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Multigranulation single valued neutrosophic covering-based rough sets and their applications to multi-criteria group decision making

J. Q. Wang[1] and X. H. Zhang[2]

[1] *College of Electrical and Control Engineering, Shaanxi University of Science and Technology, Xi'an 710021, PR China*
[2] *School of Arts and Sciences, Shaanxi University of Science and Technology, Xi'an 710021, PR China*

wangjingqianw@163.com, zhangxiaohong@sust.edu.cn

Abstract

In this paper, three types of (philosophical, optimistic and pessimistic) multigranulation single valued neutrosophic (SVN) covering-based rough set models are presented, and these three models are applied to the problem of multi-criteria group decision making (MCGDM). Firstly, a type of SVN covering-based rough set model is proposed. Based on this rough set model, three types of multigranulation SVN covering-based rough set models under the concept of multigranulation SVN β-covering approximation space are proposed, where β is a SVN number. Moreover, the connections among these four models are investigated. Secondly, some conditions under which different multigranulation SVN β-covering approximation spaces induced the same multigranulation SVN covering approximation operators are presented. Finally, three novel methods are presented to MCGDM problems under the multigranulation SVN covering-based rough set models. Furthermore, the proposed MCGDM methods are compared with other methods through a problem about paper defect diagnosis.

Keywords: Covering-based rough set, single valued neutrosophic set, multigranulation, multi-criterion decision making, paper defect diagnosis.

1 Introduction

Covering-based rough sets (CBRSs) [3, 4, 38] were presented to deal with the type of covering data, and have enriched Pawlak's rough sets [13] in many ways. In theoretical terms, covering approximation models [27] have been established, covering reduction problems [8] have been solved, and covering axiomatic systems [33] have been constructed. In application, CBRSs have been applied to knowledge reduction [25], decision rule synthesis [16], and other fields [14, 21, 29, 37]. In theory, covering-based rough set (CBRS) theory has been connected with matroid theory [23], lattice theory [32] and fuzzy set theory [7, 24].

For fuzzy CBRSs, some researchers presented different fuzzy CBRS models and studied some issues in fuzzy CBRSs, such as Ma [11] and D'eer et al. [5]. All these results were presented by only considering a fuzzy covering. Then, Zhan et al. [31] considered multigranulation in fuzzy CBRSs and presented multigranulation fuzzy rough covering models. As an extension of fuzzy set theory, intuitionistic fuzzy (IF) set theory [2] has attracted increasing attention to combine with rough set theory, such us IF relation rough sets [1] and IF CBRSs. Especially for IF CBRSs, Huang et al. [6] presented a type of IF CBRS model and a type of IF graded CBRS model. Moreover, Zhang et al. [34] proposed covering-based general multigranulation IF rough set models.

Single valued neutrosophic (SVN) sets [17, 19] can be regarded as an intuitively straightforward extension of IF sets. SVN sets and rough sets are both capable of dealing with uncertainty and partial information. Recently, some researchers combined them by SVN relations. For example, Mondal and Pramanik [12, 15] applied rough neutrosophic sets in multi-attribute decision making based on grey relational analysis and medical diagnosis; Yang et al. [26] presented a SVN relation-based rough set model under a SVN relation-based approximation space. On the other hand, SVN

Corresponding Author: X. H. Zhang
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coverings have attracted more attention. Wang and Zhang [20] proposed the first type of SVN CBRS model under a SVN β-covering approximation space. Further, Wang and Zhang [22] proposed the second type of SVN CBRS model under a new SVN β-covering approximation space, which based on a new inclusion relation. In [22], they also presented the relationship between these two SVN CBRS models, as well as the connection between these SVN CBRS models and SVN relation-based rough set models. All these work are investigated in a corresponding SVN β-covering approximation space, which contains one and only one SVN β-covering. But there will be a new covering approximation space when we consider a family of SVN β-coverings. We call it as a multigranulation SVN β-covering approximation space in this paper. Hence, a SVN β-covering approximation space is a special case of a multigranulation SVN β-covering approximation space. When a SVN β-covering approximation space is generalized to a multigranulation SVN β-covering approximation space, we consider the following statements: 1) whether some characteristics of the SVN β-covering approximation space will hold in the multigranulation SVN β-covering approximation space; 2) whether there are some new rough set models in the multigranulation SVN β-covering approximation space; 3) whether there exist some relationships among these new rough set models. Therefore, the investigation of the multigranulation SVN β-covering approximation space and some corresponding rough set models is very important. It not only can combine CBRSs with SVN sets from the viewpoint of multigranulation, but also can broaden the practical application. This is our motivation of this research.

In this paper, three types of multigranulation SVN CBRS models are presented and applied to the problem of multi-criteria group decision making (MCGDM). On one hand, a new type of SVN CBRS model is proposed in SVN β-covering approximation space. Based on this rough set model, three types of multigranulation SVN CBRS models under the concept of n-SVN β-covering approximation space are proposed. Moreover, the relationships among these multigranulation SVN CBRS models are proposed. Then, we present some conditions under which different n-SVN β-coverings can induce the same multigranulation SVN covering approximation of any SVN subset. On the other hand, many researchers have applied (fuzzy or IF) rough set models in decision making (DM) problems [18, 26, 30]. By the characterizations of the problem of MCGDM, we present three novel methods to deal with MCGDM problems under three types of multigranulation SVN CBRS models proposed in this paper. Moreover, the proposed MCGDM methods are compared with the methods which are presented by Liu [10] and Li et al. [9] respectively.

The rest of this paper is arranged as follows. Section 2 recalls some basic definitions about SVN sets and SVN β-covering approximation spaces. In Section 3, the concept of n-SVN β-covering approximation space is presented. In Section 4, three types of multigranulation SVN CBRS models under the concept of n-SVN β-covering approximation space are proposed. In Section 5, we present some conditions under which different n-SVN β-coverings can induce the same multigranulation SVN covering approximation of any SVN subset. In Section 6, we present three novel approaches to the paper defect diagnosis problem of MCGDM under different multigranulation SVN CBRS models. Moreover, the proposed MCGDM methods are compared with other methods. Section 7 gives the main conclusions and further research topics.

2 Basic definitions

In this section, we review some basic definitions about SVN sets and SVN β-covering approximation spaces. A finite and nonempty set is called as a universe.

Definition 2.1. [19] *Let U be a universe. A SVN set A in U is defined as an object of the following form:*

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in U\},$$

where $T_A(x) : U \to [0,1]$ is a truth-membership function, $I_A(x) : U \to [0,1]$ is an indeterminacy-membership function and $F_A(x) : U \to [0,1]$ is a falsity-membership function for any $x \in U$. They satisfy $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in U$. The family of all SVN sets in U is denoted by $SVN(U)$, called as the SVN power set. For convenience, a SVN number is represented by $\beta = \langle a, b, c\rangle$, where $a, b, c \in [0,1]$ and $a + b + c \leq 3$.

Specially, for two SVN numbers $\alpha = \langle a, b, c\rangle$ and $\beta = \langle d, e, f\rangle$, $\alpha \leq \beta \Leftrightarrow a \leq d$, $b \geq e$ and $c \geq f$. Some basic operations on $SVN(U)$ are shown as follows [2, 26]: $A, B \in SVN(U)$,
(1) $A \subseteq B$ iff $T_A(x) \leq T_B(x)$, $I_B(x) \leq I_A(x)$ and $F_B(x) \leq F_A(x)$ for all $x \in U$;
(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
(3) $A \cap B = \{\langle x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x)\rangle : x \in U\}$;
(4) $A \cup B = \{\langle x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x)\rangle : x \in U\}$;
(5) $A' = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle : x \in U\}$;
(6) $A \oplus B = \{\langle x, T_A(x) + T_B(x) - T_A(x) \cdot T_B(x), I_A(x) \cdot I_B(x), F_A(x) \cdot F_B(x)\rangle : x \in U\}$.

To combine CBRSs with SVN sets, Wang and Zhang [20] presented the concept of SVN β-covering approximation space.

Definition 2.2. [20] *Let U be a universe. For a SVN number β, we call $\widehat{\boldsymbol{C}} = \{C_1, C_2, \cdots, C_l\}$, with $C_i \in SVN(U)(i = 1, 2, ..., l)$, as a SVN β-covering of U, if for all $x \in U$, there exists $C_i \in \widehat{\boldsymbol{C}}$ such that $C_i(x) \geq \beta$. We also call $(U, \widehat{\boldsymbol{C}})$ as a SVN β-covering approximation space.*

Definition 2.3. [20] *Let $\widehat{\boldsymbol{C}}$ be a SVN β-covering of U and $\widehat{\boldsymbol{C}} = \{C_1, C_2, \ldots, C_l\}$. For each $x \in U$, the SVN β-neighborhood $\widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}}(x)}$ of x about $\widehat{\boldsymbol{C}}$ can be defined as: $\widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}}(x)} = \cap\{C_i \in \widehat{\boldsymbol{C}} : C_i(x) \geq \beta\}$.*

Proposition 2.4. [20] *Let $\widehat{\boldsymbol{C}}$ be a SVN β-covering of U. For any $x, y \in U$, $\widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}}(x)}(y) \geq \beta$ if and only if $\widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}}(y)} \subseteq \widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}}(x)}$.*

3 Multigranulation SVN β-covering approximation spaces

We present the notion of multigranulation SVN β-covering approximation space and some characteristics of it in this section. Based on Definition 2.2, the concept of n-SVN β-covering is given in the following definition.

Definition 3.1. *Let $\widehat{\boldsymbol{C}_k}$ $(k = 1, 2, ..., n)$ be SVN β-coverings of U. Then we call Γ_β as a n-SVN β-covering of U, where $\Gamma_\beta = \{\widehat{\boldsymbol{C}_1}, \widehat{\boldsymbol{C}_2}, \cdots, \widehat{\boldsymbol{C}_n}\}$.*

The pair (U, Γ_β) is called as a n-SVN β-covering approximation space. We also call (U, Γ_β) as a multigranulation SVN β-covering approximation space. Then the notion of SVN β-neighborhood in a SVN β-covering can be extended to a new notion in a n-SVN β-covering.

Definition 3.2. *Let (U, Γ_β) be a n-SVN β-covering approximation space. For any $x \in U$, $\widetilde{\mathbb{N}}_{\Gamma_\beta(x)} = \bigcap\{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}}(x)} \in Cov(\widehat{\boldsymbol{C}}) : \widehat{\boldsymbol{C}} \in \Gamma_\beta\}$ is called as the SVN β-neighborhood of x about Γ_β, where $Cov(\widehat{\boldsymbol{C}}) = \{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}}(x)} : x \in U\}$.*

Example 3.3. *Let (U, Γ_β) be a 2-SVN β-coverings approximation space, where $U = \{x_1, x_2, x_3, x_4, x_5\}$, $\Gamma_\beta = \{\widehat{\boldsymbol{C}_1}, \widehat{\boldsymbol{C}_2}\}$ and $\beta = \langle 0.6, 0.3, 0.7\rangle$. $\widehat{\boldsymbol{C}_1}$ and $\widehat{\boldsymbol{C}_2}$ are shown in Tables 1 and 2, respectively.*

Table 1: $\widehat{\mathbf{C}_1}$.

$\boldsymbol{U}$	C_{11}	C_{12}	C_{13}	C_{14}
x_1	$\langle 0.7, 0.3, 0.5\rangle$	$\langle 0.6, 0.2, 0.4\rangle$	$\langle 0.5, 0.2, 0.5\rangle$	$\langle 0.2, 0.5, 0.6\rangle$
x_2	$\langle 0.6, 0.3, 0.4\rangle$	$\langle 0.5, 0.3, 0.8\rangle$	$\langle 0.4, 0.5, 0.4\rangle$	$\langle 0.6, 0.2, 0.7\rangle$
x_3	$\langle 0.4, 0.6, 0.3\rangle$	$\langle 0.7, 0.3, 0.6\rangle$	$\langle 0.5, 0.3, 0.6\rangle$	$\langle 0.6, 0.3, 0.4\rangle$
x_4	$\langle 0.6, 0.2, 0.7\rangle$	$\langle 0.5, 0.5, 0.6\rangle$	$\langle 0.3, 0.6, 0.5\rangle$	$\langle 0.5, 0.6, 0.2\rangle$
x_5	$\langle 0.3, 0.3, 0.7\rangle$	$\langle 0.7, 0.3, 0.5\rangle$	$\langle 0.6, 0.3, 0.5\rangle$	$\langle 0.8, 0.3, 0.2\rangle$

Table 2: $\widehat{\mathbf{C}_2}$.

$\boldsymbol{U}$	C_{21}	C_{22}	C_{23}	C_{24}
x_1	$\langle 0.8, 0.2, 0.5\rangle$	$\langle 0.6, 0.2, 0.4\rangle$	$\langle 0.8, 0.2, 0.4\rangle$	$\langle 0.3, 0.5, 0.8\rangle$
x_2	$\langle 0.6, 0.3, 0.3\rangle$	$\langle 0.5, 0.3, 0.8\rangle$	$\langle 0.4, 0.6, 0.7\rangle$	$\langle 0.7, 0.2, 0.5\rangle$
x_3	$\langle 0.9, 0.6, 0.4\rangle$	$\langle 0.7, 0.3, 0.6\rangle$	$\langle 0.5, 0.3, 0.7\rangle$	$\langle 0.8, 0.2, 0.4\rangle$
x_4	$\langle 0.6, 0.2, 0.5\rangle$	$\langle 0.8, 0.2, 0.6\rangle$	$\langle 0.4, 0.6, 0.3\rangle$	$\langle 0.5, 0.6, 0.7\rangle$
x_5	$\langle 0.4, 0.5, 0.9\rangle$	$\langle 0.7, 0.3, 0.4\rangle$	$\langle 0.6, 0.3, 0.6\rangle$	$\langle 0.8, 0.2, 0.3\rangle$

Then $\widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}_1}(x_1)} = C_{11} \cap C_{12}, \widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}_1}(x_2)} = C_{11} \cap C_{14}, \widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}_1}(x_3)} = C_{12} \cap C_{14}, \widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}_1}(x_4)} = C_{11}, \widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}_1}(x_5)} = C_{12} \cap C_{13} \cap C_{14}$, $\widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}_2}(x_1)} = C_{21} \cap C_{22} \cap C_{23}, \widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}_2}(x_2)} = C_{21} \cap C_{24}, \widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}_2}(x_3)} = C_{22} \cap C_{24}, \widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}_2}(x_4)} = C_{21} \cap C_{22}, \widetilde{\mathbb{N}}^{\beta}_{\widehat{\boldsymbol{C}_2}(x_5)} = C_{22} \cap C_{23} \cap C_{24}$. *Hence, all SVN β-neighborhoods induced by $\widehat{\boldsymbol{C}_1}$ and $\widehat{\boldsymbol{C}_2}$ are shown in Tables 3 and 4, respectively.*

Table 3: $\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}_1}(x_i)}$ $(i = 1, 2, 3, 4, 5)$.

$\widetilde{\mathbb{N}}^{\boldsymbol{\beta}}_{\widehat{\mathbf{C}_1}(\boldsymbol{x_i})}$	x_1	x_2	x_3	x_4	x_5
$\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}_1}(x_1)}$	$\langle 0.6, 0.3, 0.5\rangle$	$\langle 0.5, 0.3, 0.8\rangle$	$\langle 0.4, 0.6, 0.6\rangle$	$\langle 0.5, 0.5, 0.7\rangle$	$\langle 0.3, 0.3, 0.7\rangle$
$\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}_1}(x_2)}$	$\langle 0.2, 0.5, 0.6\rangle$	$\langle 0.6, 0.3, 0.7\rangle$	$\langle 0.4, 0.6, 0.4\rangle$	$\langle 0.5, 0.6, 0.7\rangle$	$\langle 0.3, 0.3, 0.7\rangle$
$\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}_1}(x_3)}$	$\langle 0.2, 0.5, 0.6\rangle$	$\langle 0.5, 0.3, 0.8\rangle$	$\langle 0.6, 0.3, 0.6\rangle$	$\langle 0.5, 0.6, 0.6\rangle$	$\langle 0.7, 0.3, 0.5\rangle$
$\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}_1}(x_4)}$	$\langle 0.7, 0.3, 0.5\rangle$	$\langle 0.6, 0.3, 0.4\rangle$	$\langle 0.4, 0.6, 0.3\rangle$	$\langle 0.6, 0.2, 0.7\rangle$	$\langle 0.3, 0.3, 0.7\rangle$
$\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}_1}(x_5)}$	$\langle 0.2, 0.5, 0.6\rangle$	$\langle 0.4, 0.5, 0.8\rangle$	$\langle 0.5, 0.3, 0.6\rangle$	$\langle 0.3, 0.6, 0.6\rangle$	$\langle 0.6, 0.3, 0.5\rangle$

Table 4: $\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}_2}(x_i)}$ $(i = 1, 2, 3, 4, 5)$.

$\widetilde{\mathbb{N}}^{\boldsymbol{\beta}}_{\widehat{\mathbf{C}_2}(\boldsymbol{x_i})}$	x_1	x_2	x_3	x_4	x_5
$\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}_2}(x_1)}$	$\langle 0.6, 0.2, 0.5\rangle$	$\langle 0.4, 0.6, 0.8\rangle$	$\langle 0.5, 0.6, 0.7\rangle$	$\langle 0.4, 0.6, 0.6\rangle$	$\langle 0.4, 0.5, 0.9\rangle$
$\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}_2}(x_2)}$	$\langle 0.3, 0.5, 0.8\rangle$	$\langle 0.6, 0.3, 0.5\rangle$	$\langle 0.8, 0.6, 0.4\rangle$	$\langle 0.5, 0.6, 0.7\rangle$	$\langle 0.4, 0.5, 0.9\rangle$
$\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}_2}(x_3)}$	$\langle 0.3, 0.5, 0.8\rangle$	$\langle 0.5, 0.3, 0.8\rangle$	$\langle 0.7, 0.3, 0.6\rangle$	$\langle 0.5, 0.6, 0.7\rangle$	$\langle 0.7, 0.3, 0.4\rangle$
$\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}_2}(x_4)}$	$\langle 0.6, 0.2, 0.5\rangle$	$\langle 0.5, 0.3, 0.8\rangle$	$\langle 0.7, 0.6, 0.6\rangle$	$\langle 0.6, 0.2, 0.6\rangle$	$\langle 0.4, 0.5, 0.9\rangle$
$\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}_2}(x_5)}$	$\langle 0.3, 0.5, 0.8\rangle$	$\langle 0.4, 0.6, 0.8\rangle$	$\langle 0.5, 0.3, 0.7\rangle$	$\langle 0.4, 0.6, 0.7\rangle$	$\langle 0.6, 0.3, 0.6\rangle$

Based on Tables 3 and 4, all SVN β-neighborhoods induced by Γ_β are shown in Table 5.

Table 5: $\widetilde{\mathbb{N}}_{\Gamma_\beta(x_i)}$ $(i = 1, 2, 3, 4, 5)$.

$\widetilde{\mathbb{N}}_{\Gamma_\beta(x_i)}$	x_1	x_2	x_3	x_4	x_5
$\widetilde{\mathbb{N}}_{\Gamma_\beta(x_1)}$	$\langle 0.6, 0.3, 0.5\rangle$	$\langle 0.4, 0.6, 0.8\rangle$	$\langle 0.4, 0.6, 0.7\rangle$	$\langle 0.4, 0.6, 0.7\rangle$	$\langle 0.3, 0.5, 0.9\rangle$
$\widetilde{\mathbb{N}}_{\Gamma_\beta(x_2)}$	$\langle 0.2, 0.5, 0.8\rangle$	$\langle 0.6, 0.3, 0.7\rangle$	$\langle 0.4, 0.6, 0.4\rangle$	$\langle 0.5, 0.6, 0.7\rangle$	$\langle 0.3, 0.5, 0.9\rangle$
$\widetilde{\mathbb{N}}_{\Gamma_\beta(x_3)}$	$\langle 0.2, 0.5, 0.8\rangle$	$\langle 0.5, 0.3, 0.8\rangle$	$\langle 0.6, 0.3, 0.6\rangle$	$\langle 0.5, 0.6, 0.7\rangle$	$\langle 0.7, 0.3, 0.5\rangle$
$\widetilde{\mathbb{N}}_{\Gamma_\beta(x_4)}$	$\langle 0.6, 0.3, 0.5\rangle$	$\langle 0.5, 0.3, 0.8\rangle$	$\langle 0.4, 0.6, 0.6\rangle$	$\langle 0.4, 0.2, 0.7\rangle$	$\langle 0.3, 0.5, 0.9\rangle$
$\widetilde{\mathbb{N}}_{\Gamma_\beta(x_5)}$	$\langle 0.2, 0.5, 0.8\rangle$	$\langle 0.4, 0.6, 0.8\rangle$	$\langle 0.5, 0.3, 0.7\rangle$	$\langle 0.3, 0.6, 0.7\rangle$	$\langle 0.6, 0.3, 0.6\rangle$

Then some properties of the SVN β-neighborhood of x about Γ_β are presented.

Proposition 3.4. *Let (U, Γ_β) be a n-SVN β-covering approximation space. Then the following statements hold:*

(1) $\forall x \in U$, $\widetilde{\mathbb{N}}_{\Gamma_\beta(x)}(x) \geq \beta$;

(2) $\forall x, y, z \in U$, *if* $\widetilde{\mathbb{N}}_{\Gamma_\beta(x)}(y) \geq \beta$, $\widetilde{\mathbb{N}}_{\Gamma_\beta(y)}(z) \geq \beta$, *then* $\widetilde{\mathbb{N}}_{\Gamma_\beta(x)}(z) \geq \beta$;

(3) *For two SVN numbers β_1 and β_2, if $\beta_1 \leq \beta_2 \leq \beta$, then $\widetilde{\mathbb{N}}_{\Gamma_{\beta_1}(x)} \subseteq \widetilde{\mathbb{N}}_{\Gamma_{\beta_2}(x)}$ for all $x \in U$;*

(4) $\forall x, y \in U$, $\widetilde{\mathbb{N}}_{\Gamma_\beta(x)}(y) \geq \beta$ *if and only if* $\widetilde{\mathbb{N}}_{\Gamma_\beta(y)} \subseteq \widetilde{\mathbb{N}}_{\Gamma_\beta(x)}$.

According to Propositions 3.5 and 3.7, it is easily seen that $\widetilde{\mathbb{N}}_{\Gamma_\beta(x)}$ is a SVN β-neighborhood of x about the β-covering $Cov(\Gamma_\beta)$. Hence, Proposition 3.4 follows immediately from Section 3 in [20]. So, its proof can be omitted.

In Definition 3.2, $Cov(\widehat{\mathbf{C}})$ is also a SVN β-covering of U for any $\widehat{\mathbf{C}} \in \Gamma_\beta$. And another SVN β-covering of U can be induced by Γ_β.

Proposition 3.5. *Let (U, Γ_β) be a n-SVN β-covering approximation space and $Cov(\Gamma_\beta) = \{\widetilde{\mathbb{N}}_{\Gamma_\beta(x)} : x \in U\}$. Then $Cov(\Gamma_\beta)$ is a SVN β-covering of U. We call $Cov(\Gamma_\beta)$ as the induced SVN β-covering of Γ_β.*

Proof. By (1) in Proposition 3.4, for all $x \in U$, there exists $\widetilde{\mathbb{N}}_{\Gamma_\beta(x)} \in Cov(\Gamma_\beta)$ such that $\widetilde{\mathbb{N}}_{\Gamma_\beta(x)}(x) \geq \beta$. Hence, $Cov(\Gamma_\beta)$ is a SVN β-covering of U. □

Example 3.6. *(Continued from Example 3.3) All SVN β-neighborhoods induced by Γ_β are shown in Table 5 in Example 3.3. By Table 5, we know $Cov(\Gamma_\beta)$ is a SVN β-covering of U.*

Finally, the SVN β-neighborhood of x about Γ_β can be represented by the SVN β-neighborhood of x about $Cov(\Gamma_\beta)$.

Proposition 3.7. *Let (U, Γ_β) be a n-SVN β-covering approximation space and $Cov(\Gamma_\beta)$ be the induced SVN β-covering of Γ_β. Then for any $x \in U$, $\widetilde{\mathbb{N}}^\beta_{(Cov(\Gamma_\beta))(x)} = \widetilde{\mathbb{N}}_{\Gamma_\beta(x)}$.*

Proof. Since $Cov(\Gamma_\beta) = \{\widetilde{\mathbb{N}}_{\Gamma_\beta(x)} : x \in U\}$ is the induced SVN β-covering of Γ_β, $\widetilde{\mathbb{N}}^\beta_{(Cov(\Gamma_\beta))(x)} = \bigcap\{\widetilde{\mathbb{N}}_{\Gamma_\beta(x')} \in Cov(\Gamma_\beta) : x' \in U \wedge \widetilde{\mathbb{N}}_{\Gamma_\beta(x')}(x) \geq \beta\}$. By (1) in Proposition 3.4, for all $x \in U$, $\widetilde{\mathbb{N}}_{\Gamma_\beta(x)}(x) \geq \beta$. Hence, $\widetilde{\mathbb{N}}^\beta_{(Cov(\Gamma_\beta))(x)} \subseteq \widetilde{\mathbb{N}}_{\Gamma_\beta(x)}$. According to (4) in Proposition 3.4, for any $x' \in U - \{x\}$, if $\widetilde{\mathbb{N}}_{\Gamma_\beta(x')}(x) \geq \beta$, then $\widetilde{\mathbb{N}}_{\Gamma_\beta(x)} \subseteq \widetilde{\mathbb{N}}_{\Gamma_\beta(x')}$. Therefore, for any $x \in U$, $\widetilde{\mathbb{N}}^\beta_{(Cov(\Gamma_\beta))(x)} = \widetilde{\mathbb{N}}_{\Gamma_\beta(x)}$. □

4 Multigranulation SVN CBRSs

In this section, we present three types of multigranulation SVN CBRS models based on the concept of n-SVN β-covering approximation space. Firstly, a new type of SVN CBRS model based on the concept of SVN β-covering approximation space is proposed and some characterizations of it are investigated. Secondly, inspired by this new rough set model, three types of multigranulation SVN CBRS models are presented in the n-SVN β-covering approximation space. Finally, some properties of them and some relationships among them are investigated.

4.1 A type of SVN CBRS model

A new type of rough set model based on the concept of SVN β-covering approximation space is presented for any SVN subset.

Definition 4.1. *Let* $(U, \widehat{C})$ *be a SVN* β*-covering approximation space with* $U = \{x_1, x_2, \cdots, x_m\}$*. For each* $A \in SVN(U)$*, where* $A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle : 1 \le i \le m\}$*, we define the SVN covering upper approximation* $\overline{\mathbb{C}}(A)$ *and lower approximation* $\underline{\mathbb{C}}(A)$ *of* A *as*

$$\overline{\mathbb{C}}(A) = \{\langle x_i, T_{\overline{\mathbb{C}}(A)}(x_i), I_{\overline{\mathbb{C}}(A)}(x_i), T_{\overline{\mathbb{C}}(A)}(x_i)\rangle : 1 \le i \le m\},\ \underline{\mathbb{C}}(A) = \{\langle x_i, T_{\underline{\mathbb{C}}(A)}(x_i), I_{\underline{\mathbb{C}}(A)}(x_i), T_{\underline{\mathbb{C}}(A)}(x_i)\rangle : 1 \le i \le m\},$$

where

$$T_{\overline{\mathbb{C}}(A)}(x_i) = \bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C}(x_i)}}(x_j) \wedge T_A(x_j)],\quad T_{\underline{\mathbb{C}}(A)}(x_i) = \bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C}(x_i)}}(x_j) \vee T_A(x_j)]\ (1 \le i \le m),$$
$$I_{\overline{\mathbb{C}}(A)}(x_i) = \bigwedge_{j=1}^{m}[I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C}(x_i)}}(x_j) \vee I_A(x_j)],\quad I_{\underline{\mathbb{C}}(A)}(x_i) = \bigvee_{j=1}^{m}[(1 - I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C}(x_i)}}(x_j)) \wedge I_A(x_j)]\ (1 \le i \le m),$$
$$F_{\overline{\mathbb{C}}(A)}(x_i) = \bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C}(x_i)}}(x_j) \vee F_A(x_j)],\quad F_{\underline{\mathbb{C}}(A)}(x_i) = \bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C}(x_i)}}(x_j) \wedge F_A(x_j)]\ (1 \le i \le m).$$

If $\overline{\mathbb{C}}(A) \ne \underline{\mathbb{C}}(A)$, then A is called as a SVN CBRS. $\overline{\mathbb{C}}$ and $\underline{\mathbb{C}}$ are called as SVN covering upper and lower approximation operators about $\widehat{\mathbf{C}}$.

Example 4.2. *(Continued from Example 3.3)* $\widehat{C_1}$ *is a SVN* β*-covering of* U *in Example 3.3, where* $\beta = \langle 0.6, 0.3, 0.7\rangle$*. Table 1 gives* $\widehat{C_1}$*, and all SVN* β*-neighborhoods induced by* $\widehat{C_1}$ *are shown in Table 3 in Example 3.3. Suppose* $A = \{\langle x_1, 0.7, 0.3, 0.5\rangle, \langle x_2, 0.5, 0.4, 0.2\rangle, \langle x_3, 0.3, 0.2, 0.6\rangle, \langle x_4, 0.6, 0.5, 0.4\rangle, \langle x_5, 0.7, 0.3, 0.2\rangle\}$*. Then*

$$\overline{\mathbb{C}}_1(A) = \{\langle x_1, 0.6, 0.3, 0.5\rangle, \langle x_2, 0.5, 0.3, 0.6\rangle, \langle x_3, 0.7, 0.3, 0.5\rangle, \langle x_4, 0.7, 0.3, 0.4\rangle, \langle x_5, 0.6, 0.3, 0.5\rangle\},$$
$$\underline{\mathbb{C}}_1(A) = \{\langle x_1, 0.6, 0.5, 0.5\rangle, \langle x_2, 0.4, 0.4, 0.4\rangle, \langle x_3, 0.6, 0.4, 0.6\rangle, \langle x_4, 0.3, 0.5, 0.5\rangle, \langle x_5, 0.6, 0.4, 0.5\rangle\},$$

where $\overline{\mathbb{C}}_1$ *and* $\underline{\mathbb{C}}_1$ *are SVN covering upper and lower approximation operators about* $\widehat{C_1}$*.*

Let the SVN universe set be $U = \{\langle x, 1, 0, 0\rangle : x \in U\}$ and the SVN empty set be $\emptyset = \{\langle x, 0, 1, 1\rangle : x \in U\}$. Some basic properties of the SVN covering upper and lower approximation operators are given in the following proposition.

Proposition 4.3. *Let* $\widehat{C}$ *be a SVN* β*-covering of* $U = \{x_1, x_2, \cdots, x_m\}$*. Then for any* $A, B \in SVN(U)$*, the following statements hold:*
(1) $\overline{\mathbb{C}}(\emptyset) = \emptyset$; $\underline{\mathbb{C}}(U) = U$;
(2) $\overline{\mathbb{C}}(A') = (\underline{\mathbb{C}}(A))'$, $\underline{\mathbb{C}}(A') = (\overline{\mathbb{C}}(A))'$;
(3) *If* $A \subseteq B$*, then* $\overline{\mathbb{C}}(A) \subseteq \overline{\mathbb{C}}(B)$, $\underline{\mathbb{C}}(A) \subseteq \underline{\mathbb{C}}(B)$;
(4) $\overline{\mathbb{C}}(A \bigcup B) = \overline{\mathbb{C}}(A) \bigcup \overline{\mathbb{C}}(B)$, $\underline{\mathbb{C}}(A \bigcap B) = \underline{\mathbb{C}}(A) \bigcap \underline{\mathbb{C}}(B)$;
(5) $\overline{\mathbb{C}}(A \bigcap B) \subseteq \overline{\mathbb{C}}(A) \bigcap \overline{\mathbb{C}}(B)$, $\underline{\mathbb{C}}(A \bigcup B) \supseteq \underline{\mathbb{C}}(A) \bigcup \underline{\mathbb{C}}(B)$.

Proof. Since (3) and (5) are following immediately from Definition 4.1, we only prove (1), (2) and (4).
(1) Since the SVN universe set is $U = \{\langle x_i, 1, 0, 0\rangle : 1 \le i \le m\}$ and the SVN empty set is $\emptyset = \{\langle x_i, 0, 1, 1\rangle : 1 \le i \le m\}$, for any i $(1 \le i \le m)$,

$$T_{\underline{\mathbb{C}}(U)}(x_i) = \bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee T_U(x_j)] = \bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee 1] = 1,$$
$$I_{\underline{\mathbb{C}}(U)}(x_i) = \bigvee_{j=1}^{m}[(1 - I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j)) \wedge I_U(x_j)] = \bigvee_{j=1}^{m}[(1 - I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j)) \wedge 0] = 0,$$
$$F_{\underline{\mathbb{C}}(U)}(x_i) = \bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \wedge F_U(x_j)] = \bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \wedge 0] = 0,$$
$$T_{\overline{\mathbb{C}}(\emptyset)}(x_i) = \bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \wedge T_\emptyset(x_j)] = \bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \wedge 0] = 0,$$
$$I_{\overline{\mathbb{C}}(\emptyset)}(x_i) = \bigwedge_{j=1}^{m}[I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee I_\emptyset(x_j)] = \bigwedge_{j=1}^{m}[I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee 1] = 1,$$
$$F_{\overline{\mathbb{C}}(\emptyset)}(x_i) = \bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee F_\emptyset(x_j)] = \bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee 1] = 1.$$

Hence, $\underline{\mathbb{C}}(U) = U$ and $\overline{\mathbb{C}}(\emptyset) = \emptyset$;
(2) Let $A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle : 1 \le i \le m\}$. Then $A' = \{\langle x_i, F_A(x_i), 1 - I_A(x_i), T_A(x_i)\rangle : 1 \le i \le m\}$. For

any i $(1 \leq i \leq m)$,

$$\begin{aligned}
T_{\overline{\mathbb{C}}(A')}(x_i) &= \bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \wedge T_{A'}(x_j)] = \bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \wedge F_A(x_j)] = F_{\underline{\mathbb{C}}(A)}(x_i),\\
I_{\overline{\mathbb{C}}(A')}(x_i) &= \bigwedge_{j=1}^{m}[I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee I_{A'}(x_j)] = 1-\bigvee_{j=1}^{m}[(1-I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j)) \wedge I_A(x_j)] = 1-I_{\underline{\mathbb{C}}(A)}(x_i),\\
F_{\overline{\mathbb{C}}(A')}(x_i) &= \bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee F_{A'}(x_j)] = \bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee T_A(x_j)] = T_{\underline{\mathbb{C}}(A)}(x_i),\\
T_{\underline{\mathbb{C}}(A')}(x_i) &= \bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee T_{A'}(x_j)] = \bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee F_A(x_j)] = F_{\overline{\mathbb{C}}(A)}(x_i),\\
I_{\underline{\mathbb{C}}(A')}(x_i) &= \bigvee_{j=1}^{m}[(1-I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j)) \wedge I_{A'}(x_j)] = \bigvee_{j=1}^{m}[(1-I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j)) \wedge (1-I_A(x_j))] = 1-I_{\overline{\mathbb{C}}(A)}(x_i),\\
F_{\underline{\mathbb{C}}(A')}(x_i) &= \bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \wedge F_{A'}(x_j)] = \bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \wedge T_A(x_j)] = T_{\overline{\mathbb{C}}(A)}(x_i).
\end{aligned}$$

Hence,

$$\begin{aligned}
\overline{\mathbb{C}}(A') &= \{\langle T_{\overline{\mathbb{C}}(A')}(x_i), I_{\overline{\mathbb{C}}(A')}(x_i), F_{\overline{\mathbb{C}}(A')}(x_i)\rangle : 1 \leq i \leq m\} = \{\langle F_{\underline{\mathbb{C}}(A)}(x_i), 1-I_{\underline{\mathbb{C}}(A)}(x_i), T_{\underline{\mathbb{C}}(A)}(x_i)\rangle : 1 \leq i \leq m\}\\
&= \{\langle T_{(\underline{\mathbb{C}}(A))'}(x_i), I_{(\underline{\mathbb{C}}(A))'}(x_i), F_{(\underline{\mathbb{C}}(A))'}(x_i)\rangle : 1 \leq i \leq m\} = (\overline{\mathbb{C}}(A))',\\
\underline{\mathbb{C}}(A') &= \{\langle T_{\underline{\mathbb{C}}(A')}(x_i), I_{\underline{\mathbb{C}}(A')}(x_i), F_{\underline{\mathbb{C}}(A')}(x_i)\rangle : 1 \leq i \leq m\} = \{\langle F_{\overline{\mathbb{C}}(A)}(x_i), 1-I_{\overline{\mathbb{C}}(A)}(x_i), F_{\overline{\mathbb{C}}(A)}(x_i)\rangle : 1 \leq i \leq m\}\\
&= \{\langle T_{(\overline{\mathbb{C}}(A))'}(x_i), I_{(\overline{\mathbb{C}}(A))'}(x_i), F_{(\overline{\mathbb{C}}(A))'}(x_i)\rangle : 1 \leq i \leq m\} = (\overline{\mathbb{C}}(A))';
\end{aligned}$$

(4) Since

$$\begin{aligned}
T_{\underline{\mathbb{C}}(A\bigcap B)}(x_i) &= \bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee T_{A\bigcap B}(x_j)] = \bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee (T_A(x_j) \wedge T_B(x_j))]\\
&= \bigwedge_{j=1}^{m}[(F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee T_A(x_j)) \wedge (F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee T_B(x_j))]\\
&= [\bigwedge_{j=1}^{m}(F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee T_A(x_j))] \wedge [\bigwedge_{j=1}^{m}(F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \vee T_B(x_j))] = T_{\underline{\mathbb{C}}(A)}(x_i) \wedge T_{\underline{\mathbb{C}}(B)}(x_i),\\
I_{\underline{\mathbb{C}}(A\bigcap B)}(x_i) &= \bigvee_{j=1}^{m}[(1-I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j)) \wedge I_{A\bigcap B}(x_j)] = \bigvee_{j=1}^{m}[(1-I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j)) \wedge (I_A(x_j) \vee I_B(x_j))]\\
&= [\bigvee_{j=1}^{m}(1-I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j)) \wedge I_A(x_j)] \vee [\bigvee_{j=1}^{m}(1-I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j)) \wedge I_B(x_j)] = I_{\underline{\mathbb{C}}(A)}(x_i) \vee I_{\underline{\mathbb{C}}(B)}(x_i),\\
F_{\underline{\mathbb{C}}(A\bigcap B)}(x_i) &= \bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \wedge F_{A\bigcap B}(x_j)] = \bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \wedge (F_A(x_j) \vee F_B(x_j))]\\
&= [\bigvee_{j=1}^{m}T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \wedge F_A(x_j)] \vee [\bigvee_{j=1}^{m}T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}(x_i)}}(x_j) \wedge F_B(x_j)] = F_{\underline{\mathbb{C}}(A)}(x_i) \vee F_{\underline{\mathbb{C}}(B)}(x_i).
\end{aligned}$$

Hence, $\underline{\mathbb{C}}(A\bigcap B) = \underline{\mathbb{C}}(A)\bigcap\underline{\mathbb{C}}(B)$. Similarly, we can obtain $\overline{\mathbb{C}}(A\bigcup B) = \overline{\mathbb{C}}(A)\bigcup\overline{\mathbb{C}}(B)$. □

4.2 Three types of multigranulation SVN CBRS models

In this subsection, three types of (philosophical, optimistic and pessimistic) multigranulation SVN CBRS models are presented. Firstly, the type of philosophical multigranulation SVN CBRS model is shown.

Definition 4.4. *Let (U, Γ_β) be a n-SVN β-covering approximation space with $U = \{x_1, x_2, \cdots, x_m\}$. For each $A \in SVN(U)$, where $A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle : 1 \leq i \leq m\}$, the philosophical multigranulation SVN covering upper approximation $\overline{\mathfrak{N}_{\Gamma_\beta}}(A)$ and lower approximation $\underline{\mathfrak{N}_{\Gamma_\beta}}(A)$ of A are defined as:*

$$\begin{aligned}
\overline{\mathfrak{N}_{\Gamma_\beta}}(A) &= \{\langle x_i, T_{\overline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i), I_{\overline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i), F_{\overline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i)\rangle : 1 \leq i \leq m\},\\
\underline{\mathfrak{N}_{\Gamma_\beta}}(A) &= \{\langle x_i, T_{\underline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i), I_{\underline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i), F_{\underline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i),\rangle : 1 \leq i \leq m\},
\end{aligned}$$

where

$$T_{\overline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i) = \bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}_{\Gamma_\beta(x_i)}}(x_j) \wedge T_A(x_j)], \quad T_{\underline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i) = \bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}_{\Gamma_\beta(x_i)}}(x_j) \vee T_A(x_j)] \ (1 \le i \le m),$$
$$I_{\overline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i) = \bigwedge_{j=1}^{m}[I_{\widetilde{\mathbb{N}}_{\Gamma_\beta(x_i)}}(x_j) \vee I_A(x_j)], \quad I_{\underline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i) = \bigvee_{j=1}^{m}[(1 - I_{\widetilde{\mathbb{N}}_{\Gamma_\beta(x_i)}}(x_j)) \wedge I_A(x_j)] \ (1 \le i \le m),$$
$$F_{\overline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i) = \bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}_{\Gamma_\beta(x_i)}}(x_j) \vee F_A(x_j)], \quad F_{\underline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i) = \bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}_{\Gamma_\beta(x_i)}}(x_j) \wedge F_A(x_j)] \ (1 \le i \le m).$$

If $\overline{\mathfrak{N}_{\Gamma_\beta}}(A) \ne \underline{\mathfrak{N}_{\Gamma_\beta}}(A)$, then A is called as a philosophical multigranulation SVN CBRS.

Example 4.5. *(Continued from Example 3.3) All SVN β-neighborhoods induced by Γ_β are shown in Table 5 in Example 3.3. For $A = \{\langle x_1, 0.7, 0.3, 0.5\rangle, \langle x_2, 0.5, 0.4, 0.2\rangle, \langle x_3, 0.3, 0.2, 0.6\rangle, \langle x_4, 0.6, 0.5, 0.4\rangle, \langle x_5, 0.7, 0.3, 0.2\rangle\}$, the philosophical multigranulation SVN covering upper approximation $\overline{\mathfrak{N}_{\Gamma_\beta}}(A)$ and lower approximation $\underline{\mathfrak{N}_{\Gamma_\beta}}(A)$ of A are calculated as follows:*

$$\overline{\mathfrak{N}_{\Gamma_\beta}}(A) = \{\langle x_1, 0.6, 0.3, 0.5\rangle, \langle x_2, 0.5, 0.4, 0.6\rangle, \langle x_3, 0.7, 0.3, 0.5\rangle, \langle x_4, 0.6, 0.3, 0.5\rangle, \langle x_5, 0.6, 0.3, 0.6\rangle\},$$
$$\underline{\mathfrak{N}_{\Gamma_\beta}}(A) = \{\langle x_1, 0.7, 0.4, 0.5\rangle, \langle x_2, 0.4, 0.4, 0.4\rangle, \langle x_3, 0.6, 0.4, 0.6\rangle, \langle x_4, 0.6, 0.5, 0.5\rangle, \langle x_5, 0.7, 0.4, 0.5\rangle\}.$$

The philosophical multigranulation SVN covering approximation operators in Definition 4.4 can be represented by the SVN covering approximation operators in Definition 4.1.

Proposition 4.6. *Let (U, Γ_β) be a n-SVN β-covering approximation space. Then for each $A \in SVN(U)$,*

$$\overline{\mathfrak{N}_{\Gamma_\beta}}(A) = \overline{Cov(\Gamma_\beta)}(A), \ \underline{\mathfrak{N}_{\Gamma_\beta}}(A) = \underline{Cov(\Gamma_\beta)}(A),$$

where $\overline{Cov(\Gamma_\beta)}$ and $\underline{Cov(\Gamma_\beta)}$ are SVN covering upper and lower approximation operators about $Cov(\Gamma_\beta)$.

Proof. By Proposition 3.5, $Cov(\Gamma_\beta) = \{\widetilde{\mathbb{N}}_{\Gamma_\beta(x)} : x \in U\}$ is the induced SVN β-covering of Γ_β. According to Proposition 3.7, for any $x \in U$, $\widetilde{\mathbb{N}}^{\beta}_{(Cov(\Gamma_\beta))(x)} = \widetilde{\mathbb{N}}_{\Gamma_\beta(x)}$. Hence, $\overline{\mathfrak{N}_{\Gamma_\beta}}(A) = \overline{Cov(\Gamma_\beta)}(A)$ and $\underline{\mathfrak{N}_{\Gamma_\beta}}(A) = \underline{Cov(\Gamma_\beta)}(A)$. □

Some basic properties of the philosophical multigranulation SVN covering upper and lower approximation operators are given in the following proposition.

Proposition 4.7. *Let (U, Γ_β) be a n-SVN β-covering approximation space. Then for all $A, B \in SVN(U)$, the following statements hold:*
(1) $\underline{\mathfrak{N}_{\Gamma_\beta}}(U) = U$, $\overline{\mathfrak{N}_{\Gamma_\beta}}(\emptyset) = \emptyset$;
(2) $\overline{\mathfrak{N}_{\Gamma_\beta}}(A') = (\underline{\mathfrak{N}_{\Gamma_\beta}}(A))'$, $\underline{\mathfrak{N}_{\Gamma_\beta}}(A') = (\overline{\mathfrak{N}_{\Gamma_\beta}}(A))'$;
(3) *If $A \subseteq B$, then* $\underline{\mathfrak{N}_{\Gamma_\beta}}(A) \subseteq \underline{\mathfrak{N}_{\Gamma_\beta}}(B)$, $\overline{\mathfrak{N}_{\Gamma_\beta}}(A) \subseteq \overline{\mathfrak{N}_{\Gamma_\beta}}(B)$;
(4) $\underline{\mathfrak{N}_{\Gamma_\beta}}(A \bigcap B) = \underline{\mathfrak{N}_{\Gamma_\beta}}(A) \bigcap \underline{\mathfrak{N}_{\Gamma_\beta}}(B)$, $\overline{\mathfrak{N}_{\Gamma_\beta}}(A \bigcup B) = \overline{\mathfrak{N}_{\Gamma_\beta}}(A) \bigcup \overline{\mathfrak{N}_{\Gamma_\beta}}(B)$;
(5) $\underline{\mathfrak{N}_{\Gamma_\beta}}(A \bigcup B) \supseteq \underline{\mathfrak{N}_{\Gamma_\beta}}(A) \bigcup \underline{\mathfrak{N}_{\Gamma_\beta}}(B)$, $\overline{\mathfrak{N}_{\Gamma_\beta}}(A \bigcap B) \subseteq \overline{\mathfrak{N}_{\Gamma_\beta}}(A) \bigcap \overline{\mathfrak{N}_{\Gamma_\beta}}(B)$.

Proof. According to the proof of Proposition 4.6, it is straightforward. □

Then a type of optimistic multigranulation SVN CBRS model is proposed.

Definition 4.8. *Let (U, Γ_β) be a n-SVN β-covering approximation space with $U = \{x_1, x_2, \cdots, x_m\}$ and $\Gamma_\beta = \{\widehat{C}_1, \widehat{C}_2, \cdots, \widehat{C}_n\}$. For each $A \in SVN(U)$, where $A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle : 1 \le i \le m\}$, the optimistic multigranulation SVN covering upper approximation $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)$ and lower approximation $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)$ of A are defined as:*

$$\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A) = \{\langle x_i, T_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i), I_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i), F_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i)\rangle : 1 \le i \le m\},$$
$$\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A) = \{\langle x_i, T_{\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i), I_{\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i), F_{\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i), \rangle : 1 \le i \le m\},$$

where

$$T_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i) = \bigwedge_{k=1}^{n}\bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \wedge T_A(x_j)], \quad T_{\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i) = \bigvee_{k=1}^{n}\bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \vee T_A(x_j)] \ (1 \le i \le m),$$
$$I_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i) = \bigvee_{k=1}^{n}\bigwedge_{j=1}^{m}[I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \vee I_A(x_j)], \quad I_{\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i) = \bigwedge_{k=1}^{n}\bigvee_{j=1}^{m}[(1 - I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j)) \wedge I_A(x_j)] \ (1 \le i \le m),$$
$$F_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i) = \bigvee_{k=1}^{n}\bigwedge_{j=1}^{m}[F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \vee F_A(x_j)], \quad F_{\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i) = \bigwedge_{k=1}^{n}\bigvee_{j=1}^{m}[T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \wedge F_A(x_j)] \ (1 \le i \le m).$$

If $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A) \neq \underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)$, then A is called as an optimistic multigranulation SVN CBRS.

Example 4.9. *(Continued from Example 3.3) Suppose* $A = \{\langle x_1, 0.7, 0.3, 0.5\rangle, \langle x_2, 0.5, 0.4, 0.2\rangle, \langle x_3, 0.3, 0.2, 0.6\rangle, \langle x_4, 0.6, 0.5, 0.4\rangle, \langle x_5, 0.7, 0.3, 0.2\rangle\}$*. Then based on Tables 3 and 4, the optimistic multigranulation SVN covering upper approximation* $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)$ *and lower approximation* $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)$ *of* A *defined in Definition 4.8 are calculated as follows:*

$$\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A) = \{\langle x_1, 0.6, 0.3, 0.5\rangle, \langle x_2, 0.5, 0.4, 0.6\rangle, \langle x_3, 0.7, 0.3, 0.5\rangle, \langle x_4, 0.6, 0.3, 0.5\rangle, \langle x_5, 0.6, 0.3, 0.6\rangle\},$$
$$\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A) = \{\langle x_1, 0.6, 0.4, 0.5\rangle, \langle x_2, 0.4, 0.4, 0.4\rangle, \langle x_3, 0.6, 0.4, 0.6\rangle, \langle x_4, 0.6, 0.5, 0.5\rangle, \langle x_5, 0.7, 0.4, 0.5\rangle\}.$$

The optimistic multigranulation SVN covering approximation operators in Definition 4.8 can be represented by the SVN covering approximation operators in Definition 4.1.

Proposition 4.10. *Let* (U, Γ_β) *be a n-SVN* β*-covering approximation space with* $U = \{x_1, x_2, \cdots, x_m\}$ *and* $\Gamma_\beta = \{\widehat{C_1}, \widehat{C_2}, \cdots, \widehat{C_n}\}$*. Then for each* $A \in SVN(U)$, $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A) = \bigcap_{k=1}^{n} \overline{\mathbb{C}}_k(A)$ *and* $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A) = \bigcup_{k=1}^{n} \underline{\mathbb{C}}_k(A)$*, where* $\overline{\mathbb{C}}_k$ *and* $\underline{\mathbb{C}}_k$ *are SVN covering upper and lower approximation operators about* $\widehat{C_k}$ $(1 \leq k \leq n)$*.*

Proof. By Definitions 4.1 and 4.8, it is straightforward. □

Some basic properties of the optimistic multigranulation SVN covering upper and lower approximation operators are given in the following proposition.

Proposition 4.11. *Let* (U, Γ_β) *be a n-SVN* β*-covering approximation space with* $U = \{x_1, x_2, \cdots, x_m\}$ *and* $\Gamma_\beta = \{\widehat{C_1}, \widehat{C_2}, \cdots, \widehat{C_n}\}$*. Then the optimistic multigranulation SVN covering approximation operators in Definition 4.8 satisfy the following properties: for all* $A, B \in SVN(U)$*,*

(1) $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A') = (\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A))'$, $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A') = (\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A))'$;

(2) *If* $A \subseteq B$*, then* $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A) \subseteq \overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(B)$, $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A) \subseteq \underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(B)$;

(3) $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A \bigcup B) = \overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A) \bigcup \overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(B)$, $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A \bigcap B) = \underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A) \bigcap \underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(B)$;

(4) $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A \bigcap B) \subseteq \overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A) \bigcap \overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(B)$, $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A \bigcup B) \supseteq \underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A) \bigcup \underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(B)$.

Proof. By Propositions 4.3 and 4.10, it is straightforward. □

Finally, a type of pessimistic multigranulation SVN CBRS model is proposed.

Definition 4.12. *Let* (U, Γ_β) *be a n-SVN* β*-covering approximation space with* $U = \{x_1, x_2, \cdots, x_m\}$ *and* $\Gamma_\beta = \{\widehat{C_1}, \widehat{C_2}, \cdots, \widehat{C_n}\}$*. For each* $A \in SVN(U)$*, where* $A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle : 1 \leq i \leq m\}$*, the pessimistic multigranulation SVN covering upper approximation* $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)$ *and lower approximation* $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)$ *of* A *are defined as:*

$$\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A) = \{\langle x_i, T_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)}(x_i), I_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)}(x_i), F_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)}(x_i)\rangle : 1 \leq i \leq m\},$$
$$\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A) = \{\langle x_i, T_{\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)}(x_i), I_{\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)}(x_i), F_{\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)}(x_i),\rangle : 1 \leq i \leq m\},$$

where

$$T_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)}(x_i) = \bigvee_{k=1}^{n} \bigvee_{j=1}^{m} [T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \wedge T_A(x_j)], \quad T_{\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)}(x_i) = \bigwedge_{k=1}^{n} \bigwedge_{j=1}^{m} [F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \vee T_A(x_j)] \ (1 \leq i \leq m),$$
$$I_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)}(x_i) = \bigwedge_{k=1}^{n} \bigwedge_{j=1}^{m} [I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \vee I_A(x_j)], \quad I_{\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)}(x_i) = \bigvee_{k=1}^{n} \bigvee_{j=1}^{m} [(1 - I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j)) \wedge I_A(x_j)] \ (1 \leq i \leq m),$$
$$F_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)}(x_i) = \bigwedge_{k=1}^{n} \bigwedge_{j=1}^{m} [F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \vee F_A(x_j)], \quad F_{\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)}(x_i) = \bigvee_{k=1}^{n} \bigvee_{j=1}^{m} [T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \wedge F_A(x_j)] \ (1 \leq i \leq m).$$

If $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A) \neq \underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)$, then A is called as a pessimistic multigranulation SVN CBRS.

Example 4.13. *(Continued from Example 4.9) Based on Tables 3 and 4, the pessimistic multigranulation SVN covering upper approximation* $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)$ *and lower approximation* $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)$ *of* A *defined in Definition 4.12 are calculated as follows:*

$$\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A) = \{\langle x_1, 0.6, 0.3, 0.5\rangle, \langle x_2, 0.5, 0.3, 0.5\rangle, \langle x_3, 0.7, 0.3, 0.4\rangle, \langle x_4, 0.7, 0.3, 0.4\rangle, \langle x_5, 0.6, 0.3, 0.5\rangle\},$$
$$\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A) = \{\langle x_1, 0.6, 0.5, 0.5\rangle, \langle x_2, 0.4, 0.4, 0.6\rangle, \langle x_3, 0.6, 0.4, 0.6\rangle, \langle x_4, 0.3, 0.5, 0.6\rangle, \langle x_5, 0.6, 0.4, 0.5\rangle\}.$$

Inspired by Proposition 4.10, the pessimistic multigranulation SVN covering approximation operators in Definition 4.12 can be represented by SVN covering approximation operators in Definition 4.1.

Proposition 4.14. *Let $U = \{x_1, x_2, \cdots, x_m\}$ be a universe and (U, Γ_β) be a n-SVN β-covering approximation space with $\Gamma_\beta = \{\widehat{C_1}, \widehat{C_2}, \cdots, \widehat{C_n}\}$. Then for each $A \in SVN(U)$, $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A) = \bigcup_{k=1}^{n} \overline{\mathbb{C}}_k(A)$ and $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A) = \bigcap_{k=1}^{n} \underline{\mathbb{C}}_k(A)$.*

Proof. By Definitions 4.1 and 4.12, it is straightforward. □

Some basic properties of the pessimistic multigranulation SVN covering approximation operators are given in the following proposition.

Proposition 4.15. *Let $U = \{x_1, x_2, \cdots, x_m\}$ be a universe and (U, Γ_β) be a n-SVN β-covering approximation space with $\Gamma_\beta = \{\widehat{C_1}, \widehat{C_2}, \cdots, \widehat{C_n}\}$. Then the pessimistic multigranulation SVN covering approximation operators in Definition 4.12 satisfy the following properties: for all $A, B \in SVN(U)$,*

(1) $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A') = (\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A))'$, $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A') = (\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A))'$;

(2) *If $A \subseteq B$, then* $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A) \subseteq \overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(B)$, $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A) \subseteq \underline{\Gamma_\beta}^{(p)}(B)$;

(3) $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A \bigcup B) = \overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A) \bigcup \overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(B)$, $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A \bigcap B) = \underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A) \bigcap \underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(B)$;

(4) $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A \bigcap B) \subseteq \overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A) \bigcap \overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(B)$, $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A \bigcup B) \supseteq \underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A) \bigcup \underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(B)$.

Proof. By Propositions 4.3 and 4.14, it is straightforward. □

For these three multigranulation SVN covering upper approximation operators, we find that the philosophical multigranulation SVN covering upper approximation operator and the optimistic multigranulation SVN covering upper approximation operator are the same.

Proposition 4.16. *Let $U = \{x_1, x_2, \cdots, x_m\}$ be a universe and (U, Γ_β) be a n-SVN β-covering approximation space with $\Gamma_\beta = \{\widehat{C_1}, \widehat{C_2}, \cdots, \widehat{C_n}\}$. Then for each $A \in SVN(U)$, $\overline{\mathfrak{N}_{\Gamma_\beta}}(A) = \overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)$.*

Proof. For any $x_i \in U$,

$$
\begin{aligned}
T_{\overline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i) &= \bigvee_{j=1}^{m} [T_{\widetilde{\mathbb{N}}_{\Gamma_\beta(x_i)}}(x_j) \wedge T_A(x_j)] = \bigvee_{j=1}^{m} [(\bigwedge_{k=1}^{n} T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j)) \wedge T_A(x_j)] \\
&= \bigvee_{j=1}^{m} [\bigwedge_{k=1}^{n} (T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \wedge T_A(x_j))] = \bigvee_{j=1}^{m} \bigwedge_{k=1}^{n} [T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \wedge T_A(x_j)] \\
&= \bigwedge_{k=1}^{n} \bigvee_{j=1}^{m} [T_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \wedge T_A(x_j)] = T_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i), \\
I_{\overline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i) &= \bigwedge_{j=1}^{m} [I_{\widetilde{\mathbb{N}}_{\Gamma_\beta(x_i)}}(x_j) \vee I_A(x_j)] = \bigwedge_{j=1}^{m} [(\bigvee_{k=1}^{n} I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j)) \vee I_A(x_j)] \\
&= \bigwedge_{j=1}^{m} \bigvee_{k=1}^{n} [I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \vee I_A(x_j)] = \bigvee_{k=1}^{n} \bigwedge_{j=1}^{m} [I_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \vee I_A(x_j)] = I_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i), \\
F_{\overline{\mathfrak{N}_{\Gamma_\beta}}(A)}(x_i) &= \bigwedge_{j=1}^{m} [F_{\widetilde{\mathbb{N}}_{\Gamma_\beta(x_i)}}(x_j) \vee F_A(x_j)] = \bigwedge_{j=1}^{m} [(\bigvee_{k=1}^{n} F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j)) \vee F_A(x_j)] \\
&= \bigwedge_{j=1}^{m} \bigvee_{k=1}^{n} [F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \vee F_A(x_j)] = \bigvee_{k=1}^{n} \bigwedge_{j=1}^{m} [F_{\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}}(x_j) \vee F_A(x_j)] = F_{\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)}(x_i).
\end{aligned}
$$

□

According to Propositions 4.6, 4.10, 4.14 and 4.16, we can show all relationships among SVN covering approximation operators in Figure 1. It shows that multigranulation SVN CBRS models are extended from a SVN CBRS model after generalizing a SVN β-covering approximation space to a multigranulation SVN β-covering approximation space.

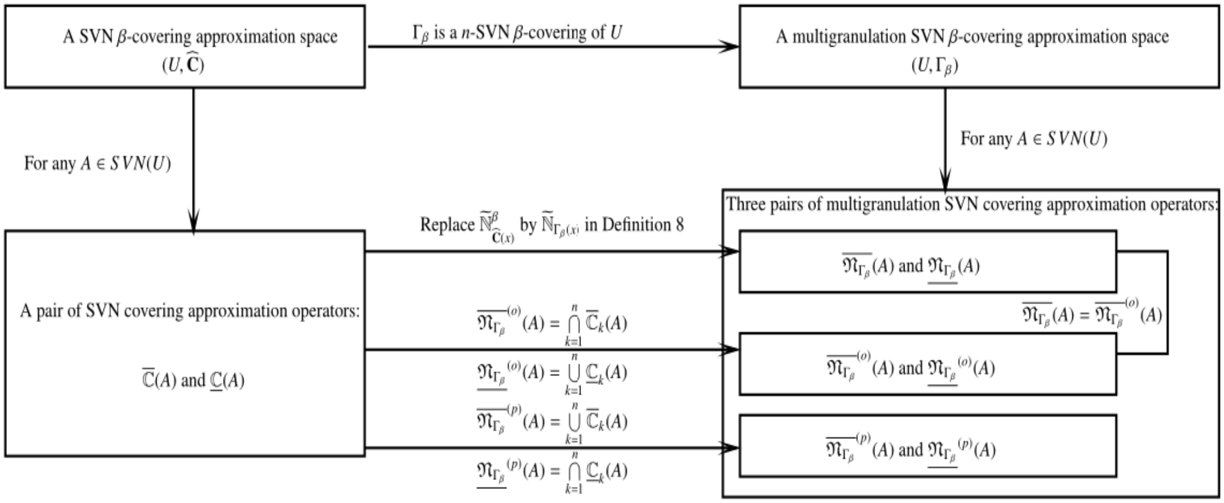

Figure 1: The relationships among SVN covering approximation operators.

5 Conditions for different n-SVN β-coverings produce the same multigranulation SVN covering approximation operators

There are three pairs of multigranulation SVN covering approximation operators in Section 4. In this section, we present some conditions under which different n-SVN β-coverings can induce the same multigranulation SVN covering approximation of any SVN subset. Because multigranulation SVN CBRS models are presented based on SVN CBRS model, we find conditions under which different SVN β-coverings can induce the same SVN covering approximation firstly. Then these conditions are generalized to the statement of multigranulation SVN covering approximation of any SVN subset.

Proposition 5.1. *Let $\widehat{\boldsymbol{C}_1}$, $\widehat{\boldsymbol{C}_2}$ be two SVN β-coverings of U. If $\widetilde{\mathbb{N}}^\beta_{\widehat{\boldsymbol{C}_1}(x)} = \widetilde{\mathbb{N}}^\beta_{\widehat{\boldsymbol{C}_2}(x)}$ for any $x \in U$, then*

$$\overline{\mathbb{C}}_1(A) = \overline{\mathbb{C}}_2(A) \text{ and } \underline{\mathbb{C}}_1(A) = \underline{\mathbb{C}}_2(A) \text{ for any } A \in SVN(U).$$

Proof. According to Definition 4.1, it is straightforward. □

Inspired by Proposition 5.1, we generalize the notion of SVN β-neighborhood to SVN β-neighborhood system.

Definition 5.2. *Let $\widehat{\boldsymbol{C}}$ be a SVN β-covering of U and $\widehat{\boldsymbol{C}} = \{C_1, C_2, \dots, C_l\}$. For each $x \in U$, the SVN β-neighborhood system $\widehat{\mathbb{N}}^\beta_{\widehat{\boldsymbol{C}}}(x)$ of x about $\widehat{\boldsymbol{C}}$ can be defined as:*

$$\widehat{\mathbb{N}}^\beta_{\widehat{\boldsymbol{C}}}(x) = \{C_i \in \widehat{\boldsymbol{C}} : C_i(x) \geq \beta\}.$$

According to Definition 2.3, we know $\widetilde{\mathbb{N}}^\beta_{\widehat{\mathbf{C}}(x)} = \cap \widehat{\mathbb{N}}^\beta_{\widehat{\mathbf{C}}}(x)$ for each $x \in U$. Hence, another condition can be shown as follows.

Proposition 5.3. *Let $\widehat{\boldsymbol{C}_1}$, $\widehat{\boldsymbol{C}_2}$ be two SVN β-coverings of U. If $\widehat{\mathbb{N}}^\beta_{\widehat{\boldsymbol{C}_1}}(x) = \widehat{\mathbb{N}}^\beta_{\widehat{\boldsymbol{C}_2}}(x)$ for any $x \in U$, then*

$$\overline{\mathbb{C}}_1(A) = \overline{\mathbb{C}}_2(A) \text{ and } \underline{\mathbb{C}}_1(A) = \underline{\mathbb{C}}_2(A) \text{ for any } A \in SVN(U).$$

Proof. According to Proposition 5.1 and Definition 5.2, it is straightforward. □

Let $\widehat{\mathbf{C}_1}$, $\widehat{\mathbf{C}_2}$ be two SVN β-coverings of U. If $\widetilde{\mathbb{N}}^\beta_{\widehat{\mathbf{C}_1}}(x) = \widetilde{\mathbb{N}}^\beta_{\widehat{\mathbf{C}_2}}(x)$ for each $x \in U$, then $\widehat{\mathbf{C}_1}$ is not necessarily equal to $\widehat{\mathbf{C}_2}$. The following example is presented to illustrate it.

Example 5.4. *(Continued from Example 3.3) Suppose* $\widehat{C_3} = \widehat{C_1} \cup \{C_{15}\}$, *where* $C_{15} = \frac{(0.2,0.7,0.5)}{x_1} + \frac{(0.7,0.4,0.8)}{x_2} + \frac{(0.3,0.7,0.8)}{x_3} + \frac{(0.6,0.8,0.9)}{x_4} + \frac{(0.4,0.9,1)}{x_5}$. *Then we have*

$$\widehat{\mathbb{N}}^{\beta}_{\widehat{C_1}}(x_1) = \{C_{11}, C_{12}\}, \widehat{\mathbb{N}}^{\beta}_{\widehat{C_1}}(x_2) = \{C_{11}, C_{14}\}, \widehat{\mathbb{N}}^{\beta}_{\widehat{C_1}}(x_3) = \{C_{12}, C_{14}\}, \widehat{\mathbb{N}}^{\beta}_{\widehat{C_1}}(x_4) = \{C_{11}\}, \widehat{\mathbb{N}}^{\beta}_{\widehat{C_1}}(x_5) = \{C_{12}, C_{13}, C_{14}\},$$
$$\widehat{\mathbb{N}}^{\beta}_{\widehat{C_3}}(x_1) = \{C_{11}, C_{12}\}, \widehat{\mathbb{N}}^{\beta}_{\widehat{C_3}}(x_2) = \{C_{11}, C_{14}\}, \widehat{\mathbb{N}}^{\beta}_{\widehat{C_3}}(x_3) = \{C_{12}, C_{14}\}, \widehat{\mathbb{N}}^{\beta}_{\widehat{C_3}}(x_4) = \{C_{11}\}, \widehat{\mathbb{N}}^{\beta}_{\widehat{C_3}}(x_5) = \{C_{12}, C_{13}, C_{14}\}.$$

Hence, $\widehat{\mathbb{N}}^{\beta}_{\widehat{C_1}}(x_i) = \widehat{\mathbb{N}}^{\beta}_{\widehat{C_3}}(x_i)$ *for any* $i \in \{1, 2, 3, 4, 5\}$, *but* $\widehat{C_1} \neq \widehat{C_3}$.

Inspired by Example 5.4, we shall consider under what conditions two SVN β-coverings can induce the same β-neighborhood system for each element in the universe.

Definition 5.5. *Let* $\widehat{C}$ *be a SVN* β*-covering of* U *and* $C \in \widehat{C}$. *We call* C *as a SVN* β*-relevant element of* $\widehat{C}$, *if there exists* $x \in U$ *such that* $C(x) \geq \beta$; *otherwise, we call* C *as a SVN* β*-irrelevant element of* $\widehat{C}$. *If all elements in* $\widehat{C}$ *are SVN* β*-relevant elements, then* $\widehat{C}$ *is SVN* β*-relevant; otherwise,* $\widehat{C}$ *is SVN* β*-irrelevant.*

Example 5.6. *(Continued from Example 5.4) Let* $\beta = \langle 0.6, 0.3, 0.7\rangle$. *Since* C_{15} *is a SVN* β*-irrelevant element of* $\widehat{C_3}$ *and* C_{1j} $(j \in \{1, 2, 3, 4\})$ *are SVN* β*-relevant elements of* $\widehat{C_3}$, $\widehat{C_3}$ *is SVN* β*-irrelevant and* $\widehat{C_1}$ *is SVN* β*-relevant.*

Then two properties of SVN β-irrelevant elements are given in the following two propositions.

Proposition 5.7. *Let* $\widehat{C}$ *be a SVN* β*-covering of* U *and* $C \in \widehat{C}$. *If* C *is a SVN* β*-irrelevant element of* $\widehat{C}$, *then* $\widehat{C} - \{C\}$ *is still a SVN* β*-covering of* U.

Proof. By Definition 5.5, it is straightforward. □

Proposition 5.8. *Let* $\widehat{C}$ *be a SVN* β*-covering of* U, C *be a SVN* β*-irrelevant element of* $\widehat{C}$ *and* $C_1 \in \widehat{C} - \{C\}$. *Then* C_1 *is a SVN* β*-irrelevant element of* $\widehat{C}$ *if and only if* C_1 *is a SVN* β*-irrelevant element of* $\widehat{C} - \{C\}$.

Proof. By Definition 5.5 and Proposition 5.8, it is straightforward. □

After deleting all SVN β-irrelevant elements of a SVN β-covering, we call the new SVN β-covering as SVN β-base.

Definition 5.9. *Let* $\widehat{C}$ *be a SVN* β*-covering of* U *and* $\widehat{B} \subseteq \widehat{C}$. *If* $\widehat{C} - \widehat{B}$ *is the family of all SVN* β*-irrelevant elements of* $\widehat{C}$, *then we call* $\widehat{B}$ *as the SVN* β*-base of* $\widehat{C}$, *which is denoted by* $\Lambda^{\beta}(\widehat{C})$.

A SVN β-covering and its SVN β-base have the same SVN β-neighborhood system for any $x \in U$.

Proposition 5.10. *Let* $\widehat{C}$ *be a SVN* β*-covering of* U. *For any* $x \in U$, $\widehat{\mathbb{N}}^{\beta}_{\widehat{C}}(x) = \widehat{\mathbb{N}}^{\beta}_{\Lambda^{\beta}(\widehat{C})}(x)$.

Proof. It is straightforward. □

According to Propositions 5.3 and 5.10, another condition under which different SVN β-coverings can induce the same SVN covering approximation is presented.

Proposition 5.11. *Let* $\widehat{C_1}$, $\widehat{C_2}$ *be two SVN* β*-coverings of* U. *If* $\Lambda^{\beta}(\widehat{C_1}) = \Lambda^{\beta}(\widehat{C_2})$, *then*

$$\overline{\mathbb{C}}_1(A) = \overline{\mathbb{C}}_2(A) \text{ and } \underline{\mathbb{C}}_1(A) = \underline{\mathbb{C}}_2(A) \text{ for any } A \in SVN(U).$$

Proof. According to Proposition 5.10, we have

$$\Lambda^{\beta}(\widehat{\mathbf{C}_1}) = \Lambda^{\beta}(\widehat{\mathbf{C}_2}) \Rightarrow \widehat{\mathbb{N}}^{\beta}_{\Lambda^{\beta}(\widehat{\mathbf{C}_1})}(x) = \widehat{\mathbb{N}}^{\beta}_{\Lambda^{\beta}(\widehat{\mathbf{C}_2})}(x) \Rightarrow \widehat{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}_1}}(x) = \widehat{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}_2}}(x) \text{ for any } x \in U.$$

Then by Proposition 5.3, we know $\overline{\mathbb{C}}_1(A) = \overline{\mathbb{C}}_2(A)$ and $\underline{\mathbb{C}}_1(A) = \underline{\mathbb{C}}_2(A)$ for any $A \in SVN(U)$. □

To find conditions under which different SVN β-coverings can induce the same multigranulation SVN covering approximation, these above conditions are generalized. Let (U, Γ_β) be a n-SVN β-covering approximation space with $\Gamma_\beta = \{\widehat{\mathbf{C}_1}, \widehat{\mathbf{C}_2}, \cdots, \widehat{\mathbf{C}_n}\}$. Then we denote $\widetilde{\mathbb{N}}_{\Gamma_\beta}(x)$ and $\widetilde{\mathbb{N}}_{\Lambda(\Gamma_\beta)}(x)$ by:

$$\widetilde{\mathbb{N}}_{\Gamma_\beta}(x) = \{\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}_i(x)} : 1 \leq i \leq n\} \text{ and } \widetilde{\mathbb{N}}_{\Lambda(\Gamma_\beta)}(x) = \{\widetilde{\mathbb{N}}^{\beta}_{\Lambda^{\beta}(\widehat{\mathbf{C}}_i)(x)} : 1 \leq i \leq n\}.$$

Proposition 5.12. *Let Γ^1_β and Γ^2_β be two n-SVN β-coverings of U. If $\widetilde{\mathbb{N}}_{\Gamma^1_\beta}(x) = \widetilde{\mathbb{N}}_{\Gamma^2_\beta}(x)$ for any $x \in U$, then for any $A \in SVN(U)$,*

$$\overline{\mathfrak{N}_{\Gamma^1_\beta}}(A) = \overline{\mathfrak{N}_{\Gamma^2_\beta}}(A),\ \underline{\mathfrak{N}_{\Gamma^1_\beta}}(A) = \underline{\mathfrak{N}_{\Gamma^2_\beta}}(A),$$
$$\overline{\mathfrak{N}_{\Gamma^1_\beta}}^{(o)}(A) = \overline{\mathfrak{N}_{\Gamma^2_\beta}}^{(o)}(A),\ \underline{\mathfrak{N}_{\Gamma^1_\beta}}^{(o)}(A) = \underline{\mathfrak{N}_{\Gamma^2_\beta}}^{(o)}(A),$$
$$\overline{\mathfrak{N}_{\Gamma^1_\beta}}^{(p)}(A) = \overline{\mathfrak{N}_{\Gamma^2_\beta}}^{(p)}(A),\ \underline{\mathfrak{N}_{\Gamma^1_\beta}}^{(p)}(A) = \underline{\mathfrak{N}_{\Gamma^2_\beta}}^{(p)}(A).$$

Proof. According to Propositions 4.10, 4.14 and 5.1, it is straightforward. □

Proposition 5.13. *Let Γ^1_β and Γ^2_β be two n-SVN β-coverings of U. If $\widetilde{\mathbb{N}}_{\Lambda(\Gamma^1_\beta)}(x) = \widetilde{\mathbb{N}}_{\Lambda(\Gamma^2_\beta)}(x)$ for any $x \in U$, then*

$$\overline{\mathfrak{N}_{\Gamma^1_\beta}}(A) = \overline{\mathfrak{N}_{\Gamma^2_\beta}}(A),\ \underline{\mathfrak{N}_{\Gamma^1_\beta}}(A) = \underline{\mathfrak{N}_{\Gamma^2_\beta}}(A),$$
$$\overline{\mathfrak{N}_{\Gamma^1_\beta}}^{(o)}(A) = \overline{\mathfrak{N}_{\Gamma^2_\beta}}^{(o)}(A),\ \underline{\mathfrak{N}_{\Gamma^1_\beta}}^{(o)}(A) = \underline{\mathfrak{N}_{\Gamma^2_\beta}}^{(o)}(A),$$
$$\overline{\mathfrak{N}_{\Gamma^1_\beta}}^{(p)}(A) = \overline{\mathfrak{N}_{\Gamma^2_\beta}}^{(p)}(A),\ \underline{\mathfrak{N}_{\Gamma^1_\beta}}^{(p)}(A) = \underline{\mathfrak{N}_{\Gamma^2_\beta}}^{(p)}(A).$$

Proof. According to Propositions 4.10, 4.14 and 5.11, it is straightforward. □

6 Applications to the problem of MCGDM

In this section, we present three approaches to the problem of MCGDM under the (philosophical, optimistic and pessimistic) multigranulation SVN CBRS models which are presented in Section 4. Then, a comparative study with other methods is shown.

6.1 The Problem of MCGDM

In the paper production process, there are many on-line detection systems for paper defects. But when the printers buy the paper, they have to judge whether the paper has any paper defects and whether these paper defects affect the printing. At this time, it can only be detected manually. In this subsection, we will introduce this issue.

Let $U = \{x_i : i = 1, 2, \cdots, m\}$ be the set of some papers and $V = \{y_j | j = 1, 2, \cdots, l\}$ be the l main symptoms (such as spot, steak and so on) for a paper defect B. Suppose that the printer X invites experts R_k $(k = 1, 2, \cdots, n)$ to evaluate every paper x_i $(i = 1, 2, \cdots, m)$.

Suppose that every expert R_k $(k = 1, 2, \cdots, n)$ believes the paper $x_i \in U$ $(i = 1, 2, \cdots, m)$ has a symptom value C_{kj} $(j = 1, 2, \cdots, l)$, denoted by $C_{kj}(x_i) = \langle T_{C_{kj}}(x_i), I_{C_{kj}}(x_i, F_{C_{kj}}(x_i)\rangle$, where $T_{C_{kj}}(x_i) \in [0, 1]$ is the degree that expert R_k confirms paper x_i has symptom y_j, $I_{C_{kj}}(x_i) \in [0, 1]$ is the degree that expert R_k is not sure paper x_i has symptom y_j, $F_{C_{kj}}(x_i) \in [0, 1]$ is the degree that expert R_k confirms paper x_i does not have symptom y_j, and $T_{C_{kj}}(x_i) + I_{C_{kj}}(x_i) + F_{C_{kj}}(x_i) \leq 3$.

Let $\beta = \langle a, b, c\rangle$ be the critical value. If for any paper $x_i \in U$, there exists at least one symptom $y_j \in V$ such that the symptom value C_{kj} for paper x_i is not less than β, respectively, then $\Gamma_\beta = \{\widehat{\mathbf{C}}_1, \widehat{\mathbf{C}}_2, \cdots, \widehat{\mathbf{C}}_n\}$, where $\widehat{\mathbf{C}}_k = \{C_{k1}, C_{k2}, \cdots, C_{kl}\}$, for all $1 \leq k \leq n$, is a n-SVN β-covering of U for some SVN value β.

If d is a possible degree, e is an indeterminacy degree and f is an impossible degree of the paper defect B of every paper $x_i \in U$ that is diagnosed by the printer X, denoted by $A(x_k) = \langle d, e, f\rangle$, then the decision maker (the printer X) for the MCGDM problem needs to know how to evaluate whether or not the papers $x_i \in U$ has the paper defect B.

Remark 6.1. *Let $\Gamma = \{\widehat{\boldsymbol{C}}_1, \widehat{\boldsymbol{C}}_2, \cdots, \widehat{\boldsymbol{C}}_n\}$, where $\widehat{\boldsymbol{C}}_k \subseteq SVN(U)$. It is an important problem to determine the critical value β such that Γ is a n-SVN β-covering of U. We can use the following method to find the range of β. Firstly, we suppose a SVN number $\alpha =< \bigwedge_{k=1}^{|\Gamma|} (\bigwedge_{j=1}^{|\widehat{\boldsymbol{C}}_k|} \bigvee_{i=1}^{|U|} T_{C_{kj}}(x_i)), \bigvee_{k=1}^{|\Gamma|} (\bigvee_{j=1}^{|\widehat{\boldsymbol{C}}_k|} \bigwedge_{i=1}^{|U|} I_{C_{kj}}(x_i)), \bigvee_{k=1}^{|\Gamma|} (\bigvee_{j=1}^{|\widehat{\boldsymbol{C}}_k|} \bigwedge_{i=1}^{|U|} F_{C_{kj}}(x_i)) >$. Secondly, if Γ is a n-SVN α-covering of U, then Γ is a n-SVN β-covering of U for any β $(\beta \leq \alpha)$. Finally, if Γ is not a n-SVN α-covering of U, then we should change some numbers of α with the minimal step, denoted by α_1, such that it is a n-SVN α_1-covering of U. Hence, Γ is a n-SVN β-covering of U for any β $(\beta \leq \alpha_1)$. In Example 3.3, we suppose $\alpha =< 0.6, 0.2, 0.4 >$ according to this method. But Γ is not a n-SVN α-covering of U. After changing α as $\alpha_1 =< 0.6, 0.3, 0.5 >$ with the step 0.1, we find Γ is a n-SVN α_1-covering of U. Hence, for any $\beta \leq \alpha_1$, Γ is a n-SVN β-covering of U.*

6.2 MCGDM algorithms

In this subsection, we present the Algorithm 1 of MCGDM under the framework of the philosophical multigranulation SVN CBRS model.

Algorithm 1 The MCGDM algorithm based on the philosophical multigranulation SVN CBRS model.

Input: SVN decision information system $(U, \beta, \Gamma_\beta, A)$.
Output: The score ordering for all alternatives.

1: Compute SVN β-neighborhood $\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}_k(x)}$ of x about $\widehat{\mathbf{C}}$, for all $x \in U$ and $k = 1, 2, \cdots, n$, according to Definition 2.3;
2: Compute SVN β-neighborhood $\widetilde{\mathbb{N}}_{\Gamma_\beta(x)}$ of x about Γ_β, for any $x \in U$, according to Definition 3.2;
3: Compute the philosophical multigranulation SVN covering upper approximation $\overline{\mathfrak{N}_{\Gamma_\beta}}(A)$ and lower approximation $\underline{\mathfrak{N}_{\Gamma_\beta}}(A)$ of A, according to Definition 4.4;
4: Compute $\overline{R}_A = \overline{\mathfrak{N}_{\Gamma_\beta}}(A) \oplus \underline{\mathfrak{N}_{\Gamma_\beta}}(A)$ according to (6) in the basic operations on $\overline{SVN}(U)$;
5: For each alternative x, compute

$$s(x) = \frac{T_{\tilde{R}_A}(x)}{\sqrt{(T_{\tilde{R}_A}(x))^2 + (I_{\tilde{R}_A}(x))^2 + (F_{\tilde{R}_A}(x))^2}};$$

6: Rank all $s(x)$ and select the paper which is more likely to have the paper defect B.

In Step 5, $s(x)$ is the cosine similarity measure between $\widetilde{R}_A(x)$ and the ideal solution $(1, 0, 0)$, which was proposed by Ye [28]. In Step 6, we rank all $s(x)$ by the principle of numerical size. Inspired by Algorithm 1, we present the Algorithm 2 of MCGDM under the framework of the optimistic multigranulation SVN CBRS model.

Algorithm 2 The MCGDM algorithm based on the optimistic multigranulation SVN CBRS model.

Input: SVN decision information system $(U, \beta, \Gamma_\beta, A)$.
Output: The score ordering for all alternatives.

1: Compute SVN β-neighborhood $\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}_k(x)}$ of x about $\widehat{\mathbf{C}}$, for all $x \in U$ and $k = 1, 2, \cdots, n$, according to Definition 2.3;
2: Compute the optimistic multigranulation SVN covering upper approximation $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)$ and lower approximation $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)$ of A, according to Definition 4.8;
3: Compute $\overline{R}^{(o)}_A = \overline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A) \oplus \underline{\mathfrak{N}_{\Gamma_\beta}}^{(o)}(A)$ according to (6) in the basic operations on $\overline{SVN}(U)$;
4: For each alternative x, compute

$$s(x) = \frac{T_{\tilde{R}^{(o)}_A}(x)}{\sqrt{(T_{\tilde{R}^{(o)}_A}(x))^2 + (I_{\tilde{R}^{(o)}_A}(x))^2 + (F_{\tilde{R}^{(o)}_A}(x))^2}};$$

5: Rank all $s(x)$ and select the paper which is more likely to have the paper defect B.

Algorithm 2 is presented based on the optimistic multigranulation SVN CBRS model. If **Steps 2, 3, 4** in Algorithm 2 are replaced by the pessimistic multigranulation SVN covering upper approximation $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)$ and lower approximation $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)$, then Algorithm 3 is presented.

Algorithm 3 The MCGDM algorithm based on the pessimistic multigranulation SVN CBRS model.

Input: SVN decision information system $(U, \beta, \Gamma_\beta, A)$.
Output: The score ordering for all alternatives.

1: Compute SVN β-neighborhood $\widetilde{\mathbb{N}}^{\beta}_{\widehat{\mathbf{C}}_k(x)}$ of x about $\widehat{\mathbf{C}}$, for all $x \in U$ and $k = 1, 2, \cdots, n$, according to Definition 2.3;
2: Compute the pessimistic multigranulation SVN covering upper approximation $\overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)$ and lower approximation $\underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)$ of A, according to Definition 4.12;
3: Compute $\overline{R}^{(p)}_A = \overline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A) \oplus \underline{\mathfrak{N}_{\Gamma_\beta}}^{(p)}(A)$ according to (6) in the basic operations on $\overline{SVN}(U)$;
4: For each alternative x, compute

$$s(x) = \frac{T_{\tilde{R}^{(p)}_A}(x)}{\sqrt{(T_{\tilde{R}^{(p)}_A}(x))^2 + (I_{\tilde{R}^{(p)}_A}(x))^2 + (F_{\tilde{R}^{(p)}_A}(x))^2}};$$

5: Rank all $s(x)$ and select the paper which is more likely to have the paper defect B.

6.3 An applied example

Example 6.2. *Assume that* $U = \{x_1, x_2, x_3, x_4, x_5\}$ *is a set of papers. By the paper defect's symptoms, we write* $V = \{y_1, y_2, y_3, y_4\}$ *to be four main symptoms (spot, steak, crater and fracture) for a paper defect* B*. Suppose that the printer* X *invites two experts* R_k $(k = 1, 2)$ *to evaluate every paper* x_i $(i = 1, 2, \cdots, 5)$ *as in Tables 1 and 2.*

Let $\beta = \langle 0.6, 0.3, 0.7\rangle$ *be the critical value. Then,* $\Gamma_\beta = \{\widehat{\boldsymbol{C}_1}, \widehat{\boldsymbol{C}_2}\}$*, where* $\widehat{\boldsymbol{C}_k} = \{C_{k1}, C_{k2}, C_{k3}, C_{k4}\}$*, for all* $k = 1, 2$*, is a 2-SVN* β*-coverings of* U*. Assume that the printer* X *diagnosed the value* $A = \frac{(0.6,0.3,0.5)}{x_1} + \frac{(0.4,0.5,0.1)}{x_2} + \frac{(0.3,0.2,0.6)}{x_3} + \frac{(0.5,0.3,0.4)}{x_4} + \frac{(0.7,0.2,0.3)}{x_5}$ *of the paper defect* B *of every paper. Algorithm 1 is first used for Example 6.2.*

Step 1*: All* $\widetilde{\mathbb{N}}^{\beta}_{\widehat{C_k}(x_i)}$ $(k = 1, 2)$ *for any* $1 \leq i \leq 5$ *are shown in Tables 3 and 4.*

Step 2*: All* $\widetilde{\mathbb{N}}_{\Gamma_\beta(x_i)}$ *for any* $1 \leq i \leq 5$ *are shown in Table 5.*

Step 3*:*

$$\overline{\mathfrak{N}_{\Gamma_\beta}}(A) = \{\langle x_1, 0.6, 0.3, 0.5\rangle, \langle x_2, 0.5, 0.4, 0.6\rangle, \langle x_3, 0.7, 0.3, 0.5\rangle, \langle x_4, 0.6, 0.3, 0.5\rangle, \langle x_5, 0.6, 0.3, 0.6\rangle\},$$
$$\underline{\mathfrak{N}_{\Gamma_\beta}}(A) = \{\langle x_1, 0.7, 0.4, 0.5\rangle, \langle x_2, 0.4, 0.4, 0.4\rangle, \langle x_3, 0.6, 0.4, 0.6\rangle, \langle x_4, 0.6, 0.5, 0.5\rangle, \langle x_5, 0.7, 0.4, 0.5\rangle\}.$$

Step 4*:*

$$\begin{aligned}\widetilde{R}_A &= \overline{\mathfrak{N}_{\Gamma_\beta}}(A) \oplus \underline{\mathfrak{N}_{\Gamma_\beta}}(A)\\ &= \{\langle x_1, 0.88, 0.12, 0.25\rangle, \langle x_2, 0.70, 0.16, 0.24\rangle, \langle x_3, 0.88, 0.12, 0.30\rangle, \langle x_4, 0.84, 0.15, 0.25\rangle, \langle x_5, 0.88, 0.12, 0.30\rangle\}.\end{aligned}$$

Step 5*: We can obtain* $s(x_i)$ $(i = 1, 2, \cdots, 5)$ *as follows:*

$$s(x_1) = 0.954,\ s(x_2) = 0.925,\ s(x_3) = 0.939,\ s(x_4) = 0.945,\ s(x_5) = 0.939.$$

Step 6*: According to the order of* $s(x_i)$ $(i = 1, 2, \cdots, 5)$*, we have:*

$$x_2 < x_3 \approx x_5 < x_4 < x_1.$$

Therefore, the printer X *diagnoses that the paper* x_1 *is more likely to have the paper defect* B*.*

Then, Algorithm 2 is used for Example 6.2. We can obtain $s(x_i)$ $(i = 1, 2, \cdots, 5)$ *by Algorithm 2.*

$$s(x_1) = 0.950,\ s(x_2) = 0.925,\ s(x_3) = 0.939,\ s(x_4)0.945,\ s(x_5) = 0.939.$$

According to the order of $s(x_i)$ $(i = 1, 2, \cdots, 5)$*, we have:*

$$x_2 < x_3 \approx x_5 < x_4 < x_1.$$

Therefore, the printer X *diagnoses that the paper* x_1 *is more likely to have the paper defect* B*.*

Or Algorithm 3 is also used for Example 6.2. We can obtain $s(x_i)$ $(i = 1, 2, \cdots, 5)$ *by Algorithm 3.*

$$s(x_1) = 0.945,\ s(x_2) = 0.908,\ s(x_3) = 0.957,\ s(x_4)0.941,\ s(x_5) = 0.950.$$

According to the order of $s(x_i)$ $(i = 1, 2, \cdots, 5)$*, we have:*

$$x_2 < x_4 < x_1 < x_5 < x_3.$$

Therefore, the printer X *diagnoses that the paper* x_3 *is more likely to have the paper defect* B*.*

6.4 A comparison analysis

In order to validate the feasibility of the proposed decision making method, a comparative study was conducted with other methods. These methods which were introduced in Liu [10] and Li et al. [9] are compared with the proposed approach using SVN information system.

Liu's method can be used for Example 6.2. Tables 1 and 2 are SVN decision matrices D_1 and D_2, respectively. We suppose the weight vector $\omega = (0.3, 0, 2, 0.4, 0.1)$ and $\delta = (0.6, 0.4)$. We get

$$s(n_1) = 0.768,\ s(n_2) = 0.565,\ s(n_3) = 0.707,\ s(n_4) = 0.623,\ s(n_5) = 0.679.$$

According to the cosine similarity degrees $s(n_i)$ $(i = 1, 2, \cdots, 5)$, we obtain $x_2 < x_4 < x_5 < x_3 < x_1$. Hence, paper x_1 is more likely to have the paper defect B.

Li's method can be used for Example 6.2. Tables 1 and 2 are SVN decision matrices D_1 and D_2, respectively. We suppose the weight vectors $\omega = (0.3, 0, 2, 0.4, 0.1)$, $\delta = (0.6, 0.4)$ and $p = q = 1$. We get

$$s(n_1) = 0.054,\ s(n_2) = 0.017,\ s(n_3) = 0.040,\ s(n_4) = 0.016,\ s(n_5) = 0.031.$$

According to the cosine similarity degrees $s(n_i)$ $(i = 1, 2, \cdots, 5)$, we obtain $x_4 < x_2 < x_5 < x_3 < x_1$. Therefore, paper x_1 is more likely to have the paper defect B.

All results are shown in Table 6 and Figure 2. It can be seen from Table 6 and Figure 2 that the decision making result of our proposed methods (Algorithms 1 and 2), Lius method and Li's method is consistent, that is, x_1 is most sick with the paper defect B. This phenomenon illustrates the effectiveness of our proposed method. At the same time, we can see that graphs of our methods (Algorithms 1, 2 and 3) are above the other methods from Figure 2. The graphs for Algorithms 1 and 2 are very close.

Table 6: The results utilizing the different methods of Example 6.2

Methods	The final ranking	The paper is most sick with the paper defect B
Liu [10]	$x_2 < x_4 < x_5 < x_3 < x_1$	x_1
Li et al. [9]	$x_4 < x_2 < x_5 < x_3 < x_1$	x_1
Algorithm 1	$x_2 < x_3 \approx x_5 < x_4 < x_1$	x_1
Algorithm 2	$x_2 < x_3 \approx x_5 < x_4 < x_1$	x_1
Algorithm 3	$x_2 < x_4 < x_1 < x_5 < x_3$	x_3

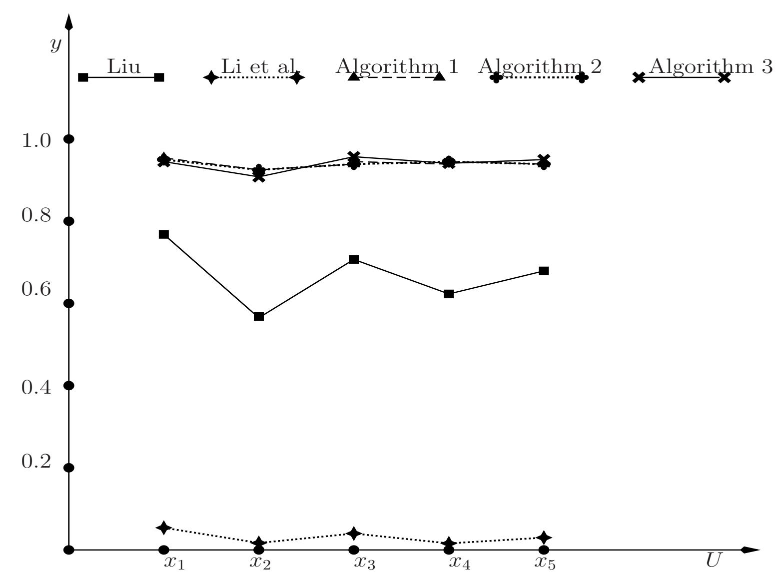

Figure 2: The chat of different values of papers in utilizing different methods in Example 6.2.

Liu [10] and Li et al. [9] presented the methods by SVN theory. In their methods, the results will be different by different ω and δ. In Example 6.2, we set $\mathbf{w} = (0.3, 0.2, 0.4, 0.1)$ and $\delta = (0.6, 0.4)$. Then, Liu's method [10] is based on the Hammer SVN number aggregation (HSVNNWA) operator. Using Liu's method, we obtain $x_2 < x_4 < x_5 < x_3 < x_1$. That is to say, paper x_1 is more likely to be sick with the paper defect B. The method developed by Li et al. [9] is based on the SVN number improved generalized weighted Heronian mean (NNIGWHM) operator. Then, we find that paper x_1 is more likely to be sick with the paper defect B by using Li's method.

We use three types of (philosophical, optimistic and pessimistic) multigranulation SVN CBRS models to MCGDM problems, respectively. Hence, there are three methods for MCGDM problems, which used corresponding multigranulation SVN covering approximation operators. In Algorithms 1, 2 and 3, the cosine similarity measure is the main operation. The contributions of our proposed methods are summarized as follows:

(1) This is the first time to use multigranulation SVN covering rough sets in the MCGDM problems. According to Algorithm 3, we obtain that paper x_3 is more likely to be sick with the paper defect B. This is different from other methods. Maybe this is our new choice. Hence, our proposed methods complement the existing methods.

(2) There are two uncertainties which are $\mathbf{w}$ and δ in Liu's method, and four uncertainties which are $\mathbf{w}$, δ, p and q. But there is only one uncertainty β in our methods. In many cases, the smaller the number of uncertainties, the more accurate the decision.

(3) It is a new viewpoint to use multigranulation SVN covering rough sets in paper defect diagnosis.

By the analysis of our three algorithms, we can readily verify that the time complexity of them are both $O(|U||\Gamma_\beta|^2|V|)$. Hence, it is up to the user to select a suitable decision-making method from three methods for the MCGDM problem. Moreover, the obtained results may be different when the user select different methods. In order to achieve the most accurate results, the further diagnosis is necessary in combination with other hybrid methods.

7 Conclusions

This paper is a bridge linking CBRSs, SVN sets and multigranulation rough sets. By introducing the definition of SVN β-covering approximation space, we present a new type of SVN CBRS model. Then the concept of n-SVN β-covering approximation space is presented, and three types of multigranulation SVN CBRS models under this new approximation space are proposed. Moreover, the (paper defect diagnosis) problem of MCGDM is managed under the multigranulation SVN CBRS models. The main conclusions of this paper are listed as follows:

1. Three types of (philosophical, optimistic and pessimistic) multigranulation SVN CBRS models are firstly presented.

2. Some conditions under which different n-SVN β-coverings induced the same (philosophical, optimistic and pessimistic) multigranulation SVN covering approximation operators are presented. By these conditions, one can recognize the new rough set models further.

3. We propose three novel methodologies to the MCGDM problem about paper defect diagnosis under (philosophical, optimistic and pessimistic) multigranulation SVN CBRS models. The comparison analysis is very interesting to show the difference between the proposed methods and other methods.

In the future, the following research topics will be deserved. We will find a way to select a suitable decision-making method from different methods. Some issues in SVN covering information systems will be solved, such as reductions and the method of data compression. The relationship between SVN covering information systems and SVN covering approximation spaces will be presented. The combination of some algebraic structures [36, 35] with the main content of this paper will be investigated.

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Multigranulation single valued neutrosophic covering-based rough sets and their applications to multi-criteria group decision making

J. Q. Wang and X. H. Zhang

مجموعه‌های تقریبی پوشش مبنا مقایسه ناپذیر تک مقداری چند دانه‌ای و کاربرد آن‌ها در تصمیم‌گیری گروهی چند-معیاره

چکیده. در این مقاله، سه نوع (فیلسوفانه، خوش‌بینانه و بدبینانه) از مدل‌های مجموعه تقریبی پوشش مبنای مقایسه ناپذیر تک مقداری چنددانه‌ای (SVN) ارائه گردیده، و این سه مدل در مورد مسئله تصمیم‌گیری گروهی چند معیاره (MCGDM) بکار برده می‌شود. ابتدا، نوعی از مدل مجموعه تقریبی پوشش مبنا SVN پیشنهاد شده‌است. بر اساس این مدل مجموعه تقریبی، سه نوع از مدل‌های مجموعه‌ی تقریبی پوشش مبنا SVN چنددانه‌ای تحت مفهوم فضای تقریب، β - پوشش SVN چنددانه‌ای پیشنهاد گردیده، که β یک عدد SVN است. بعلاوه، ارتباط بین این چهار مدل بررسی شده‌است. سپس، برخی از شرایط که تحت آن‌ها فضاهای تقریب β-پوشش SVN چنددانه‌ای متفاوت عملگرهای تقریب پوشش SVN چنددانه‌ای یکسانی را القاء می‌کنند، ارائه گردیده‌اند. بالاخره، برای مسائل MCGDM سه روش جدید تحت مدل‌های مجموعه تقریبی پوشش مبنا SVN چنددانه‌ای ارائه گردیده‌اند. علاوه‌برآن، روش‌های MCGDM پیشنهاد شده از طریق یک مسئله مربوط به تشخیص نقصان مقاله، با روش‌های دیگر مقایسه شده‌اند.