

Building fuzzy approach with linearization technique for fully rough multi-objective multi-level linear fractional programming problem

E. Fathy¹

¹*Department of Mathematics, Faculty of Science, Helwan University, Cairo, Egypt*

eman.fathy@science.helwan.edu.eg

Abstract

This paper presents a suitable solution procedure to solve the fully rough multi-objective multi-level linear fractional programming (FRMMFP) problem. First, an extension of interval method is presented to deal with roughness of the stated problem. Then, an iterative technique is proposed for linearization of fractional objectives. Finally, a modification of fuzzy approach is provided in the environment of the fully rough to solve the linear model. An example is provided for understanding the solution procedure of the proposed method.

Keywords: Fully rough programming, multi-level programming, multi-objective programming, fractional programming, fuzzy approach.

1 Introduction

A multi-objective linear fractional programming (MOLFP) problem is an optimization problem that contains several linear fractional objective functions under some linear limitations. In literature, there are many different studies to solve various models of MOLFP problems; some of them deal with theory [14, 15] and some of them concern solution methods [10, 24] and applications [19, 28]. Pramy and Islam [26] proposed a method to find an efficient solution for optimising MOLFP problems. Ammar and Khalifa [4] formulated an MOLFP problem involving fuzzy parameters in the objective functions and constraints (IVF-MOLFP). The variable transformation method and fuzzy programming approach are used to deal with the IVF-MOLFP problem. Acharya et al. [2] investigated a compromise solution to the multi-objective probabilistic fractional programming problem. Nayak and Ojha [20] used the parametric functions to introduce an iterative technique to obtain the preferred optimal solution of an MOLFP problem. Arya and Singh [5] proposed an iterative fuzzy approach to search fuzzy efficient solution set for multi-objective linear fractional programming problem. Multi-level programming techniques are formulated to solve decentralized problems with multiple decision makers who are organized in hierarchical levels [18]. In order to overcome such problems, several approaches have been proposed in literature [7, 21]. Osman et al. [22] modified Shi and Xias [32] work such as to develop an interactive approach which yields a compromise solution for optimising multi-level multi-objective fractional programming problem under hybrid uncertainty. By applying fuzzy goal programming, Kassa and Tsegay [16] proposed an algorithm for solving tri-level quadratic fractional programming problems. Sadeghi and Moslemi [30] developed a new algorithm to find an optimal solution for the linear bi-level multi-follower programming problem. Based on the bounded decomposition method and the separable programming, Abd Alhakim et al., [1] proposed an effective method to find the fuzzy optimal solution for a fully fuzzy multi-level quadratically constrained quadratic programming problem. Lachhwani [17] presented a new modified method for solving multi-level multi-objective linear fractional programming problems based on fuzzy goal programming approach.

Rebolledo [27] proposed the rough interval programming (RIP) based on the interval analysis to deal with partially unknown or ill-defined parameters and variables. Several authors [13, 23] studied the RIP. Pandian et al., [25] proposed a new method, namely the slice-sum method, to obtain the rough integer optimal solution for fully rough interval

transportation problems. Based on the TOPSIS method and the lower and upper approximation method, El-Feky and Abou-El-Enien [11] introduced a computational hybrid algorithm to generate compromise solutions for rough multi-level multi-objective integer decision making problems. Saad and Fathy [29] proposed a method to find a fuzzy rough solution for fuzzy variables linear programming problems with rough intervals coefficients. Akilbasha et al., [3] proposed a new method, namely, separation method based on zero point method for finding an optimal solution for integer transportation problems where transportation cost, supply and demand are intervals. Biswas and Pal [8] presented a priority-based fuzzy goal programming (FGP) method for solving the congestion management problem in electric power transmission lines by employing genetic algorithm. Sharma et al., [31] presented interesting approach with application of rough sets in forecasting models. They compared the performance of forecasting models using rough set methods, Total Roughness, Min-Min Roughness and Maximum Dependency of attributes.

Up to the best of our knowledge, no work so far has investigated. This main research gap leads to tow questions that need to be answered:

- Is there any method that consider FRMMFP problem in fully rough environment?
- If there is such method, can we use it to solve complicated applications?

Searching for the optimal solution of FRMMFP problem is a very difficult task. Various procedures have been developed for solving the FRMMFP problem with rough coefficients and crisp decision variables. Each of these approaches is Osman et al. [23]. They introduced an algorithm for solving MMFP problems where all of its coefficients in objective functions and constraints are rough interval while the decision variables are crisp. The proposed methodology in our paper is only one of the techniques that introduced for solving the MMFP problems in fully rough data where the coefficients in the objective functions and constraints are and also the decision variables are rough intervals.

This paper continues the pioneering work of the authors previous article [12] for applying the fuzzy approach to the fully fuzzy multi-objective multi-level integer quadratic programming problems. In this paper, the author extends the same idea by explaining new methodological developments in fully rough data. The proposed concept will be helpful in solving real life problems involving linear fractional programming problems in agriculture, production planning, financial and corporate planning, health care, hospital management, etc. The paper is organized as follows: The next section outlines the methodology, introduces the formulation of fully rough multi-objective multi-level linear fractional programming (FRMMFP) problem, the processes of the crisping and linearization, and presents the extended fuzzy approach for the solution of an FRMMFP problem. Section3, presented a flowchart to illustrate the decision making process of the proposed problem. Section 4, provides a numerical example of the proposed method. Advantages of the proposed approach are highlighted in Section 5. Finally, Section 6 provides the conclusion.

2 Methodology

2.1 Formulation of a fully rough multi-objective multi-level linear fractional programming problem

Consider an FRMMFP problem with p number of decision makers DMs at p different levels. Let the vector of decision rough variables $x^R \in R^n$ be partitioned among these DMs as $x_1^R \in R^{n_1}$, $x_2^R \in R^{n_2}$, ..., $x_p^R \in R^{n_p}$, where $\sum_{i=1}^p n_i = n$. Suppose that DM_i denotes the decision maker at the i th-level ($i = 1, 2, \dots, p$) that has control over x_i^R , and each level DMs has their own objective functions.

Hence an FRMMFP problem can be formulated as follows:

$$\begin{aligned}
 & \text{FRMMFP :} & (1) \\
 & \text{First-level decision maker}[DM_1] : \\
 & \max_{x_1^R} F_1^R(x^R) = \max_{x_1^R} (f_{11}^R(x^R), f_{12}^R(x^R), \dots, f_{1m_1}^R(x^R)) \\
 & \text{where } x_2^R, x_3^R, \dots, x_p^R \text{ solve} \\
 & \text{Second-level decision maker}[DM_2] : \\
 & \max_{x_2^R} F_2^R(x^R) = \max_{x_2^R} (f_{21}^R(x^R), f_{22}^R(x^R), \dots, f_{2m_2}^R(x^R))
 \end{aligned}$$

where $x_3^R, x_4^R, \dots, x_p^R$ solve

⋮

where x_p^R solve

Pth-level decision maker $[DM_p]$:

$$\max_{x_p^R} F_p^R(x^R) = \max_{x_p^R} (f_{p1}^R(x^R), f_{p2}^R(x^R), \dots, f_{pm_p}^R(x^R))$$

$$\text{subject to } x^R \in G^R = \{x^R \in R^n \mid \sum_{j=1}^n a_{sj}^R \otimes x_j^R \leq b_s^R; x_j^R \geq 0\},$$

where G^R is the rough feasible choice, a_{sj}^R is an $m \times n$ rough-coefficients matrix, $b_s^R, (s = 1, 2, \dots, m)$ is a rough vector, and $F_i^R(x^R) = (f_{i1}^R(x^R), f_{i2}^R(x^R), \dots, f_{im_i}^R(x^R)); (i = 1, 2, \dots, p)$ is the vector of m_i distinct rough fractional objective functions for the i th-level decision maker,

$$f_{ir}^R(x^R) = \frac{\sum_{j=1}^n c_{ir}^{jR} \otimes x_j^R \oplus \alpha_{ir}^R}{\sum_{j=1}^n d_{ir}^{jR} \otimes x_j^R \oplus \beta_{ir}^R}, (i = 1, 2, \dots, p), \tag{2}$$

$r = 1, 2, \dots, m_1$ for the first-level objective functions,

$r = 1, 2, \dots, m_2$ for the second-level objective functions,

⋮

$r = 1, 2, \dots, m_p$ for the p th-level objective functions,

where c_{ir}^{jR} and d_{ir}^{jR} are n -vectors representing the coefficients of decision rough variables x_j^R , and α_{ir}^R and β_{ir}^R are n -vectors of constants. It is customary to assume that $D_{ir}^R = \sum_{j=1}^n d_{ir}^{jR} \otimes x_j^R \oplus \beta_{ir}^R > 0$ for all $x^R \in G^R$.

2.2 Equivalent crisp problem

The first step to solve the FRMMFP problem described by equation (1) is to convert its rough parameters and rough variables into their respective crisp equivalents. Hamzheeh et al.[13] developed a method to find optimal solutions of linear programming problem with rough interval coefficients. In this paper, the author extends the same idea to solve the FRMMFP problem.

Theorem 2.1. Let $A = ([a^2, a^3], [a^1, a^4])$ be a two-dimensional constant interval vector on real numbers R . The reciprocal of A^R is defined as follows:

$$\frac{1}{A} = ([\frac{1}{a^3}, \frac{1}{a^2}], [\frac{1}{a^4}, \frac{1}{a^1}]), \text{ this assumes } A \neq 0.$$

Let all of the rough parameters and the rough variables for both objective functions and constraints for each level in equations (1)-(2) be represented by rough intervals, each of these has the following values: $F_i^R(x^R) = ([F_i^2(x), F_i^3(x)], [F_i^1(x), F_i^4(x)])$, $f_{ir}^R(x^R) = ([f_{ir}^2(x), f_{ir}^3(x)], [f_{ir}^1(x), f_{ir}^4(x)])$, $c_{ir}^{jR} = ([c_{ir}^{j2}, c_{ir}^{j3}], [c_{ir}^{j1}, c_{ir}^{j4}])$, $d_{ir}^{jR} = ([d_{ir}^{j2}, d_{ir}^{j3}], [d_{ir}^{j1}, d_{ir}^{j4}])$, $x_j^R = ([x_j^2, x_j^3], [x_j^1, x_j^4])$, $\alpha_{ir}^R = ([\alpha_{ir}^2, \alpha_{ir}^3], [\alpha_{ir}^1, \alpha_{ir}^4])$, $\beta_{ir}^R = ([\beta_{ir}^2, \beta_{ir}^3], [\beta_{ir}^1, \beta_{ir}^4])$, $a_{sj}^R = ([a_{sj}^2, a_{sj}^3], [a_{sj}^1, a_{sj}^4])$ and $b_s^R = ([b_s^2, b_s^3], [b_s^1, b_s^4])$

Conforming to the method in [13], an FRMMFP problem is decomposed into two interval multi-objective multi-level linear fractional programming (IMMFP) problems for each level, as follows:

i th-level decision maker $[DM_i](i = 1, 2, \dots, p)$:

$$(U_i - IMMFP) : \max_{x_i^1, x_i^4} [F_i^1(x^1, x^4), F_i^4(x^1, x^4)] \tag{3}$$

$$= \max_{x_i^1, x_i^4} ([f_{i1}^1(x^1, x^4), f_{i1}^4(x^1, x^4)], [f_{i2}^1(x^1, x^4), f_{i2}^4(x^1, x^4)], \dots, [f_{im_i}^1(x^1, x^4), f_{im_i}^4(x^1, x^4)])$$

$$= \max_{x_i^1, x_i^4} ([\frac{\sum_{j=1}^n c_{i1}^{j1} x_j^1 + \alpha_{i1}^1}{\sum_{j=1}^n d_{i1}^{j4} x_j^4 + \beta_{i1}^4}, \frac{\sum_{j=1}^n c_{i1}^{j4} x_j^4 + \alpha_{i1}^4}{\sum_{j=1}^n d_{i1}^{j1} x_j^1 + \beta_{i1}^1}], \dots, [\frac{\sum_{j=1}^n c_{im_i}^{j1} x_j^1 + \alpha_{im_i}^1}{\sum_{j=1}^n d_{im_i}^{j4} x_j^4 + \beta_{im_i}^4}, \frac{\sum_{j=1}^n c_{im_i}^{j4} x_j^4 + \alpha_{im_i}^4}{\sum_{j=1}^n d_{im_i}^{j1} x_j^1 + \beta_{im_i}^1}]),$$

where $x_{i+1}^1, x_{i+1}^4, \dots, x_p^1, x_p^4$ solve

subject to $x^1, x^4 \in \bar{G} = \{x^1, x^4 \in R^n \mid \sum_{j=1}^n [a_{sj}^1 x_j^1, a_{sj}^4 x_j^4] \leq [b_s^1, b_s^4]; x_j^1, x_j^4 \geq 0\}$

$$(L_i - IMMFP) : \max_{x_i^2, x_i^3} [F_i^2(x^2, x^3), F_i^3(x^2, x^3)] \tag{4}$$

$$= \max_{x_i^2, x_i^3} ([f_{i1}^2(x^2, x^3), f_{i1}^3(x^2, x^3)], [f_{i2}^2(x^2, x^3), f_{i2}^3(x^2, x^3)], \dots, [f_{im_i}^2(x^2, x^3), f_{im_i}^3(x^2, x^3)])$$

$$= \max_{x_i^2, x_i^3} \left(\left[\frac{\sum_{j=1}^n c_{i1}^{j2} x_j^2 + \alpha_{i1}^2}{\sum_{j=1}^n d_{i1}^{j3} x_j^3 + \beta_{i1}^3}, \frac{\sum_{j=1}^n c_{i1}^{j3} x_j^3 + \alpha_{i1}^3}{\sum_{j=1}^n d_{i1}^{j2} x_j^2 + \beta_{i1}^2} \right], \dots, \left[\frac{\sum_{j=1}^n c_{im_i}^{j2} x_j^2 + \alpha_{im_i}^2}{\sum_{j=1}^n d_{im_i}^{j3} x_j^3 + \beta_{im_i}^3}, \frac{\sum_{j=1}^n c_{im_i}^{j3} x_j^3 + \alpha_{im_i}^3}{\sum_{j=1}^n d_{im_i}^{j2} x_j^2 + \beta_{im_i}^2} \right] \right)$$

where $x_{i+1}^2, x_{i+1}^3, \dots, x_p^2, x_p^3$ solve

subject to $x^2, x^3 \in \underline{G} = \{x^2, x^3 \in R^n \mid \sum_{j=1}^n [a_{sj}^2 x_j^2, a_{sj}^3 x_j^3] \leq [b_s^2, b_s^3]; x_j^2, x_j^3 \geq 0\}$.

Definition 2.2. In equation(3), the author defines the following sets:

- $U^1 = \{x^4 \in R^n \mid \sum_{j=1}^n a_{sj}^4 x_j^4 \leq b_s^4, x_j^4 \geq 0\}$,
- $U^2 = \{x^4 \in R^n \mid \sum_{j=1}^n a_{sj}^4 x_j^4 \leq b_s^4, x_j^4 \geq 0\}$,
- $U^3 = \{x^1 \in R^n \mid \sum_{j=1}^n a_{sj}^1 x_j^1 \leq b_s^1, x_j^1 \geq 0\}$,
- $U^4 = \{x^1 \in R^n \mid \sum_{j=1}^n a_{sj}^1 x_j^1 \leq b_s^1, x_j^1 \geq 0\}$,

where $U^1 \subseteq U^2 \subseteq U^3 \subseteq U^4$.

Definition 2.3. In equation(4), the author defines the following sets:

- $L^1 = \{x^3 \in R^n \mid \sum_{j=1}^n a_{sj}^3 x_j^3 \leq b_s^3, x_j^3 \geq 0\}$,
- $L^2 = \{x^3 \in R^n \mid \sum_{j=1}^n a_{sj}^3 x_j^3 \leq b_s^3, x_j^3 \geq 0\}$,
- $L^3 = \{x^2 \in R^n \mid \sum_{j=1}^n a_{sj}^2 x_j^2 \leq b_s^2, x_j^2 \geq 0\}$,
- $L^4 = \{x^2 \in R^n \mid \sum_{j=1}^n a_{sj}^2 x_j^2 \leq b_s^2, x_j^2 \geq 0\}$,

where $L^1 \subseteq L^2 \subseteq L^3 \subseteq L^4$.

Definition 2.4. The objective functions in equation (3) have most favorable value and least favorable value at the feasible sets $U^2 \cup U^3$ and $U^1 \cup U^4$, respectively.

Definition 2.5. The objective functions in equation (4) have most favorable value and least favorable value at the feasible sets $L^2 \cup L^3$ and $L^1 \cup L^4$, respectively.

Theorem 2.6. Consider the inequality $\sum_{j=1}^n [a_{ij}, \bar{a}_{ij}] x_j \geq [\underline{b}_i, \bar{b}_i]$ where $x_j \geq 0 (j = 1, 2, \dots, n)$. Then, $\sum_{j=1}^n \bar{a}_{ij} x_j \geq \underline{b}_i$ and $\sum_{j=1}^n a_{ij} x_j \leq \bar{b}_i$ are the minimum value range and maximum value range inequalities, respectively.

With the help of Definitions 2.4 and 2.5, equations (3) and (4) are transformed into four crisp multi-objective multi-level linear fractional programming (MMFP) problems, as follows:

ith – level decision maker $[DM_i] (i = 1, 2, \dots, p) :$ (5)

$$(\bar{L}_i - MMFP) : \max_{x_i^1, x_i^4} F_i^1(x^1, x^4) = \max_{x_i^1, x_i^4} (f_{i1}^1(x^1, x^4), \dots, f_{im_i}^1(x^1, x^4)) = \left(\frac{\sum_{j=1}^n c_{i1}^{j1} x_j^1 + \alpha_{i1}^1}{\sum_{j=1}^n d_{i1}^{j4} x_j^4 + \beta_{i1}^4}, \dots, \frac{\sum_{j=1}^n c_{im_i}^{j1} x_j^1 + \alpha_{im_i}^1}{\sum_{j=1}^n d_{im_i}^{j4} x_j^4 + \beta_{im_i}^4} \right),$$

where $x_{i+1}^1, x_{i+1}^4, \dots, x_p^1, x_p^4$ solve

subject to $x^1, x^4 \in G^1 = \{x^1, x^4 \in R^n \mid \sum_{j=1}^n a_{sj}^4 x_j^4 \leq b_s^4, \sum_{j=1}^n a_{sj}^1 x_j^1 \leq b_s^1; x^1, x^4 \geq 0\}$,

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$$(\underline{L}_i - MMFP) : \max_{x^2, x^3} F_i^2(x^2, x^3) = \max_{x^2, x^3} (f_{i1}^2(x^2, x^3), \dots, f_{im_i}^2(x^2, x^3)) = \left(\frac{\sum_{j=1}^n c_{i1}^{j2} x_j^2 + \alpha_{i1}^2}{\sum_{j=1}^n d_{i1}^{j3} x_j^3 + \beta_{i1}^3}, \dots, \frac{\sum_{j=1}^n c_{im_i}^{j2} x_j^2 + \alpha_{im_i}^2}{\sum_{j=1}^n d_{im_i}^{j3} x_j^3 + \beta_{im_i}^3} \right),$$

where $x_{i+1}^2, x_{i+1}^3, \dots, x_p^2, x_p^3$ solve

$$\text{subject to } x^2, x^3 \in G^2 = \{x^2, x^3 \in R^n \mid \sum_{j=1}^n a_{sj}^3 x_j^3 \leq b_s^2, \sum_{j=1}^n a_{sj}^2 x_j^2 \leq b_s^3; \quad x^2, x^3 \geq 0\},$$

$$(\underline{U}_i - MMFP) : \max_{x^2, x^3} F_i^3(x^2, x^3) = \max_{x^2, x^3} (f_{i1}^3(x^2, x^3), \dots, f_{im_i}^3(x^2, x^3)) = \left(\frac{\sum_{j=1}^n c_{i1}^{j3} x_j^3 + \alpha_{i1}^3}{\sum_{j=1}^n d_{i1}^{j2} x_j^2 + \beta_{i1}^2}, \dots, \frac{\sum_{j=1}^n c_{im_i}^{j3} x_j^3 + \alpha_{im_i}^3}{\sum_{j=1}^n d_{im_i}^{j2} x_j^2 + \beta_{im_i}^2} \right),$$

where $x_{i+1}^2, x_{i+1}^3, \dots, x_p^2, x_p^3$ solve

$$\text{subject to } x^2, x^3 \in G^3 = \{x^2, x^3 \in R^n \mid \sum_{j=1}^n a_{sj}^2 x_j^2 \leq b_s^2, \sum_{j=1}^n a_{sj}^3 x_j^3 \leq b_s^3; \quad x^2, x^3 \geq 0\},$$

$$(\overline{U}_i - MMFP) : \max_{x^1, x^4} F_i^4(x^1, x^4) = \max_{x^1, x^4} (f_{i1}^4(x^1, x^4), \dots, f_{im_i}^4(x^1, x^4)) = \left(\frac{\sum_{j=1}^n c_{i1}^{j4} x_j^4 + \alpha_{i1}^4}{\sum_{j=1}^n d_{i1}^{j1} x_j^1 + \beta_{i1}^1}, \dots, \frac{\sum_{j=1}^n c_{im_i}^{j4} x_j^4 + \alpha_{im_i}^4}{\sum_{j=1}^n d_{im_i}^{j1} x_j^1 + \beta_{im_i}^1} \right),$$

where $x_{i+1}^1, x_{i+1}^4, \dots, x_p^1, x_p^4$ solve

$$\text{subject to } x^1, x^4 \in G^4 = \{x^1, x^4 \in R^n \mid \sum_{j=1}^n a_{sj}^1 x_j^1 \leq b_s^1, \sum_{j=1}^n a_{sj}^4 x_j^4 \leq b_s^4; \quad x^1, x^4 \geq 0\}.$$

2.3 Equivalent linear programming problem

In this paper, the authors main contribution is the establishment of a transformation technique for the linearization of fractional objective functions. With the help of this technique, all of fractional objective functions of an MMFP problem become a linear objective. Furthermore, the MMFP problem is converted into multi-objective multi-level linear programming (MMLP) problem. Thus, the computations and the complexity in solving the MMFP problem reduce in a certain amount. Thus, the author assumes the positivity of the denominator of the objective functions and excludes the case $\beta_{ir}^\ell = 0, (\ell = 1, 2, 3, 4)$.

To simplify, the author applies the steps of the linearization technique to objective functions $f_{ir}^1(x^1, x^4)$. In a similar way, objective functions $f_{ir}^2(x^2, x^3), f_{ir}^3(x^2, x^3)$, and $f_{ir}^4(x^1, x^4)$ are transformed into the corresponding linear form. Let $x_j^4 = x_j^4 - \varepsilon_j^4$, then

$$f_{ir}^1(x^1, x^4) = \frac{\sum_{j=1}^n c_{ir}^{j1} (x_j^4 - \varepsilon_j^4) + \alpha_{ir}^1}{\sum_{j=1}^n d_{ir}^{j4} x_j^4 + \beta_{ir}^4}. \tag{6}$$

Multiplying both the denominator and the numerator of equation (6) by β_{ir}^4 , the result is as follows:

$$\begin{aligned} f_{ir}^1(x^1, x^4) &= \frac{\beta_{ir}^4 (\sum_{j=1}^n c_{ir}^{j1} (x_j^4 - \varepsilon_j^4) + \alpha_{ir}^1) + \alpha_{ir}^1 \sum_{j=1}^n d_{ir}^{j4} x_j^4 - \alpha_{ir}^1 \sum_{j=1}^n d_{ir}^{j4} x_j^4}{\beta_{ir}^4 (\sum_{j=1}^n d_{ir}^{j4} x_j^4 + \beta_{ir}^4)} \\ &= \frac{\alpha_{ir}^1 (\sum_{j=1}^n d_{ir}^{j4} x_j^4 + \beta_{ir}^4) - \beta_{ir}^4 \sum_{j=1}^n c_{ir}^{j1} \varepsilon_j^4 + \sum_{j=1}^n (c_{ir}^{j1} \beta_{ir}^4 - \alpha_{ir}^1 d_{ir}^{j4}) x_j^4}{\beta_{ir}^4 (\sum_{j=1}^n d_{ir}^{j4} x_j^4 + \beta_{ir}^4)} \\ &= \frac{\alpha_{ir}^1}{\beta_{ir}^4} - \frac{\sum_{j=1}^n c_{ir}^{j1} \varepsilon_j^4}{\sum_{j=1}^n d_{ir}^{j4} x_j^4 + \beta_{ir}^4} + \left(\sum_{j=1}^n (c_{ir}^{j1} - \frac{\alpha_{ir}^1}{\beta_{ir}^4} d_{ir}^{j4}) \right) \left(\frac{x_j^4}{\sum_{j=1}^n d_{ir}^{j4} x_j^4 + \beta_{ir}^4} \right). \end{aligned}$$

In order to simplify the notations, set $\varepsilon_{ir}^{j4} = \rho_{ir}^1 \varepsilon_j^4$, $\rho_{ir}^1 = \frac{1}{\sum_{j=1}^n d_{ir}^{j4} x_j^4 + \beta_{ir}^4}$, $y_{ir}^{j4} = \frac{x_j^4}{\sum_{j=1}^n d_{ir}^{j4} x_j^4 + \beta_{ir}^4}$. Therefore, the fractional objective functions $f_{ir}^1(x^1, x^4)$ can be simplified into the linear form as below:

$$f_{ir}^1(y_{ir}^4) = \frac{\alpha_{ir}^1}{\beta_{ir}^4} - \sum_{j=1}^n c_{ir}^{j1} \varepsilon_{ir}^{j4} + \sum_{j=1}^n (c_{ir}^{j1} - \frac{\alpha_{ir}^1}{\beta_{ir}^4} d_{ir}^{j4}) y_{ir}^{j4}.$$

Hence, equations (5) are transformed into four crisp MMLP problems, as follows:

ith – level decision maker $[DM_i](i = 1, 2, \dots, p)$: (7)

$$\begin{aligned} (\bar{L}_i - MMLP) : \max_{y_{ir}^4} F_i^1(y_{ir}^4) &= \max_{y_{ir}^4} (f_{i1}^1(y_{i1}^4), \dots, f_{im_i}^1(y_{im_i}^4)) \\ &= \left(\frac{\alpha_{i1}^1}{\beta_{i1}^4} - \sum_{j=1}^n c_{ij}^{j1} \varepsilon_{i1}^{j4} + \sum_{j=1}^n (c_{ij}^{j1} - \frac{\alpha_{i1}^1}{\beta_{i1}^4} d_{i1}^{j4}) y_{i1}^{j4}, \dots, \frac{\alpha_{im_i}^1}{\beta_{im_i}^4} - \sum_{j=1}^n c_{im_i}^{j1} \varepsilon_{im_i}^{j4} + \sum_{j=1}^n (c_{im_i}^{j1} - \frac{\alpha_{im_i}^1}{\beta_{im_i}^4} d_{im_i}^{j4}) y_{im_i}^{j4} \right), \end{aligned}$$

where $y_{ir}^4 (j = i + 1, 2, \dots, p), (r = 1, 2, \dots, m_i)$ solve

subject to $y_{ir}^4 \in G_{ir}^1 \cup G_{\rho_{ir}^1}^1$,

$$G_{ir}^1 = \left\{ \sum_{j=1}^n a_{sj}^4 y_{ir}^{j4} \leq \rho_{ir}^1 b_s^1, \sum_{j=1}^n a_{sj}^1 (y_{ir}^{j4} - \varepsilon_{ir}^{j4}) \leq \rho_{ir}^1 b_s^4; \quad y_{ir}^{j4} \geq 0 \right\},$$

$$G_{\rho_{ir}^1}^1 = \left\{ \sum_{j=1}^n d_{ir}^{j4} y_{ir}^{j4} + \beta_{ir}^4 \rho_{ir}^1 = 1 \right\}$$

$$(\underline{L}_i - MMLP) : \max_{y_{ir}^3} F_i^2(y_{ir}^3) = \max_{y_{ir}^3} (f_{i1}^2(y_{i1}^3), \dots, f_{im_i}^2(y_{im_i}^3)),$$

$$= \left(\frac{\alpha_{i1}^2}{\beta_{i1}^3} - \sum_{j=1}^n c_{ij}^{j2} \varepsilon_{i1}^{j3} + \sum_{j=1}^n (c_{ij}^{j2} - \frac{\alpha_{i1}^2}{\beta_{i1}^3} d_{i1}^{j3}) y_{i1}^{j3}, \dots, \frac{\alpha_{im_i}^2}{\beta_{im_i}^3} - \sum_{j=1}^n c_{im_i}^{j2} \varepsilon_{im_i}^{j3} + \sum_{j=1}^n (c_{im_i}^{j2} - \frac{\alpha_{im_i}^2}{\beta_{im_i}^3} d_{im_i}^{j3}) y_{im_i}^{j3} \right),$$

where $y_{ir}^3 (j = i + 1, 2, \dots, p), (r = 1, 2, \dots, m_i)$ solve

subject to $y_{ir}^3 \in G_{ir}^2 \cup G_{\rho_{ir}^2}^2$,

$$G_{ir}^2 = \left\{ \sum_{j=1}^n a_{sj}^3 y_{ir}^{j3} \leq \rho_{ir}^2 b_s^2, \sum_{j=1}^n a_{sj}^2 (y_{ir}^{j3} - \varepsilon_{ir}^{j3}) \leq \rho_{ir}^2 b_s^3; \quad y_{ir}^{j3} \geq 0 \right\},$$

$$G_{\rho_{ir}^2}^2 = \left\{ \sum_{j=1}^n d_{ir}^{j3} y_{ir}^{j3} + \beta_{ir}^3 \rho_{ir}^2 = 1 \right\}$$

$$(\underline{U}_i - MMLP) : \max_{y_{ir}^2} F_i^3(y_{ir}^2) = \max_{y_{ir}^2} (f_{i1}^3(y_{i1}^2), \dots, f_{im_i}^3(y_{im_i}^2)), \quad (8)$$

$$= \left(\frac{\alpha_{i1}^3}{\beta_{i1}^2} + \sum_{j=1}^n c_{ij}^{j3} \varepsilon_{i1}^{j2} + \sum_{j=1}^n (c_{ij}^{j3} - \frac{\alpha_{i1}^3}{\beta_{i1}^2} d_{i1}^{j2}) y_{i1}^{j2}, \dots, \frac{\alpha_{im_i}^3}{\beta_{im_i}^2} + \sum_{j=1}^n c_{im_i}^{j3} \varepsilon_{im_i}^{j2} + \sum_{j=1}^n (c_{im_i}^{j3} - \frac{\alpha_{im_i}^3}{\beta_{im_i}^2} d_{im_i}^{j2}) y_{im_i}^{j2} \right),$$

where $y_{ir}^2 (j = i + 1, 2, \dots, p), (r = 1, 2, \dots, m_i)$ solve

subject to $y_{ir}^2 \in G_{ir}^3 \cup G_{\rho_{ir}^3}^3$,

$$G_{ir}^3 = \left\{ \sum_{j=1}^n a_{sj}^2 y_{ir}^{j2} \leq \rho_{ir}^3 b_s^2, \sum_{j=1}^n a_{sj}^3 (y_{ir}^{j2} + \varepsilon_{ir}^{j2}) \leq \rho_{ir}^3 b_s^3; \quad y_{ir}^{j2} \geq 0 \right\},$$

$$G_{\rho_{ir}^3}^3 = \left\{ \sum_{j=1}^n d_{ir}^{j2} y_{ir}^{j2} + \beta_{ir}^2 \rho_{ir}^3 = 1 \right\},$$

$$(\bar{U}_i - MMLP) : \max_{y_{ir}^1} F_i^4(y_{ir}^1) = \max_{y_{ir}^1} (f_{i1}^4(y_{i1}^1), \dots, f_{im_i}^4(y_{im_i}^1)),$$

$$= \left(\frac{\alpha_{i1}^4}{\beta_{i1}^1} + \sum_{j=1}^n c_{ij}^{j4} \varepsilon_{i1}^{j1} + \sum_{j=1}^n (c_{ij}^{j4} - \frac{\alpha_{i1}^4}{\beta_{i1}^1} d_{i1}^{j1}) y_{i1}^{j1}, \dots, \frac{\alpha_{im_i}^4}{\beta_{im_i}^1} + \sum_{j=1}^n c_{im_i}^{j4} \varepsilon_{im_i}^{j1} + \sum_{j=1}^n (c_{im_i}^{j4} - \frac{\alpha_{im_i}^4}{\beta_{im_i}^1} d_{im_i}^{j1}) y_{im_i}^{j1} \right),$$

where $y_{ir}^1 (j = i + 1, 2, \dots, p), (r = 1, 2, \dots, m_i)$ solve

subject to $y_{ir}^1 \in G_{ir}^4 \cup G_{\rho_{ir}^4}^4$,

$$G_{ir}^4 = \left\{ \sum_{j=1}^n a_{sj}^1 y_{ir}^{j1} \leq \rho_{ir}^4 b_s^1, \sum_{j=1}^n a_{sj}^4 (y_{ir}^{j1} + \varepsilon_{ir}^{j1}) \leq \rho_{ir}^4 b_s^4; \quad y_{ir}^{j1} \geq 0 \right\},$$

$$G_{\rho_{ir}^4}^4 = \left\{ \sum_{j=1}^n d_{ir}^{j1} y_{ir}^{j1} + \beta_{ir}^1 \rho_{ir}^4 = 1 \right\}.$$

2.4 The modification of fuzzy programming approach

This section is based on the concepts the author discussed in his previous work [12]. In [12], the author discussed the fuzzy programming approach to deal with the fully fuzzy multi-objective and multi-level integer quadratic programming problems. In this paper, the author extends this idea by explaining new methodological developments in fully rough data.

2.4.1 Construction of the membership functions

Let $f_{ir}^{b\ell}$ and $f_{ir}^{w\ell}$, ($r = 1, 2, \dots, m_i; i = 1, 2, \dots, p; \ell = 1, 2, 3, 4$) be the individual best solutions and the individual worst solutions for each objective function, respectively, where

$$\begin{aligned} f_{ir}^{b1} &= \max_{y_{ir}^4 \in G_{ir}^1 \cup G_{\rho_{ir}^1}^1} f_{ir}^1 & f_{ir}^{w1} &= \min_{y_{ir}^4 \in G_{ir}^1 \cup G_{\rho_{ir}^1}^1} f_{ir}^1 \\ f_{ir}^{b2} &= \max_{y_{ir}^3 \in G_{ir}^2 \cup G_{\rho_{ir}^2}^2} f_{ir}^2 & f_{ir}^{w2} &= \min_{y_{ir}^3 \in G_{ir}^2 \cup G_{\rho_{ir}^2}^2} f_{ir}^2 \\ f_{ir}^{b3} &= \max_{y_{ir}^2 \in G_{ir}^3 \cup G_{\rho_{ir}^3}^3} f_{ir}^3 & f_{ir}^{w3} &= \min_{y_{ir}^2 \in G_{ir}^3 \cup G_{\rho_{ir}^3}^3} f_{ir}^3 \\ f_{ir}^{b4} &= \max_{y_{ir}^1 \in G_{ir}^4 \cup G_{\rho_{ir}^4}^4} f_{ir}^4 & f_{ir}^{w4} &= \min_{y_{ir}^1 \in G_{ir}^4 \cup G_{\rho_{ir}^4}^4} f_{ir}^4 \end{aligned}$$

According to Zimmermann [33], the associated membership function for the r th-objective function ($r = 1, 2, \dots, m_i$) which has to be optimized at DM_i can be formulated using the individual optimal solution in the following manner:

$$(\bar{L}_i - MF) : \mu_{f_{ir}^1}^1 (f_{ir}^1(y_{ir}^4)) = \begin{cases} 1, & \text{if } f_{ir}^1(y_{ir}^4) \geq f_{ir}^{b1} \\ \frac{f_{ir}^1(y_{ir}^4) - f_{ir}^{w1}}{f_{ir}^{b1} - f_{ir}^{w1}}, & \text{if } f_{ir}^{w1} < f_{ir}^1(y_{ir}^4) < f_{ir}^{b1} \\ 0, & \text{if } f_{ir}^{w1} \geq f_{ir}^1(y_{ir}^4). \end{cases} \quad (9)$$

$$(\underline{L}_i - MF) : \mu_{f_{ir}^2}^2 (f_{ir}^2(y_{ir}^3)) = \begin{cases} 1, & \text{if } f_{ir}^2(y_{ir}^3) \geq f_{ir}^{b2} \\ \frac{f_{ir}^2(y_{ir}^3) - f_{ir}^{w2}}{f_{ir}^{b2} - f_{ir}^{w2}}, & \text{if } f_{ir}^{w2} < f_{ir}^2(y_{ir}^3) < f_{ir}^{b2} \\ 0, & \text{if } f_{ir}^{w2} \geq f_{ir}^2(y_{ir}^3). \end{cases} \quad (10)$$

$$(\underline{U}_i - MF) : \mu_{f_{ir}^3}^3 (f_{ir}^3(y_{ir}^2)) = \begin{cases} 1, & \text{if } f_{ir}^3(y_{ir}^2) \geq f_{ir}^{b3} \\ \frac{f_{ir}^3(y_{ir}^2) - f_{ir}^{w3}}{f_{ir}^{b3} - f_{ir}^{w3}}, & \text{if } f_{ir}^{w3} < f_{ir}^3(y_{ir}^2) < f_{ir}^{b3} \\ 0, & \text{if } f_{ir}^{w3} \geq f_{ir}^3(y_{ir}^2). \end{cases} \quad (11)$$

$$(\bar{U}_i - MF) : \mu_{f_{ir}^4}^4 (f_{ir}^4(y_{ir}^1)) = \begin{cases} 1, & \text{if } f_{ir}^4(y_{ir}^1) \geq f_{ir}^{b4} \\ \frac{f_{ir}^4(y_{ir}^1) - f_{ir}^{w4}}{f_{ir}^{b4} - f_{ir}^{w4}}, & \text{if } f_{ir}^{w4} < f_{ir}^4(y_{ir}^1) < f_{ir}^{b4} \\ 0, & \text{if } f_{ir}^{w4} \geq f_{ir}^4(y_{ir}^1). \end{cases} \quad (12)$$

Then, it is possible to obtain the solution of the DM_i s problem by solving the following Tchebycheff problems: ($r = 1, 2, \dots, m_i; i = 1, 2, \dots, p; j = 1, 2, \dots, n$)

$$\begin{array}{llll}
 (\bar{L}_i - Tch) : \max \lambda_{i1} & (\underline{L}_i - Tch) : \max \lambda_{i2} & (\underline{U}_i - Tch) : \max \lambda_{i3} & (\bar{U}_i - Tch) : \max \lambda_{i4} \\
 \text{subject to} & \text{subject to} & \text{subject to} & \text{subject to} \\
 y_{ir}^4 \in G_{ir}^1 \cup G_{\rho_{ir}^1}^1 & y_{ir}^3 \in G_{ir}^2 \cup G_{\rho_{ir}^2}^2 & y_{ir}^2 \in G_{ir}^3 \cup G_{\rho_{ir}^3}^3 & y_{ir}^1 \in G_{ir}^4 \cup G_{\rho_{ir}^4}^4 \\
 \mu_{f_{ir}^1}^1(f_{ir}^1(y_{ir}^4)) \geq \lambda_{i1} & \mu_{f_{ir}^2}^2(f_{ir}^2(y_{ir}^3)) \geq \lambda_{i2} & \mu_{f_{ir}^3}^3(f_{ir}^3(y_{ir}^2)) \geq \lambda_{i3} & \mu_{f_{ir}^4}^4(f_{ir}^4(y_{ir}^1)) \geq \lambda_{i4} \\
 \frac{y_{i1}^4}{\rho_{i1}^4} = \frac{y_{i2}^4}{\rho_{i2}^4} = \dots = \frac{y_{ir}^4}{\rho_{ir}^4} & \frac{y_{i1}^3}{\rho_{i1}^3} = \frac{y_{i2}^3}{\rho_{i2}^3} = \dots = \frac{y_{ir}^3}{\rho_{ir}^3} & \frac{y_{i1}^2}{\rho_{i1}^2} = \frac{y_{i2}^2}{\rho_{i2}^2} = \dots = \frac{y_{ir}^2}{\rho_{ir}^2} & \frac{y_{i1}^1}{\rho_{i1}^1} = \frac{y_{i2}^1}{\rho_{i2}^1} = \dots = \frac{y_{ir}^1}{\rho_{ir}^1} \\
 \frac{\varepsilon_{i1}^4}{\rho_{i1}^4} = \frac{\varepsilon_{i2}^4}{\rho_{i2}^4} = \dots = \frac{\varepsilon_{ir}^4}{\rho_{ir}^4} & \frac{\varepsilon_{i1}^3}{\rho_{i1}^3} = \frac{\varepsilon_{i2}^3}{\rho_{i2}^3} = \dots = \frac{\varepsilon_{ir}^3}{\rho_{ir}^3} & \frac{\varepsilon_{i1}^2}{\rho_{i1}^2} = \frac{\varepsilon_{i2}^2}{\rho_{i2}^2} = \dots = \frac{\varepsilon_{ir}^2}{\rho_{ir}^2} & \frac{\varepsilon_{i1}^1}{\rho_{i1}^1} = \frac{\varepsilon_{i2}^1}{\rho_{i2}^1} = \dots = \frac{\varepsilon_{ir}^1}{\rho_{ir}^1} \\
 \lambda_{i1} \in [0, 1] & \lambda_{i2} \in [0, 1] & \lambda_{i3} \in [0, 1] & \lambda_{i4} \in [0, 1]
 \end{array} \tag{13}$$

The solution of equations (13) is found to be $y_{i1}^{*j\ell}, \varepsilon_{i1}^{*j\ell}, \rho_{i1}^{*j\ell}, \lambda_{i\ell}^*$, ($j = 1, 2, \dots, n; i = 1, 2, \dots, p; \ell = 1, 2, 3, 4$). Using these values, the optimum solution of the DM_i is obtained as below, with optimal objective values $f_{ir}^{i\ell}$:

$$x_j^{i1} = \max\left(\frac{y_{i1}^{*j1}}{\rho_{i1}^{*j4}}, \frac{y_{i1}^{*j4} - \varepsilon_{i1}^{*j4}}{\rho_{i1}^{*j1}}\right) \leq x_j^{i2} = \max\left(\frac{y_{i1}^{*j2}}{\rho_{i1}^{*j3}}, \frac{y_{i1}^{*j3} - \varepsilon_{i1}^{*j3}}{\rho_{i1}^{*j2}}\right) \leq x_j^{i3} = \max\left(\frac{y_{i1}^{*j3}}{\rho_{i1}^{*j2}}, \frac{y_{i1}^{*j2} + \varepsilon_{i1}^{*j2}}{\rho_{i1}^{*j3}}\right) \leq x_j^{i4} = \max\left(\frac{y_{i1}^{*j4}}{\rho_{i1}^{*j1}}, \frac{y_{i1}^{*j1} + \varepsilon_{i1}^{*j1}}{\rho_{i1}^{*j4}}\right).$$

2.4.2 Fuzzy programming model

Assume that the solution obtained from equations (13) is $x_j^i = ([x_j^{i2}, x_j^{i3}], [x_j^{i1}, x_j^{i4}])$, ($j = 1, 2, \dots, n; i = 1, 2, \dots, p$). If this solution is not satisfactory to any of the upper decision makers, then it is necessary to find a mechanism to update the current solution. To this end, restrict the optimal value of $f_{ir}^1(x), f_{ir}^2(x), f_{ir}^3(x)$ and $f_{ir}^4(x)$ ($r = 1, 2, \dots, m_i; i = 1, 2, \dots, p-1$) at the DM_i on the interval $[f_{ir}^1(x^i), f_{ir}^2(x^{i+1})], [f_{ir}^1(x^{i+1}), f_{ir}^2(x^i)], [f_{ir}^2(x^{i+1}), f_{ir}^3(x^i)],$ and $[f_{ir}^3(x^{i+1}), f_{ir}^4(x^i)]$, ($i = 1, 2, \dots, p-1$) respectively, such that at $i = p$ then $f_{ir}^\ell(x^{i+1}) = f_{ir}^\ell(x^{i-1})$, ($\ell = 1, 2, 3$). Then, the membership functions of the DM_i ($i = 1, 2, \dots, p$) can be assumed as follows:

$$\mu_{\bar{L}}(f_{ir}^1(x)) = \begin{cases} 1, & \text{if } f_{ir}^1(x) \geq f_{ir}^2(x^{i+1}) \\ \frac{f_{ir}^1(x) - f_{ir}^1(x^i)}{f_{ir}^2(x^{i+1}) - f_{ir}^1(x^i)}, & \text{if } f_{ir}^1(x^i) < f_{ir}^1(x) < f_{ir}^2(x^{i+1}) \\ 0, & \text{if } f_{ir}^1(x) \leq f_{ir}^1(x^i). \end{cases} \tag{14}$$

$$\mu_{\underline{L}}(f_{ir}^2(x)) = \begin{cases} 1, & \text{if } f_{ir}^2(x) \geq f_{ir}^2(x^i) \\ \frac{f_{ir}^2(x) - f_{ir}^1(x^{i+1})}{f_{ir}^2(x^i) - f_{ir}^1(x^{i+1})}, & \text{if } f_{ir}^1(x^{i+1}) < f_{ir}^2(x) < f_{ir}^2(x^i) \\ 0, & \text{if } f_{ir}^2(x) \leq f_{ir}^1(x^{i+1}). \end{cases} \tag{15}$$

$$\mu_{\underline{U}}(f_{ir}^3(x)) = \begin{cases} 1, & \text{if } f_{ir}^3(x) \geq f_{ir}^3(x^i) \\ \frac{f_{ir}^3(x) - f_{ir}^2(x^{i+1})}{f_{ir}^3(x^i) - f_{ir}^2(x^{i+1})}, & \text{if } f_{ir}^2(x^{i+1}) < f_{ir}^3(x) < f_{ir}^3(x^i) \\ 0, & \text{if } f_{ir}^3(x) \leq f_{ir}^2(x^{i+1}). \end{cases} \tag{16}$$

$$\mu_{\bar{U}}(f_{ir}^4(x)) = \begin{cases} 1, & \text{if } f_{ir}^4(x) \geq f_{ir}^4(x^i) \\ \frac{f_{ir}^4(x) - f_{ir}^3(x^{i+1})}{f_{ir}^4(x^i) - f_{ir}^3(x^{i+1})}, & \text{if } f_{ir}^3(x^{i+1}) < f_{ir}^4(x) < f_{ir}^4(x^i) \\ 0, & \text{if } f_{ir}^4(x) \leq f_{ir}^3(x^{i+1}). \end{cases} \tag{17}$$

The DM_i control the decision variable x_j^ℓ , ($i = 1, 2, \dots, p; \ell = 1, 2, 3, 4$) to get a compromise solution, all the DM_i ($i = 1, 2, \dots, p-1$) have to give a range for their controlled decision variables. Let $t_{i\ell}^-$ and $t_{i\ell}^+$ be the negative and positive

tolerance values on the decision vectors x_i^ℓ recognized by the DM_i respectively, where $t_{i\ell}^-, t_{i\ell}^+ \in R^{n_i}$. Hence, it is possible to construct the membership functions for the decision vectors x_i^ℓ as follows:

$$\mu_{x_i^\ell}(x_i^\ell) = \begin{cases} \frac{x_i^\ell - (x_i^{i\ell} - t_{i\ell}^-)}{t_{i\ell}^-}, & \text{if } x_i^{i\ell} - t_{i\ell}^- \leq x_i^\ell \leq x_i^{i\ell} \\ \frac{(x_i^{i\ell} + t_{i\ell}^+) - x_i^\ell}{t_{i\ell}^+}, & \text{if } x_i^{i\ell} \leq x_i^\ell < x_i^{i\ell} + t_{i\ell}^+. \end{cases} \quad (18)$$

Then, it is possible to apply the max-min decision model [6] and the Tchebycheff model [9] to construct the following satisfactory level model for the linear problem: ($r = 1, 2, \dots, m_i$)

<p>$(\bar{L} - Tch) : \max \lambda_1$</p> <p>subject to</p> <p>$y_{ir}^4 \in G_{ir}^1 \cup G_{\rho_{ir}^1}^1 (i = 1, 2, \dots, p)$</p> <p>$f_{ir}^1(y_{ir}^4) - f_{ir}^1(x^i) \geq \lambda_1(f_{ir}^2(x^{i+1}) - f_{ir}^1(x^i))(i = 1, 2, \dots, p)$</p> <p>$y_{ir}^{j4} - \rho_{ir}^1(x_i^{i4} - t_{i4}^-) \geq \rho_{ir}^1 t_{i4}^- \lambda_1 (i = j = 1, 2, \dots, p - 1)$</p> <p>$\rho_{ir}^1(x_i^{i4} + t_{i4}^+) - y_{ir}^{j4} \geq \rho_{ir}^1 t_{i4}^+ \lambda_1 (i = j = 1, 2, \dots, p - 1)$</p> <p>$\frac{y_{i1}^{j4}}{\rho_{i1}^1} = \frac{y_{i2}^{j4}}{\rho_{i2}^1} = \dots = \frac{y_{ir}^{j4}}{\rho_{ir}^1}, (j = 1, 2, \dots, n)$</p> <p>$\frac{\varepsilon_{i1}^{j4}}{\rho_{i1}^1} = \frac{\varepsilon_{i2}^{j4}}{\rho_{i2}^1} = \dots = \frac{\varepsilon_{ir}^{j4}}{\rho_{ir}^1}, (j = 1, 2, \dots, n)$</p> <p>$t_{i4}^-, t_{i4}^+ > 0$</p> <p>$\lambda_1 \in [0, 1]$</p>	<p>$(\underline{L} - Tch) : \max \lambda_2$</p> <p>subject to</p> <p>$y_{ir}^3 \in G_{ir}^2 \cup G_{\rho_{ir}^2}^2 (i = 1, 2, \dots, p)$</p> <p>$f_{ir}^2(y_{ir}^3) - f_{ir}^1(x^{i+1}) \geq \lambda_2(f_{ir}^2(x^i) - f_{ir}^1(x^{i+1}))(i = 1, 2, \dots, p)$</p> <p>$y_{ir}^{j3} - \rho_{ir}^2(x_i^{i3} - t_{i3}^-) \geq \rho_{ir}^2 t_{i3}^- \lambda_2 (i = j = 1, 2, \dots, p - 1)$</p> <p>$\rho_{ir}^2(x_i^{i3} + t_{i3}^+) - y_{ir}^{j3} \geq \rho_{ir}^2 t_{i3}^+ \lambda_2 (i = j = 1, 2, \dots, p - 1)$</p> <p>$\frac{y_{i1}^{j3}}{\rho_{i1}^2} = \frac{y_{i2}^{j3}}{\rho_{i2}^2} = \dots = \frac{y_{ir}^{j3}}{\rho_{ir}^2}, (j = 1, 2, \dots, n)$</p> <p>$\frac{\varepsilon_{i1}^{j3}}{\rho_{i1}^2} = \frac{\varepsilon_{i2}^{j3}}{\rho_{i2}^2} = \dots = \frac{\varepsilon_{ir}^{j3}}{\rho_{ir}^2}, (j = 1, 2, \dots, n)$</p> <p>$t_{i3}^-, t_{i3}^+ > 0$</p> <p>$\lambda_2 \in [0, 1]$</p>
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<p>$(\underline{U} - Tch) : \max \lambda_3$</p> <p>subject to</p> <p>$y_{ir}^2 \in G_{ir}^3 \cup G_{\rho_{ir}^3}^3 (i = 1, 2, \dots, p)$</p> <p>$f_{ir}^3(y_{ir}^2) - f_{ir}^2(x^{i+1}) \geq \lambda_3(f_{ir}^3(x^i) - f_{ir}^2(x^{i+1}))(i = 1, 2, \dots, p)$</p> <p>$y_{ir}^{j2} - \rho_{ir}^3(x_i^{i2} - t_{i2}^-) \geq \rho_{ir}^3 t_{i2}^- \lambda_3 (i = j = 1, 2, \dots, p - 1)$</p> <p>$\rho_{ir}^3(x_i^{i2} + t_{i2}^+) - y_{ir}^{j2} \geq \rho_{ir}^3 t_{i2}^+ \lambda_3 (i = j = 1, 2, \dots, p - 1)$</p> <p>$\frac{y_{i1}^{j2}}{\rho_{i1}^3} = \frac{y_{i2}^{j2}}{\rho_{i2}^3} = \dots = \frac{y_{ir}^{j2}}{\rho_{ir}^3}, (j = 1, 2, \dots, n)$</p> <p>$\frac{\varepsilon_{i1}^{j2}}{\rho_{i1}^3} = \frac{\varepsilon_{i2}^{j2}}{\rho_{i2}^3} = \dots = \frac{\varepsilon_{ir}^{j2}}{\rho_{ir}^3}, (j = 1, 2, \dots, n)$</p> <p>$t_{i2}^-, t_{i2}^+ > 0$</p> <p>$\lambda_3 \in [0, 1]$</p>	<p>$(\bar{U} - Tch) : \max \lambda_4$</p> <p>subject to</p> <p>$y_{ir}^1 \in G_{ir}^4 \cup G_{\rho_{ir}^4}^4 (i = 1, 2, \dots, p)$</p> <p>$f_{ir}^4(y_{ir}^1) - f_{ir}^3(x^i) \geq \lambda_4(f_{ir}^4(x^{i+1}) - f_{ir}^3(x^i))(i = 1, 2, \dots, p)$</p> <p>$y_{ir}^{j1} - \rho_{ir}^4(x_i^{i1} - t_{i1}^-) \geq \rho_{ir}^4 t_{i1}^- \lambda_4 (i = j = 1, 2, \dots, p - 1)$</p> <p>$\rho_{ir}^4(x_i^{i1} + t_{i1}^+) - y_{ir}^{j1} \geq \rho_{ir}^4 t_{i1}^+ \lambda_4 (i = j = 1, 2, \dots, p - 1)$</p> <p>$\frac{y_{i1}^{j1}}{\rho_{i1}^4} = \frac{y_{i2}^{j1}}{\rho_{i2}^4} = \dots = \frac{y_{ir}^{j1}}{\rho_{ir}^4}, (j = 1, 2, \dots, n)$</p> <p>$\frac{\varepsilon_{i1}^{j1}}{\rho_{i1}^4} = \frac{\varepsilon_{i2}^{j1}}{\rho_{i2}^4} = \dots = \frac{\varepsilon_{ir}^{j1}}{\rho_{ir}^4}, (j = 1, 2, \dots, n)$</p> <p>$t_{i1}^-, t_{i1}^+ > 0$</p> <p>$\lambda_4 \in [0, 1]$</p>
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If the solution is not satisfactory for any of the DMs, the membership function for all the upper level DMs objective and controlled decision variable with new tolerance level or the tolerances of the DMs are re-adjusted. Again, this process can be continued until a satisfactory solution is achieved.

3 A flowchart for the proposed method

This section presented a flowchart to illustrate the decision making process of the proposed problem as Figure 1.

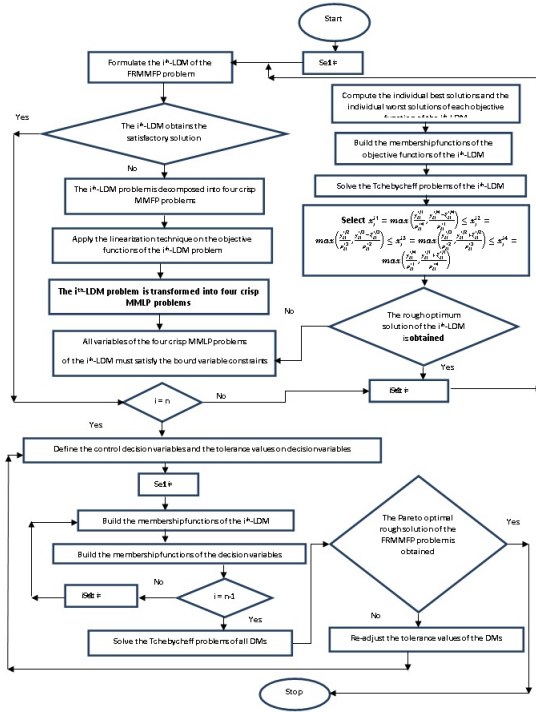


Figure 1: A flowchart for the proposed method.

4 Numerical example

In view of proper illustration of the developed method, the author presents the following example:

[FRMMFP] :

First – level decision maker [DM₁]

$$\max_{x_1^R} F_1^R(x^R) = \max_{x_1^R} (f_{11}^R(x^R), f_{12}^R(x^R), f_{13}^R(x^R))$$

where x_2^R, x_3^R solve

Second – level decision maker [DM₂] :

$$\max_{x_2^R} F_2^R(x^R) = \max_{x_2^R} (f_{21}^R(x^R), f_{22}^R(x^R), f_{23}^R(x^R))$$

where x_3^R solve

Third – level decision maker [DM₃]

$$\max_{x_3^R} F_3^R(x^R) = \max_{x_3^R} (f_{31}^R(x^R), f_{32}^R(x^R), f_{33}^R(x^R))$$

subject to

$$\begin{aligned} G^R &= \{([3, 6], [1, 7]) \otimes x_1^R \oplus ([3, 5], [2, 6]) \otimes x_2^R \leq ([12, 14], [10, 16]), \\ &([3, 5], [1, 8]) \otimes x_1^R \oplus ([5, 7], [2, 9]) \otimes x_2^R \oplus ([6, 7], [3, 9]) \otimes x_3^R \leq ([9, 11], [8, 14]), \\ &([5, 7], [5, 9]) \otimes x_1^R \oplus ([3, 5], [3, 8]) \otimes x_2^R \geq ([4, 8], [3, 16]), \\ &x_1^R, x_2^R, x_3^R \geq 0\} \end{aligned}$$

where

$$f_{11}^R(x^R) = \frac{([1, 1], [1, 4]) \otimes x_1^R \oplus ([1, 3], [1, 6]) \otimes x_2^R \oplus ([2, 2], [2, 5]) \otimes x_3^R \oplus ([2, 5], [1, 7])}{([2, 3], [1, 6]) \otimes x_1^R \oplus ([1, 1], [0, 3]) \otimes x_2^R \oplus ([2, 4], [1, 5]) \otimes x_3^R \oplus ([7, 7], [4, 8])}$$

$$\begin{aligned}
 f_{12}^R(x^R) &= \frac{([2, 3], [1, 4]) \otimes x_1^R \oplus ([5, 7], [3, 8]) \otimes x_2^R \oplus ([3, 5], [2, 8]) \otimes x_3^R \oplus ([5, 7], [2, 10])}{([4, 5], [2, 7]) \otimes x_1^R \oplus ([7, 9], [3, 10]) \otimes x_2^R \oplus ([2, 5], [1, 6]) \otimes x_3^R \oplus ([3, 9], [3, 10])}, \\
 f_{13}^R(x^R) &= \frac{([3, 5], [2, 7]) \otimes x_1^R \oplus ([4, 5], [3, 6]) \otimes x_2^R \oplus ([5, 8], [3, 9]) \otimes x_3^R \oplus ([3, 6], [1, 8])}{([5, 6], [3, 8]) \otimes x_1^R \oplus ([3, 5], [2, 6]) \otimes x_2^R \oplus ([4, 6], [3, 7]) \otimes x_3^R \oplus ([5, 6], [2, 9])}, \\
 f_{21}^R(x^R) &= \frac{([5, 7], [4, 8]) \otimes x_1^R \oplus ([6, 8], [2, 11]) \otimes x_2^R \oplus ([3, 5], [1, 8]) \otimes x_3^R \oplus ([5, 7], [3, 10])}{([5, 7], [3, 8]) \otimes x_1^R \oplus ([3, 4], [1, 6]) \otimes x_2^R \oplus ([2, 3], [1, 5]) \otimes x_3^R \oplus ([6, 8], [5, 10])}, \\
 f_{22}^R(x^R) &= \frac{([1, 2], [0, 4]) \otimes x_1^R \oplus ([3, 4], [1, 7]) \otimes x_2^R \oplus ([4, 6], [2, 6]) \otimes x_3^R \oplus ([2, 4], [1, 4])}{([2, 2], [1, 5]) \otimes x_1^R \oplus ([5, 7], [3, 9]) \otimes x_2^R \oplus ([3, 5], [1, 8]) \otimes x_3^R \oplus ([8, 9], [8, 9])}, \\
 f_{23}^R(x^R) &= \frac{([6, 8], [3, 10]) \otimes x_1^R \oplus ([7, 9], [4, 11]) \otimes x_2^R \oplus ([7, 8], [5, 12]) \otimes x_3^R \oplus ([8, 10], [5, 11])}{([4, 7], [2, 9]) \otimes x_1^R \oplus ([5, 7], [3, 10]) \otimes x_2^R \oplus ([5, 6], [2, 9]) \otimes x_3^R \oplus ([7, 9], [7, 12])}, \\
 f_{31}^R(x^R) &= \frac{([3, 5], [2, 8]) \otimes x_1^R \oplus ([5, 7], [3, 8]) \otimes x_2^R \oplus ([2, 3], [1, 4]) \otimes x_3^R \oplus ([8, 9], [6, 12])}{([2, 3], [1, 4]) \otimes x_1^R \oplus ([3, 5], [2, 7]) \otimes x_2^R \oplus ([4, 5], [2, 6]) \otimes x_3^R \oplus ([6, 8], [2, 10])}, \\
 f_{32}^R(x^R) &= \frac{([4, 6], [3, 8]) \otimes x_1^R \oplus ([3, 5], [2, 9]) \otimes x_2^R \oplus ([3, 7], [2, 10]) \otimes x_3^R \oplus ([7, 12], [4, 16])}{([3, 5], [1, 8]) \otimes x_1^R \oplus ([2, 3], [1, 5]) \otimes x_2^R \oplus ([4, 6], [3, 7]) \otimes x_3^R \oplus ([5, 6], [2, 9])}, \\
 f_{33}^R(x^R) &= \frac{([3, 4], [1, 6]) \otimes x_1^R \oplus ([4, 5], [1, 7]) \otimes x_2^R \oplus ([5, 6], [2, 8]) \otimes x_3^R \oplus ([7, 10], [4, 12])}{([2, 3], [2, 6]) \otimes x_1^R \oplus ([3, 5], [1, 5]) \otimes x_2^R \oplus ([6, 7], [3, 9]) \otimes x_3^R \oplus ([5, 8], [3, 11])}.
 \end{aligned}$$

The first step in the procedure is to transform the FRMMFP problem into an MMFP problem with crisp coefficients and crisp decision variables. Let $F_i^R(x^R) = ([F_i^2(x), F_i^3(x)], [F_i^1(x), F_i^4(x)])$, $x_j^R = ([x_j^2, x_j^3], [x_j^1, x_j^4])$, and $f_{ir}^R(x^R) = ([f_{ir}^2(x), f_{ir}^3(x)], [f_{ir}^1(x), f_{ir}^4(x)])$, $(i, j, r = 1, 2, 3)$

First, the first-level decision maker (FLDM) can be re-written, as follows:

First – level decision maker[DM₁] :

$$\begin{aligned}
 (\bar{L}_1 - MMFP) : \max_{x_1^1, x_1^4} F_1^1(x^1, x^4) & \quad (\underline{L}_1 - MMFP) : \max_{x_1^2, x_1^3} F_1^2(x^2, x^3) \\
 = \max_{x_1^1, x_1^4} (f_{11}^1(x^1, x^4), f_{12}^1(x^1, x^4), f_{13}^1(x^1, x^4)) & \quad = \max_{x_1^2, x_1^3} (f_{11}^2(x^2, x^3), f_{12}^2(x^2, x^3), f_{13}^2(x^2, x^3)) \\
 = \max_{x_1^1, x_1^4} \left(\frac{x_1^1 + x_2^1 + 2x_3^1 + 1}{6x_1^4 + 3x_2^4 + 5x_3^4 + 3}, \right. & \quad = \max_{x_1^2, x_1^3} \left(\frac{x_1^2 + x_2^2 + 2x_3^2 + 2}{3x_1^3 + x_2^3 + 4x_3^3 + 7}, \right. \\
 \left. \frac{x_1^1 + 3x_2^1 + 2x_3^1 + 2}{7x_1^4 + 10x_2^4 + 6x_3^4 + 10}, \frac{2x_1^1 + 3x_2^1 + 3x_3^1 + 1}{8x_1^4 + 6x_2^4 + 7x_3^4 + 9} \right) & \quad \left. \frac{2x_1^2 + 5x_2^2 + 3x_3^2 + 5}{5x_1^3 + 9x_2^3 + 5x_3^3 + 9}, \frac{3x_1^2 + 4x_2^2 + 5x_3^2 + 3}{6x_1^3 + 5x_2^3 + 6x_3^3 + 6} \right) \\
 \text{where } x_2^1, x_3^1, x_2^4, x_3^4 \text{ solve,} & \quad \text{where } x_2^2, x_3^2, x_2^3, x_3^3 \text{ solve} \\
 \text{subject to} & \quad \text{subject to} \\
 x^1, x^4 \in G^1 = \{x_1^1 + 2x_2^1 \leq 16, & \quad x^2, x^3 \in G^2 = \{3x_1^2 + 3x_2^2 \leq 14, \\
 x_1^1 - 9x_2^4 + 3x_3^1 \leq 14, & \quad 3x_1^2 - 7x_2^3 + 6x_3^2 \leq 11, \\
 5x_1^1 + 3x_2^1 \leq 16, & \quad 5x_1^2 + 3x_2^2 \leq 8, \\
 7x_1^4 + 6x_2^4 \leq 10, & \quad 6x_1^3 + 5x_2^3 \leq 12, \\
 8x_1^4 - 2x_2^1 + 9x_3^4 \leq 8, & \quad 5x_1^3 - 5x_2^2 + 7x_3^3 \leq 9, \\
 9x_1^4 + 8x_2^4 \geq 3, & \quad 7x_1^3 + 5x_2^3 \geq 4, \\
 x_1^1, x_2^1, x_3^1, x_1^4, x_2^4, x_3^4 \geq 0\}. & \quad x_1^2, x_2^2, x_3^2, x_1^3, x_2^3, x_3^3 \geq 0\}. \\
 (\bar{U}_1 - MMFP) : \max_{x_1^1, x_1^4} F_1^4(x^1, x^4) & \quad (\underline{U}_1 - MMFP) : \max_{x_1^2, x_1^3} F_1^3(x^2, x^3) \\
 = \max_{x_1^1, x_1^4} (f_{11}^4(x^1, x^4), f_{12}^4(x^1, x^4), f_{13}^4(x^1, x^4)) & \quad = \max_{x_1^2, x_1^3} (f_{11}^3(x^2, x^3), f_{12}^3(x^2, x^3), f_{13}^3(x^2, x^3)) \\
 = \max_{x_1^1, x_1^4} \left(\frac{4x_1^4 + 6x_2^4 + 5x_3^4 + 7}{x_1^1 + x_3^1 + 4}, \right. & \quad = \max_{x_1^2, x_1^3} \left(\frac{x_1^3 + 3x_2^3 + 2x_3^3 + 5}{2x_1^2 + x_2^2 + 2x_3^2 + 7}, \right.
 \end{aligned}$$

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$$\frac{4x_1^4 + 8x_2^4 + 8x_3^4 + 10}{2x_1^3 + 3x_2^3 + x_3^3 + 3}, \frac{7x_1^4 + 6x_2^4 + 9x_3^4 + 8}{3x_1^3 + 2x_2^3 + 3x_3^3 + 2}$$

where $x_2^1, x_3^1, x_2^4, x_3^4$ solve,

subject to

$$x^1, x^4 \in G^4 = \{x_1^1 + 2x_2^1 \leq 10,$$

$$x_1^1 - 9x_2^4 + 3x_3^1 \leq 8,$$

$$5x_1^1 + 3x_2^1 \geq 3,$$

$$7x_1^4 + 6x_2^4 \leq 16,$$

$$8x_1^4 - 2x_2^1 + 9x_3^4 \leq 14,$$

$$9x_1^4 + 8x_2^4 \leq 16,$$

$$x_1^1, x_2^1, x_3^1, x_1^4, x_2^4, x_3^4 \geq 0\}.$$

$$\frac{3x_1^3 + 7x_2^3 + 5x_3^3 + 7}{4x_1^2 + 7x_2^2 + 2x_3^2 + 3}, \frac{5x_1^3 + 5x_2^3 + 8x_3^3 + 6}{5x_1^2 + 3x_2^2 + 4x_3^2 + 5}$$

where $x_2^2, x_3^2, x_2^3, x_3^3$ solve

subject to

$$x^2, x^3 \in G^3 = \{3x_1^2 + 3x_2^2 \leq 12,$$

$$3x_1^2 - 7x_2^3 + 6x_3^2 \leq 9,$$

$$5x_1^2 + 3x_2^2 \geq 4,$$

$$6x_1^3 + 5x_2^3 \leq 14,$$

$$5x_1^3 - 5x_2^2 + 7x_3^3 \leq 11,$$

$$7x_1^3 + 5x_2^3 \leq 8,$$

$$x_1^2, x_2^2, x_3^2, x_1^3, x_2^3, x_3^3 \geq 0\}.$$

Then, using equations (7) and (8) the *FLDM*'s problem can be transformed into an *MMLP* problem, as below:

$$\begin{aligned} (\bar{L}_1 - MMLP) : \max_{y_{1r}^4} F_1^4(y_{1r}^4) \quad (r = 1, 2, 3) \\ = \max_{y_{1r}^4} (f_{11}^4(y_{11}^4), f_{12}^4(y_{12}^4), f_{13}^4(y_{13}^4)) \\ = \max_{y_{1r}^4} \left(\frac{1}{8} - \xi_{11}^{14} - \xi_{11}^{24} - 2\xi_{11}^{34} + \frac{2}{8}y_{11}^{14} + \frac{5}{8}y_{11}^{24} + \frac{11}{8}y_{11}^{34}, \right. \\ \left. \frac{1}{5} - \xi_{12}^{14} - 3\xi_{12}^{24} - 2\xi_{12}^{34} - \frac{2}{5}y_{12}^{14} + y_{12}^{24} + \frac{4}{5}y_{12}^{34}, \right. \\ \left. \frac{1}{9} - 2\xi_{13}^{14} - 3\xi_{13}^{24} - 3\xi_{13}^{34} + \frac{10}{9}y_{13}^{14} + \frac{21}{9}y_{13}^{24} + \frac{20}{9}y_{13}^{34} \right) \end{aligned}$$

where y_{1r}^{24}, y_{1r}^{34} solve ($r = 1, 2, 3$)

subject to

$$y_{1r}^4 \in G_{1r}^1 \cup G_{\rho_{1r}^1}^1, \quad (r = 1, 2, 3)$$

$$G_{1r}^1 = \{7y_{1r}^{14} + 6y_{1r}^{24} \leq 10\rho_{1r}^1,$$

$$8y_{1r}^{14} - 2y_{1r}^{24} + 9y_{1r}^{34} + 2\xi_{1r}^{24} \leq 8\rho_{1r}^1,$$

$$9y_{1r}^{14} + 8y_{1r}^{24} \geq 3\rho_{1r}^1,$$

$$y_{1r}^{14} + 2y_{1r}^{24} - \xi_{1r}^{14} - 2\xi_{1r}^{24} \leq 16\rho_{1r}^1,$$

$$y_{1r}^{14} - 9y_{1r}^{24} + 3y_{1r}^{34} - \xi_{1r}^{14} - 3\xi_{1r}^{34} \leq 14\rho_{1r}^1,$$

$$5y_{1r}^{14} + 3y_{1r}^{24} - 5\xi_{1r}^{14} - 3\xi_{1r}^{24} \leq 16\rho_{1r}^1,$$

$$y_{1r}^{14}, y_{1r}^{24}, y_{1r}^{34}, \xi_{1r}^{14}, \xi_{1r}^{24}, \xi_{1r}^{34} \geq 0, \rho_{1r}^1 > 0\},$$

$$G_{\rho_{11}^1}^1 = \{6y_{11}^{14} + 3y_{11}^{24} + 5y_{11}^{34} + 8\rho_{11}^1 = 1\}$$

$$G_{\rho_{12}^1}^1 = \{7y_{12}^{14} + 10y_{12}^{24} + 6y_{12}^{34} + 10\rho_{12}^1 = 1\}$$

$$G_{\rho_{13}^1}^1 = \{8y_{13}^{14} + 6y_{13}^{24} + 7y_{13}^{34} + 9\rho_{13}^1 = 1\}$$

$$(\bar{U}_1 - MMLP) : \max_{y_{1r}^{11}} F_1^4(y_{1r}^4) \quad (r = 1, 2, 3)$$

$$= \max_{y_{1r}^{11}} (f_{11}^4(y_{11}^1), f_{12}^4(y_{12}^1), f_{13}^4(y_{13}^1))$$

$$= \max_{y_{1r}^{11}} \left(\frac{7}{4} + 4\xi_{11}^{11} + 6\xi_{11}^{21} + 5\xi_{11}^{31} + \frac{9}{4}y_{11}^{11} + 6y_{11}^{21} + \frac{13}{4}y_{11}^{31}, \right.$$

$$\left. \frac{10}{3} + 4\xi_{12}^{11} + 8\xi_{12}^{21} + 8\xi_{12}^{31} - \frac{8}{3}y_{12}^{11} - 2y_{12}^{21} + \frac{14}{3}y_{12}^{31}, \right.$$

$$\left. 4 + 7\xi_{13}^{11} + 6\xi_{13}^{21} + 9\xi_{13}^{31} - 5y_{13}^{11} - 2y_{13}^{21} - 3y_{13}^{31} \right)$$

$$(\underline{L}_1 - MMLP) : \max_{y_{1r}^{13}} F_1^2(y_{1r}^3) \quad (r = 1, 2, 3)$$

$$= \max_{y_{1r}^{13}} (f_{11}^2(y_{11}^3), f_{12}^2(y_{12}^3), f_{13}^2(y_{13}^3))$$

$$= \max_{y_{1r}^{13}} \left(\frac{2}{7} - \xi_{11}^{13} - \xi_{11}^{23} - 2\xi_{11}^{33} + \frac{1}{7}y_{11}^{13} + \frac{5}{7}y_{11}^{23} + \frac{6}{7}y_{11}^{33}, \right.$$

$$\left. \frac{5}{9} - 2\xi_{12}^{13} - 5\xi_{12}^{23} - 3\xi_{12}^{33} - \frac{7}{9}y_{12}^{13} + \frac{2}{9}y_{12}^{33}, \right.$$

$$\left. \frac{1}{2} - 3\xi_{13}^{13} - 4\xi_{13}^{23} - 5\xi_{13}^{33} + \frac{3}{2}y_{13}^{23} + 2y_{13}^{33} \right)$$

where y_{1r}^{23}, y_{1r}^{33} solve ($r = 1, 2, 3$)

subject to

$$y_{1r}^3 \in G_{1r}^2 \cup G_{\rho_{1r}^2}^2, \quad (r = 1, 2, 3)$$

$$G_{1r}^2 = \{6y_{1r}^{13} + 5y_{1r}^{23} \leq 12\rho_{1r}^2,$$

$$5y_{1r}^{13} - 5y_{1r}^{23} + 7y_{1r}^{33} + 5\xi_{1r}^{23} \leq 9\rho_{1r}^2,$$

$$7y_{1r}^{13} + 5y_{1r}^{23} \geq 4\rho_{1r}^2,$$

$$3y_{1r}^{13} + 3y_{1r}^{23} - 3\xi_{1r}^{13} - 3\xi_{1r}^{23} \leq 14\rho_{1r}^2,$$

$$3y_{1r}^{13} - 7y_{1r}^{23} + 6y_{1r}^{33} - 3\xi_{1r}^{13} - 6\xi_{1r}^{33} \leq 11\rho_{1r}^2,$$

$$5y_{1r}^{13} + 3y_{1r}^{23} - 5\xi_{1r}^{13} - 3\xi_{1r}^{23} \leq 8\rho_{1r}^2,$$

$$y_{1r}^{13}, y_{1r}^{23}, y_{1r}^{33}, \xi_{1r}^{13}, \xi_{1r}^{23}, \xi_{1r}^{33} \geq 0, \rho_{1r}^2 > 0\},$$

$$G_{\rho_{11}^2}^2 = \{3y_{11}^{13} + y_{11}^{23} + 4y_{11}^{33} + 7\rho_{11}^2 = 1\}$$

$$G_{\rho_{12}^2}^2 = \{5y_{12}^{13} + 9y_{12}^{23} + 5y_{12}^{33} + 9\rho_{12}^2 = 1\}$$

$$G_{\rho_{13}^2}^2 = \{6y_{13}^{13} + 5y_{13}^{23} + 6y_{13}^{33} + 6\rho_{13}^2 = 1\}$$

$$(\underline{U}_1 - MMLP) : \max_{y_{1r}^{12}} F_1^3(y_{1r}^2) \quad (r = 1, 2, 3)$$

$$= \max_{y_{1r}^{12}} (f_{11}^3(y_{11}^2), f_{12}^3(y_{12}^2), f_{13}^3(y_{13}^2))$$

$$= \max_{y_{1r}^{12}} \left(\frac{5}{7} + \xi_{11}^{12} + 3\xi_{11}^{22} + 2\xi_{11}^{32} - \frac{3}{7}y_{11}^{12} + \frac{16}{7}y_{11}^{22} + \frac{4}{7}y_{11}^{32}, \right.$$

$$\left. \frac{7}{3} + 3\xi_{12}^{12} + 7\xi_{12}^{22} + 5\xi_{12}^{32} - \frac{19}{3}y_{12}^{12} - \frac{28}{3}y_{12}^{22} + \frac{1}{3}y_{12}^{32}, \right.$$

$$\left. \frac{6}{5} + 5\xi_{13}^{12} + 5\xi_{13}^{22} + 8\xi_{13}^{32} - y_{13}^{12} + \frac{7}{5}y_{13}^{22} + \frac{16}{5}y_{13}^{32} \right)$$

where y_{1r}^{21}, y_{1r}^{31} solve ($r = 1, 2, 3$)

subject to

$$\begin{aligned} y_{1r}^1 &\in G_{1r}^4 \cup G_{\rho_{1r}^4}, (r = 1, 2, 3) \\ G_{1r}^4 &= \{y_{1r}^{11} + 2y_{1r}^{21} \leq 10\rho_{1r}^4, \\ y_{1r}^{11} - 9y_{1r}^{21} + 3y_{1r}^{31} - 9\xi_{1r}^{21} &\leq 8\rho_{1r}^4, \\ 5y_{1r}^{11} + 3y_{1r}^{21} &\geq 3\rho_{1r}^4, \\ 7y_{1r}^{11} + 6y_{1r}^{21} + 7\xi_{1r}^{11} + 6\xi_{1r}^{21} &\leq 16\rho_{1r}^4, \\ 8y_{1r}^{11} - 2y_{1r}^{21} + 9y_{1r}^{31} + 8\xi_{1r}^{11} + 9\xi_{1r}^{31} &\leq 14\rho_{1r}^4, \\ 9y_{1r}^{11} + 8y_{1r}^{21} + 9\xi_{1r}^{11} + 8\xi_{1r}^{21} &\leq 16\rho_{1r}^4, \\ y_{1r}^{11}, y_{1r}^{21}, y_{1r}^{31}, \xi_{1r}^{11}, \xi_{1r}^{21}, \xi_{1r}^{31} &\geq 0, \rho_{1r}^4 > 0\}, \\ G_{\rho_{11}^4} &= \{y_{11}^{11} + y_{11}^{31} + 4\rho_{11}^4 = 1\} \\ G_{\rho_{12}^4} &= \{2y_{12}^{11} + 3y_{12}^{21} + y_{12}^{31} + 3\rho_{12}^4 = 1\} \\ G_{\rho_{13}^4} &= \{3y_{13}^{11} + 2y_{13}^{21} + 3y_{13}^{31} + 2\rho_{13}^4 = 1\} \end{aligned}$$

where y_{1r}^{22}, y_{1r}^{32} solve ($r = 1, 2, 3$)

subject to

$$\begin{aligned} y_{1r}^2 &\in G_{1r}^3 \cup G_{\rho_{1r}^3}, (r = 1, 2, 3) \\ G_{1r}^3 &= \{3y_{1r}^{12} + 3y_{1r}^{22} \leq 12\rho_{1r}^3, \\ 3y_{1r}^{12} - 7y_{1r}^{22} + 6y_{1r}^{32} - 7\xi_{1r}^{22} &\leq 9\rho_{1r}^3, \\ 5y_{1r}^{12} + 3y_{1r}^{22} &\geq 4\rho_{1r}^3, \\ 6y_{1r}^{12} + 5y_{1r}^{22} + 6\xi_{1r}^{12} + 5\xi_{1r}^{22} &\leq 14\rho_{1r}^3, \\ 5y_{1r}^{12} - 5y_{1r}^{22} + 7y_{1r}^{32} + 5\xi_{1r}^{12} + 7\xi_{1r}^{32} &\leq 11\rho_{1r}^3, \\ 7y_{1r}^{12} + 5y_{1r}^{22} + 7\xi_{1r}^{12} + 5\xi_{1r}^{22} &\leq 8\rho_{1r}^3, \\ y_{1r}^{12}, y_{1r}^{22}, y_{1r}^{32}, \xi_{1r}^{12}, \xi_{1r}^{22}, \xi_{1r}^{32} &\geq 0, \rho_{1r}^3 > 0\}, \\ G_{\rho_{11}^3} &= \{2y_{11}^{12} + y_{11}^{22} + 2y_{11}^{32} + 7\rho_{11}^3 = 1\} \\ G_{\rho_{12}^3} &= \{4y_{12}^{12} + 7y_{12}^{22} + 2y_{12}^{32} + 3\rho_{12}^3 = 1\} \\ G_{\rho_{13}^3} &= \{5y_{13}^{12} + 3y_{13}^{22} + 4y_{13}^{32} + 5\rho_{13}^3 = 1\} \end{aligned}$$

The individual optimal rough solution of the *FLDM*'s objectives is $f_{11}^{bR} = ([0.4860, 1.7817], [0.2687, 7.250])$, $f_{11}^{wR} = ([0.000, 0.6615], [0.000, 2.0435])$, $f_{12}^{bR} = ([0.5717, 2.8645], [0.2781, 7.4233])$, $f_{12}^{wR} = ([0.000, 1.2817], [0.000, 2.6148])$, $f_{13}^{bR} = ([0.7667, 3.7989], [0.3515, 9.0000])$ and $f_{13}^{wR} = ([0.000, 1.0933], [0.000, 2.7931])$.

Using equations (9)-(13), the optimum rough solution of the *FLDM*'s problem is obtained by solving the following problems:

$$(\bar{L}_1 - Tch) : \max \lambda_{11},$$

subject to

$$\begin{aligned} y_{1r}^4 &\in G_{1r}^1 \cup G_{\rho_{1r}^1}, (r = 1, 2, 3) \\ \frac{1}{8} - \xi_{11}^{14} - \xi_{11}^{24} - 2\xi_{11}^{34} + \frac{2}{8}y_{11}^{14} + \frac{5}{8}y_{11}^{24} + \frac{11}{8}y_{11}^{34} &\geq 0.269\lambda_{11}, \\ \frac{1}{5} - \xi_{12}^{14} - 3\xi_{12}^{24} - 2\xi_{12}^{34} - \frac{2}{5}y_{12}^{14} + y_{12}^{24} + \frac{4}{5}y_{12}^{34} &\geq 0.278\lambda_{11}, \\ \frac{1}{9} - 2\xi_{13}^{14} - 3\xi_{13}^{24} - 3\xi_{13}^{34} + \frac{10}{9}y_{13}^{14} + \frac{21}{9}y_{13}^{24} + \frac{20}{9}y_{13}^{34} &\geq 0.352\lambda_{11}, \\ \frac{y_{11}^{14}}{\rho_{11}^1} = \frac{y_{12}^{14}}{\rho_{12}^1} = \frac{y_{13}^{14}}{\rho_{13}^1}, \frac{y_{11}^{24}}{\rho_{11}^1} = \frac{y_{12}^{24}}{\rho_{12}^1} = \frac{y_{13}^{24}}{\rho_{13}^1}, \\ \frac{y_{11}^{34}}{\rho_{11}^1} = \frac{y_{12}^{34}}{\rho_{12}^1} = \frac{y_{13}^{34}}{\rho_{13}^1}, \frac{\xi_{11}^{14}}{\rho_{11}^1} = \frac{\xi_{12}^{14}}{\rho_{12}^1} = \frac{\xi_{13}^{14}}{\rho_{13}^1}, \\ \frac{\xi_{11}^{24}}{\rho_{11}^1} = \frac{\xi_{12}^{24}}{\rho_{12}^1} = \frac{\xi_{13}^{24}}{\rho_{13}^1}, \frac{\xi_{11}^{34}}{\rho_{11}^1} = \frac{\xi_{12}^{34}}{\rho_{12}^1} = \frac{\xi_{13}^{34}}{\rho_{13}^1}, \\ \lambda_{11} &\in [0, 1]. \end{aligned}$$

$$(\underline{L}_1 - Tch) : \max \lambda_{12},$$

subject to

$$\begin{aligned} y_{1r}^3 &\in G_{1r}^2 \cup G_{\rho_{1r}^2}, (r = 1, 2, 3) \\ \frac{2}{7} - \xi_{11}^{13} - \xi_{11}^{23} - 2\xi_{11}^{33} + \frac{1}{7}y_{11}^{13} + \frac{5}{7}y_{11}^{23} + \frac{6}{7}y_{11}^{33} &\geq 0.49\lambda_{12}, \\ \frac{5}{9} - 2\xi_{12}^{13} - 5\xi_{12}^{23} - 3\xi_{12}^{33} - \frac{7}{9}y_{12}^{13} + \frac{2}{9}y_{12}^{33} &\geq 0.57\lambda_{12}, \\ \frac{1}{2} - 3\xi_{13}^{13} - 4\xi_{13}^{23} - 5\xi_{13}^{33} + \frac{3}{2}y_{13}^{23} + 2y_{13}^{33} &\geq 0.77\lambda_{12}, \end{aligned}$$

$$(\underline{U}_1 - Tch) : \max \lambda_{13},$$

subject to

$$\begin{aligned} y_{1r}^2 &\in G_{1r}^3 \cup G_{\rho_{1r}^3}, (r = 1, 2, 3) \\ 0.053 + \xi_{11}^{12} + 3\xi_{11}^{22} + 2\xi_{11}^{32} - \frac{3}{7}y_{11}^{12} + \frac{16}{7}y_{11}^{22} + \frac{4}{7}y_{11}^{32} &\geq 1.12\lambda_{13}, \\ 1.05 + 3\xi_{12}^{12} + 7\xi_{12}^{22} + 5\xi_{12}^{32} - \frac{19}{3}y_{12}^{12} - \frac{28}{3}y_{12}^{22} + \frac{1}{3}y_{12}^{32} &\geq 1.58\lambda_{13}, \\ 0.107 + 5\xi_{13}^{12} + 5\xi_{13}^{22} + 8\xi_{13}^{32} - y_{13}^{12} + \frac{7}{5}y_{13}^{22} + \frac{16}{5}y_{13}^{32} &\geq 2.706\lambda_{13} \\ \frac{y_{11}^{12}}{\rho_{11}^3} = \frac{y_{12}^{12}}{\rho_{12}^3} = \frac{y_{13}^{12}}{\rho_{13}^3}, \frac{y_{11}^{22}}{\rho_{11}^3} = \frac{y_{12}^{22}}{\rho_{12}^3} = \frac{y_{13}^{22}}{\rho_{13}^3}, \\ \frac{y_{11}^{32}}{\rho_{11}^3} = \frac{y_{12}^{32}}{\rho_{12}^3} = \frac{y_{13}^{32}}{\rho_{13}^3}, \frac{\xi_{11}^{12}}{\rho_{11}^3} = \frac{\xi_{12}^{12}}{\rho_{12}^3} = \frac{\xi_{13}^{12}}{\rho_{13}^3}, \\ \frac{\xi_{11}^{22}}{\rho_{11}^3} = \frac{\xi_{12}^{22}}{\rho_{12}^3} = \frac{\xi_{13}^{22}}{\rho_{13}^3}, \frac{\xi_{11}^{32}}{\rho_{11}^3} = \frac{\xi_{12}^{32}}{\rho_{12}^3} = \frac{\xi_{13}^{32}}{\rho_{13}^3}, \\ \lambda_{13} &\in [0, 1]. \end{aligned}$$

$$(\bar{U}_1 - Tch) : \max \lambda_{14},$$

subject to

$$\begin{aligned} y_{1r}^1 &\in G_{1r}^4 \cup G_{\rho_{1r}^4}, (r = 1, 2, 3) \\ -0.29 + 4\xi_{11}^{11} + 6\xi_{11}^{21} + 5\xi_{11}^{31} + \frac{9}{4}y_{11}^{11} + 6y_{11}^{21} + \frac{13}{4}y_{11}^{31} &\geq 5.21\lambda_{14}, \\ 0.72 + 4\xi_{12}^{11} + 8\xi_{12}^{21} + 8\xi_{12}^{31} - \frac{8}{3}y_{12}^{11} - 2y_{12}^{21} + \frac{14}{3}y_{12}^{31} &\geq 4.81\lambda_{14}, \\ 1.21 + 7\xi_{13}^{11} + 6\xi_{13}^{21} + 9\xi_{13}^{31} - 5y_{13}^{11} - 2y_{13}^{21} - 3y_{13}^{31} &\geq 6.21\lambda_{14}, \end{aligned}$$

$$\begin{aligned}
 \frac{y_{11}^{13}}{\rho_{11}^2} &= \frac{y_{12}^{13}}{\rho_{12}^2} = \frac{y_{13}^{13}}{\rho_{13}^2}, \quad \frac{y_{11}^{23}}{\rho_{11}^2} = \frac{y_{12}^{23}}{\rho_{12}^2} = \frac{y_{13}^{23}}{\rho_{13}^2}, & \frac{y_{11}^{11}}{\rho_{11}^4} &= \frac{y_{12}^{11}}{\rho_{12}^4} = \frac{y_{13}^{11}}{\rho_{13}^4}, \quad \frac{y_{11}^{21}}{\rho_{11}^4} = \frac{y_{12}^{21}}{\rho_{12}^4} = \frac{y_{13}^{21}}{\rho_{13}^4}, \\
 \frac{y_{11}^{33}}{\rho_{11}^2} &= \frac{y_{12}^{33}}{\rho_{12}^2} = \frac{y_{13}^{33}}{\rho_{13}^2}, \quad \frac{\xi_{11}^{13}}{\rho_{11}^2} = \frac{\xi_{12}^{13}}{\rho_{12}^2} = \frac{\xi_{13}^{13}}{\rho_{13}^2}, & \frac{y_{11}^{31}}{\rho_{11}^4} &= \frac{y_{12}^{31}}{\rho_{12}^4} = \frac{y_{13}^{31}}{\rho_{13}^4}, \quad \frac{\xi_{11}^{11}}{\rho_{11}^4} = \frac{\xi_{12}^{11}}{\rho_{12}^4} = \frac{\xi_{13}^{11}}{\rho_{13}^4}, \\
 \frac{\xi_{11}^{23}}{\rho_{11}^2} &= \frac{\xi_{12}^{23}}{\rho_{12}^2} = \frac{\xi_{13}^{23}}{\rho_{13}^2}, \quad \frac{\xi_{11}^{33}}{\rho_{11}^2} = \frac{\xi_{12}^{33}}{\rho_{12}^2} = \frac{\xi_{13}^{33}}{\rho_{13}^2}, & \frac{\xi_{11}^{21}}{\rho_{11}^4} &= \frac{\xi_{12}^{21}}{\rho_{12}^4} = \frac{\xi_{13}^{21}}{\rho_{13}^4}, \quad \frac{\xi_{11}^{31}}{\rho_{11}^4} = \frac{\xi_{12}^{31}}{\rho_{12}^4} = \frac{\xi_{13}^{31}}{\rho_{13}^4}, \\
 \lambda_{12} &\in [0, 1]. & \lambda_{14} &\in [0, 1].
 \end{aligned}$$

Table 1 illustrates the solution of these equations. Hence, the rough optimum solution of the FLDMs problem is $x_1^F = ([0.0000, 0.0000], [0.0000, 0.1031])$, $x_2^F = ([0.9172, 1.2505], [0.8281, 1.8840])$, $x_3^F = ([0.0000, 1.5991], [0.0000, 1.6479])$, $f_{11}^F = ([0.1992, 1.5093], [0.0812, 6.7390])$, $f_{12}^F = ([0.3393, 2.3087], [0.1137, 7.0505])$, and $f_{13}^F = ([0.3053, 2.0638], [0.1067, 9.5335])$. In a similar way, the second-level decision maker (SLDM) solves his/her problem as follows:

Second – level decision maker[DM₂] :

$$\begin{aligned}
 (\bar{L}_2 - MMFP) : \max_{x_2^1, x_2^4} F_2^1(x^1, x^4) &= \max_{x_2^1, x_2^4} (f_{21}^1(x^1, x^4), f_{22}^1(x^1, x^4), f_{23}^1(x^1, x^4)) \\
 &= \max_{x_2^1, x_2^4} \left(\frac{4x_1^1 + 2x_2^1 + x_3^1 + 3}{8x_1^4 + 6x_2^4 + 5x_3^4 + 10}, \right. \\
 &\quad \left. \frac{x_2^1 + 2x_3^1 + 1}{5x_1^4 + 9x_2^4 + 8x_3^4 + 9}, \frac{3x_1^1 + 4x_2^1 + 5x_3^1 + 5}{9x_1^4 + 10x_2^4 + 9x_3^4 + 12} \right) \\
 &\text{where } x_3^1, x_3^4 \text{ solve,} \\
 &\text{subject to} \\
 &x^1, x^4 \in G^1 \\
 (\bar{U}_2 - MMFP) : \max_{x_2^1, x_2^4} F_2^4(x^1, x^4) &= \max_{x_2^1, x_2^4} (f_{21}^4(x^1, x^4), f_{22}^4(x^1, x^4), f_{23}^4(x^1, x^4)) \\
 &= \max_{x_2^1, x_2^4} \left(\frac{8x_1^4 + 11x_2^4 + 8x_3^4 + 10}{3x_1^1 + x_2^1 + x_3^1 + 5}, \right. \\
 &\quad \left. \frac{4x_1^4 + 7x_2^4 + 6x_3^4 + 4}{x_1^1 + 3x_2^1 + x_3^1 + 8}, \frac{10x_1^4 + 11x_2^4 + 12x_3^4 + 11}{2x_1^1 + 3x_2^1 + 2x_3^1 + 3} \right) \\
 &\text{where } x_3^1, x_3^4 \text{ solve,} \\
 &\text{subject to} \\
 &x^1, x^4 \in G^4 \\
 (\underline{L}_2 - MMFP) : \max_{x_2^2, x_2^3} F_2^2(x^2, x^3) &= \max_{x_2^2, x_2^3} (f_{21}^2(x^2, x^3), f_{22}^2(x^2, x^3), f_{23}^2(x^2, x^3)) \\
 &= \max_{x_2^2, x_2^3} \left(\frac{5x_1^2 + 6x_2^2 + 3x_3^2 + 5}{7x_1^3 + 4x_2^3 + 3x_3^3 + 8}, \right. \\
 &\quad \left. \frac{x_2^2 + 3x_3^2 + 4x_3^2 + 2}{2x_1^3 + 7x_2^3 + 5x_3^3 + 9}, \frac{6x_1^2 + 7x_2^2 + 7x_3^2 + 8}{7x_1^3 + 7x_2^3 + 6x_3^3 + 9} \right) \\
 &\text{where } x_3^2, x_3^3 \text{ solve} \\
 &\text{subject to} \\
 &x^2, x^3 \in G^2 \\
 (\underline{U}_2 - MMFP) : \max_{x_2^2, x_2^3} F_2^3(x^2, x^3) &= \max_{x_2^2, x_2^3} (f_{21}^3(x^2, x^3), f_{22}^3(x^2, x^3), f_{23}^3(x^2, x^3)) \\
 &= \max_{x_2^2, x_2^3} \left(\frac{7x_1^3 + 8x_2^3 + 5x_3^3 + 7}{5x_1^2 + 3x_2^2 + 2x_3^2 + 6}, \right. \\
 &\quad \left. \frac{2x_1^3 + 4x_2^3 + 6x_3^3 + 4}{2x_1^2 + 5x_2^2 + 3x_3^2 + 8}, \frac{8x_1^3 + 9x_2^3 + 8x_3^3 + 10}{4x_1^2 + 5x_2^2 + 5x_3^2 + 7} \right) \\
 &\text{where } x_3^2, x_3^3 \text{ solve} \\
 &\text{subject to} \\
 &x^2, x^3 \in G^3
 \end{aligned}$$

The results in Table 1 evidence that the rough optimum solution of the SLDM's problem is $x_1^S = ([0.0000, 0.0000], [0.0000, 0.3889])$, $x_2^S = ([1.8754, 1.8754], [1.0000, 2.0000])$, $x_3^S = ([1.6447, 1.6447], [0.5432, 1.7778])$, $f_{21}^S = ([1.0367, 2.0265], [0.1630, 7.5397])$, $f_{22}^S = ([0.4680, 0.9578], [0.0715, 2.6182])$, and $f_{23}^S = ([1.60202, 1.6275], [0.2275, 8.2162])$. Third, the TLDM's problem can be formulated in a similar way as follows:

Third – level decision maker[DM₃] :

$$\begin{aligned}
 (\bar{L}_3 - MMFP) : \max_{x_3^1, x_3^4} F_3^1(x^1, x^4) &= \max_{x_3^1, x_3^4} (f_{31}^1(x^1, x^4), f_{32}^1(x^1, x^4), f_{33}^1(x^1, x^4)) \\
 &= \max_{x_3^1, x_3^4} \left(\frac{2x_1^1 + 3x_2^1 + x_3^1 + 6}{4x_1^4 + 7x_2^4 + 6x_3^4 + 10}, \right. \\
 &\quad \left. \frac{3x_1^1 + 2x_2^1 + 2x_3^1 + 4}{8x_1^4 + 5x_2^4 + 7x_3^4 + 9}, \frac{x_1^1 + x_2^1 + 2x_3^1 + 4}{6x_1^4 + 5x_2^4 + 9x_3^4 + 11} \right) \\
 (\underline{L}_3 - MMFP) : \max_{x_3^2, x_3^3} F_3^2(x^2, x^3) &= \max_{x_3^2, x_3^3} (f_{31}^2(x^2, x^3), f_{32}^2(x^2, x^3), f_{33}^2(x^2, x^3)) \\
 &= \max_{x_3^2, x_3^3} \left(\frac{3x_1^2 + 5x_2^2 + 2x_3^2 + 8}{3x_1^3 + 5x_2^3 + 5x_3^3 + 8}, \right. \\
 &\quad \left. \frac{4x_1^2 + 3x_2^2 + 3x_3^2 + 7}{5x_1^3 + 3x_2^3 + 6x_3^3 + 6}, \frac{3x_1^2 + 4x_2^2 + 5x_3^2 + 7}{3x_1^3 + 5x_2^3 + 7x_3^3 + 8} \right)
 \end{aligned}$$

subject to

$$x^1, x^4 \in G^1$$

$$\begin{aligned}
 (\bar{U}_3 - MMFP) : & \max_{x^1, x^4} F_3^4(x^1, x^4) \\
 = & \max_{x^1, x^4} (f_{31}^4(x^1, x^4), f_{32}^4(x^1, x^4), f_{33}^4(x^1, x^4)) \\
 = & \max_{x^1, x^4} \left(\frac{8x_1^4 + 8x_2^4 + 4x_3^4 + 12}{x_1^1 + 2x_2^1 + 2x_3^1 + 2}, \right. \\
 & \left. \frac{8x_1^4 + 9x_2^4 + 10x_3^4 + 16}{x_1^1 + x_2^1 + 3x_3^1 + 2}, \frac{6x_1^4 + 7x_2^4 + 8x_3^4 + 12}{2x_1^1 + x_2^1 + 3x_3^1 + 3} \right)
 \end{aligned}$$

subject to

$$x^1, x^4 \in G^4$$

subject to

$$x^2, x^3 \in G^2$$

$$\begin{aligned}
 (\underline{U}_3 - MMFP) : & \max_{x^2, x^3} F_3^3(x^2, x^3) \\
 = & \max_{x^2, x^3} (f_{31}^3(x^2, x^3), f_{32}^3(x^2, x^3), f_{33}^3(x^2, x^3)) \\
 = & \max_{x^2, x^3} \left(\frac{5x_1^3 + 7x_2^3 + 3x_3^3 + 9}{2x_1^2 + 3x_2^2 + 4x_3^2 + 6}, \right. \\
 & \left. \frac{6x_1^3 + 5x_2^3 + 7x_3^3 + 12}{3x_1^2 + 2x_2^2 + 4x_3^2 + 5}, \frac{4x_1^3 + 5x_2^3 + 6x_3^3 + 10}{2x_1^2 + 3x_2^2 + 6x_3^2 + 5} \right)
 \end{aligned}$$

subject to

$$x^2, x^3 \in G^3$$

The TLDM's problem can be treated in a similar way as the author discussed in the FLDM. Then, the rough optimum solution of the TLDM's problem is found to be $x_1^T = ([0.3404, 0.3404], [0.2316, 0.3608])$, $x_2^T = ([0.3234, 0.3234], [0.1144, 1.5941])$, $x_3^T = ([0.0000, 0.0000], [0.0000, 1.3235])$, $f_{31}^T = ([1.000, 1.6947], [0.2229, 13.3848])$, $f_{32}^T = ([1.0761, 2.3484], [0.1691, 19.8070])$, and $f_{33}^T = ([0.8756, 1.9514], [0.1315, 10.0377])$. Table 1 shows the optimum results.

	FLDM, i=1				SLDM, i=2				TLDM, i=3			
	r = 1	r = 2	r = 3	r = 4	r = 1	r = 2	r = 3	r = 4	r = 1	r = 2	r = 3	r = 4
	\bar{L}_1	\underline{L}_1	\underline{U}_1	\bar{U}_1	\bar{L}_2	\underline{L}_2	\underline{U}_2	\bar{U}_2	\bar{L}_3	\underline{L}_3	\underline{U}_3	\bar{U}_3
y_{i1}^{11}	0.0000	0.0000	0.0297	0.0251	0.0246	0.0186	0.0000	0.0000	0.0197	0.0320	0.0000	0.1142
y_{i1}^{21}	0.0864	0.1108	0.1090	0.2018	0.0000	0.0798	0.1333	0.1667	0.0098	0.0304	0.1333	0.1262
y_{i1}^{31}	0.0653	0.1398	0.0000	0.0000	0.0343	0.0700	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ξ_{i1}^{11}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ξ_{i1}^{21}	0.0000	0.0000	0.0396	0.2573	0.0000	0.0000	0.0267	0.1667	0.0000	0.0000	0.0267	0.3785
ξ_{i1}^{31}	0.0000	0.0000	0.1900	0.4016	0.0000	0.0000	0.2524	0.2963	0.0000	0.0000	0.2524	0.4191
ρ_{i1}^r	0.0518	0.0472	0.1188	0.2437	0.0632	0.0426	0.1000	0.1667	0.0853	0.0940	0.1000	0.3166
y_{ir}	1.0000	0.9973	0.6757	0.8693	0.8284	0.9024	1.0000	0.9993	0.9867	0.9498	1.0000	0.8025

Table 1: Optimum solutions of the FLDM's, SLDM's and TLDM's Tchebycheff problems

Let the FLDM decide to give tolerance limit for his/her controlled variables $x_1^{1F} = 0.0000, x_1^{2F} = 0.0000, x_1^{3F} = 0.0000$, and $x_1^{4F} = 0.1031$ as $(t_{11}^-, t_{11}^+) = (0.1000, 0.1000), (t_{12}^-, t_{12}^+) = (0.1000, 0.2000), (t_{13}^-, t_{13}^+) = (0.1000, 0.1000)$, and $(t_{14}^-, t_{14}^+) = (0.1000, 0.1000)$, respectively. Similarly, the SLDM chooses the tolerance limit for $x_2^{1F} = 1.0000, x_2^{2F} = 1.8754, x_2^{3F} = 1.8754$, and $x_2^{4F} = 2.0000$ as $(t_{21}^-, t_{21}^+) = (0.1000, 0.1000), (t_{22}^-, t_{22}^+) = (2.0000, 2.0000), (t_{23}^-, t_{23}^+) = (0.1000, 0.1000)$, and $(t_{24}^-, t_{24}^+) = (2.0000, 2.0000)$, respectively. Hence, it is possible to use equations (14)-(20) and construct the fuzzy programming model for the TLDM as follows:

$$(\bar{L}\text{-Tch}) : \max \lambda_1,$$

subject to

$$y_{ir}^4 \in G_{ir}^1 \cup G_{\rho_{ir}^1}, (i = 1, 2, 3; r = 1, 2, 3)$$

$$0.0438 - \xi_{11}^{14} - \xi_{11}^{24} - 2\xi_{11}^{34} + \frac{2}{8}y_{11}^{14} + \frac{5}{8}y_{11}^{24} + \frac{11}{8}y_{11}^{34} \geq 0.382\lambda_1,$$

$$0.086 - \xi_{12}^{14} - 3\xi_{12}^{24} - 2\xi_{12}^{34} - \frac{2}{5}y_{12}^{14} + y_{12}^{24} + \frac{4}{5}y_{12}^{34} \geq 0.453\lambda_1,$$

$$0.004 - 2\xi_{13}^{14} - 3\xi_{13}^{24} - 3\xi_{13}^{34} + \frac{10}{9}y_{13}^{14} + \frac{21}{9}y_{13}^{24} + \frac{20}{9}y_{13}^{34} \geq 0.64\lambda_1,$$

$$0.137 - 4\xi_{21}^{14} - 2\xi_{21}^{24} - \xi_{21}^{34} + \frac{8}{5}y_{21}^{14} + \frac{1}{5}y_{21}^{24} - \frac{1}{2}y_{21}^{34} \geq 0.577\lambda_1,$$

$$0.0396 - \xi_{22}^{24} - 2\xi_{22}^{34} - \frac{5}{9}y_{22}^{14} + \frac{10}{9}y_{22}^{34} \geq 0.2057\lambda_1,$$

$$(\underline{U}\text{-Tch}) : \max \lambda_3,$$

subject to

$$y_{ir}^2 \in G_{ir}^3 \cup G_{\rho_{ir}^3}, (i = 1, 2, 3; r = 1, 2, 3)$$

$$0.25 + \xi_{11}^{12} + 3\xi_{11}^{22} + 2\xi_{11}^{32} - \frac{3}{7}y_{11}^{12} + \frac{16}{7}y_{11}^{22} + \frac{4}{7}y_{11}^{32} \geq 1.046\lambda_3,$$

$$1.767 + 3\xi_{12}^{12} + 7\xi_{12}^{22} + 5\xi_{12}^{32} - \frac{19}{3}y_{12}^{12} - \frac{28}{3}y_{12}^{22} + \frac{1}{3}y_{12}^{32} \geq 1.74\lambda_3,$$

$$0.458 + 5\xi_{13}^{12} + 5\xi_{13}^{22} + 8\xi_{13}^{32} - y_{13}^{12} + \frac{7}{5}y_{13}^{22} + \frac{16}{5}y_{13}^{32} \geq 1.32\lambda_3$$

$$0.427 + 7\xi_{21}^{12} + 8\xi_{21}^{22} + 5\xi_{21}^{32} + \frac{7}{6}y_{21}^{12} + \frac{9}{2}y_{21}^{22} + \frac{8}{3}y_{21}^{32} \geq 1.286\lambda_3,$$

$$0.22 + 2\xi_{22}^{12} + 4\xi_{22}^{22} + 6\xi_{22}^{32} + y_{22}^{12} + \frac{3}{2}y_{22}^{22} + \frac{9}{2}y_{22}^{32} \geq 0.86\lambda_3,$$

$$0.189 - 3\xi_{23}^{14} - 4\xi_{23}^{24} - 5\xi_{23}^{34} - \frac{3}{4}y_{23}^{14} - \frac{1}{6}y_{23}^{24} + \frac{5}{4}y_{23}^{34} \geq 0.674\lambda_1, \quad 0.53 + 8\xi_{23}^{12} + 9\xi_{23}^{22} + 8\xi_{23}^{32} + \frac{16}{7}y_{23}^{12} + \frac{13}{7}y_{23}^{22} + \frac{6}{7}y_{23}^{32} \geq 0.73\lambda_3$$

$$0.377 - 2\xi_{31}^{14} - 3\xi_{31}^{24} - \xi_{31}^{34} - \frac{2}{5}y_{31}^{14} - \frac{6}{5}y_{31}^{24} - \frac{13}{5}y_{31}^{34} \geq 0.58\lambda_1, \quad 0.69 + 5\xi_{31}^{12} + 7\xi_{31}^{22} + 3\xi_{31}^{32} + 2y_{31}^{12} + \frac{5}{2}y_{31}^{22} - 3y_{31}^{32} \geq 0.887\lambda_3,$$

$$0.275 - 3\xi_{32}^{14} - 2\xi_{32}^{24} - 2\xi_{32}^{34} - \frac{5}{9}y_{32}^{14} - \frac{2}{9}y_{32}^{24} - \frac{10}{9}y_{32}^{34} \geq 0.65\lambda_1, \quad 1.58 + 6\xi_{32}^{12} + 5\xi_{32}^{22} + 7\xi_{32}^{32} - \frac{6}{5}y_{32}^{12} + \frac{1}{5}y_{32}^{22} - \frac{13}{5}y_{32}^{32} \geq 1.53\lambda_3,$$

$$0.23 - \xi_{33}^{14} - \xi_{33}^{24} - 2\xi_{33}^{34} - \frac{13}{11}y_{33}^{14} - \frac{9}{11}y_{33}^{24} - \frac{14}{11}y_{33}^{34} \geq 0.655\lambda_1, \quad 1.21 + 4\xi_{33}^{12} + 5\xi_{33}^{22} + 6\xi_{33}^{32} - y_{33}^{22} - 6y_{33}^{32} \geq 1.16\lambda_3$$

$$y_{11}^{14} + 0.1\rho_{11}^1 - 0.103\rho_{11}^1 \geq 0.1\rho_{11}^1\lambda_1,$$

$$y_{11}^{12} + 0.1\rho_{11}^3 \geq 0.1\rho_{11}^3\lambda_3,$$

$$0.1\rho_{11}^1 + 0.103\rho_{11}^1 - y_{11}^{14} \geq 0.1\rho_{11}^1\lambda_1,$$

$$0.2\rho_{11}^3 - y_{11}^{12} \geq 0.2\rho_{11}^3\lambda_3,$$

$$y_{11}^{24} + 2\rho_{11}^1 - 2\rho_{11}^1 \geq 2\rho_{11}^1\lambda_1,$$

$$y_{11}^{22} + 2\rho_{11}^3 - 1.875\rho_{11}^2 \geq 2\rho_{11}^3\lambda_3,$$

$$2\rho_{11}^1 + 2\rho_{11}^1 - y_{11}^{24} \geq 2\rho_{11}^1\lambda_1,$$

$$2\rho_{11}^3 + 1.875\rho_{11}^3 - y_{11}^{22} \geq 2\rho_{11}^3\lambda_3,$$

$$\frac{y_{i1}^{14}}{\rho_{i1}^1} = \frac{y_{i2}^{14}}{\rho_{i2}^1} = \frac{y_{i3}^{14}}{\rho_{i3}^1}, \quad \frac{y_{i1}^{24}}{\rho_{i1}^1} = \frac{y_{i2}^{24}}{\rho_{i2}^1} = \frac{y_{i3}^{24}}{\rho_{i3}^1},$$

$$\frac{y_{i1}^{12}}{\rho_{i1}^3} = \frac{y_{i2}^{12}}{\rho_{i2}^3} = \frac{y_{i3}^{12}}{\rho_{i3}^3}, \quad \frac{y_{i1}^{22}}{\rho_{i1}^3} = \frac{y_{i2}^{22}}{\rho_{i2}^3} = \frac{y_{i3}^{22}}{\rho_{i3}^3},$$

$$\frac{y_{i1}^{34}}{\rho_{i1}^1} = \frac{y_{i2}^{34}}{\rho_{i2}^1} = \frac{y_{i3}^{34}}{\rho_{i3}^1}, \quad \frac{\xi_{i1}^{14}}{\rho_{i1}^1} = \frac{\xi_{i2}^{14}}{\rho_{i2}^1} = \frac{\xi_{i3}^{14}}{\rho_{i3}^1},$$

$$\frac{y_{i1}^{32}}{\rho_{i1}^3} = \frac{y_{i2}^{32}}{\rho_{i2}^3} = \frac{y_{i3}^{32}}{\rho_{i3}^3}, \quad \frac{\xi_{i1}^{12}}{\rho_{i1}^3} = \frac{\xi_{i2}^{12}}{\rho_{i2}^3} = \frac{\xi_{i3}^{12}}{\rho_{i3}^3},$$

$$\frac{\xi_{i1}^{24}}{\rho_{i1}^1} = \frac{\xi_{i2}^{24}}{\rho_{i2}^1} = \frac{\xi_{i3}^{24}}{\rho_{i3}^1}, \quad \frac{\xi_{i1}^{34}}{\rho_{i1}^1} = \frac{\xi_{i2}^{34}}{\rho_{i2}^1} = \frac{\xi_{i3}^{34}}{\rho_{i3}^1},$$

$$\frac{\xi_{i1}^{22}}{\rho_{i1}^3} = \frac{\xi_{i2}^{22}}{\rho_{i2}^3} = \frac{\xi_{i3}^{22}}{\rho_{i3}^3}, \quad \frac{\xi_{i1}^{32}}{\rho_{i1}^3} = \frac{\xi_{i2}^{32}}{\rho_{i2}^3} = \frac{\xi_{i3}^{32}}{\rho_{i3}^3},$$

$$\lambda_1 \in [0, 1].$$

$$\lambda_3 \in [0, 1].$$

($\underline{L} - Tch$) : max λ_2 ,
subject to

($\overline{U} - Tch$) : max λ_4 ,
subject to

$$y_{ir}^3 \in G_{ir}^2 \cup G_{\rho_{ir}^2}, \quad (i = 1, 2, 3; r = 1, 2, 3)$$

$$y_{ir}^1 \in G_{ir}^4 \cup G_{\rho_{ir}^4}, \quad (i = 1, 2, 3; r = 1, 2, 3)$$

$$0.163 - \xi_{11}^{13} - \xi_{11}^{23} - 2\xi_{11}^{33} + \frac{1}{7}y_{11}^{13} + \frac{5}{7}y_{11}^{23} + \frac{6}{7}y_{11}^{33} \geq 0.077\lambda_2,$$

$$0.61 + 4\xi_{11}^{11} + 6\xi_{11}^{21} + 5\xi_{11}^{31} + \frac{9}{4}y_{11}^{11} + 6y_{11}^{21} + \frac{13}{4}y_{11}^{31} \geq 5.6\lambda_4,$$

$$0.415 - 2\xi_{12}^{13} - 5\xi_{12}^{23} - 3\xi_{12}^{33} - \frac{7}{9}y_{12}^{13} + \frac{2}{9}y_{12}^{33} \geq 0.199\lambda_2,$$

$$1.87 + 4\xi_{12}^{11} + 8\xi_{12}^{21} + 8\xi_{12}^{31} - \frac{8}{3}y_{12}^{11} - 2y_{12}^{21} + \frac{14}{3}y_{12}^{31} \geq 5.6\lambda_4,$$

$$0.346 - 3\xi_{13}^{13} - 4\xi_{13}^{23} - 5\xi_{13}^{33} + \frac{3}{2}y_{13}^{23} + 2y_{13}^{33} \geq 0.151\lambda_2,$$

$$2.34 + 7\xi_{13}^{11} + 6\xi_{13}^{21} + 9\xi_{13}^{31} - 5y_{13}^{11} - 2y_{13}^{21} - 3y_{13}^{31} \geq 7.875\lambda_4,$$

$$0.48 - 5\xi_{11}^{13} - 6\xi_{11}^{23} - 3\xi_{11}^{33} + \frac{5}{8}y_{11}^{13} + \frac{7}{2}y_{11}^{23} + \frac{9}{8}y_{11}^{33} \geq 0.89\lambda_2,$$

$$0.62 + 8\xi_{11}^{11} + 11\xi_{11}^{21} + 8\xi_{11}^{31} + 2y_{11}^{11} + 9y_{11}^{21} + 6y_{11}^{31} \geq 6.16\lambda_4,$$

$$0.19 - \xi_{12}^{13} - 3\xi_{12}^{23} - 4\xi_{12}^{33} + \frac{5}{9}y_{12}^{13} + \frac{13}{9}y_{12}^{23} + \frac{26}{9}y_{12}^{33} \geq 0.44\lambda_2,$$

$$-0.08 + 4\xi_{12}^{11} + 7\xi_{12}^{21} + 6\xi_{12}^{31} + \frac{7}{2}y_{12}^{11} + \frac{11}{2}y_{12}^{21} + \frac{11}{2}y_{12}^{31} \geq 2.04\lambda_4,$$

$$0.75 - 6\xi_{13}^{13} - 7\xi_{13}^{23} - 7\xi_{13}^{33} - \frac{2}{9}y_{13}^{13} + \frac{7}{9}y_{13}^{23} + \frac{5}{3}y_{13}^{33} \geq 0.88\lambda_2,$$

$$0.005 + 10\xi_{13}^{11} + 11\xi_{13}^{21} + 12\xi_{13}^{31} + \frac{48}{7}y_{13}^{11} + \frac{44}{7}y_{13}^{21} + \frac{62}{7}y_{13}^{31} \geq 6.65\lambda_4,$$

$$0.7365 - 3\xi_{11}^{13} - 5\xi_{11}^{23} - 2\xi_{11}^{33} - 3y_{11}^{33} \geq 0.7365\lambda_2,$$

$$4.51 + 8\xi_{11}^{11} + 8\xi_{11}^{21} + 4\xi_{11}^{31} + 2y_{11}^{11} - 4y_{11}^{21} - 8y_{11}^{31} \geq 11.9\lambda_4,$$

$$0.96 - 4\xi_{12}^{13} - 3\xi_{12}^{23} - 3\xi_{12}^{33} - \frac{11}{6}y_{12}^{13} - \frac{1}{2}y_{12}^{23} - 4y_{12}^{33} \geq 0.87\lambda_2,$$

$$5.58 + 8\xi_{12}^{11} + 9\xi_{12}^{21} + 10\xi_{12}^{31} + y_{12}^{21} - 14y_{12}^{31} \geq 17.66\lambda_4,$$

$$0.72 - 3\xi_{13}^{13} - 4\xi_{13}^{23} - 5\xi_{13}^{33} + \frac{3}{8}y_{13}^{13} - \frac{3}{8}y_{13}^{23} - \frac{9}{8}y_{13}^{33} \geq 0.72\lambda_2,$$

$$2.57 + 6\xi_{13}^{11} + 7\xi_{13}^{21} + 8\xi_{13}^{31} - 2y_{13}^{11} + 3y_{13}^{21} - 4y_{13}^{31} \geq 8.6\lambda_4,$$

$$y_{11}^{13} + 0.1\rho_{11}^2 \geq 0.1\rho_{11}^2\lambda_2,$$

$$y_{11}^{11} + 0.1\rho_{11}^4 \geq 0.1\rho_{11}^4\lambda_4,$$

$$0.1\rho_{11}^2 - y_{11}^{13} \geq 0.1\rho_{11}^2\lambda_2,$$

$$0.1\rho_{11}^4 - y_{11}^{11} \geq 0.1\rho_{11}^4\lambda_4,$$

$$y_{11}^{23} + 0.1\rho_{11}^2 - 1.8754\rho_{11}^2 \geq 0.1\rho_{11}^2\lambda_2,$$

$$y_{11}^{21} + 0.1\rho_{11}^4 - \rho_{11}^4 \geq 0.1\rho_{11}^4\lambda_4,$$

$$0.1\rho_{11}^2 + 1.8754\rho_{11}^2 - y_{11}^{23} \geq 0.1\rho_{11}^2\lambda_2,$$

$$0.1\rho_{11}^4 + \rho_{11}^4 - y_{11}^{21} \geq 0.1\rho_{11}^4\lambda_4,$$

$$\frac{y_{i1}^{13}}{\rho_{i1}^2} = \frac{y_{i2}^{13}}{\rho_{i2}^2} = \frac{y_{i3}^{13}}{\rho_{i3}^2}, \quad \frac{y_{i1}^{23}}{\rho_{i1}^2} = \frac{y_{i2}^{23}}{\rho_{i2}^2} = \frac{y_{i3}^{23}}{\rho_{i3}^2},$$

$$\frac{y_{i1}^{11}}{\rho_{i1}^4} = \frac{y_{i2}^{11}}{\rho_{i2}^4} = \frac{y_{i3}^{11}}{\rho_{i3}^4}, \quad \frac{y_{i1}^{21}}{\rho_{i1}^4} = \frac{y_{i2}^{21}}{\rho_{i2}^4} = \frac{y_{i3}^{21}}{\rho_{i3}^4},$$

$$\frac{y_{i1}^{33}}{\rho_{i1}^2} = \frac{y_{i2}^{33}}{\rho_{i2}^2} = \frac{y_{i3}^{33}}{\rho_{i3}^2}, \quad \frac{\xi_{i1}^{13}}{\rho_{i1}^2} = \frac{\xi_{i2}^{13}}{\rho_{i2}^2} = \frac{\xi_{i3}^{13}}{\rho_{i3}^2},$$

$$\frac{y_{i1}^{31}}{\rho_{i1}^4} = \frac{y_{i2}^{31}}{\rho_{i2}^4} = \frac{y_{i3}^{31}}{\rho_{i3}^4}, \quad \frac{\xi_{i1}^{11}}{\rho_{i1}^4} = \frac{\xi_{i2}^{11}}{\rho_{i2}^4} = \frac{\xi_{i3}^{11}}{\rho_{i3}^4},$$

$$\frac{\xi_{i1}^{23}}{\rho_{i1}^2} = \frac{\xi_{i2}^{23}}{\rho_{i2}^2} = \frac{\xi_{i3}^{23}}{\rho_{i3}^2}, \quad \frac{\xi_{i1}^{33}}{\rho_{i1}^2} = \frac{\xi_{i2}^{33}}{\rho_{i2}^2} = \frac{\xi_{i3}^{33}}{\rho_{i3}^2},$$

$$\frac{\xi_{i1}^{21}}{\rho_{i1}^4} = \frac{\xi_{i2}^{21}}{\rho_{i2}^4} = \frac{\xi_{i3}^{21}}{\rho_{i3}^4}, \quad \frac{\xi_{i1}^{31}}{\rho_{i1}^4} = \frac{\xi_{i2}^{31}}{\rho_{i2}^4} = \frac{\xi_{i3}^{31}}{\rho_{i3}^4},$$

$$\lambda_2 \in [0, 1]$$

$$\lambda_4 \in [0, 1]$$

Then, the Pareto optimal rough solution of an FRMMFP problem which is satisfactory to all DMs is obtained as $x_1^* = ([0.0000, 0.0000], [0.0000, 0.1109])$, $x_2^* = ([1.6000, 1.6000], [1.0000, 2.0000])$, $x_3^* = ([0.6216, 0.6216], [0.2856, 1.7778])$ with $\lambda^* = ([0.5811, 0.8540], [0.2573, 0.9541])$, where $f_{11}^* = ([0.4369, 1.1219], [0.1092, 6.6111])$, $f_{12}^* = ([0.5608, 1.3798], [0.1344, 6.4700])$, $f_{13}^* = ([0.7055, 1.5442], [0.1415, 7.5721])$, $f_{21}^* = ([1.0123, 1.9021], [0.1663, 7.4948])$, $f_{22}^* = ([0.3984, 0.7909], [0.0615, 2.5794])$, $f_{23}^* = ([0.9842, 1.6221], [0.2128, 8.4372])$, $f_{31}^* = ([0.9024, 1.6607], [0.2645, 7.8750])$, $f_{32}^* = ([0.9405, 2.2787], [0.2032, 13.6551])$, and $f_{33}^* = ([0.8112, 1.6061], [0.1479, 8.4186])$. Table 2 illustrates the optimum result of the fuzzy programming model.

	$\underline{L} - Tch(r=1)$	$\underline{L} - Tch(r=2)$	$\underline{U} - Tch(r=3)$	$\overline{U} - Tch(r=4)$
y_{11}^{11}	0.0088	0.0000	0.0000	0.0000
y_{11}^{21}	0.0653	0.2019	0.1860	0.2500
y_{11}^{31}	0.0227	0.0523	0.0000	0.0000
ξ_{11}^{11}	0.0000	0.0000	0.0000	0.0000
ξ_{11}^{21}	0.0000	0.0000	0.0000	0.2500
ξ_{11}^{31}	0.0000	0.0000	0.0000	0.4444
ρ_{11}^r	0.0797	0.0841	0.1163	0.2500
λ_r	0.2573	0.8540	0.9541	0.5811

Table 2: The optimum result of the fuzzy programming model

5 Discussion

FRMMFP has become widely used tools in many fields such as engineering, finance, economics and other disciplines. The FRMMFP problem is very complex to solve by conventional methods because of

- The fractional form of objective functions.
- A hierarchical decision structure with independent and often conflicting objectives.
- The decision variables and the coefficients in the objective functions and in constraints are rough intervals.

To overcome these reasons, this paper presented a methodology to solve the FRMMFP problem based on the fuzzy approach and linearization technique.

The main features of the proposed method could be summarized as follows:

1. Our approach can deal with non-continues and non differentiable functions which are actually existed in practical optimization problems.
2. Our approach suitable to handle FRMMFP problem, simple concepts, easy implementations, less execution efforts, more flexible and adaptive to awide variety of problems and robust than the convential methods.
3. We can convert the linear fractional programming (LFP) problem into an LP problem easily by using som steps.

6 Conclusions

This paper presented an approach to acquire the solution of MMFP problem in which all the parameters and decision variables are rough intervals. The author first transformed the problem into its respective crisp equivalent using extended interval method. Then, all of fractional objective functions of crisp model become a linear objective using the linearization technique. Furthermore, the crisp MMFP problem was simply reduced to MMLP problem. The MMLP problem then gets reduced to four single linear programming problems by applying modified fuzzy approach. Finally, the linear programming problems were solved to find a rough compromise solution for the FRMMFP problem. The author provided an example for demonstrating the applicability of the proposed procedure for solving the FRMMFP problem. The proposed approach is an excellent for solving the FRMMFP problem because it can serve DMs by providing an appropriate best solution to a variety of MMFP models having crisp or rough interval parameters and variables in a simple and effective manner. The main advantage of the proposed approach is that computational complexity is reduced by defining a linearization technique for the fractional objective functions. This procedure can be extended for solving fully fuzzy multi-level multi objective linear fractional programming problems. Also, this modified approach can be

extended for solving fully fuzzy rough multi-level multi objective linear fractional programming problems. We further conclude that the proposed concept will be helpful in solving real life problems involving linear fractional programming problems in agriculture, production planning, financial and corporate planning, health care, hospital management, etc.

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**Building fuzzy approach with linearization technique for fully
rough multi-objective multi-level linear fractional
programming problem**

E. Fathy

**ساخت یک روش فازی، با تکنیک خطی سازی برای مسئله برنامه نویسی کسری خطی
چند مرحله ای چند منظوره کاملاً طبیعی**

چکیده. این مقاله یک روند حل مناسب برای مسئله برنامه نویسی کسری خطی چند مرحله ای چند منظوره کاملاً طبیعی (FRMMFP) ارائه می دهد. ابتدا، توسیعی از روش فاصله برای مقابله با ناهمواری مسئله بیان شده ارائه می شود. سپس، یک تکنیک فاصله برای خطی سازی اهداف کسری پیشنهاد شده است. سرانجام، یک اصلاح از رویکرد فازی در محیط کاملاً طبیعی برای حل مدل خطی ارائه می شود. برای درک روند حل روش پیشنهادی، یک مثال ارائه شده است.