Iranian Journal of Fuzzy Systems
Volume 18, Number 2, (2021), pp. 159-173
Original paper

Synchronization of BAM Cohen–Grossberg FCNNs with mixed time delays

M. Manikandan[1], K. Ratnavelu[2], P. Balasubramaniam[3] and S. H. Ong[4]

[1,4] *Department of Actuarial Science and Applied Statistics, Faculty of Business & Management, UCSI University, Kuala Lumpur, Malaysia*
[2] *Institute of Computer Science and Digital Innovation, UCSI University, Kuala Lumpur, Malaysia*
[1,2] *ISM, Faculty of Science, University of Malaya - 50603, Kuala Lumpur, Malaysia*
[3] *Department of Mathematics, The Gandhigram Rural Institute, Deemed to be University, Gandhigram - 624 302, Dindigul, Tamilnadu, India*

manikandangru@gmail.com, kurunathan@ucsiuniversity.edu.my, balugru@gmail.com, ongsh@ucsiuniversity.edu.my

Abstract

This paper deals with the synchronization problem of bidirectional associative memory (BAM) Cohen-Grossberg fuzzy cellular neural networks (CGFCNNs) with discrete time-varying and unbounded distributed delays. Some sufficient conditions are obtained to guarantee the robust synchronization of BAM CGFCNNs with discrete time-varying and unbounded distributed delays subjected to parametric uncertainty by using Lyapunov-Krasovskii (LK) functional and Linear matrix inequality (LMI) approach. Sufficient criteria ensure that the error dynamics of considered system is globally asymptotically stable. Finally, numerical examples with simulations are given to show the efficacy of the derived results.

Keywords: Linear matrix inequality, fuzzy cellular neural networks, delay, synchronization, Cohen-Grossberg neural networks.

1 Introduction

The past few decades have witnessed tremendous developments in the research field of neural networks (NNs). Various NNs, such as cellular neural networks (CNNs), bidirectional associative memory (BAM) NNs [6, 16, 19, 33], Hopfield NNs and BAM Cohen-Grossberg neural networks (CGNNs), have been widely investigated and successfully applied in many areas. Among them, the renowned BAM CGNNs [5] has recently gained considerable research attention, due to their potential applications in various fields such as signal processing, parallel computation, pattern recognition, classification of patterns and so on [1, 18]. Up to now, BAM CGNNs with various types of delays have been widely investigated by many authors [17, 24]. In mathematical modelling of real world problems, the uncertainty or vagueness is unavoidable. In order to take vagueness into consideration, fuzzy theory is considered as a suitable method. Yang et al. [32] integrated fuzzy set theory into the CNNs paradigm to give birth to the fuzzy cellular neural networks (FCNNs). It is a very useful tool in image processing and pattern recognition problems [21, 25]. Recently, various interesting results on the stability, synchronization and other behaviors of FCNNs have been reported [12, 13, 31]. In addition, time delay and parameter uncertainties which often break the stability of systems can be commonly encountered because of the modeling inaccuracies and changes in the environment of the model. In model of electronic NNs, time delays are likely to be present due to the finite switching speed of amplifiers or finite speed of information processing which are the core elements for implementing artificial neurons. A time delay that occurred in the interaction between neurons will affect the stability of a network by creating oscillation or unstable phenomena. Therefore, it is important to ensure that the model is stable with respect to the time delay and parameter uncertainties. In this regard, many researchers have focused on stability and synchronization issues of delayed NNs with an existence of uncertain parameters [30].

Corresponding Author: S. H. Ong
Received: February 2020; Revised: September 2020; Accepted: November 2020.

Since the drive-response (master-slave) concept introduced by Pecora and Carroll [23], there has been increasing focus on the master-slave synchronization of chaotic NNs with delay. The control and synchronization problems of chaotic systems have been intensively investigated due to their potential applications in various fields [9]. Based on nonlinear transformation and Fillipov discontinuous theory, the problem of fixed-time synchronization of memristive CGNNs with impulsive effects was investigated in [28]. Hu et al. [11] analyzed the problem of adaptive exponential synchronization of complex-valued CGNNs with known and unknown parameters. Finite-time stability theorem and generalized Lyapunov approach are used to realize the synchronization of discontinuous CGNNs with mixed time delays [4]. In [26], we have discussed the synchronization problem of fuzzy bidirectional associative memory neural networks with various time delays. Also, He and Chu [10] considered the delayed fuzzy BAM CGNNs with impulses for the problem of exponential stability analysis via Lyapunov function. Nevertheless, the synchronization of BAM CGFCNNs with discrete time-varying and unbounded distributed delays has yet to be addressed. Motivated by the above discussions, the main purpose of this paper is to analyze the synchronization of BAM CGFCNNs with delays via Lyapunov-Krasovskii (LK) functional and Linear matrix inequality (LMI) technique. Further, we consider the robust synchronization problem for delayed BAM CGFCNNs with parametric uncertainty. The uncertainty is assumed to be norm-bounded. By employing LK functional and free weighting matrix method, the synchronization and robust synchronization criterions are established in terms of LMI, which ensures that the error systems are globally asymptotically stable and globally robustly asymptotically stable, respectively.

Notations: Throughout this paper, the superscript "T" stands for matrix or vector transposition. A real $n \times n$ matrix Y, $Y > 0$ ($Y \geq 0, Y < 0, Y \leq 0$) implies that Y is symmetric and positive definite (semi-positive definite, negative definite, semi-negative definite, respectively). I is the identity matrix with appropriate dimension. We use an asterisk ($*$) to represent a term that is induced by symmetry. $diag(d_1, ..., d_n)$ denotes the diagonal matrix with diagonal entries $d_1, ..., d_n$. Let $\mathbb{R}$, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote respectively the set of real numbers, n-dimensional Euclidean space and the set of all $n \times m$ real matrices. Further let us denote $\mathcal{H} = \{1, ..., n\}$ & $\bar{\mathcal{H}} = \{1, ..., m\}$.

2 Preliminaries and model description

Consider the BAM CGFCNNs with delays as drive system [10] follows:

$$\begin{cases} \dot{x}_i(t) &= d_i[-\tilde{a}_{1i}(x_i(t)) + \sum_{j=1}^m b_{ij}\bar{f}_j(y_j(t)) + \sum_{j=1}^m c_{ij}\bar{f}_j(y_j(t-\rho_1(t))) \\ &\quad + \bigwedge_{j=1}^m \xi_{ij}\int_{-\infty}^t q_j(t-r)\bar{f}_j(y_j(t))dr + \bigvee_{j=1}^m \mu_{ij}\int_{-\infty}^t q_j(t-r)\bar{f}_j(y_j(t))dr \\ &\quad + I_i + \bigwedge_{j=1}^m \xi_{ij}\bar{f}_j(y_j(t-\rho_1(t))) + \bigvee_{j=1}^m \mu_{ij}\bar{f}_j(y_j(t-\rho_1(t)))], \\ \dot{y}_j(t) &= \bar{d}_j[-\bar{a}_{2j}(y_j(t)) + \sum_{i=1}^n \bar{b}_{ji}\bar{g}_i(x_i(t)) + \sum_{i=1}^n \bar{c}_{ji}\bar{g}_i(x_i(t-\rho_2(t))) \\ &\quad + \bigwedge_{i=1}^n \bar{\xi}_{ji}\int_{-\infty}^t q_i(t-r)\bar{g}_i(x_i(t))dr + \bigvee_{i=1}^n \bar{\mu}_{ji}\int_{-\infty}^t q_i(t-r)\bar{g}_i(x_i(t))dr \\ &\quad + \bar{I}_j + \bigwedge_{i=1}^n \bar{\xi}_{ji}\bar{g}_i(x_i(t-\rho_2(t))) + \bigvee_{i=1}^n \bar{\mu}_{ji}\bar{g}_i(x_i(t-\rho_2(t)))], \\ & x_i(r) = \nu_i(r), \quad y_j(r) = \zeta_j(r), \quad r \in (-\infty, 0], \quad i \in \mathcal{H}, \quad j \in \bar{\mathcal{H}}, \end{cases} \tag{1}$$

where $x(t) = (x_1(t), .., x_n(t))^T \in \mathbb{R}^n$, $y(t) = (y_1(t), .., y_m(t))^T \in \mathbb{R}^m$. $x_i(t)$ and $y_j(t)$ are the states of the i^{th} & j^{th} neurons, respectively; $\nu_i(\cdot) \in \mathbb{C}((-\infty, 0], \mathbb{R})$, $\zeta_j(\cdot) \in \mathbb{C}((-\infty, 0], \mathbb{R})$, where $\mathbb{C}((-\infty, 0], \mathbb{R})$ denotes the set of all continuous functions from $(-\infty, 0]$ to $\mathbb{R}$; $\bar{g}_i(\cdot)$ and $\bar{f}_j(\cdot)$ stand for the signal functions of the i^{th} & j^{th} neurons, respectively; d_i and $\bar{d}_j$ are positive constants represent the amplification gain. $\tilde{a}_{1i}(x_i(t))$ and $\bar{a}_{2j}(y_j(t))$ are appropriately behaved function at time t: I_i and $\bar{I}_j$ denote the bias of the ith neuron and jth neuron, respectively; $q_i(r) \geq 0$ and $q_j(r) \geq 0$ are the feedback kernels and satisfy $\int_0^\infty q_i(r)dr = 1$, $\int_0^\infty q_j(r)dr = 1$; $b_{ij}, c_{ij}, \bar{b}_{ji}$ and $\bar{c}_{ji}$ are elements of feedback template; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively; $\xi_{ij}, \bar{\xi}_{ji}$ are the elements of fuzzy feedback MIN template; $\mu_{ij}, \bar{\mu}_{ji}$ are the elements of fuzzy feedback MAX template.

The response system [10] is described by the following equations:

$$\begin{cases} \dot{\hat{x}}_i(t) &= d_i[-\tilde{a}_{1i}(\hat{x}_i(t)) + \sum_{j=1}^m b_{ij}\bar{f}_j(\hat{y}_j(t)) + \sum_{j=1}^m c_{ij}\bar{f}_j(\hat{y}_j(t-\rho_1(t))) \\ &\quad + \bigwedge_{j=1}^m \xi_{ij}\int_{-\infty}^t q_j(t-r)\bar{f}_j(\hat{y}_j(r))dr + \bigvee_{j=1}^m \mu_{ij}\int_{-\infty}^t q_j(t-r)\bar{f}_j(\hat{y}_j(r))dr \\ &\quad + I_i + \bigwedge_{j=1}^m \xi_{ij}\bar{f}_j(\hat{y}_j(t-\rho_1(t))) + \bigvee_{j=1}^m \mu_{ij}\bar{f}_j(\hat{y}_j(t-\rho_1(t)))] + M_{1i}(t), \\ \dot{\hat{y}}_j(t) &= \bar{d}_j[-\bar{a}_{2j}(\hat{y}_j(t)) + \sum_{i=1}^n \bar{b}_{ji}\bar{g}_i(\hat{x}_i(t)) + \sum_{i=1}^n \bar{c}_{ji}\bar{g}_i(\hat{x}_i(t-\rho_2(t))) \\ &\quad + \bigwedge_{i=1}^n \bar{\xi}_{ji}\int_{-\infty}^t q_i(t-r)\bar{g}_i(\hat{x}_i(r))dr + \bigvee_{i=1}^n \bar{\mu}_{ji}\int_{-\infty}^t q_i(t-r)\bar{g}_i(\hat{x}_i(r))dr \\ &\quad + \bar{I}_j + \bigwedge_{i=1}^n \bar{\xi}_{ji}\bar{g}_i(\hat{x}_i(t-\rho_2(t))) + \bigvee_{i=1}^n \bar{\mu}_{ji}\bar{g}_i(\hat{x}_i(t-\rho_2(t)))] + M_{2j}(t), \\ & \hat{x}_i(r) = \varphi_i(r), \quad \hat{y}_j(r) = \varsigma_j(r), \quad r \in (-\infty, 0], \quad i \in \mathcal{H}, \quad j \in \bar{\mathcal{H}}, \end{cases} \tag{2}$$

where $M_{1i}(t)$ and $M_{2j}(t)$ are the control inputs; $\hat{x}_i(t)$ and $\hat{y}_j(t)$ are the states of the i^{th} & j^{th} neurons, respectively. $\varphi_i(\cdot) \in \mathbb{C}((-\infty,0],\mathbb{R})$, $\varsigma_j(\cdot) \in \mathbb{C}((-\infty,0],\mathbb{R})$. All coefficients in (2) which are the same as in (1). The apposite control inputs are:

$$M_{1i}(t) = m_{1i}e_{1i}(t),\ M_{2j}(t) = m_{2j}e_{2j}(t),\ i \in \mathcal{H},\ j \in \bar{\mathcal{H}} \tag{3}$$

To prove the drive system (1) synchronizing with the response system (2), we define the synchronization errors $e_{1i}(t) = x_i(t) - \hat{x}_i(t),\ e_{2j}(t) = y_j(t) - \hat{y}_j(t)$. Thus, the error dynamics between the systems (1) and (2) can be expressed as

$$\begin{cases} \dot{e}_{1i}(t) & = d_i[-\tilde{a}_{1i}(e_{1i}(t)) + \sum_{j=1}^m b_{ij}f_j(e_{2j}(t)) + \sum_{j=1}^m c_{ij}f_j(e_{2j}(t-\rho_1(t))) \\ & \quad + \bigwedge_{j=1}^m \xi_{ij}\int_{-\infty}^t q_j(t-r)\bar{f}_j(y_j(r))dr - \bigwedge_{j=1}^m \xi_{ij}\int_{-\infty}^t q_j(t-r)\bar{f}_j(\hat{y}_j(r))dr \\ & \quad + \bigvee_{j=1}^m \mu_{ij}\int_{-\infty}^t q_j(t-r)\bar{f}_j(y_j(r))dr - \bigvee_{j=1}^m \mu_{ij}\int_{-\infty}^t q_j(t-r)\bar{f}_j(\hat{y}_j(r))dr \\ & \quad + \bigwedge_{j=1}^m \xi_{ij}\bar{f}_j(y_j(t-\rho_1(t))) - \bigwedge_{j=1}^m \xi_{ij}\bar{f}_j(\hat{y}_j(t-\rho_1(t))) \\ & \quad + \bigvee_{j=1}^m \mu_{ij}\bar{f}_j(y_j(t-\rho_1(t))) - \bigvee_{j=1}^m \mu_{ij}\bar{f}_j(\hat{y}_j(t-\rho_1(t)))] - m_{1i}e_{1i}(t), \\ \dot{e}_{2j}(t) & = \bar{d}_j[-\bar{a}_{2j}(e_{2j}(t)) + \sum_{i=1}^n \bar{b}_{ji}g_i(e_{1i}(t)) + \sum_{i=1}^n \bar{c}_{ji}g_i(e_{1i}(t-\rho_2(t))) \\ & \quad + \bigwedge_{i=1}^n \bar{\xi}_{ji}\int_{-\infty}^t q_i(t-r)\bar{g}_i(x_i(r))dr - \bigwedge_{i=1}^n \bar{\xi}_{ji}\int_{-\infty}^t q_i(t-r)\bar{g}_i(\hat{x}_i(r))dr \\ & \quad + \bigvee_{i=1}^n \bar{\mu}_{ji}\int_{-\infty}^t q_i(t-r)\bar{g}_i(x_i(r))dr - \bigvee_{i=1}^n \bar{\mu}_{ji}\int_{-\infty}^t q_i(t-r)\bar{g}_i(\hat{x}_i(r))dr \\ & \quad + \bigwedge_{i=1}^n \bar{\xi}_{ji}\bar{g}_i(x_i(t-\rho_2(t))) - \bigwedge_{i=1}^n \bar{\xi}_{ji}\bar{g}_i(\hat{x}_i(t-\rho_2(t))) \\ & \quad + \bigvee_{i=1}^n \bar{\mu}_{ji}\bar{g}_i(x_i(t-\rho_2(t))) - \bigvee_{i=1}^n \bar{\mu}_{ji}\bar{g}_i(\hat{x}_i(t-\rho_2(t)))] - m_{2j}e_{2j}(t), \\ & e_{1i}(r) = \nu_i(r) - \varphi_i(r),\ e_{2j}(r) = \zeta_j(r) - \varsigma_j(r),\ r \in (-\infty,0],\ i \in \mathcal{H},\ j \in \bar{\mathcal{H}}, \end{cases} \tag{4}$$

where $g_i(e_{1i}(\cdot)) = \bar{g}_i(x_i(\cdot)) - \bar{g}_i(\hat{x}_i(\cdot)),\ f_j(e_{2j}(\cdot)) = \bar{f}_j(y_j(\cdot)) - \bar{f}_j(\hat{y}_j(\cdot))$.

The following assumptions and lemmas are vital to prove our results.

Assumption (A_1) [14]. $\tilde{a}_{1i}(\cdot),\ \tilde{a}_{1i}^{-1}(\cdot),\ \bar{a}_{2j}(\cdot)$ and $\bar{a}_{2j}^{-1}(\cdot)$ are locally Lipschitz continuous, and assume that there exist positive constants $\delta_i > 0\ (i = 1,2,...n),\ \bar{\delta}_j > 0\ (j = 1,2,...m)$ such that $\tilde{a}_{1i}(p) \geq \delta_i,\ \bar{a}_{2j}(q) \geq \bar{\delta}_j$ for all $p,q \in \mathbb{R}$.

Assumption (A_2). The neuron activation functions f_j and g_i satisfy the following condition

$$\Big\{F_j^- \leq \frac{f_j(\eta_1)-f_j(\eta_2)}{\eta_1-\eta_2} \leq F_j^+\ ,\ G_i^- \leq \frac{g_i(\eta_1)-g_i(\eta_2)}{\eta_1-\eta_2} \leq G_i^+, \tag{5}$$

$i \in \mathcal{H},\ j \in \bar{\mathcal{H}}$ for all $\eta_1,\eta_2 \in \mathbb{R},\ \eta_1 \neq \eta_2$, where F_j^-, F_j^+, G_i^- and G_i^+ are constants.

Assumption (A_3). The delays $\rho_1(t)$ and $\rho_2(t)$ are the discrete time-varying delays and satisfy $0 \leq \rho_1(t) \leq \rho_1,\ 0 \leq \rho_2(t) \leq \rho_2$, where ρ_1 and ρ_2 are the positive constants.

Lemma 2.1. [29] *Let M,N and δ be matrix of appropriate dimensions , and $\delta^T\delta \leq I$, then for any scalar $\epsilon > 0$, $M\delta N + N^T\delta^T M^T \leq \epsilon MM^T + \epsilon^{-1}N^TN$.*

Lemma 2.2. [2] *For any $x,y \in \mathbb{R}^{n\times m}$, any positive definite matrix $Z \in \mathbb{R}^{n\times n}$ the following inequality $2x^Ty \leq x^TZ^{-1}x + y^TZy$ holds.*

Lemma 2.3. [3] *Given constant matrices Ω_1, Ω_2 and Ω_3 with appropriate dimensions, where $\Omega_1^T = \Omega_1$ and $\Omega_2^T = \Omega_2$, then $\Omega_1 + \Omega_3^T\Omega_2^{-1}\Omega_3 < 0$ if and only if $\begin{bmatrix}\Omega_1 & \Omega_3^T \\ * & -\Omega_2\end{bmatrix} < 0$.*

Lemma 2.4. [8] *Any constant matrix $\tau \in \mathbb{R}^{n\times n},\ \tau = \tau^T > 0$, scalar $\gamma > 0$, function $\Lambda : [0,\gamma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, then $\left(\int_0^\gamma \Lambda(r)dr\right)^T \tau \left(\int_0^\gamma \Lambda(r)dr\right) \leq \gamma\int_0^\gamma \Lambda^T(r)\ \tau\ \Lambda(r)dr$.*

3 Synchronization criteria for BAM Cohen-Grossberg FCNNs with delays

Theorem 3.1. *For given scalars $\epsilon,\ \bar{\epsilon},\ \rho_1,\ \rho_2$, and by assumptions A_1, A_2 and A_3, the error system (4) with time-varying delays $\rho_1(t), \rho_2(t)$ is globally asymptotically stable, if there exist some symmetric matrices $N_1 > 0,\ N_2 >$*

0, $L_1 > 0$, $L_2 > 0$, positive diagonal matrices V_ι, where $\iota = 1,2,3,4$, some positive scalars ω_1, ω_2 and the $2n \times 2n$ matrices $\begin{bmatrix} U_{11} & U_{12} \\ * & U_{22} \end{bmatrix} > 0$, $\begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{bmatrix} > 0$, such that the following LMIs hold:

$$i)\Xi^\star = \begin{bmatrix} \Xi & \Gamma_1^T & \Gamma_2^T & \Gamma_3^T & \Gamma_4^T & \Gamma_5^T \\ * & -\omega_1 I & 0 & 0 & 0 & 0 \\ * & * & -\omega_1 I & 0 & 0 & 0 \\ * & * & * & -\omega_1 I & 0 & 0 \\ * & * & * & * & -\omega_1 I & 0 \\ * & * & * & * & * & -\omega_1 I \end{bmatrix} < 0, ii)\Sigma^\star = \begin{bmatrix} \Sigma & \bar\Gamma_1^T & \bar\Gamma_2^T & \bar\Gamma_3^T & \bar\Gamma_4^T & \bar\Gamma_5^T \\ * & -\omega_2 I & 0 & 0 & 0 & 0 \\ * & * & -\omega_2 I & 0 & 0 & 0 \\ * & * & * & -\omega_2 I & 0 & 0 \\ * & * & * & * & -\omega_2 I & 0 \\ * & * & * & * & * & -\omega_2 I \end{bmatrix} < 0, \tag{6}$$

$$\begin{aligned}
\textit{where } \Xi &= (\Xi_{ij})_{6\times 6},\ \Sigma = (\Sigma_{ij})_{6\times 6},\ \textit{with} \\
\Xi_{11} &= -2N_1 D\epsilon - 2H_1 + 4N_1 - V_1\Omega_{11},\ \Xi_{1,2} = U_{12}^T,\ \Xi_{13} = -\epsilon^T D^T N_1^T - H_1^T, \\
\Xi_{14} &= V_1\Omega_{12},\ \Xi_{22} = \rho_2 U_{11} - 2U_{12}^T - V_2\Omega_{11},\ \Xi_{25} = V_2\Omega_{12},\ \Xi_{33} = \rho_2 U_{22} - 2N_1, \\
\Xi_{44} &= \bar B^T \bar D^T L_1 \bar D \bar B + L_2 - V_1 + \omega_2,\ \Xi_{55} = \bar C^T \bar D^T L_1 \bar D \bar C + S_2^T \bar D^T L_1 \bar D S_2 - V_2 + 2\omega_2, \\
\Xi_{66} &= S_2^T \bar D^T L_1 \bar D S_2 - L_2 + \omega_2,\ \Gamma_1^T = [0\ \ 0\ \ N_1 DB\ \ 0\ \ 0\ \ 0]^T, \\
\Gamma_2^T &= [0\ \ 0\ \ N_1 DC\ \ 0\ \ 0\ \ 0]^T,\ \Gamma_3^T = [0\ \ 0\ \ N_1 DS_1\ \ 0\ \ 0\ \ 0]^T,\ \Gamma_4^T = [0\ \ 0\ \ N_1 DS_1\ \ 0\ \ 0\ \ 0]^T, \\
\Gamma_5^T &= [0\ \ 0\ \ H_1\ \ 0\ \ 0\ \ 0]^T,\ \Sigma_{1,1} = -2L_1 \bar D \bar\epsilon - 2H_2 + 4L_1 - V_3\Omega_{21},\ \Sigma_{1,2} = Z_{12}^T, \\
\Sigma_{13} &= -\bar\epsilon^T \bar D^T L_1^T - H_2^T,\ \Sigma_{14} = V_3\Omega_{22},\ \Sigma_{22} = \rho_1 Z_{11} - 2Z_{12}^T - V_3\Omega_{21},\ \Sigma_{25} = V_4\Omega_{22}, \\
\Sigma_{33} &= \rho_1 Z_{22} - 2L_1,\ \Sigma_{44} = B^T D^T N_1 DB + N_2 - V_3 + \omega_1,\ \Sigma_{66} = S_1^T D^T N_1 DS_1 - N_2 + \omega_1, \\
\Sigma_{55} &= C^T D^T N_1 DC + S_1^T D^T N_1 DS_1 - V_4 + 2\omega_1,\ S_1 = |\varrho|_s + |\varpi|_s,\ S_2 = |\bar\varrho|_s + |\bar\varpi|_s, \\
\bar\Gamma_1^T &= [0\ \ 0\ \ L_1 \bar D \bar B\ \ 0\ \ 0\ \ 0]^T,\ \bar\Gamma_2^T = [0\ \ 0\ \ L_1 \bar D \bar C\ \ 0\ \ 0\ \ 0]^T,\ \bar\Gamma_3^T = [0\ \ 0\ \ L_1 \bar D S_2\ \ 0\ \ 0\ \ 0]^T, \\
\bar\Gamma_4^T &= [0\ \ 0\ \ L_1 \bar D S_2\ \ 0\ \ 0\ \ 0]^T,\ \bar\Gamma_5^T = [0\ \ 0\ \ H_2\ \ 0\ \ 0\ \ 0]^T,\ |\varrho|_s = diag\Big(\sum_{i=1}^n |\xi_{i1}|, ..., \sum_{i=1}^n |\xi_{im}|\Big), \\
|\varpi|_s &= diag\Big(\sum_{i=1}^n |\mu_{i1}|, ..., \sum_{i=1}^n |\mu_{im}|\Big),\ |\bar\varrho|_s = diag\Big(\sum_{j=1}^m |\bar\xi_{j1}|, ..., \sum_{j=1}^m |\bar\xi_{jn}|\Big),\ |\bar\varpi|_s = diag\Big(\sum_{j=1}^m |\bar\mu_{j1}|, ..., \sum_{j=1}^m |\bar\mu_{jn}|\Big).
\end{aligned}$$

Then, the controller gain matrices are $m_1 = N_1^{-1} H_1$ and $m_2 = L_1^{-1} H_2$.

Proof. Consider the LK functional as follows:

$$V(t) = \sum_{\upsilon=1}^{4} V_\upsilon(t), \tag{7}$$

where

$$\begin{aligned}
V_1(t) &= e_1^T(t) N_1 e_1(t) + e_2^T(t) L_1 e_2(t), \\
V_2(t) &= \sum_{j=1}^m (n_2)_j \int_0^\infty q_j(\vartheta) \int_{t-\vartheta}^t f_j^2(e_{2j}(r))\, dr\, d\vartheta + \sum_{i=1}^n (l_2)_i \int_0^\infty q_i(\vartheta) \int_{t-\vartheta}^t g_i^2(e_{1i}(r))\, dr\, d\vartheta, \\
V_3(t) &= \int_0^t \int_{\vartheta-\rho_2(\vartheta)}^{\vartheta} \begin{pmatrix} e_1(\vartheta-\rho_2(\vartheta)) \\ \dot e_1(r) \end{pmatrix}^T \begin{pmatrix} U_{11} & U_{12} \\ * & U_{22} \end{pmatrix} \begin{pmatrix} e_1(\vartheta-\rho_2(\vartheta)) \\ \dot e_1(r) \end{pmatrix} dr\, d\vartheta \\
&\quad + \int_0^t \int_{\vartheta-\rho_1(\vartheta)}^{\vartheta} \begin{pmatrix} e_2(\vartheta-\rho_1(\vartheta)) \\ \dot e_2(r) \end{pmatrix}^T \begin{pmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{pmatrix} \begin{pmatrix} e_2(\vartheta-\rho_1(\vartheta)) \\ \dot e_2(r) \end{pmatrix} dr\, d\vartheta, \\
V_4(t) &= \int_{-\rho_2}^0 \int_{t+\vartheta}^t \dot e_1^T(r) U_{22} \dot e_1(r)\, dr\, d\vartheta + \int_{-\rho_1}^0 \int_{t+\vartheta}^t \dot e_2^T(r) Z_{22} \dot e_2(r)\, dr\, d\vartheta,
\end{aligned}$$

and also $B = (b_{ij})_{m\times m}, C = (c_{ij})_{m\times m}, \xi = (\xi_{ij})_{m\times m}, \mu = (\mu_{ij})_{m\times m}, \bar B = (\bar b_{ji})_{n\times n}, \bar C = (\bar c_{ji})_{n\times n}, \bar\xi = (\bar\xi_{ji})_{n\times n}$,

$\bar{\mu} = (\bar{\mu}_{ji})_{n\times n}$. The time derivatives of $V_\upsilon(t)$ with error system (4), Lemma 2.1 & 2.2 are given below:

$$\begin{aligned}
D^+V_1(t) \;\le\;& -2e_1^T(t)N_1[D\epsilon - M_1]e_1(t) + 4e_1^T(t)N_1e_1(t) + f^T(e_2(t))B^TD^TN_1DBf(e_2(t)) \\
&+f^T(e_2(t-\rho_1(t)))C^TD^TN_1DCf(e_2(t-\rho_1(t))) + \int_{-\infty}^{t} Q(t-r)f^T(e_2(r))dr \\
&\times S_1^TD^TN_1DS_1\int_{-\infty}^{t} Q(t-r)f(e_2(r))dr + f^T(e_2(t-\rho_1(t)))S_1^TD^TN_1DS_1 \\
&\times f(e_2(t-\rho_1(t))) - 2e_2^T(t)L_1[\bar{D}\bar{\epsilon} - M_2]e_2(t) + 4e_2^T(t)L_1e_2(t) \\
&+g^T(e_1(t))\bar{B}^T\bar{D}^TL_1\bar{D}\bar{B}g(e_1(t)) + g^T(e_1(t-\rho_2(t)))\bar{C}^T\bar{D}^TL_1\bar{D}\bar{C}g(e_1(t-\rho_2(t))) \\
&+\int_{-\infty}^{t} Q(t-r)g^T(e_1(r))drS_2^T\bar{D}^TL_1\bar{D}S_2\int_{-\infty}^{t} Q(t-r)g(e_1(r))dr \\
&+g^T(e_1(t-\rho_2(t)))S_2^T\bar{D}^TL_1\bar{D}S_2g(e_1(t-\rho_2(t))) - 2\dot{e}_1^T(t)\epsilon N_1De_1(t) \\
&+\dot{e}_1^T(t)H_1^T\omega_1^{-1}H_1\dot{e}_1(t) + e_1^T(t)\omega_1e_1(t) + \dot{e}_1^T(t)N_1DB\omega_1^{-1}B^TD^TN_1^T\dot{e}_1(t) \\
&+f^T(e_2(t))\omega_1f(e_2(t)) + \dot{e}_1^T(t)N_1DC\omega_1^{-1}C^TD^TN_1^T\dot{e}_1(t) + f^T(e_2(t-\rho_1(t)))\omega_1 \\
&\times f(e_2(t-\rho_1(t))) + \dot{e}_1^T(t)N_1DS_1\omega_1^{-1}S_1^TD^TN_1^T\dot{e}_1(t) + \int_{-\infty}^{t} Q(t-r)f^T(e_2(r))dr \\
&\times\omega_1\int_{-\infty}^{t} Q(t-r)f(e_2(r))dr + \dot{e}_1^T(t)N_1DS_1\omega_1^{-1}S_1^TD^TN_1^T\dot{e}_1(t) \\
&+f^T(e_2(t-\rho_1(t)))\omega_1f(e_2(t-\rho_1(t))) - 2\dot{e}_1^T(t)N_1\dot{e}_1(t) - 2\dot{e}_2^T(t)\bar{\epsilon}L_1\bar{D}e_2(t) \\
&+\dot{e}_2^T(t)H_2^T\omega_2^{-1}H_2\dot{e}_2(t) + \dot{e}_2^T(t)L_1\bar{D}\bar{B}\omega_2^{-1}\bar{B}^T\bar{D}^TL_1^T\dot{e}_2(t) \\
&+g^T(e_1(t))\omega_2g(e_1(t)) + \dot{e}_2^T(t)L_1\bar{D}\bar{C}\omega_2^{-1}\bar{C}^T\bar{D}^TL_1^T\dot{e}_2(t) \\
&+g^T(e_1(t-\rho_2(t)))\omega_2g(e_1(t-\rho_2(t))) + \dot{e}_2^T(t)L_1\bar{D}S_2\omega_2^{-1}S_2^T\bar{D}^TL_1^T\dot{e}_2(t) \\
&+\int_{-\infty}^{t} Q(t-r)g^T(e_1(r))dr\omega_2\int_{-\infty}^{t} Q(t-r)g(e_1(r))dr + \dot{e}_2^T(t)L_1\bar{D}S_2 \\
&\times\omega_2^{-1}S_2^T\bar{D}^TL_1^T\dot{e}_2(t) + g^T(e_1(t-\rho_2(t)))\omega_2g(e_1(t-\rho_2(t))) - 2\dot{e}_2^T(t)L_1\dot{e}_2(t), \qquad (8)
\end{aligned}$$

$$\begin{aligned}
D^+V_2(t) \;=\;& f^T(e_2(t))N_2f(e_2(t)) - \int_{-\infty}^{t} Q(t-r)f^T(e_2(r))drN_2\int_{-\infty}^{t} Q(t-r)f(e_2(r))dr \\
&+g^T(e_1(t))L_2g(e_1(t)) - \int_{-\infty}^{t} Q(t-r)g^T(e_1(r))drL_2\int_{-\infty}^{t} Q(t-r)g(e_1(r))dr, \qquad (9)
\end{aligned}$$

$$\begin{aligned}
D^+V_3(t) \;=\;& e_1^T(t-\rho_2(t))\Big[\rho_2U_{11} - 2U_{12}^T\Big]e_1(t-\rho_2(t)) + 2e_1^T(t)U_{12}^Te_1(t-\rho_2(t)) \\
&+\int_{t-\rho_2(t)}^{t} \dot{e}_1^T(r)U_{22}\dot{e}_1(r)dr + e_2^T(t-\rho_1(t))\Big[\rho_1Z_{11} - 2Z_{12}^T\Big]e_2(t-\rho_1(t)) \\
&+2e_2^T(t)Z_{12}^Te_2(t-\rho_1(t)) + \int_{t-\rho_1(t)}^{t} \dot{e}_2^T(r)Z_{22}\dot{e}_2(r)dr, \qquad (10)
\end{aligned}$$

$$D^+V_4(t) \;\le\; \rho_2\dot{e}_1^T(t)U_{22}\dot{e}_1(t) - \int_{t-\rho_2}^{t} \dot{e}_1^T(r)U_{22}\dot{e}_1(r)dr + \rho_1\dot{e}_2^T(t)Z_{22}\dot{e}_2(t) - \int_{t-\rho_1}^{t} \dot{e}_2^T(r)Z_{22}\dot{e}_2(r)dr. \qquad (11)$$

In addition, for any diagonal matrices $V_1 > 0, V_2 > 0, V_3 > 0$ and $V_4 > 0$, the following inequality holds by the methods proposed in [20]

$$\begin{pmatrix} e_1(t) \\ g(e_1(t)) \end{pmatrix}^T \begin{pmatrix} -V_1\Omega_{11} & V_1\Omega_{12} \\ * & -V_1 \end{pmatrix} \begin{pmatrix} e_1(t) \\ g(e_1(t)) \end{pmatrix} + \begin{pmatrix} e_1(t-\rho_2(t)) \\ g(e_1(t-\rho_2(t))) \end{pmatrix}^T \begin{pmatrix} -V_2\Omega_{11} & V_2\Omega_{12} \\ * & -V_2 \end{pmatrix} \begin{pmatrix} e_1(t-\rho_2(t)) \\ g(e_1(t-\rho_2(t))) \end{pmatrix} \geq 0, \quad (12)$$

$$\begin{pmatrix} e_2(t) \\ f(e_2(t)) \end{pmatrix}^T \begin{pmatrix} -V_3\Omega_{21} & V_3\Omega_{22} \\ * & -V_3 \end{pmatrix} \begin{pmatrix} e_2(t) \\ f(e_2(t)) \end{pmatrix} + \begin{pmatrix} e_2(t-\rho_1(t)) \\ f(e_2(t-\rho_1(t))) \end{pmatrix}^T \begin{pmatrix} -V_4\Omega_{21} & V_4\Omega_{22} \\ * & -V_4 \end{pmatrix} \begin{pmatrix} e_2(t-\rho_1(t)) \\ f(e_2(t-\rho_1(t))) \end{pmatrix} \geq 0. \quad (13)$$

Hence, from (8)-(13) and using Lemma 2.3, one can have

$$\dot{V}(t) \leq \Theta_1^T(t)\ \Xi^\star\ \Theta_1(t) + \Theta_2^T(t)\ \Sigma^\star\ \Theta_2(t), \quad (14)$$

where

$$\Theta_1^T(t) = \Big[e_1^T(t), e_1^T(t-\rho_2(t)), \dot{e}_1^T(t), g^T(e_1(t)), g^T(e_1(t-\rho_2(t))), \int_{-\infty}^{t} Q(t-r) g^T(e_1(r)) dr\Big], \quad (15)$$

$$\Theta_2^T(t) = \Big[e_2^T(t), e_2^T(t-\rho_1(t)), \dot{e}_2^T(t), f^T(e_2(t)), f^T(e_2(t-\rho_1(t))), \int_{-\infty}^{t} Q(t-r) f^T(e_2(r)) dr\Big]. \quad (16)$$

By (6(i & ii)), we obtain

$$\dot{V}(t) \leq -\Big[\Theta_1^T(t)\ \Xi^{\star\star}\ \Theta_1(t) + \Theta_2^T(t)\ \Sigma^{\star\star}\ \Theta_2(t)\Big], \quad t > 0,$$

where $\Xi^{\star\star} = -\Xi^\star > 0$, $\Sigma^{\star\star} = -\Sigma^\star > 0$.

That means, if LMIs (6(i & ii)) hold, one can obtain $\dot{V}(t) \leq 0$. Then, the proposed error system (4) is globally asymptotically stable. This completes the proof. □

4 Robust synchronization criteria for delayed BAM Cohen-Grossberg FCNNs

In this section, the obtained synchronization result of the systems (1) and (2) in the previous section will be further extended to the delayed BAM CGFCNNs with parameter uncertainties.

Consider the system (1) with parameter uncertainties described by

$$\begin{cases} \dot{x}_i(t) &= d_i[-\tilde{a}_{1i}(x_i(t)) + \sum_{j=1}^{m}(b_{ij} + \Delta b_{ij}(t))\bar{f}_j(y_j(t)) + \sum_{j=1}^{m}(c_{ij} + \Delta c_{ij}(t))\bar{f}_j(y_j(t-\rho_1(t))) \\ & \quad + \bigwedge_{j=1}^{m} \xi_{ij} \int_{-\infty}^{t} q_j(t-r)\bar{f}_j(y_j(r))dr + \bigvee_{j=1}^{m} \mu_{ij} \int_{-\infty}^{t} q_j(t-r)\bar{f}_j(y_j(r))dr \\ & \quad + I_i + \bigwedge_{j=1}^{m} \xi_{ij}\bar{f}_j(y_j(t-\rho_1(t))) + \bigvee_{j=1}^{m} \mu_{ij}\bar{f}_j(y_j(t-\rho_1(t)))], \\ \dot{y}_j(t) &= \bar{d}_j[-\bar{a}_{2j}(y_j(t)) + \sum_{i=1}^{n}(\bar{b}_{ji} + \Delta\bar{b}_{ji}(t))\bar{g}_i(x_i(t)) + \sum_{i=1}^{n}(\bar{c}_{ji} + \Delta\bar{c}_{ji})\bar{g}_i(x_i(t-\rho_2(t))) \\ & \quad + \bigwedge_{i=1}^{n} \bar{\xi}_{ji} \int_{-\infty}^{t} q_i(t-r)\bar{g}_i(x_i(r))dr + \bigvee_{i=1}^{n} \bar{\mu}_{ji} \int_{-\infty}^{t} q_i(t-r)\bar{g}_i(x_i(r))dr \\ & \quad + \bar{I}_j + \bigwedge_{i=1}^{n} \bar{\xi}_{ji}\bar{g}_i(x_i(t-\rho_2(t))) + \bigvee_{i=1}^{n} \bar{\mu}_{ji}\bar{g}_i(x_i(t-\rho_2(t)))], \end{cases} \quad (17)$$

where $\Delta b_{ij}(t), \Delta c_{ij}(t), \Delta\bar{b}_{ji}(t)$ and $\Delta\bar{c}_{ji}(t)$ are unknown constants representing time-varying parameter uncertainties, which are of the following form

$$[\Delta b_{ij}(t) \ \ \Delta c_{ij}(t)] = R\ P(t)\ [T_b \ \ T_c],$$

$$[\Delta\bar{b}_{ji}(t) \ \ \Delta\bar{c}_{ji}(t)] = \bar{R}\ \bar{P}(t)\ [\bar{T}_b \ \ \bar{T}_c],$$

where $R, T_b, T_c, \bar{R}, \bar{T}_b$ and $\bar{T}_c$ are appropriately dimensioned constant matrices; $P(t)$ and $\bar{P}(t)$ are an unknown time-varying matrices with Lebesgue measurable elements satisfying $P^T(t)P(t) \leq I$ and $\bar{P}^T(t)\bar{P}(t) \leq I$, respectively.

Consider the system (2) with parameter uncertainties as described aforementioned system and its denoted by:

$$\dot{\hat{x}}_i(t)\ \&\ \dot{\hat{y}}_j(t) \quad (18)$$

then the error system as follows:

$$\begin{cases} \dot{e}_{1i}(t) &= d_i[-\tilde{a}_{1i}(e_{1i}(t)) + \sum_{j=1}^{m}(b_{ij}+\Delta b_{ij}(t))f_j(e_{2j}(t)) + \sum_{j=1}^{m}(c_{ij}+\Delta c_{ij})f_j(e_{2j}(t-\rho_1(t))) \\ &\quad + \bigwedge_{j=1}^{m}\xi_{ij}\int_{-\infty}^{t}q_j(t-r)\bar{f}_j(y_j(r))dr - \bigwedge_{j=1}^{m}\xi_{ij}\int_{-\infty}^{t}q_j(t-r)\bar{f}_j(\hat{y}_j(r))dr \\ &\quad + \bigvee_{j=1}^{m}\mu_{ij}\int_{-\infty}^{t}q_j(t-r)\bar{f}_j(y_j(r))dr - \bigvee_{j=1}^{m}\mu_{ij}\int_{-\infty}^{t}q_j(t-r)\bar{f}_j(\hat{y}_j(r))dr \\ &\quad + \bigwedge_{j=1}^{m}\xi_{ij}\bar{f}_j(y_j(t-\rho_1(t))) - \bigwedge_{j=1}^{m}\xi_{ij}\bar{f}_j(\hat{y}_j(t-\rho_1(t))) \\ &\quad + \bigvee_{j=1}^{m}\mu_{ij}\bar{f}_j(y_j(t-\rho_1(t))) - \bigvee_{j=1}^{m}\mu_{ij}\bar{f}_j(\hat{y}_j(t-\rho_1(t)))] - m_{1i}e_{1i}(t), \\ \dot{e}_{2j}(t) &= \bar{d}_j[-\bar{a}_{2j}(e_{2j}(t)) + \sum_{i=1}^{n}(\bar{b}_{ji}+\Delta\bar{b}_{ji})g_i(e_{1i}(t)) + \sum_{i=1}^{n}(\bar{c}_{ji}+\Delta\bar{c}_{ji})g_i(e_{1i}(t-\rho_2(t))) \\ &\quad + \bigwedge_{i=1}^{n}\bar{\xi}_{ji}\int_{-\infty}^{t}q_i(t-r)\bar{g}_i(x_i(r))dr - \bigwedge_{i=1}^{n}\bar{\xi}_{ji}\int_{-\infty}^{t}q_i(t-r)\bar{g}_i(\hat{x}_i(r))dr \\ &\quad + \bigvee_{i=1}^{n}\bar{\mu}_{ji}\int_{-\infty}^{t}q_i(t-r)\bar{g}_i(x_i(r))dr - \bigvee_{i=1}^{n}\bar{\mu}_{ji}\int_{-\infty}^{t}q_i(t-r)\bar{g}_i(\hat{x}_i(r))dr \\ &\quad + \bigwedge_{i=1}^{n}\bar{\xi}_{ji}\bar{g}_i(x_i(t-\rho_2(t))) - \bigwedge_{i=1}^{n}\bar{\xi}_{ji}\bar{g}_i(\hat{x}_i(t-\rho_2(t))) \\ &\quad + \bigvee_{i=1}^{n}\bar{\mu}_{ji}\bar{g}_i(x_i(t-\rho_2(t))) - \bigvee_{i=1}^{n}\bar{\mu}_{ji}\bar{g}_i(\hat{x}_i(t-\rho_2(t)))] - m_{2j}e_{2j}(t), \\ & e_{1i}(r) = \nu_i(r) - \varphi_i(r),\ e_{2j}(r) = \zeta_j(r) - \varsigma_j(r),\ r\in(-\infty,0],\ i\in\mathcal{H},\ j\in\bar{\mathcal{H}}. \end{cases} \tag{19}$$

Theorem 4.1. *For given scalars $\epsilon,\ \bar{\epsilon},\ \rho_1,\ \rho_2$, and by assumptions A_1, A_2 and A_3, the uncertain error system (19) with time-varying delays $\rho_1(t), \rho_2(t)$ is globally robustly asymptotically stable, if there exist some symmetric matrices $N_1>0,\ N_2>0,\ L_1>0,\ L_2>0$, positive diagonal matrices V_ι, where $\iota=1,2,3,4$, some positive scalars ω_1,ω_2 and $2n\times 2n$ matrices $\begin{bmatrix} U_{11} & U_{12} \\ * & U_{22}\end{bmatrix}>0, \begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{22}\end{bmatrix}>0$, such that the following LMIs hold:*

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2N_1DR \\ * & \Xi_{22} & 0 & 0 & \Xi_{25} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & N_1DB & N_1DC & N_1DS_1 & N_1DS_1 & H_1 & 0 \\ * & * & * & \chi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \chi_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Xi_{66} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\omega_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\omega_1 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\omega_1 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\omega_1 I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\omega_1 I & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -\eta_1-\eta_2 \end{bmatrix} < 0, \tag{20}$$

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2L_1\bar{D}\bar{R} \\ * & \Sigma_{22} & 0 & 0 & \Sigma_{25} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Sigma_{33} & 0 & 0 & 0 & L_1\bar{D}\bar{B} & L_1\bar{D}\bar{C} & L_1\bar{D}S_2 & L_1\bar{D}S_2 & H_2 & 0 \\ * & * & * & \bar{\chi}_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \bar{\chi}_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Sigma_{66} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\omega_2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\omega_2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\omega_2 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\omega_2 I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\omega_2 I & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -\bar{\eta}_1-\bar{\eta}_2 \end{bmatrix} < 0, \tag{21}$$

where $\chi_{44} = \Xi_{44} + \bar{\eta}_1\bar{T}_{\bar{b}}^T\bar{T}_{\bar{b}},\ \chi_{55} = \Xi_{55} + \bar{\eta}_2\bar{T}_{\bar{c}}^T\bar{T}_{\bar{c}},\ \bar{\chi}_{44} = \Sigma_{44} + \eta_1 T_b^T T_b,\ \bar{\chi}_{55} = \Sigma_{55} + \eta_2 T_c^T T_c.$

Proof. Consider the LK functional as used in Theorem 3.1. The time derivatives of $V_\upsilon(t)$ with system (19) are given below:

$$\begin{aligned} D^+V_1(t) \leq\ & -2e_1^T(t)N_1[D\epsilon - M_1]e_1(t) + 4e_1^T(t)N_1e_1(t) + f^T(e_2(t))B^TD^TN_1DBf(e_2(t)) \\ & +f^T(e_2(t-\rho_1(t)))C^TD^TN_1DCf(e_2(t-\rho_1(t))) + \int_{-\infty}^t Q(t-r)f^T(e_2(r))dr \\ & \times S_1^TD^TN_1DS_1\int_{-\infty}^t Q(t-r)f(e_2(r))dr + f^T(e_2(t-\rho_1(t)))S_1^TD^TN_1DS_1 \\ & \times f(e_2(t-\rho_1(t))) - 2e_2^T(t)L_1[\bar{D}\bar{\epsilon} - M_2]e_2(t) + 4e_2^T(t)L_1e_2(t) \\ & +g^T(e_1(t))\bar{B}^T\bar{D}^TL_1\bar{D}\bar{B}g(e_1(t)) + g^T(e_1(t-\rho_2(t)))\bar{C}^T\bar{D}^TL_1\bar{D}\bar{C}g(e_1(t-\rho_2(t))) \\ & +\int_{-\infty}^t Q(t-r)g^T(e_1(r))drS_2^T\bar{D}^TL_1\bar{D}S_2\int_{-\infty}^t Q(t-r)g(e_1(r))dr \\ & +g^T(e_1(t-\rho_2(t)))S_2^T\bar{D}^TL_1\bar{D}S_2g(e_1(t-\rho_2(t))) - 2\dot{e}_1^T(t)\epsilon N_1De_1(t) \\ & +\dot{e}_1^T(t)H_1^T\omega_1^{-1}H_1\dot{e}_1(t) + e_1^T(t)\omega_1e_1(t) + \dot{e}_1^T(t)N_1DB\omega_1^{-1}B^TD^TN_1^T\dot{e}_1(t) \\ & +f^T(e_2(t))\omega_1f(e_2(t)) + \dot{e}_1^T(t)N_1DC\omega_1^{-1}C^TD^TN_1^T\dot{e}_1(t) + f^T(e_2(t-\rho_1(t)))\omega_1 \\ & \times f(e_2(t-\rho_1(t))) + \dot{e}_1^T(t)N_1DS_1\omega_1^{-1}S_1^TD^TN_1^T\dot{e}_1(t) + \int_{-\infty}^t Q(t-r)f^T(e_2(r))dr \\ & \times\omega_1\int_{-\infty}^t Q(t-r)f(e_2(r))dr + \dot{e}_1^T(t)N_1DS_1\omega_1^{-1}S_1^TD^TN_1^T\dot{e}_1(t) \\ & +f^T(e_2(t-\rho_1(t)))\omega_1f(e_2(t-\rho_1(t))) - 2\dot{e}_1^T(t)N_1\dot{e}_1(t) \\ & +\eta_1^{-1}e_1^T(t)N_1DRR^TD^TN_1^Te_1(t) + \eta_1f^T(e_2(t))T_b^TT_bf(e_2(t)) \\ & +\eta_2^{-1}e_1^T(t)N_1DRR^TD^TN_1^Te_1(t) + \eta_2f^T(e_2(t-\rho_1(t)))T_c^TT_cf(e_2(t-\rho_1(t))) \\ & -2\dot{e}_2^T(t)\bar{\epsilon}L_1\bar{D}e_2(t) + \dot{e}_2^T(t)H_2^T\omega_2^{-1}H_2\dot{e}_2(t) + \dot{e}_2^T(t)L_1\bar{D}\bar{B}\omega_2^{-1}\bar{B}^T\bar{D}^TL_1^T\dot{e}_2(t) \\ & +g^T(e_1(t))\omega_2g(e_1(t)) + \dot{e}_2^T(t)L_1\bar{D}\bar{C}\omega_2^{-1}\bar{C}^T\bar{D}^TL_1^T\dot{e}_2(t) + g^T(e_1(t-\rho_2(t)))\omega_2 \\ & \times g(e_1(t-\rho_2(t))) + \dot{e}_2^T(t)L_1\bar{D}S_2\omega_2^{-1}S_2^T\bar{D}^TL_1^T\dot{e}_2(t) + \int_{-\infty}^t Q(t-r)g^T(e_1(r))dr \\ & \times\omega_2\int_{-\infty}^t Q(t-r)g(e_1(r))dr + \dot{e}_2^T(t)L_1\bar{D}S_2\omega_2^{-1}S_2^T\bar{D}^TL_1^T\dot{e}_2(t) + g^T(e_1(t-\rho_2(t))) \\ & \times\omega_2g(e_1(t-\rho_2(t))) - 2\dot{e}_2^T(t)L_1\dot{e}_2(t) + \bar{\eta}_1^{-1}e_2^T(t)L_1\bar{D}\bar{R}\bar{R}^T\bar{D}^TL_1^Te_2(t) \\ & +\bar{\eta}_1g^T(e_1(t))\bar{T}_b^T\bar{T}_bg(e_1(t)) + \bar{\eta}_2^{-1}e_2^T(t)L_1\bar{D}\bar{R}\bar{R}^T\bar{D}^TL_1^Te_2(t) \\ & +\bar{\eta}_2g^T(e_1(t-\rho_2(t)))\bar{T}_c^T\bar{T}_cg(e_1(t-\rho_2(t))). \end{aligned} \quad (22)$$

Hence, from (9)-(13) and (22), we can obtain the globally robustly asymptotically stable result of the considered uncertain error dynamical system (19) by following the proof of Theorem 3.1. □

5 Numerical examples and simulations

Secure communication using synchronization between complex networks is a new concept. In this concept, we expected that any two units communicate and other than that is impossible to realize. In this point of view, encryption methods are designed to evade message eavesdropped and intercepted. Complex networks encryption is an efficient scheme to build better safety of communication. We can consider a communication networks in which the sender and receiver corresponds to the drive system and response system, respectively. The BAM Cohen-Grossberg FCNNs and mixed delay terms are intricate, which means more composite and difficulties of unmasking the encrypted data.

In this application, an information signal $M(t)$ contains the transmitted messages such as text, image and audio, that can be masked with BAM CGFCNNs and mixed delays into the carrier $\dot{x}(t)$ and $\dot{y}(t)$. Define $N(t)$ is the transmitted message, this is acquired for communication. Some strategies can be used the actual transmitted signal $N(t)$ as broadband as possible by following the given structure.

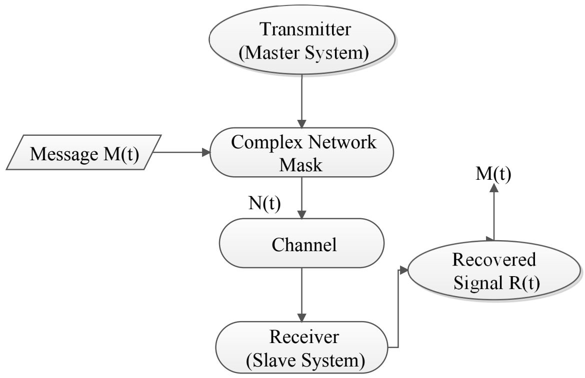

Figure 1: Complex networking mask.

In order to implement the networking in Figure 1 and to illustrate the effectiveness of the proposed theoretical results, the following two counter examples are provided.

Example 5.1. *Consider the error dynamical system (4) with the following parameters:*

$$D = 0.03\ I\ ,\ B = \begin{bmatrix} 7.0954 & -10.6969 & -8.1006 \\ 6.7462 & 5.0089 & -9.2744 \\ -3.5736 & 11.8303 & 9.9090 \end{bmatrix},\quad C = \begin{bmatrix} -0.4402 & -7.5178 & 2.6151 \\ 8.4433 & -6.0672 & 6.6536 \\ 7.4379 & -10.6995 & 0.2655 \end{bmatrix},$$

$$\xi = \mu = \begin{bmatrix} 0.149 & 0.149 & 0.149 \\ 0.149 & 0.149 & 0.149 \\ 0.149 & 0.149 & 0.149 \end{bmatrix},\ \bar{D} = 0.06\ I\ ,\ \bar{B} = \begin{bmatrix} -11.3340 & -4.0321 & 1.8032 \\ 11.7692 & -7.8268 & 6.0236 \\ 0.0226 & 3.0153 & -8.3155 \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} -4.4371 & -3.8911 & 7.2459 \\ -8.5452 & -5.3953 & -0.0622 \\ 8.4146 & -11.8557 & 0.9082 \end{bmatrix},\ \bar{\xi} = \bar{\mu} = \begin{bmatrix} 0.34 & 0.34 & 0.34 \\ 0.34 & 0.34 & 0.34 \\ 0.34 & 0.34 & 0.34 \end{bmatrix},\ \epsilon = 0.6I,\ \bar{\epsilon} = 3.99I,$$

$$\bar{f}_j(y_j(t)) = \frac{1}{2}(|y_j(t)+1| - |y_j(t)-1|),\ \bar{g}_i(x_i(t)) = \frac{1}{2}(|x_i(t)+1| - |x_i(t)-1|),\ i = j = 1,2,3.$$

For the given activation function, one can easily verify that $\bar{f}_j(y_j(t))$ and $\bar{g}_i(x_i(t))$ satisfy assumption (A2), and $F_j^- = -0.5$, $F_j^+ = 0.5$, $G_i^- = -0.4$, $G_i^+ = 0.4$, then we obtain $\Omega_{11} = -0.25$, $\Omega_{12} = 0$, $\Omega_{21} = -0.16$, $\Omega_{22} = 0$. $\rho_1(t) = \rho_2(t) = 0.1|\sin t|$, $\rho_1 = \rho_2 = 0.1$. By solving the LMIs (6(i & ii)) in Theorem 3.1, it is found that the error dynamical system (4) is globally asymptotically stable. By utilizing robust control toolbox LMI solver, we solve the LMIs (6(i & ii)) and the feasible solutions can be obtained through controller gain matrices:

$$(i)\ m_1 = \begin{bmatrix} 4.2019 & 0.2664 & -0.0779 \\ 0.2527 & 3.9683 & -0.1424 \\ -0.0832 & -0.1568 & 4.1458 \end{bmatrix},\qquad (ii)\ m_2 = \begin{bmatrix} 1.4030 & -0.0940 & -0.1089 \\ -0.0871 & 1.2860 & -0.0240 \\ -0.1175 & -0.0301 & 1.3897 \end{bmatrix}. \tag{23}$$

From computation it is found that the allowable upper bounds ρ_2 of Theorem 3.1 for the various values of ρ_1 and the corresponding results which are listed in Table 1.

Remark 5.2. *In the simulations of Example 5.1, the initial values $\nu_i(r) = [1, -0.1, -0.5]$, $\zeta_j(r) = [0.8, 0.5, -1.8]$, and $\varphi_i(r) = [-1.5, 0.25, -0.3]$, $\varsigma_j(r) = [0.82, 0.6, -1]$ are chosen for the systems (1) and (2), respectively. The results demonstrated in Figure 2 represents the time response of drive $x_i(t)$, response $\hat{x}_i(t)$ systems and synchronization error $e_{1i}(t)$ $(i = 1,2,3)$ with control input (23(i)). Figure 3 represents the time response of drive $y_j(t)$, response $\hat{y}_j(t)$ systems and synchronization error $e_{2j}(t)$ $(j = 1,2,3)$ with control input (23(ii)). The simulation results illustrated in*

Figure 4 and 5 represent the time response of drive $x_i(t),\ y_j(t)$, response $\hat{x}_i(t),\ \hat{y}_j(t)$ systems and synchronization error $e_{1i}(t)\ (i = 1, 2, 3),\ e_{2j}(t)\ (j = 1, 2, 3)$ without control inputs.

Table 1: The allowable upper bound ρ_2 for different ρ_1.

	$\rho_1 = 0.01$	$\rho_1 = 0.05$	$\rho_1 = 0.1$	$\rho_1 = 0.2$	$\rho_1 = 0.3$	$\rho_1 = 0.4$
ρ_2	0.3606	0.3158	0.2612	0.1556	0.0544	Infeasible

Example 5.3. *Consider the error dynamical system (19) with the following parameters:*

$$D = 0.03I \ , \ B = \begin{bmatrix} -9.9945 & -8.9382 & -5.1770 \\ -8.4129 & 2.4300 & -1.5524 \\ -5.4280 & -4.2915 & 9.6902 \end{bmatrix}, \quad C = \begin{bmatrix} 0.2025 & 5.2623 & -10.8832 \\ 0.1270 & -12.02 & -1.8592 \\ 3.0620 & 1.7984 & -0.7744 \end{bmatrix},$$

$$T_b = \begin{bmatrix} 0.1862 & 0 & 0 \\ 0 & 0.5074 & 0 \\ 0 & 0 & 0.1476 \end{bmatrix}, \ T_c = \begin{bmatrix} 0.9207 & 0 & 0 \\ 0 & 0.9295 & 0 \\ 0 & 0 & 0.1368 \end{bmatrix}, \ R = \begin{bmatrix} 0.310 & 0.10 & 0.20 \\ 0.198 & 0.291 & 0.11 \\ 0.10 & 0.201 & 0.30 \end{bmatrix},$$

$$\xi = \mu = \begin{bmatrix} 0.22 & 0.22 & 0.22 \\ 0.22 & 0.22 & 0.22 \\ 0.22 & 0.22 & 0.22 \end{bmatrix}, \ \bar{D} = \begin{bmatrix} 0.07 & 0 & 0 \\ 0 & 0.07 & 0 \\ 0 & 0 & 0.07 \end{bmatrix}, \ \bar{B} = \begin{bmatrix} -10.7045 & 2.3671 & 2.8284 \\ -8.5052 & -9.2633 & -8.3149 \\ -3.3555 & 7.1099 & -8.3373 \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} 0.2025 & 5.2623 & -10.8832 \\ -0.1270 & -11.4261 & -1.8592 \\ 3.0620 & 1.7984 & -2.12 \end{bmatrix}, \ \bar{T}_b = \begin{bmatrix} 2.0 & 0 & 0.3 \\ 0.5 & 0.98 & 0.1 \\ 0 & 0 & 1.0 \end{bmatrix}, \ \bar{T}_c = \begin{bmatrix} 0.40 & 0.22 & -1.0 \\ 0.35 & 0.20 & 0.2 \\ 0 & 0 & 0.30 \end{bmatrix},$$

$$\bar{R} = \begin{bmatrix} 0.8 & 0.7 & 0.91 \\ 2.0 & 1.5 & 1.86 \\ 0.5 & 0.22 & 0.49 \end{bmatrix}, \quad \bar{\xi} = \bar{\mu} = \begin{bmatrix} 2.0 & 0 & 0.3 \\ 0.5 & 0.98 & 0.1 \\ 0 & 0 & 1.0 \end{bmatrix}, \ \epsilon = 0.7I, \ \bar{\epsilon} = 3.5I.$$

The activation functions are $\bar{f}_j(y_j(t)) = \frac{1}{2}(|y_j(t)+1|-|y_j(t)-1|),\ \bar{g}_i(x_i(t)) = \frac{1}{2}(|x_i(t)+1|-|x_i(t)-1|)$, satisfy assumption (A2) with $F_j^- = -0.5,\ F_j^+ = 0.5,\ G_i^- = -0.4,\ G_i^+ = 0.4$, then we obtain $\Omega_{11} = -0.25,\ \Omega_{12} = 0,\ \Omega_{21} = -0.16,\ \Omega_{22} = 0$. Consider the time-varying delays $\rho_1(t) = \rho_2(t) = 0.1|\sin t|$, with $\rho_1 = \rho_2 = 0.1$ and $P(t) = \bar{P}(t) = 0.7\cos t$. Solving the LMIs (20) and (21) in Theorem 4.1 by utilizing the robust control toolbox, we obtain the feasibility of the LMIs through the controller gain matrices:

$$(i)\ \ m_1 = \begin{bmatrix} 4.2232 & 0.1854 & -0.1361 \\ 0.1669 & 3.8576 & 0.0732 \\ -0.1617 & 0.0784 & 4.1059 \end{bmatrix}, \qquad (ii)\ \ m_2 = \begin{bmatrix} 1.2572 & 0.0014 & -0.0286 \\ 0.0032 & 1.1509 & -0.0354 \\ -0.0281 & -0.0428 & 1.3901 \end{bmatrix}. \qquad (24)$$

From Theorem 4.1, that the error dynamical system (19) is globally robustly asymptotically stable. For this example, we now compute the allowable upper bounds ρ_2 of Theorem 4.1 for different values of ρ_1 and the corresponding results are listed in Table 2.

Remark 5.4. *In the simulations of Example 5.3, we consider the same initial values of Example 5.1 for the systems (17) and (18). Figure 6 represents the state, error trajectories of uncertain dynamical system (17) with control input (24(i)). Figure 7 represents the state, error trajectories of uncertain dynamical system (18) with control input (24(ii)). Figure 8 and 9 represent the state, error trajectories of uncertain dynamical systems (17) and (18) without control inputs, respectively.*

Table 2: The allowable upper bound ρ_2 for different ρ_1.

	$\rho_1 = 0.01$	$\rho_1 = 0.1$	$\rho_1 = 0.2$	$\rho_1 = 0.3$	$\rho_1 = 0.4$	$\rho_1 = 0.5$
ρ_2	0.4073	0.3100	0.2073	0.1099	0.0171	Infeasible

Remark 5.5. *Recently, Shi and Cao [27] have analyzed the finite-time synchronization of memristor-based CGNNs with delays via nonlinear transformation. Based on the LK functional approach, the problem of synchronization for a class of discontinuous CGNNs with time-varying delays are discussed in [15]. In recent times, Meng et al. [22] investigated the periodicity of Cohen-Grossberg-type fuzzy NNs with impulses by delay differential inequality, fuzzy theory and Lyapunov method. So far no results have been proposed on the synchronization of BAM CGFCNNs with delays using LK functional and LMI technique. Thus, authors are unable to provide the comparison result over the existing results in the literature.*

Remark 5.6. *We can utilize our theoretical results into the applications in secure communication [7], secure image transmission, intelligent data analysis, biological networks, chemical reactions, image encryption/decryption [21, 25], etc.*

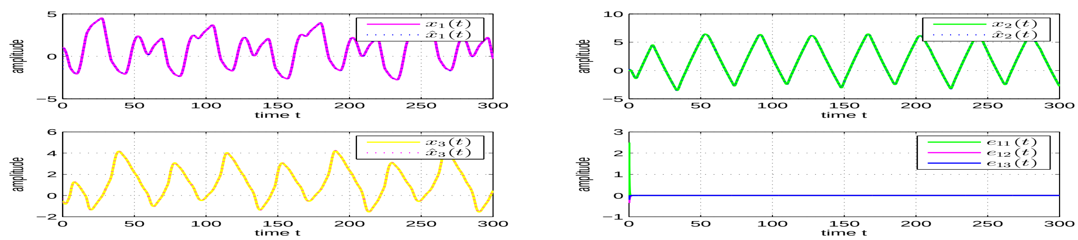

Figure 2: The time response of $x_i(t)$, $\hat{x}_i(t)$ in (1) & (2), respectively; dynamical behavior of errors $e_{1i}(t)$ $(i = 1, 2, 3)$ with control input (23(i)).

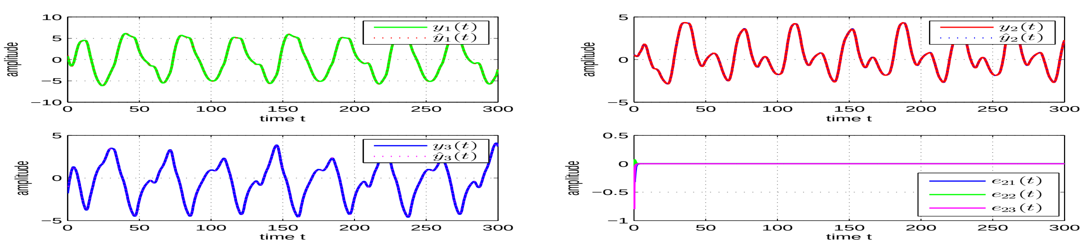

Figure 3: The time response of $y_j(t)$, $\hat{y}_j(t)$ in (1) & (2), respectively; dynamical behavior of errors $e_{2j}(t)$ $(j = 1, 2, 3)$ with control input (23(ii)).

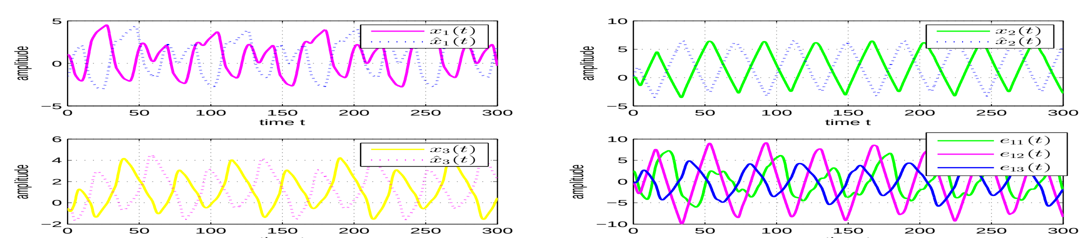

Figure 4: The time response of $x_i(t)$, $\hat{x}_i(t)$ in (1) & (2), respectively; dynamical behavior of errors $e_{1i}(t)$ $(i = 1, 2, 3)$ without control inputs.

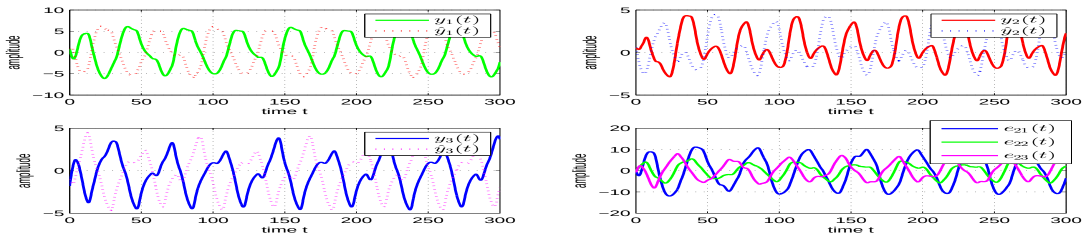

Figure 5: The time response of $y_j(t)$, $\hat{y}_j(t)$ in (1) & (2), respectively; dynamical behavior of errors $e_{2j}(t)$ $(j = 1, 2, 3)$ without control inputs.

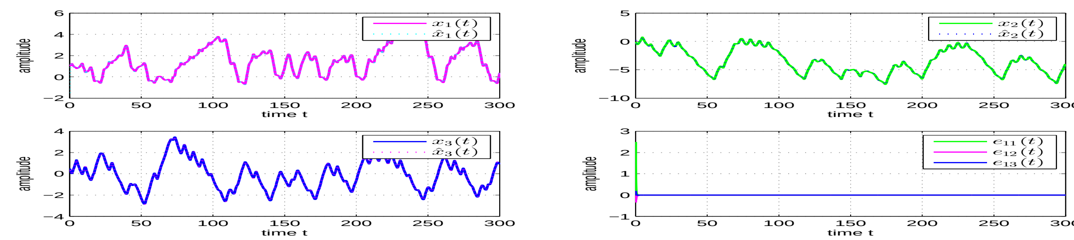

Figure 6: The time response of $x_i(t)$, $\hat{x}_i(t)$ in (17) & (18), respectively; dynamical behavior of errors $e_{1i}(t)$ $(i = 1, 2, 3)$ with control input (24(i)).

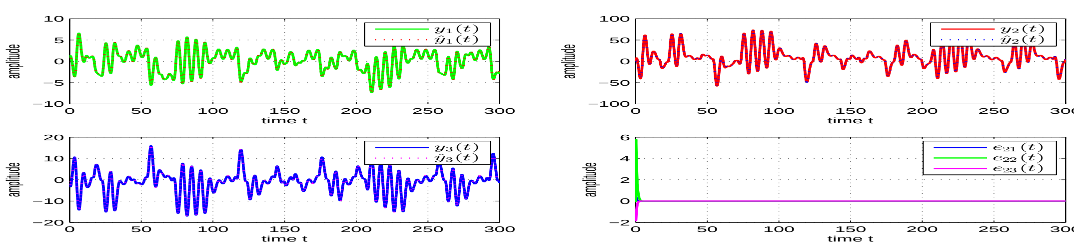

Figure 7: The time response of $y_j(t)$, $\hat{y}_j(t)$ in (17) & (18), respectively; dynamical behavior of errors $e_{2j}(t)$ $(j = 1, 2, 3)$ with control input (24(ii)).

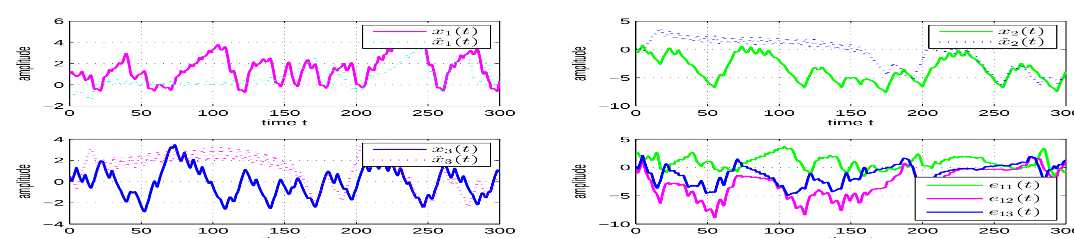

Figure 8: The time response of $x_i(t)$, $\hat{x}_i(t)$ in (17) & (18), respectively; dynamical behavior of errors $e_{1i}(t)$ $(i = 1, 2, 3)$ without control inputs.

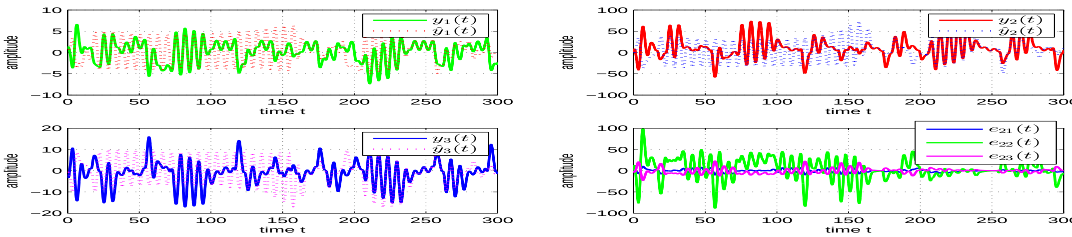

Figure 9: The time response of $y_j(t),\ \hat{y}_j(t)$ in (17) & (18), respectively; dynamical behavior of errors $e_{2j}(t)$ $(j = 1, 2, 3)$ without control inputs.

6 Conclusion

We investigated the synchronization and robust synchronization for BAM CGFCNNs with mixed time delays, where the mixed time delays include discrete time-varying and unbounded distributed delays. LMI based sufficient conditions have been derived to guarantee the global asymptotic stability of the error dynamical system by utilizing LK functionals and free weighting matrix method. Furthermore, we have considered synchronization of BAM CGFCNNs with mixed time delays in the presence of parameter uncertainties. The improved upper bounds on time delay obtained from Theorem 3.1 and 4.1 have been listed in Table 1 and 2, respectively. Numerical examples and its simulations have been demonstrated to show the efficacy of the obtained results. For future work, we will investigate the image encryption based on synchronization of BAM Cohen-Grossberg FCNNs with various delays. And we will also derive the sufficient conditions to ensure enough security strength to protect color image and make the hacker's attack infeasible. Moreover, the same synchronization problem will analyze in a finite-time.

Acknowledgement

This research work was supported by the UCSI University Research Excellence & Innovation Grant (REIG), Project Code REIG-FBM-2020/033 and the University of Malaya, Frontier Research Grant 2017, Grant No. FG037-17AFR. Dr. M. Manikandan is working as a Post-Doctoral Research Fellow at the UCSI University, Kuala Lumpur, Malaysia.

References

[1] M. S. Ali, S. Saravanan, L. Palanisamy, *Stochastic finite-time stability of reaction-diffusion Cohen-Grossberg neural networks with time-varying delays*, Chinese Journal of Physics, **57** (2019), 314-328.

[2] P. Balasubramaniam, M. Kalpana, R. Rakkiyappan, *Global asymptotic stability of BAM fuzzy cellular neural networks with time delay in the leakage term, discrete and unbounded distributed delays*, Mathematical and Computer Modelling, **53**(5-6) (2011), 839-853.

[3] S. Boyd, L. E. Ghaoui, E. Feron, V. Balakrishnan, *Linear matrix inequalities in systems and control theory*, SIAM, Philadelphia, 1994.

[4] Z. W. Cai, L. H. Huang, *Finite-time synchronization by switching state-feedback control for discontinuous Cohen-Grossberg neural networks with mixed delays*, International Journal of Machine Learning and Cybernetics, **9**(10) (2018), 1683-1695.

[5] M. A. Cohen, S. Grossberg, *Absolute stability of global pattern formation and parallel memory storage by competitive neural networks*, IEEE Transactions on Systems, Man, and Cybernetics, **13** (1983), 815-826.

[6] L. Duan, J. Jian, B. Wang, *Global exponential dissipativity of neutral-type BAM inertial neural networks with mixed time-varying delays*, Neurocomputing, **378** (2020), 399-412.

[7] M. Feki, *An adaptive chaos synchronization scheme applied to secure communication*, Chaos, Solitons and Fractals, **18**(1) (2003), 141-148.

[8] K. Gu, *An integral inequality in the stability problem of time-delay systems*, in: Proceedings of the 39th IEEE Conference on Decision and Control Sydney, Australia, **3** (2000), 2805-2810.

[9] Y. Han, J. Lian, *Synchronization of switched neural networks with time-varying delays via sampled-data control*, Asian Journal of Control, **21**(3) (2019), 1260-1269.

[10] W. He, L. Chu, *Exponential stability criteria for fuzzy bidirectional associative memory Cohen-Grossberg neural networks with mixed delays and impulses*, Advances in Difference Equations, **61**(2017) (2017), 1-16.

[11] J. Hu, C. Zeng, *Adaptive exponential synchronization of complex-valued Cohen-Grossberg neural networks with known and unknown parameters*, Neural Networks, **86** (2017), 90-101.

[12] J. Jian, L. Duan, *Finite-time synchronization for fuzzy neutral-type inertial neural networks with time-varying coefficients and proportional delays*, Fuzzy Sets and Systems, **381** (2020), 51-67.

[13] M. Kalpana, P. Balasubramaniam, K. Ratnavelu, *Direct delay decomposition approach to synchronization of chaotic fuzzy cellular neural networks with discrete, unbounded distributed delays and markovian jumping parameters*, Applied Mathematics and Computation, **254** (2015), 291-304.

[14] F. Kong, Q. Zhu, F. Liang, J. J. Nieto Roig, *Robust fixed-time synchronization of discontinuous Cohen-Grossberg neural networks with mixed time delays*, Nonlinear Analysis: Modelling and Control, **24**(4) (2019), 603-625.

[15] F. Kong, Q. Zhu, R. Rakkiyappan, *Robust synchronization control of discontinuous CGNNs with time-varying delays*, International Journal of Control, (2019), 1-17.

[16] B. Kosko, *Adaptive bidirectional associative memories*, Applied Optics, **26** (1987), 4947-4960.

[17] H. Li, C. Li, T. Huang, W. Zhang, *Fixed-time stabilization of impulsive Cohen-Grossberg BAM neural networks*, Neural Networks, **98** (2018), 203-211.

[18] K. Liang, L. Wanli, *Exponential synchronization in inertial Cohen-Grossberg neural networks with time delays*, Journal of the Franklin Institute, **356**(18) (2019), 11285-11304.

[19] J. Liu, J. Jian, B. Wang, *Stability analysis for BAM quaternion-valued inertial neural networks with time delay via nonlinear measure approach*, Mathematics and Computers in Simulation, **174** (2020), 134-152.

[20] Y. Liu, Z. Wang, X. Liu, *Design of exponential state estimators for neural networks with mixed time delays*, Physics Letters A, **364**(5) (2007), 401-412.

[21] P. Mani, R. Rajan, L. Shanmugam, Y. H. Joo, *Adaptive control for fractional order induced chaotic fuzzy cellular neural networks and its application to image encryption*, Information Sciences, **491** (2019), 74-89.

[22] F. Meng, K. Li, Q. Song, Y. Liu, F. E. Alsaadi, *Periodicity of Cohen-Grossberg–type fuzzy neural networks with impulses and time-varying delays*, Neurocomputing, **325** (2019), 254-259.

[23] L. M. Pecora, T. L. Carroll, *Synchronization in chaotic systems*, Physical Review Letters, **64**(8) (1990), 821-824.

[24] C. Rajivganthi, F. A. Rihan, S. Lakshmanan, P. Muthukumar, *Finite-time stability analysis for fractional-order Cohen-Grossberg BAM neural networks with time delays*, Neural Computing and Applications, **29**(12) (2018), 1309-1320.

[25] K. Ratnavelu, M. Kalpana, P. Balasubramaniam, K. Wong, P. Raveendran, *Image encryption method based on chaotic fuzzy cellular neural networks*, Signal Processing, **140** (2017), 87-96.

[26] K. Ratnavelu, M. Manikandan, P. Balasubramaniam, *Synchronization of fuzzy bidirectional associative memory neural networks with various time delays*, Applied Mathematics and Computation, **270** (2015), 582-605.

[27] Y. Shi, J. Cao, *Finite-time synchronization of memristive Cohen-Grossberg neural networks with time delays*, Neurocomputing, **377** (2020), 159-167.

[28] R. Wei, J. Cao, A. Alsaedi, *Fixed-time synchronization of memristive Cohen-Grossberg neural networks with impulsive effects*, International Journal of Control, Automation and Systems, **16**(5) (2018), 2214-2224.

[29] R. Yang, H. Gao, P. Shi, *Novel robust stability criteria for stochastic hopfield neural networks with time delays*, IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), **39**(2) (2008), 467-474.

[30] X. Yang, C. Li, T. Huang, Q. Song, J. Huang, *Synchronization of fractional-order memristor-based complex-valued neural networks with uncertain parameters and time delays*, Chaos, Solitons and Fractals, **110** (2018), 105-123.

[31] T. Yang, L. B. Yang, *The global stability of fuzzy cellular neural network*, IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, **43**(10) (1996), 880-883.

[32] T. Yang, L. B. Yang, C. W. Wu, L. O. Chua, *Fuzzy cellular neural networks: Theory*, in: Proceedings of IEEE International Workshop on Cellular Neural Networks and Applications, (1996), 181-186.

[33] F. Zhou, H. Yao, *Stability analysis for neutral-type inertial BAM neural networks with time-varying delays*, Nonlinear Dynamics, **92**(4) (2018), 1583-1598.

Iranian Journal of Fuzzy Systems
Volume 18, Number 2 (2021), pp. 215
University of Sistan and Baluchestan

Synchronization of BAM Cohen - Grossberg FCNNs with mixed time delays

M. Manikandan, K. Ratnavelu, P. Balasubramaniam and S. H. Ong

هماهنگ‌سازی BAM کوهن-گراس برگ FCNNs با تأخیرات زمانی مختلط

چکیده. این مقاله به مسئله همگام‌سازی حافظه شرکت‌پذیر دو طرفه (BAM) شبکه‌های عصبی سلولی فازی کوهن-گراس برگ (CGFCNNs) متغیر با زمان گسسته و تأخیرات توزیع شده نامحدود می‌پردازد. برخی شرایط کافی به منظور تضمین هماهنگی قوی BAM ,CGFCNNs متغیر با زمان گسسته و تأخیرات توزیع شده نامحدود، مشروط به عدم قطعیت پارامتری، با استفاده از تابعی (LK) Lyapunov-Krasovskii و روش نامساوی ماتریس خطی (LMI) بدست آمده‌اند. معیار کافی اطمینان می‌دهد که پویایی خطای سیستم در نظر گرفته شده در سطح جهانی بطور مجانبی پایدار است. سرانجام، مثال‌های عددی با شبیه‌سازی، برای نشان دادن اثربخشی نتایج بدست آمده ارائه شده‌است.