meanwhile, introducing Lemma 2.5 for the integral terms $\dot{V}_{k3}(z_k(t), i, t)$, terms plays an important role in the improvement of our stability results. Lemma 2.4 provides a new integral inequality which fully use the relationship between the terms in the $-\int_{t-\tau_1}^{t}\dot{z}_k^T(s)\boldsymbol{E}^T\boldsymbol{M}_{k1}\boldsymbol{E}\dot{z}_k(s)ds$ and $-\int_{t-\tau_1-\tau_2}^{t-\tau_1}\dot{z}_k^T(s)\boldsymbol{E}^T\boldsymbol{M}_{k2}\boldsymbol{E}\dot{z}_k(s)ds$ within the system of LMIs.*

Remark 3.5. *Changing the nonlinear part $f(x_k(t),t)$ into linear one $f(x_k(t))$, then the system (1) turns into the following system*

$$\boldsymbol{E}\dot{x}_k(t) = \boldsymbol{A}(r(t))x_k(t) + f(x_k(t)) + a_1 \sum_{w=1}^{N} g_{kw}\Gamma_1(r(t))x_w(t)$$
$$+ a_2 \sum_{w=1}^{N} g_{kw}\Gamma_2(r(t))x_w(t-\tau_1(t)-\tau_2(t)) + a_3 \sum_{w=1}^{N} g_{kw}\boldsymbol{D}x_j(t-\tau(t)), \quad k = 1,2,....,N, \tag{45}$$

The synchronization of this system with T-S fuzzy theory has already studied in [19].

Theorem 3.6. *Assume that (1) and (2) holds. Then the T-S fuzzy singular Markovian jumping error dynamical network (11) is asymptotically stable. For a given scalars $\tau_1 > 0$, $\tau_2 > 0$, $d_1 > 0$ and $d_2 > 0$, if there exist nonnegative constants α_k, nonnegative definite matrices $\boldsymbol{P}_{ki} = diag\{\boldsymbol{P}_{k1}, \boldsymbol{P}_{k2}, ..., \boldsymbol{P}_{kN}\} > 0$, $\boldsymbol{Q}_{kq} > 0$, $\boldsymbol{M}_{kq} > 0$ $(q = 1,2,3,4)$, $\boldsymbol{N}_{k1} > 0$, $\boldsymbol{N}_{k2} > 0$ and positive diagonal matrices $\boldsymbol{S}_k$ such that the following LMIs hold $\forall\, i \in S$:*

$$\boldsymbol{E}^T\boldsymbol{P}_{ki} = \boldsymbol{P}_{ki}\boldsymbol{E} \geq 0, \quad \widehat{\Psi}_i < 0, \quad i = 1,2,....,N, \tag{46}$$

$$\widehat{\Psi}_i = \begin{bmatrix} \widehat{\Upsilon}_{11} & \widehat{\Upsilon}_{12} & 0 & \widehat{\Upsilon}_{14} & 0 & \widehat{\Upsilon}_{16} & \frac{2}{\tau_1}\boldsymbol{E}^T\boldsymbol{N}_{k1}\boldsymbol{E} & \boldsymbol{P}_{ki} & \widehat{\Upsilon}_{19} \\ * & -(1-d)\boldsymbol{Q}_{k2} & 0 & 0 & 0 & 0 & 0 & 0 & \widehat{\Upsilon}_{29} \\ * & * & \widehat{\Upsilon}_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \widehat{\Upsilon}_{44} & \frac{1}{\tau_2}\boldsymbol{E}^T\boldsymbol{M}_{k2}\boldsymbol{E} & 0 & 0 & 0 & 0 \\ * & * & * & * & \widehat{\Upsilon}_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \widehat{\Upsilon}_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \widehat{\Upsilon}_{77} & 0 & 0 \\ * & * & * & * & * & * & * & -\alpha_k\boldsymbol{S}_k & I \\ * & * & * & * & * & * & * & * & -2\boldsymbol{P}_{ki}+R \end{bmatrix}, \tag{47}$$

where

$$\widehat{\Upsilon}_{11} = \boldsymbol{P}_{ki}(\boldsymbol{A}_i + c_1\lambda_k\Gamma_{1i} - c_4\sigma_k\Gamma_3) + (\boldsymbol{A}_i + c_1\lambda_k\Gamma_{1i} - c_4\sigma_k\Gamma_3)^T\boldsymbol{P}_{ki} + \sum_{j=1}^{m}\pi_{ij}\boldsymbol{P}_{kj} + \boldsymbol{Q}_{k1} + \boldsymbol{Q}_{k3}$$
$$+ \tau_2^2\boldsymbol{M}_{k3} + \tau_1^2\boldsymbol{M}_{k4} - \frac{1}{\tau_1}\boldsymbol{E}^T\boldsymbol{M}_{k1}\boldsymbol{E} - 2\boldsymbol{E}^T\boldsymbol{N}_{k2}\boldsymbol{E} + \alpha_k(N-1)\bar{l}\boldsymbol{S}_k,\ \widehat{\Upsilon}_{12} = \boldsymbol{P}_{ki}c_2\Gamma_{2i}\lambda_k + \boldsymbol{P}_{ki}c_3\boldsymbol{D}\lambda_k,$$

$$\widehat{\Upsilon}_{14} = \frac{1}{\tau_1}\boldsymbol{E}^T\boldsymbol{M}_{k1}\boldsymbol{E},\ \widehat{\Upsilon}_{16} = \frac{2}{\tau_2}E^TN_{k2}E,\ \Upsilon_{19} = (\boldsymbol{A}_i + c_1\Gamma_{1i}\lambda_k - c_4\sigma_k\Gamma_3)\boldsymbol{P}_{ki},\ \widehat{\Upsilon}_{29} = (c_2\Gamma_{2i}\lambda_k + c_3\boldsymbol{D}\lambda_k)\boldsymbol{P}_{ki},$$

$$\widehat{\Upsilon}_{33} = -(1-d_1)(\boldsymbol{Q}_{k1} - \boldsymbol{Q}_{k2}),\ \widehat{\Upsilon}_{44} = \boldsymbol{Q}_{k4} - \boldsymbol{Q}_{k3} - \frac{1}{\tau_1}\boldsymbol{E}^T\boldsymbol{M}_{k1}\boldsymbol{E} - \frac{1}{\tau_2}\boldsymbol{E}^T\boldsymbol{M}_{k2}\boldsymbol{E},\ \widehat{\Upsilon}_{55} = -\boldsymbol{Q}_{k4} - \frac{1}{\tau_2}\boldsymbol{E}^T\boldsymbol{M}_{k2}\boldsymbol{E},$$

$$\widehat{\Upsilon}_{66} = -\boldsymbol{M}_{k3} - \frac{2}{\tau_2^2}\boldsymbol{E}^T\boldsymbol{N}_{k2}\boldsymbol{E},\ \widehat{\Upsilon}_{77} = -\boldsymbol{M}_{k4} - \frac{2}{\tau_1^2}\boldsymbol{E}^T\boldsymbol{N}_{k1}\boldsymbol{E},\ R = \boldsymbol{E}^T(\tau_1\boldsymbol{M}_{k1} + \tau_2\boldsymbol{M}_{k2})\boldsymbol{E}$$
$$+ \frac{\tau_1^2}{2}\boldsymbol{E}^T\boldsymbol{N}_{k1}\boldsymbol{E} + \frac{\tau_2^2}{2}\boldsymbol{E}^T\boldsymbol{N}_{k2}\boldsymbol{E}.$$

Proof. By considering the inequality $-\mathbf{P}_{ki}R^{-1}\mathbf{P}_{ki} \leq -2\mathbf{P}_{ki} + R$, the LMI (46) becomes,

$$\widehat{\Psi}_i = \begin{bmatrix} \widehat{\Upsilon}_{11} & \widehat{\Upsilon}_{12} & 0 & \widehat{\Upsilon}_{14} & 0 & \widehat{\Upsilon}_{16} & \frac{2}{\tau_1}\mathbf{E}^T\mathbf{N}_{k1}\mathbf{E} & \mathbf{P}_{ki} & \widehat{\Upsilon}_{19} \\ * & -(1-d)\mathbf{Q}_{k2} & 0 & 0 & 0 & 0 & 0 & 0 & \Upsilon_{29} \\ * & * & \widehat{\Upsilon}_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \widehat{\Upsilon}_{44} & \frac{1}{\tau_2}\mathbf{E}^T\mathbf{M}_{k2}\mathbf{E} & 0 & 0 & 0 & 0 \\ * & * & * & * & \widehat{\Upsilon}_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \widehat{\Upsilon}_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \widehat{\Upsilon}_{77} & 0 & 0 \\ * & * & * & * & * & * & * & -\alpha_k\mathbf{S}_k & I \\ * & * & * & * & * & * & * & * & -\mathbf{P}_{ki}R^{-1}\mathbf{P}_{ki} \end{bmatrix} < 0. \tag{48}$$

Then a congruence transformation of $diag\{I,\ I,\ I,\ I,\ I,\ I,\ I,\ I, \mathbf{P}_{ki}^{-1}\}$ to the LMI (47), we obtain the LMI (20). The rest of the proof follows directly from Theorem 3.1. □

Remark 3.7. *The $\boldsymbol{P}_{ki}$ appear in Theorem 3.1 and Theorem 3.2 can be chosen with give $\boldsymbol{E}$. We only need $\boldsymbol{E}\boldsymbol{P}_{ki}^T = \boldsymbol{P}_{ki}\boldsymbol{E}$ to be positive semi-definite.*

4 Numerical example

In this section, we present a numerical model to determine the performance of the proposed synchronization criteria given in this paper.

Example 4.1. *Consider a three-dimensional Markovian jumping T-S fuzzy singular complex dynamical networks with additive time varying delays and pinning control with 3-nodes:*
Fuzzy Rule 1:

IF $\{\theta_1$ *is* $F_{11}\}$ ***and ..... and*** $\{\theta_g$ *is* $F_{1g}\}$ ***THEN***

$$\boldsymbol{E}\dot{z}_1(t) = (\boldsymbol{A}_1 + a_1\Gamma_{11}\lambda_1 - a_4\sigma_1\Gamma_3)z_1(t) + h_1(t) + (a_2\Gamma_{21}\lambda_1 + a_3\boldsymbol{D}\lambda_1)z_1(t-\tau(t)), \tag{49}$$

Fuzzy Rule 2:

IF $\{\theta_1$ *is* $F_{21}\}$ ***and ..... and*** $\{\theta_g$ *is* $F_{2g}\}$ ***THEN***

$$\boldsymbol{E}\dot{z}_2(t) = (\boldsymbol{A}_2 + a_1\Gamma_{12}\lambda_2 - a_4\sigma_2\Gamma_3)z_2(t) + h_2(t) + (a_2\Gamma_{22}\lambda_2 + a_3\boldsymbol{D}\lambda_2)z_2(t-\tau(t)), \tag{50}$$

where $z_k(t) = (z_{k1}^T, z_{k2}^T)^T$, $f_k(t) = (tanh(z_{k1}(t), t), tanh(z_{k2}(t), t))^T$ furthermore, the applicable parameters are given as follows:

$$\boldsymbol{E} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ \boldsymbol{A}_1 = \begin{bmatrix} -2 & 1 \\ -5 & 6 \end{bmatrix},\ \Gamma_{11} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},\ \Gamma_{21} = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix},$$

$$\Gamma_3 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix},\ \boldsymbol{A}_2 = \begin{bmatrix} -4 & 3 \\ 3 & -4 \end{bmatrix},\ \Gamma_{12} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix},\ \Gamma_{22} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix},$$

$$\Pi = \begin{bmatrix} -3 & 3 \\ 4 & -4 \end{bmatrix},\ \boldsymbol{D} = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix},\ U_1 = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix},\ U_2 = \begin{bmatrix} -3 & 1 \\ 1 & -1 \end{bmatrix}.$$

Let us consider $\tau_1(t) = 0.2 + 0.2sin(0.5t)$, $\tau_2(t) = 0.5 + 0.5sin(0.5t)$, $a_1 = 0$, $a_2 = 0.1$, $a_3 = 0.2$, $a_4 = 0.3$, $\sigma_1 = 4$,

Table 1: Maximum admissible upper bounds for delays τ_2 with different values of τ_1.

τ_1	1.1	1.2	1.3	1.4	1.5	1.6	1.7
τ_2	2.2564	2.2465	2.1321	2.0005	1.9005	1.8201	1.7356

$\sigma_2 = 5$, $\sigma_3 = 3$, $\tau_1 = 0.2$, $\tau_2 = 0.5$ and the eigenvalues of $\boldsymbol{D}$ are found to be $\lambda_1 = 0$, $\lambda_2 = -3$, $\lambda_3 = -1$. The maximum admissible upper bounds τ_2 obtained for different values of τ_1 are listed in Table 1. Then by utilizing the Matlab LMI Toolbox, we solve the LMIs in Theorem 3.2, we acquire the feasible solutions for $k = 1$, and $i = 1$ as follows:

$$\boldsymbol{P}_{11} = \begin{bmatrix} 0.0132 & -0.0035 \\ -0.0035 & 0.0867 \end{bmatrix}, \ \boldsymbol{Q}_{11} = \begin{bmatrix} 1.8966 & -0.0967 \\ -0.0967 & 1.9157 \end{bmatrix}, \ \boldsymbol{Q}_{12} = \begin{bmatrix} 1.1321 & -0.0693 \\ -0.0693 & 1.1997 \end{bmatrix},$$

$$\boldsymbol{Q}_{13} = \begin{bmatrix} -2.8181 & -0.1750 \\ -0.1750 & -2.4924 \end{bmatrix}, \ \boldsymbol{Q}_{14} = \begin{bmatrix} 0.6557 & 0.0774 \\ 0.0774 & 1.2471 \end{bmatrix}, \ \boldsymbol{M}_{11} = \begin{bmatrix} -0.0081 & -0.0004 \\ -0.0004 & 0.0000 \end{bmatrix},$$

$$\boldsymbol{M}_{12} = \begin{bmatrix} -0.3658 & -0.0166 \\ -0.0166 & 0.0000 \end{bmatrix}, \ \boldsymbol{M}_{13} = \begin{bmatrix} 0.5321 & -0.0138 \\ -0.0138 & 0.6550 \end{bmatrix}, \ \boldsymbol{M}_{14} = \begin{bmatrix} 0.7285 & -0.0016 \\ -0.0016 & 0.7110 \end{bmatrix},$$

$$\boldsymbol{N}_{11} = \begin{bmatrix} -0.0103 & 0.0693 \\ 0.0693 & 0.0022 \end{bmatrix}, \ \boldsymbol{N}_{12} = \begin{bmatrix} 0.0176 & 0.0023 \\ 0.0023 & 0.0000 \end{bmatrix}, \ \boldsymbol{S}_1 = \begin{bmatrix} 2.1080 & 0 \\ 0 & 2.1080 \end{bmatrix}.$$

Meanwhile, this example shows that all the conditions stated in Theorem 3.2 have been satisfied and hence system (11) achieve synchronization through the pinning control $u(t)$ with the given parameters in the sense of Definition 2.1.

5 Conclusions

In this paper, we mainly discuss the synchronization for singular complex dynamical networks with Markovian jumping and additive time-varying delays using pinning control dependent on T–S fuzzy theory. By utilizing a new Lyapunov-Krasovskii functional with contain triple integral terms and utilizing some integral inequalities we demonstrated that the stability problem of Markovian jumping singular complex dynamical networks (MJSCDNs) system is resolvable if a set of linear matrix inequalities (LMIs) are feasible. A brought together LMI approach was created to build up sufficient conditions for the MJSCDNs synchronize have been accomplished by means of pinning control. Finally a numerical example is provided to given to demonstrate the effectiveness of the proposed results. The thought and approach created in this paper can be further generalized to deal with some other problems on impulsive control, sampled data control and especially for the synchronization of singular complex dynamical networks. This will be our near future topics of our research.

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Synchronization criteria for T-S fuzzy singular complex dynamical networks with Markovian jumping parameters and mixed time-varying delays using pinning control

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محک همزمان‌سازی برای شبکه‌های دینامیکی مختلط تکین T-S فازی با پارامترهای جهشی Markovian و تأخیرات با تغییرات –زمانی مرکب با استفاده از کنترل الحاقی

چکیده. در این مقاله، درباره همزمان‌سازی شبکه‌های دینامیکی مختلط تکین با پارامترهای جهشی Markovian و تأخیرات با تغییرات – زمانی جمعی از طریق کنترل الحاقی و با استفاده از نظریه T-S فازی Takagi-Sugeno بحث می‌کنیم. سیستم‌های دینامیکی مختلط شامل m گره و سیستم‌ها از یک طریق به طریق دیگر تعویض می‌شوند، یک زنجیر Markovian با احتمال انتقال شگرف استراتژی‌ها براساس کنترل طراحی و شبکه‌های دینامیکی مختلط تکین همزمان‌سازی شده‌اند. یک خانواده جدید از تابع Lyapunov-Krasovskii که شامل عبارات انتگرال می‌باشد، جهت هدایت محک پایداری ساخته شده‌است. برخی از شرایط کافی برای همزمان‌سازی، به صورت روش نامساوی ماتریس خطی (LMI) ارائه گردیده و در پایان، جهت پشتیبانی نتایج اصلی این مقاله، مثال‌های عددی ارائه شده‌است.