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Original paper



# A novel method for ranking generalized fuzzy numbers with two different heights and its application in fuzzy risk analysis

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**Abstract**

Due to the large use of fuzzy numbers, the ranking of these numbers is very important. In this paper, we propose a new method for ranking generalized fuzzy numbers with different left and right heights. The proposed method, at first obtains the centers of gravity of fuzzy numbers and left and right side crisp numbers; then by computing left and right areas associated with them, ranks the fuzzy numbers. The proposed method can overcome the flaws and defects of some ranking methods, and the provided examples are evidence of this. Finally this method is applied to the fuzzy risk analysis problem.

*Keywords:* Generalized fuzzy numbers, ranking method, center of gravity, fuzzy risk analysis.

## 1 Introduction

In various sciences such as medicine, engineering, industry, natural sciences, psychology, etc., we deal with cases that are not precise and definite. Zadeh [24] presented the concept of fuzzy sets to encounter such issues. D. Dubois and H. Prade [10] defined the fuzzy number as a fuzzy subset in the real numbers line.

One of the important issues related to fuzzy numbers is their ranking, which has many applications in data analysis, optimization, approximate reasoning, economic, social systems, and so on. R. Jain presented the first method for ranking fuzzy numbers [13]. There are several methods like [1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 14, 16, 17, 19, 20, 21, 22, 23] which ranked fuzzy numbers based on specific features and suitable in certain situations.

T.C. Chu and C.T. Tsao [9] ranked the generalized fuzzy numbers by using the area between the centroid point and Y.J. Wang and H.Sh. Lee [19] revised Chu and Tsao's method. S. Abbasbandy and T. Hajjari [1] ranked the normal trapezoidal fuzzy numbers based on left and right $\alpha-$level of fuzzy numbers and also Y.M. Wang and Y. Luo [20] ranked them based on positive and negative ideal points and two risks indices. S.M. Chen and J.H. Chen [5] calculated the score of generalized trapezoidal fuzzy numbers based on defuzzified value, heights and spreads. But if the defuzzified value be equal to zero then the score will be zero, and as a result, the height and dispersion will be ineffective. S.M. Chen and K. Sanguansat [7], based on areas on the positive side, the areas on the negative side and the heights, has ranked the generalized trapezoidal fuzzy numbers. However, as in Chen and Chen's method, if defuzzified value be equal to zero, then the height will be ineffective. T. Hajjari [12] by presenting the new magnitude ($MagN$) ranked the generalized trapezoidal fuzzy numbers. S.M. Chen, et al. [6] based on the areas of the positive and negative side and centroid value, and W. Jiang [14] based on the areas of the positive side, the areas of the negative side and spread value ranked the generalized fuzzy numbers with different left and right heights. In these methods, like methods presented in [5] and [7], score of generalized trapezoidal fuzzy numbers with zero defuzzified value is equal zero. D. Wu [21] based on ordered weighted averaging (OWA) operator and consideration of the different importance of the three scoring factors defuzzified value, height and spread ranked the generalized fuzzy numbers with different left and right heights. In this method, by changing $\alpha$, the weights also change. In [21], $\alpha = 0.7$ is considered and may not provide appropriate ranking in some situations.

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In this paper, for ranking generalized fuzzy numbers with different left and right heights, the centers of gravity of the generalized fuzzy numbers and center of gravity of left and right side crisp numbers obtained; then the fuzzy numbers are ranked by using the left and right areas associated with them. The purpose of this article is to provide a method that, in addition to the standards defined in the ranking fuzzy numbers, covers the defects and shortcomings of some above mentioned methods.

The paper has been organized as follows: Section 2 presents some definitions of generalized fuzzy numbers. In Section 3, the proposed method for ranking generalized fuzzy numbers is discussed. In Section 4, a comparison between some ranking methods with the proposed method is presented. In Section 5, the proposed ranking method for fuzzy risk analysis problem has been used and the conclusion is given in Section 6.

## 2 Preliminaries

Generalized fuzzy numbers concept with different left and right heights ($w_L, w_R \in [0,1]$ respectively), was presented in [6, 25]. This fuzzy number indicated as $\tilde{A} = (v_1, v_2, v_3, v_4; w_L; w_R)$ that can be seen in Fig. 1. If $0 \le v_1 \le v_2 \le v_3 \le v_4 \le 1$, then we say $\tilde{A}$ is a standard generalized fuzzy number. The membership function $\mu_{\tilde{A}}(x)$ is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} f_L(x) & v_1 \le x \le v_2, \\ f_C(x) & v_2 \le x \le v_3, \\ f_R(x) & v_3 \le x \le v_4, \\ 0 & otherwise, \end{cases}$$

where $f_L(x) = \frac{w_L(x-v_1)}{v_2-v_1}$, $f_C(x) = \frac{w_L(v_3-v_2)+(w_R-w_L)(x-v_2)}{v_3-v_2}$ and $f_R(x) = \frac{w_R(x-v_4)}{v_3-v_4}$.

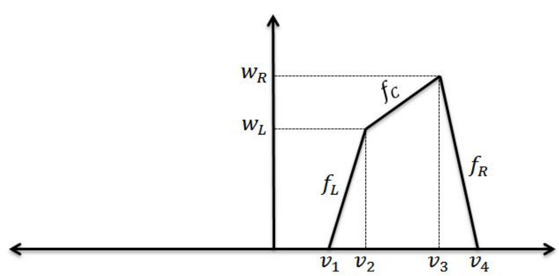


Figure 1: Generalized fuzzy number with different left and right heights

If $w_L = w_R = w$, then $\tilde{A}$ is a generalized trapezoidal fuzzy number and membership function as:

$$\mu_{\tilde{A}}(x) = \begin{cases} w\frac{x-v_1}{v_2-v_1} & v_1 \le x \le v_2, \\ w & v_2 \le x \le v_3, \\ w\frac{v_4-x}{v_4-v_3} & v_3 \le x \le v_4, \\ 0 & otherwise. \end{cases}$$

If $w_L = w_R = w$ and $v_2 = v_3$, then $\tilde{A}$ is a triangular fuzzy number. In mentioned states, if $w_L = w_R = 1$, then the fuzzy number is normal; and in the end if $v_1 = v_2 = v_3 = v_4$ and $w_L = w_R$, then we have crisp number.

## 3 A proposed method for ranking generalized fuzzy numbers

In this section, firstly some definitions of the centers of gravity of triangle and quadrilateral are presented, then the novel method for ranking generalized fuzzy numbers based on center of gravity of fuzzy numbers and areas associated with them has been introduced.

**Definition 3.1.** [15] *The centroid of a triangle is the point of intersection of its medians where it denoted by G in Fig. 2.*

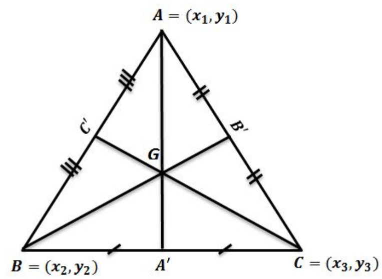


Figure 2: The center of gravity of triangle $ABC$.

**Lemma 3.2.** *The Cartesian coordinate of $G$ in Fig. 2 is* $(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3})$.

*Proof.* Cartesian coordinates of points $A'$, $B'$, and $C'$ are $(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2})$, $(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2})$ and $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ respectively. Thus, by obtaining the equation of the lines passing through the pair of points $(A, A'; B, B'; C, C')$ and finds the point of intersection of the two lines of these three lines, we can easily attain the Cartesian coordinate of $G$. □

**Definition 3.3.** [15] *In Fig. 3, suppose that $u, v, w$ and $x$ are the centers of triangles $B_0B_1B_2$, $B_0B_2B_3$, $B_0B_1B_3$, and $B_1B_2B_3$ respectively, then the location of the intersection of $uv$ and $wx$ lines is the center of gravity of quadrilateral $B_0B_1B_2B_3$ where it is marked with $G$.*

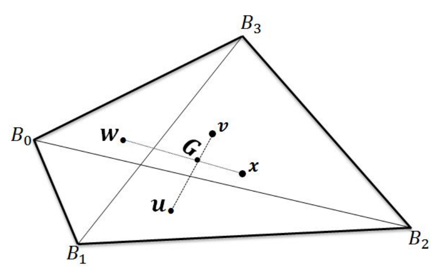


Figure 3: The center of gravity of quadrilateral $B_0B_1B_2B_3$.

Now suppose that $n$ fuzzy numbers $\tilde{A}_i = (a_{1i}, a_{2i}, a_{3i}, a_{4i}; w_{Li}; w_{Ri})$, $i = 1, 2, 3, ..., n$ to be ranked. The steps of the proposed method are as follows:

**Step 1: Convert generalized fuzzy numbers to standard generalized fuzzy numbers:**
Standard generalized fuzzy number $Ai$ is obtained as follow:

$$A_i = (\frac{a_{1i}}{m}, \frac{a_{2i}}{m}, \frac{a_{3i}}{m}, \frac{a_{4i}}{m}; w_{Li}; w_{Ri}) = (v_{1i}, v_{1i}, v_{3i}, v_{4i}; w_{Li}; w_{Ri})$$

where $m = \max\{\lceil|a_{1i}|\rceil, \lceil|a_{2i}|\rceil, \lceil|a_{3i}|\rceil, \lceil|a_{4i}|\rceil, 1\}$.

**Step 2: Determine the geometric center of gravity of standard generalized fuzzy numbers:**
If $A_i = (v_{1i}, v_{2i}, v_{3i}, v_{4i}; w_{Li}; w_{Ri})$ be a triangle fuzzy number in which $v_{2i} = v_{3i} = {v_i}^*$ and $w_{Li} = w_{Ri} = w_i$ then, by employing Lemma 3.2 the center of gravity of $A_i$ is $G_i = (G_{xi}, G_{yi}) = (\frac{v_{1i}+{v_i}^*+v_{4i}}{3}, \frac{w_i}{3})$.
If $A_i = (v_{1i}, v_{2i}, v_{3i}, v_{4i}; w_{Li}; w_{Ri})$ be generalized fuzzy number in which $v_{1i} < v_{2i} < v_{3i} < v_{4i}$, then by employing Definition 3.3 the center of gravity of $A_i$ is obtained by solving the following system

$$\begin{cases} G_{yi} - \frac{w_{Ri}}{v_{3i}-v_{1i}}(G_{xi} - \frac{v_{1i}+v_{2i}+v_{4i}}{3}) = \frac{w_{Li}}{3} \\ G_{yi} - \frac{w_{Li}}{v_{2i}-v_{4i}}(G_{xi} - \frac{v_{1i}+v_{2i}+v_{3i}}{3}) = \frac{w_{Li}+w_{Ri}}{3}. \end{cases}$$

If $A_i = (v_{1i}, v_{2i}, v_{3i}, v_{4i}; w_{Li}; w_{Ri})$ be a fuzzy number in which $v_{1i} = v_{2i} = v_{3i} = v_{4i} = v_i$ and $w_{Li} = w_{Ri} = w_i$ then it is obvious that the center of gravity of $A_i$ is $G_i = (G_{xi}, G_{yi}) = (v_i, \frac{w_i}{2})$

**Step 3: Obtain the left and right side areas of standard generalized fuzzy numbers:**
If $G_i = (G_{xi}, G_{yi})$, $G_a = (-1, \frac{1}{2})$ and $G_b = (1, \frac{1}{2})$ be the centers of gravity of $A_i$, $(-1,-1,-1,-1;1;1)$ and $(1,1,1,1;1;1)$ respectively and also $g_{1i}$ and $g_{2i}$ be lines joining $G_a, G_i$ and $G_b, G_i$ respectively, then the left and right side areas are as fallow:

$$S_{1i} = \int_{-1}^{G_{xi}} g_{1i}(x)dx, \quad S_{2i} = \int_{G_{xi}}^{1} g_{2i}(x)dx$$

, where $S_{1i}$ is the left area and $S_{2i}$ is the right area and that are shown in Fig. 4.

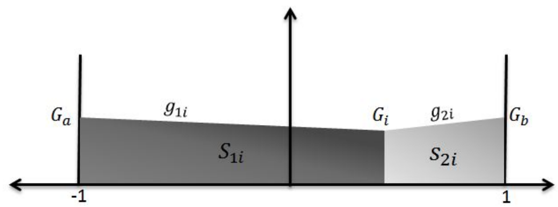


Figure 4: The areas $S_{1i}, S_{2i}$ for standard fuzzy number $A_i$.

**Step 4: Calculate $score(A_i)$ and ranked standard generalized fuzzy numbers:**
If we set $score(A_i) = S_{1i} - S_{2i}$, then the ranking of fuzzy numbers $A_i$ is determined as follow:
if $score(A_i) < score(A_j)$, then $A_i \prec A_j$,
if $score(A_i) > score(A_j)$, then $A_i \succ A_j$,
if $score(A_i) = score(A_j) = 0$ and $G_{yi} < G_{yj}$, then $A_i \prec A_j$,
if $score(A_i) = score(A_j) \neq 0$, then $A_i \approx A_j$.

In the following, some properties of proposed ranking method are presented:
**Property 1.** Suppose that $A = (v_1, v_2, v_3, v_4; w_L; w_R)$, $B = (v_1+a, v_2+a, v_3+a, v_4+a; w_L; w_R)$ standard generalized fuzzy numbers and $a \in \mathbb{R}$. If $a > 0$, then $B \succ A$, and if $a < 0$, then $B \prec A$.

*Proof.* Because the fuzzy number $B$ is transmitted to the fuzzy number $A$ in the direction of horizontal axis, we have $G_{xB} = a + G_{xA}$ and $G_{yA} = G_{yB}$. At first we obtain the $score(A)$:

$$g_{1A}(x) = \frac{G_{yA} - \frac{1}{2}}{G_{xA} + 1}(x+1) + \frac{1}{2}, \quad g_{2A}(x) = \frac{G_{yA} - \frac{1}{2}}{G_{xA} - 1}(x-1) + \frac{1}{2},$$

and so

$$S_{1A} = \int_{-1}^{G_{xA}} g_{1A}(x)dx = \frac{1}{2}(G_{xA}+1)(G_{yA} + \frac{1}{2}) \tag{1}$$

$$S_{2A} = \int_{G_{xA}}^{1} g_{2A}(x)dx = \frac{1}{2}(1 - G_{xA})(G_{yA} + \frac{1}{2}). \tag{2}$$

Therefore,

$$score(A) = S_{1A} - S_{2A} = G_{xA}(G_{yA} + \frac{1}{2}), \tag{3}$$

and

$$score(B) = S_{1B} - S_{2B} = (a + G_{xA})(G_{yA} + \frac{1}{2}). \tag{4}$$

From relations (3), (4) we can conclude:
if $a > 0$ then $B \succ A$, and if $a < 0$ then $B \prec A$.
**Property 2**. For $A = (1,1,1,1;1;1)$, $B = (-1,-1,-1,-1;1;1)$ we have $score(A) = 1$ and $score(B) = -1$.

*Proof.*

$$S_{1A} = \int_{-1}^{1} \frac{1}{2} dx = 1, \quad S_{2A} = 0, \quad score(A) = 1 - 0 = 1$$

$$S_{1B} = 0, \quad S_{2B} = \int_{-1}^{1} \frac{1}{2} dx = 1, \quad score(B) = 0 - 1 = -1.$$

**Property 3.** Suppose $A = (v_1, v_2, v_3, v_4; w_L; w_R)$ is a standard generalized fuzzy numbers and $B = (-v_4, -v_3, -v_2, -v_1; w_R; w_L)$, then $score(B) = -score(A)$.

*Proof.* Fuzzy numbers $A$ and $B$ are symmetric relative to the vertex line, so $G_{xB} = -G_{xA},\ G_{yA} = G_{yB}$. Since

$$g_{1A}(x) = \frac{G_{yA} - \frac{1}{2}}{G_{xA} + 1}(x + 1) + \frac{1}{2}, \quad g_{2A}(x) = \frac{G_{yA} - \frac{1}{2}}{G_{xA} - 1}(x - 1) + \frac{1}{2},$$

$$S_{1A} = \int_{-1}^{G_{xA}} g_{1A}(x)dx = \frac{1}{2}(G_{xA} + 1)(G_{yA} + \frac{1}{2}) \text{ and } S_{2A} = \int_{G_{xA}}^{1} g_{2A}(x)dx = \frac{1}{2}(1 - G_{xA})(G_{yA} + \frac{1}{2}),$$

$$score(A) = S_{1A} - S_{2A} = G_{xA}(G_{yA} + \frac{1}{2}), \tag{5}$$

we can write:

$$score(B) = S_{1B} - S_{2B} = -G_{xA}(G_{yA} + \frac{1}{2}). \tag{6}$$

Equations (5), (6) gives the result that $score(B) = -score(A)$.

**Property 4.** If $G_A = (G_{xA}, G_{yA}) = (0, G_{yA})$ is the center of gravity $A$, then $score(A) = 0$.

*Proof.*

$$S_{1A} = \int_{-1}^{0} (G_{yA} - \frac{1}{2})(x + 1) + \frac{1}{2} dx = \frac{1}{2} G_{yA} + \frac{1}{4} \text{ and } S_{2A} = \int_{0}^{1} (G_{yA} - \frac{1}{2})(-x + 1) + \frac{1}{2} dx = \frac{1}{2} G_{yA} + \frac{1}{4},$$

hence $score(A) = S_{1A} - S_{2A} = 0$.

# 4 The comparison of some ranking methods with the proposed method

In this section, we compare our scheme with the methods presented in [19, 1, 20, 5, 7, 12, 6, 14, 21] by using eight sets of fuzzy numbers represented in Fig. 5 and 6 . The results are shown in Tables 1 and 2.

For set 1 in Fig. 5, methods presented in [1, 20] only rank normal trapezoidal fuzzy numbers and method presented in [12] shows $A \approx B$ where there is no logical result but other methods and the proposed method get $A \prec B$.

For set 2 in Fig. 5, all methods presented in [19, 1, 20, 5, 7, 12, 6, 14, 21] and the proposed method get the same ranking order: $A \succ B$.

For set 3 in Fig. 5, methods presented in [19, 1, 12, 7, 6] show $A \approx B$ that ranking cannot be correct. The methods presented in [5, 14, 21] show $A \prec B$ because dispersion is a negative factor in the ranking of these methods. Method presented in [20] and proposed method get $A \succ B$ that is a logical consequence.

For set 4 in Fig. 5, the methods presented in [5, 7, 14], and [20] ($\alpha = 0.5$) get $A \approx B$ and, [1, 12, 6] get $A \prec B$ which are not true by intuition. Method presented in [19, 21],[20] ($\alpha = 1$) and the proposed method get $A \succ B$ that match the observations.

For set 5 in Fig. 6, methods presented in [19, 1, 20] are not able to rank crisp numbers $A, B$ and $C$. By using methods presented in [5, 7, 12, 6, 14] we have $A \approx B \prec C$ and Wu, et al.'s method [21] get $A \prec C \prec B$. That are wrong ranking between $A, B$ and $C$. But in proposed method we have $A \prec B \prec C$ that is a logical consequence.

For set 6 in Fig. 6, methods presented in [1, 20] are not able to calculate score for non normal fuzzy number $A$. Chen and Sanguansat's method [7] get $A \approx B$ which is incorrect ranking. methods presented in [12, 6] show $A \succ B$ that do not match the observations but the presented methods in [19, 5, 14, 21] and the proposed method show $A \prec B$ that is a logical result.

For set 7 in Fig. 6, Abbasbandy's method [1] and Hajjari's method [12] get $A \prec B \prec C$ and $B \prec C \prec A$ respectively. By using methods presented in [5, 7, 6, 14] we have $A \approx B \prec C$. Methods presented in [19, 20, 21] and the proposed method show $B \prec A \prec C$ which is true by intuition.
For set 8 in Fig. 6, the methods presented in [19, 1, 20, 5, 7, 12] are not able to calculate score for fuzzy numbers $B$ and $C$. By using method presented in [6], [14] we have $C \prec A \prec B$. Wu et al.'s method [21] and the proposed method show $C \prec B \prec A$ which is true by intuition.

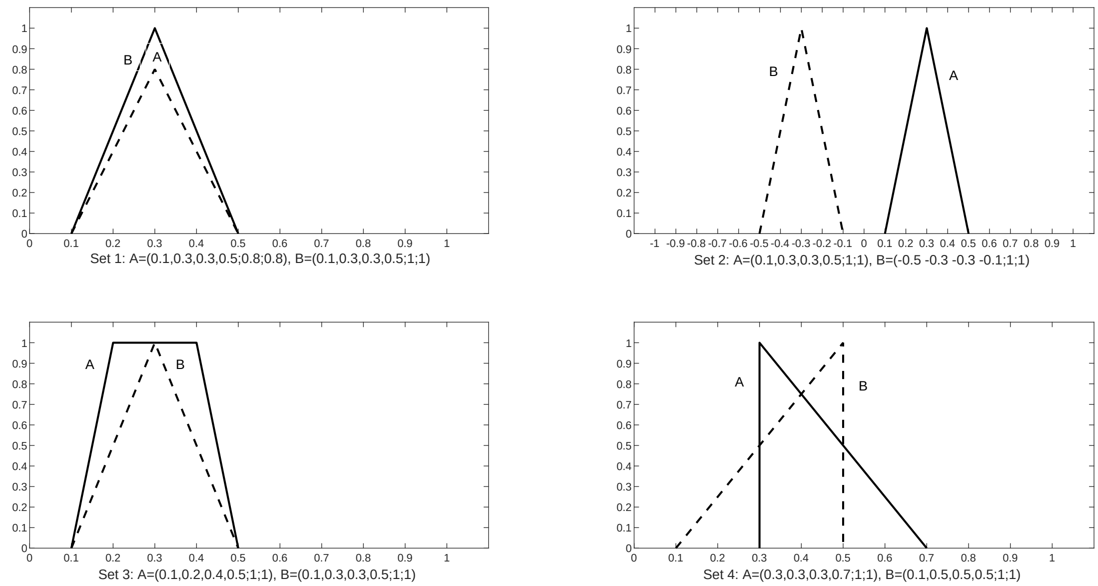


Figure 5: Sets of fuzzy numbers

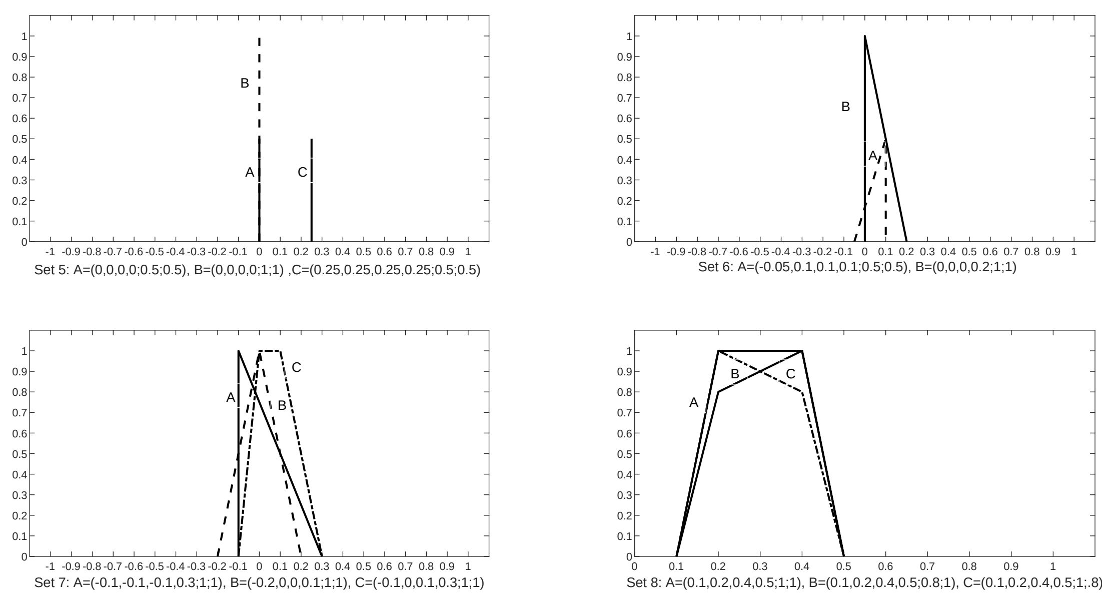


Figure 6: Sets of fuzzy numbers

Table 1: The comparison between existing methods of ranking the fuzzy numbers shown in Fig. 5

| Authors | Fuzzy numbers | Set 1 | Set 2 | Set 3 | Set 4 |
|---|---|---|---|---|---|
| Wang, Lee [19] | $A$ | | | | |
| | $B$ | | | | |
| Results | | $A \prec B$ | $A \succ B$ | $A \approx B$ | $A \succ B$ |
| Abbasbandy, Hajjari [1] | $A$ | $*$ | 0.3000 | 0.3000 | 0.3333 |
| | $B$ | 0.3000 | -0.3000 | 0.3000 | 0.4667 |
| Results | | N | $A \succ B$ | $A \approx B$ | $A \prec B$ |
| Wang, Luo [20] | $A$ | | | | |
| | $B$ | | | | |
| Results ($\alpha = 0.5, \alpha = 1$) | | N | $(A \succ B, A \succ B)$ | $(A \succ B, A \succ B)$ | $(A \approx B, A \succ B)$ |
| Chen, Chen [5] | $A$ | 0.2063 | 0.2537 | 0.2537 | 0.3333 |
| | $B$ | 0.2579 | -0.2597 | 0.2579 | 0.3333 |
| Results | | $A \prec B$ | $A \succ B$ | $A \prec B$ | $A \approx B$ |
| Chen, Sanguansat [7] | $A$ | 0.2824 | 0.3000 | 0.3000 | 0.4000 |
| | $B$ | 0.3000 | -0.3000 | 0.3000 | 0.4000 |
| Results | | $A \prec B$ | $A \succ B$ | $A \approx B$ | $A \approx B$ |
| Hajjari [12] | $A$ | 0.6000 | 1.2000 | 0.6000 | 0.8667 |
| | $B$ | 0.6000 | -1.2000 | 0.6000 | 1.1333 |
| Results | | $A \approx B$ | $A \succ B$ | $A \approx B$ | $A \prec B$ |
| Chen, et al. [6] | $A$ | 0.2462 | 0.2553 | 0.2553 | 0.3810 |
| | $B$ | 0.2553 | -0.2553 | 0.2553 | 0.3934 |
| Results | | $A \prec B$ | $A \succ B$ | $A \approx B$ | $A \prec B$ |
| Jiang [14] | $A$ | 0.2306 | 0.2882 | 0.2869 | 0.5714 |
| | $B$ | 0.2882 | -0.2882 | 0.2882 | 0.5714 |
| Results | | $A \prec B$ | $A \succ B$ | $A \prec B$ | $A \approx B$ |
| Wu, et al. [21] | $A$ | 0.5322 | 0.5906 | 0.5884 | 0.6604 |
| | $B$ | 0.5906 | -0.5906 | 0.5906 | 0.6235 |
| Results | | $A \prec B$ | $A \succ B$ | $A \prec B$ | $A \succ B$ |
| The proposed method | $A$ | 0.2300 | 0.2500 | 0.2833 | 0.3611 |
| | $B$ | 0.2500 | -0.2500 | 0.2500 | 0.3056 |
| Results | | $A \prec B$ | $A \succ B$ | $A \succ B$ | $A \succ B$ |

# 5 An application of ranking of fuzzy number

In this section, we apply the proposed ranking method for fuzzy risk analysis.

**Example 5.1.** *In Table 3, the effects of some factors (Fac) on type 2 diabetes with linguistic terms (LingT) are presented that include: age, body mass index (BMI), family history (FHis), blood triglyceride level (BTL) and hypertension that causes high blood pressure and identified by systolic pressure (SP) and diastolic pressure (DP) [18]. Linguistic terms for five persons $S_1, S_2, S_3, S_4$ and $S_5$ associated with mentioned factors and Physical weakness (PW) due to factors given in Table 4. Notice that in this example we use the fuzzy linguistic terms in [3]. The goal is to determine which persons, $S_1, S_2, S_3, S_4$ and $S_5$, have a highest risk of developing diabetes. We have also used the following abbreviations: Very Low (VL), Low (L), Fairly Low (FL), Medium (M), Fairly High (FH), High (H) and Very High (VH).*

To solve this problem, we first use the Schumacher formula, then by proposed ranking method identify the person with the highest risk.

$$\tilde{R_i} = \sum_{j=1}^{j=n} \frac{\tilde{W_{ij}} \otimes \tilde{R_{ij}}}{\tilde{W_{ij}}}$$

in which $\tilde{R_i}$ is the risk of affected diabetes for $i$th person and $\tilde{R_{ij}}$ and $\tilde{W_{ij}}$ indicate the probability of being affected and the PW due to $j$th factor for $i$th person respectively.

$$\tilde{R_1} = (0.0757, 0.1282, 0.2202, 0.2703; 0.8; 0.8),$$

$$\tilde{R_2} = (0.2083, 0.2608, 0.4010, 0.4587; 0.7; 0.7),$$

$$\tilde{R_3} = (0.3834, 0.4288, 0.5148, 0.5535; 0.8; 0.8),$$

$$\tilde{R_4} = (0.1629, 0.2273, 0.3498, 0.404; 0.7; 0.7),$$

$$\tilde{R_5} = (0.4959, 0.5571, 0.6868, 0.7395; 0.8; 0.8).$$

Now we calculate the scores of generalized fuzzy numbers $\tilde{R_1}, \tilde{R_2}, \tilde{R_3}, \tilde{R_4}$ and $\tilde{R_5}$.

$$G_{\tilde{R_1}} = (0.1735, 0.3523),\ S_{1\tilde{R_1}} = 0.5001,\ S_{2\tilde{R_1}} = 0.3522,\ score(\tilde{R_1}) = 0.1479,$$

Table 2: The comparison between existing methods of ranking the fuzzy numbers shown in Fig. 6

| Authors | Fuzzy numbers | Set 5 | Set 6 | Set 7 | Set 8 |
|---|---|---|---|---|---|
| Wang, Lee [19] | $A$ | | | | |
| | $B$ | | | | |
| | $C$ | | ........ | | |
| Results | | N | $A \prec B$ | $B \prec A \prec C$ | N |
| Abbasbandy, Hajjari [1] | $A$ | $*$ | $*$ | -0.0667 | 0.3000 |
| | $B$ | 0 | 0.0167 | -0.0083 | $*$ |
| | $C$ | $*$ | ........ | 0.0583 | $*$ |
| Results | | N | N | $A \prec B \prec C$ | N |
| Wang, Luo [20] | $A$ | | | | |
| | $B$ | | | | |
| | $C$ | | ........ | | |
| Results ($\alpha = 0.5, \alpha = 1$) | | N | N | $(B \prec A \prec C, B \prec A \prec C)$ | N |
| Chen, Chen [5] | $A$ | 0 | 0.0297 | 0 | 0.2537 |
| | $B$ | 0 | 0.4546 | 0 | $*$ |
| | $C$ | 0.1250 | ........ | 0.6410 | $*$ |
| Results | | $A \approx B \prec C$ | $A \prec B$ | $A \approx B \prec C$ | N |
| Chen, Sanguansat [7] | $A$ | 0 | 0.0500 | 0 | 0.3 |
| | $B$ | 0 | 0.0500 | 0 | $*$ |
| | $C$ | 0.2000 | ........ | 0.0750 | $*$ |
| Results | | $A \approx B \prec C$ | $A \approx B$ | $A \approx B \prec C$ | N |
| Hajjari [12] | $A$ | 0 | 0.3250 | 0.2333 | 0.6000 |
| | $B$ | 0 | 0.0833 | -0.2167 | $*$ |
| | $C$ | 0.7500 | ........ | 0.2167 | $*$ |
| Results | | $A \approx B \prec C$ | $A \succ B$ | $B \prec C \prec A$ | N |
| Chen, et al. [6] | $A$ | 0 | 0.0431 | 0 | 0.2533 |
| | $B$ | 0 | 0.0400 | 0 | 0.2687 |
| | $C$ | 0.1111 | ........ | 0.6060 | 0.2420 |
| Results | | $A \approx B \prec C$ | $A \succ B$ | $A \approx B \prec C$ | $C \prec A \prec B$ |
| Jiang [14] | $A$ | 0 | 0.0305 | 0 | 0.2869 |
| | $B$ | 0 | 0.0488 | 0 | 0.3012 |
| | $C$ | 0.1250 | ........ | 0.0719 | 0.2726 |
| Results | | $A \approx B \prec C$ | $A \prec B$ | $A \approx B \prec C$ | $C \prec A \prec B$ |
| Wu, et al. [21] | $A$ | 0.0300 | 0.3170 | 0.4388 | 0.5884 |
| | $B$ | 0.4460 | 0.4689 | 0.4244 | 0.5662 |
| | $C$ | 0.4385 | ...... | 0.4679 | 0.5454 |
| Results | | $A \prec C \prec B$ | $A \prec B$ | $B \prec A \prec C$ | $C \prec B \prec A$ |
| The proposed method | $A$ | $0_1$ | 0.0330 | 0.0278 | 0.2833 |
| | $B$ | $0_2$ | 0.5560 | 0 | 0.2774 |
| | $C$ | 0.1875 | ........ | 0.0720 | 0.2641 |
| Results | | $A \prec B \prec C$ | $A \prec B$ | $B \prec A \prec C$ | $C \prec B \prec A$ |

"$*$"and "N" denote the method cannot campute score and cannot rank the fuzzy numbers respectively.

$$G_{\tilde{R}_2} = (0.3323, 0.3171),\ S_{1\tilde{R}_2} = 0.5443,\ S_{2\tilde{R}_2} = 0.2728,\ score(\tilde{R}_2) = 0.2715,$$

$$G_{\tilde{R}_3} = (0.4699, 0.3562),\ S_{1\tilde{R}_3} = 0.6293,\ S_{2\tilde{R}_3} = 0.2269,\ score(\tilde{R}_3) = 0.4024,$$

$$G_{\tilde{R}_4} = (0.2858, 0.3119),\ S_{1\tilde{R}_4} = 0.5220,\ S_{2\tilde{R}_4} = 0.2899,\ score(\tilde{R}_4) = 0.2321,$$

$$G_{\tilde{R}_5} = (0.6196, 0.3593),\ S_{1\tilde{R}_5} = 0.6959,\ S_{2\tilde{R}_5} = 0.1634,\ score(\tilde{R}_5) = 0.5324.$$

Since $\tilde{R}_1 \prec \tilde{R}_4 \prec \tilde{R}_2 \prec \tilde{R}_3 \prec \tilde{R}_5$ hence $S_5$ person has the highest probabbility for type 2 diabetes followed by $\tilde{R}_3, \tilde{R}_2, \tilde{R}_4$ and $\tilde{R}_1$.

Table 3: The effects of factors on type 2 diabetes with linguistic terms in Example 5.1.

| Fac\LingT | VL | L | FL | M | FH | H | VH |
|---|---|---|---|---|---|---|---|
| Age | under 30 | 30-45 and over 75 | | 45-65 | | | |
| BMI: $Kg/m^2$ | under 27 | | | 27-30 | | over 30 | |
| FHis | | none | | | yes | | |
| BTL: $mg/dL$ | | under 150 | 150-250 | | | 250-499 | 500 |
| BP { SP | | { $100 - 140$ | | { $140 - 160$ | | { $over 160$ | |
| BP { DP | | $70 - 90$ | | $90 - 100$ | | $over 100$ | |

Table 4: The fuzzy linguistic terms for $S_1, S_2, S_3, S_4$ and $S_5$ in Example 5.1.

| Fac, PW | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ |
|---|---|---|---|---|---|
| Age | L | L | M | L | M |
| PW | VL | VL | FH($w_L = w_R = 0.8$) | VL | FH($w_L = w_R = 0.8$) |
| BMI | VL | M | VL | M | H |
| PW | L | FL($w_L = w_R = 0.7$) | L | FH($w_L = w_R = 0.7$) | H($w_L = w_R = 0.95$) |
| FHis | L | L | L | L | FH |
| PW | L($w_L = w_R = 0.8$) | L($w_L = w_R = 0.8$) | L($w_L = w_R = 0.8$) | L($w_L = w_R = 0.8$) | H($w_L = w_R = 0.95$) |
| BTL | FL | FL | H | FL | H |
| PW | M($w_L = w_R = 0.9$) | M($w_L = w_R = 0.9$) | VH | M($w_L = w_R = 0.9$) | VH |
| BP | L | M | L | L | L |
| PW | H($w_L = w_R = 0.95$) | VL | H($w_L = w_R = 0.95$) | H($w_L = w_R = 0.95$) | H($w_L = w_R = 0.95$) |

**Example 5.2.** *An example of fuzzy risk analysis was presented in [5], [7] and with changes in [6]. Here, we focus our attention on solving that example in the final step, namely, ranking, using our proposed ranking method. Now the assumption of the probability of failure of each component $A_i$ generated by manufactory $C_i$ is equal $\tilde{R}_i$ for $i = 1, 2, 3$.*

$$\tilde{R}_1 = (0.1765, 0.2860, 0.7244, 1.0574; 0.5; 0.6),$$

$$\tilde{R}_2 = (0.3221, 0.4949, 1.1392, 1.6373; 0.4; 0.5),$$

$$\tilde{R}_3 = (0.3290, 0.4890, 1.1737, 1.7787; 0.5; 0.6).$$

We first convert $\tilde{R}_1$, $\tilde{R}_2$ and $\tilde{R}_3$ to standard numbers $R_1$, $R_2$ and $R_3$ shown in Fig. 7.

$$R_1 = (\frac{0.1765}{2}, \frac{0.2860}{2}, \frac{0.7244}{2}, \frac{1.0574}{2}; 0.5; 0.6) = (0.0883, 0.1430, 0.3622, 0.5287; 0.5; 0.6),$$

$$R_2 = (\frac{0.3221}{2}, \frac{0.4949}{2}, \frac{1.1392}{2}, \frac{1.6373}{2}; 0.4; 0.5) = (0.1611, 0.2475, 0.5696, 0.8187; 0.4; 0.5),$$

$$R_3 = (\frac{0.3290}{2}, \frac{0.4890}{2}, \frac{1.1737}{2}, \frac{1.7787}{2}; 0.5; 0.6) = (0.1645, 0.2445, 0.5869, 0.8894; 0.5; 0.6).$$

We will have

$$G_{R_1} = (0.2901, 0.2471),\ G_{R_2} = (0.4655, 0.2024),\ G_{R_3} = (0.4883, 0.2455)$$

and

$$S_{1R_1} = 0.4819,\ S_{2R_1} = 0.2652,\ score(R_1) = 0.2167$$

$$S_{1R_2} = 0.5147,\ S_{2R_2} = 0.1877,\ score(R_2) = 0.3269$$

$$S_{1R_3} = 0.5547,\ S_{2R_2} = 0.1907,\ score(R_3) = 0.3640.$$

Therefore $R_1 \prec R_2 \prec R_3$; that is, the ranking order of the risk of manufactories $C_1, C_2$ and $C_3$ is $C_1 \prec C_2 \prec C_3$; that is the component $A_3$ generated by manufactory $C_3$ has a highest probability of failure, then $C_2, C_1$ respectively.

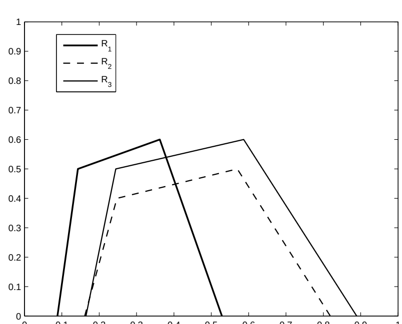


Figure 7: Set of standard fuzzy numbers $R_1, R_2, R_3$

# 6 Conclusion

This work introduces the new method for ranking generalized fuzzy numbers with different left and right heights. By using the centers of gravity of the fuzzy numbers, centers of gravity of left and right side crisp numbers and left and right areas associated with them, this method ranked the fuzzy numbers. The proposed method having standards of the fuzzy ranking numbers, can cover the defects and shortcomings of previously mentioned methods . It is hoped that this method will provide a more useful and more general future for ranking fuzzy numbers and its applications.


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# A novel method for ranking generalized fuzzy numbers with two different heights and its application in fuzzy risk analysis

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## روشی جدید برای رتبه‌بندی اعداد فازی تعمیم یافته با دو ارتفاع متفاوت و کاربرد آن در آنالیز ریسک فازی

**چکیده.** با توجه به کاربردهای فراوان اعداد فازی، رتبه‌بندی این اعداد بسیار مهم است. در این مقاله، روشی جدید برای رتبه‌بندی اعداد فازی تعمیم یافته با ارتفاع‌های چپ و راست متفاوت ارائه داده‌ایم. روش پیشنهادی، در وهله اول مرکز ثقل اعداد فازی و اعداد قطعی سمت چپ و راست را بدست آورده، سپس با محاسبه مساحت‌های چپ و راست متناظر، اعداد فازی را رتبه‌بندی می‌نماید. روش پیشنهادی می‌تواند نارسایی‌ها و نواقص برخی از روش‌های رتبه‌بندی را برطرف کند و مثال‌های ارائه شده تایید کننده این نکته می‌باشند. در پایان این روش را برای مسئله آنالیز ریسک فازی بکار برده‌ایم.