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Original paper

Reliability analysis of a warm standby series-parallel system with different switches and bi-uncertain lifetimes

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Abstract

The reliability of a warm standby series-parallel system without adequate samples is studied based on the uncertainty theory. It is assumed that the lifetimes of system elements follow independent uncertainty distributions with uncertain parameters. Three different switch models, including absolutely reliable mode, discrete mode and continuous mode, are developed for the warm standby series-parallel system. Besides, the cold standby series-parallel system is discussed as a special case. The reliability function and mean time to failure of each developed model are analyzed. A numerical example for the system with different switches is implemented to illustrate the application and efficiency of the proposed models.

Keywords: Uncertainty theory, bi-uncertain variable, warm standby system, reliability, mean time to failure.

1 Introduction

System reliability has been extensively studied so as to ensure safety and operation of systems. Maintaining a high performance of reliability or availability is usually essential, and redundancy is an effective technique, which is convenient operation and short time-consuming. Redundant methods have been used in various crucial infrastructures to improve the system reliability [13, 35, 43, 45]. Conversion switch plays an important role in the redundancy system. Switch failure can affect the system reliability even if the system elements are in operation. Owing to this, imperfect conversion switches have been considered in system and have been studied by many scholars [17, 34, 36]. Warm standby is one of the practical redundant techniques to improve system reliability in applications. Reliability analysis of warm standby system based on probability theory has been widely studied by many scholars, such as She and Pecht [32], Li et al. [19], Yuan and Meng [40], and so on.

Although probability theory has been proved to be effective for system reliability analysis, we require long-run cumulative frequency to approximate the actual value in order to estimate the probability distribution of element lifetime, which means that a large amount of observation data is needed for statistics. In practical, we cannot often obtain the complete data precisely owing to the technological or economical difficulties. There are limitations to deal with system reliability by using probability theory. In 1965, Zadeh [41] proposed fuzzy theory, and some concepts of fuzzy sets were defined. In 1975, Kaufmann [15] introduced fuzzy theory into reliability engineering. Fuzzy theory has a general application both in theory and engineering. For instance, fuzzy system reliability [12, 14, 16, 31], picture fuzzy number [2], fuzzy soft graphs [3], fuzzy logical relationship [20], and so on.

Although probability theory and fuzzy theory have been extensively applied in reliability analysis, Liu [22] claimed that some kinds of uncertainty behaves were neither randomness nor fuzziness. In order to deal with human uncertainty phenomena, uncertainty theory was founded in 2007 [22] and refined it in 2010 [24]. Nowadays, the uncertainty theory has been applied in different fields, such as uncertain reliability analysis [8, 11, 28, 37, 42, 44, 46], uncertain optimization [38], uncertain graph [21], uncertain integral [39], uncertain sequence [5] and so on.

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In uncertainty theory, uncertain reliability analysis has been extensively applied to address system reliability. Liu [25] introduced the reliability of uncertain system and presented a theorem of reliability index. The reliability and mean time to failure of non-repairable systems were studied by Liu et al. [29] under assumption that the system lifetimes were considered as uncertain variables. Gao and Yao [6] defined a concept of importance index in an uncertain reliability system and derived some formulas to calculate the importance index. Zu et al. [47] proposed an uncertain evaluating model for uncertainty metrics in reliability. Ahmadzade and Gao [1] proposed concepts of mean residual life and residual entropy to describe a failed system, and discussed their relationships with the reversed hazard function of uncertain lifetime. Liu et al. [30] studied the reliability models of repairable systems with uncertain lifetimes and repair times. Gao et al. [7] investigated weighted k-out-of-n system with uncertain variable and calculated the reliability index of system. Li et al. [18] presented an uncertain accelerated degradation model to improve the system reliability and lifetime evaluation. Sheng and Ke [33] derived some formulas to evaluate the reliability index of uncertain multi-state k-out-of-n system.

Research on the reliability of uncertain redundant systems is increasingly progressing, such as Liu [26] analyzed the reliability of redundant systems with uncertain lifetimes, and discussed absolutely reliable and non-absolutely reliable conversion switches. Hu et al. [9] established some reliability models for discrete time redundant systems based on the assumption that conversion switches are imperfect, including cold redundant systems and warm redundant systems. Cao et al. [4] gave some formulas to calculate the reliability of discrete time redundant standby series-parallel systems with uncertain parameters. Hu et al. [10] developed three redundancy optimization models of an uncertain parallel-series system with warm standby elements.

Many efforts have been devoted to the reliability analysis based on uncertainty theory under the assumption that element lifetimes are uncertain variables. Actually, the parameters of element uncertainty distribution are also uncertain in the practical engineering owing to the influence of system environment, production workshop, factor interference and other uncertain factors. There is indeed bi-uncertain phenomena in the real situations. In order to deal with various uncertain phenomena, uncertainty theory shows its advantages. Reliability modeling for a system with bi-uncertain phenomena is more challenging than a system with single-uncertain phenomena. In order to deal with this bi-uncertain phenomena, a specific concept of bi-uncertain variable is presented in this paper, the reliability function and mean time to failure of a warm standby series-parallel system with bi-uncertain lifetimes are investigated according to the uncertainty theory.

Series-parallel systems are closely related to our lives and becoming increasingly vital in various applications such as bulb lighting systems, electrical systems, drainage pipe systems, production workshop systems and so on. The warm standby is striking method to maintain the high level of system reliability. Therefore, we chose the typical representative warm standby series-parallel system for reliability analysis. The system mainly consists of three types of components: original elements, standby elements and conversion switches. Considering that these components may be innovative products with inadequate data, uncertain variables are applied to represent uncertain lifetimes. The lifetime distributions are obtained based on experts empirical data. The parameters of the uncertainty distribution are also uncertain. Therefore, the bi-uncertain variables can be modeled as the proposed bi-uncertain lifetimes.

The remainder of this paper is organized as follows. In Section 2, some basic concepts and theorems in uncertainty theory are introduced. In Section 3, the reliability function and mean time to failure of system are defined, and the warm redundant series-parallel system is described. In Section 4, three models of conversion switches in the system are established, including perfect switch, discrete switch and continuous switch. In order to illustrate the application and efficiency of the proposed system models, a numerical example is given in Section 5.

2 Preliminaries

Definition 2.1. [22] *Let Γ be a nonempty set, and $\mathcal{L}$ be a σ-algebra over Γ. A set function $\mathcal{M}$ is called an uncertain measure if it satisfies the following three axioms.*
Axiom 1 *(Normality Axiom) $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ.*
Axiom 2 *(Duality Axiom) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ.*
Axiom 3 *(Subadditivity Axiom) For every countable sequence of events $\Lambda_1, \Lambda_2, \cdots$, we have*

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} \mathcal{M}\left\{\Lambda_i\right\}.$$

Then the triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space. In addition, in order to provide the operational law, the uncertain measure on the product σ-algebra was proposed by Liu [23] as follows.

Axiom 4. *(Product Axiom) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \cdots$, the product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying*

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\left\{\Lambda_k\right\},$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1, 2, \cdots$, respectively.

Definition 2.2. [22] *An uncertain variable is a function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that $\{\xi \in B\}$ is an event for any Borel set B of real numbers.*

Definition 2.3. [22] *The uncertainty distribution Φ of an uncertain variable ξ is defined by $\Phi(x) = \mathcal{M}\{\xi \le x\}$ for any real number x.*

Definition 2.4. [24] *Let ξ be an uncertain variable with regular uncertainty distribution $\Phi(x)$, then the inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of ξ.*

Theorem 2.5. [24] *Let $\xi_1, \xi_2, \cdots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. If $f(\xi_1, \xi_2, \cdots, \xi_n)$ is strictly increasing with respect to $\xi_1, \xi_2, \cdots, \xi_m$ and strictly decreasing with respect to $\xi_{m+1}, \xi_{m+2}, \cdots, \xi_n$, then $\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$ has an inverse uncertainty distribution*

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \cdots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \cdots, \Phi_n^{-1}(1-\alpha)).$$

Theorem 2.6. [24] *Let $\xi_1, \xi_2, \cdots, \xi_n$ be independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively, then $\xi_1 \wedge \xi_2 \wedge \cdots \wedge \xi_n$ and $\xi_1 \vee \xi_2 \vee \cdots \vee \xi_n$ have uncertainty distributions $\Psi(x) = \Phi_1(x) \vee \Phi_2(x) \vee \cdots \vee \Phi_n(x)$ and $\Psi(x) = \Phi_1(x) \wedge \Phi_2(x) \wedge \cdots \wedge \Phi_n(x)$, respectively.*

Definition 2.7. [22] *Let ξ be an uncertain variable, then the expected value of ξ is defined by*

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \ge x\}\mathrm{d}x - \int_{-\infty}^0 \mathcal{M}\{\xi \le x\}\mathrm{d}x,$$

provided that at least one of the two integrals is finite.

Theorem 2.8. [24] *Let ξ be an uncertain variable with regular uncertainty distribution Φ, then*

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha)\mathrm{d}\alpha.$$

Theorem 2.9. [27] *Assume $\xi_1, \xi_2, \cdots, \xi_n$ are independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. If $f(\xi_1, \xi_2, \cdots, \xi_n)$ is strictly increasing with respect to $\xi_1, \xi_2, \cdots, \xi_m$ and strictly decreasing with respect to $\xi_{m+1}, \xi_{m+2}, \cdots, \xi_n$, then $\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$ has an expected value*

$$E[\xi] = \int_0^1 f(\Phi_1^{-1}(\alpha), \cdots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \cdots, \Phi_n^{-1}(1-\alpha))\mathrm{d}\alpha.$$

3 Problem description

3.1 Reliability index

Definition 3.1. *Let ζ be an uncertain variable with uncertainty distribution $\Phi(a_1, a_2, \cdots, a_k; t)$, whose parameters $a_i (i = 1, 2, \cdots, k)$ are independent uncertain variables with uncertainty distributions Υ_i $(i = 1, 2, \cdots, k)$. Then ζ is called a bi-uncertain variable.*

Definition 3.2. *The lifetime ξ of a system is said to bi-uncertain lifetime, if ξ is a bi-uncertain variable.*

Definition 3.3. *Let ξ, a non-negative bi-uncertain variable, be the lifetime of a system defined on the uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$. The reliability function of the system is defined by*

$$R(t) = E\left[\mathcal{M}\{\gamma \in \Gamma | \, \xi(\gamma) > t\}\right]. \tag{1}$$

Then we denote $\mathcal{M}\{\gamma \in \Gamma | \, \xi(\gamma) > t\}$ as $R^{var}(t)$, which is called uncertain reliability variable, that is

$$R(t) = E\left[\mathcal{M}\{\gamma \in \Gamma | \, \xi(\gamma) > t\}\right] = E\left[R^{var}(t)\right]. \tag{2}$$

Definition 3.4. *The mean time to failure (MTTF) of the system is defined by*

$$\text{MTTF} = \int_0^{+\infty} E\left[\mathcal{M}\{\gamma \in \Gamma | \, \xi(\gamma) > t\}\right]\mathrm{d}t = \int_0^{+\infty} R(t)\mathrm{d}t. \tag{3}$$

3.2 System description

A warm standby series-parallel system is considered with different switches and bi-uncertain lifetimes. The series-parallel system consists of $\sum_{i=1}^{n} m_i$ subsystems, each subsystem is composed of a conversion switch and a warm standby system, where the warm standby system is composed of an original element and $h_{ij}-1$ warm standby elements, as show in Figure 1. Throughout the paper, we assume that as follow for the warm standby series-parallel system.

1) All elements and conversion switches are independent of each other.
2) Elements and conversion switches are non-repairable.
3) The lifetime of each element is bi-uncertain variable.
4) The conversion of switch is instantaneous.
5) Elements and conversion switches are only in one of two states: functioning or failing.
6) The lifetime of each conversion switch is in one of two types: discrete uncertain variable or continuous bi-uncertain variable.

In order to illustrate the lifetime of the warm standby system under different switches, we initially make the necessary explanations for some notations. ξ_{ij}^p means the lifetime of subsystem A_{ij} with perfect conversion switch. ξ_{ij}^d denotes the lifetime of subsystem A_{ij} with discrete conversion switch. ξ_{ij}^c expresses the lifetime of subsystem A_{ij} with continuous conversion switch. ξ_{ijv} is the lifetime of the vth element in subsystem A_{ij} at time 0. ξ_{ijv}^w indicates the working lifetime of the vth element in subsystem A_{ij}.

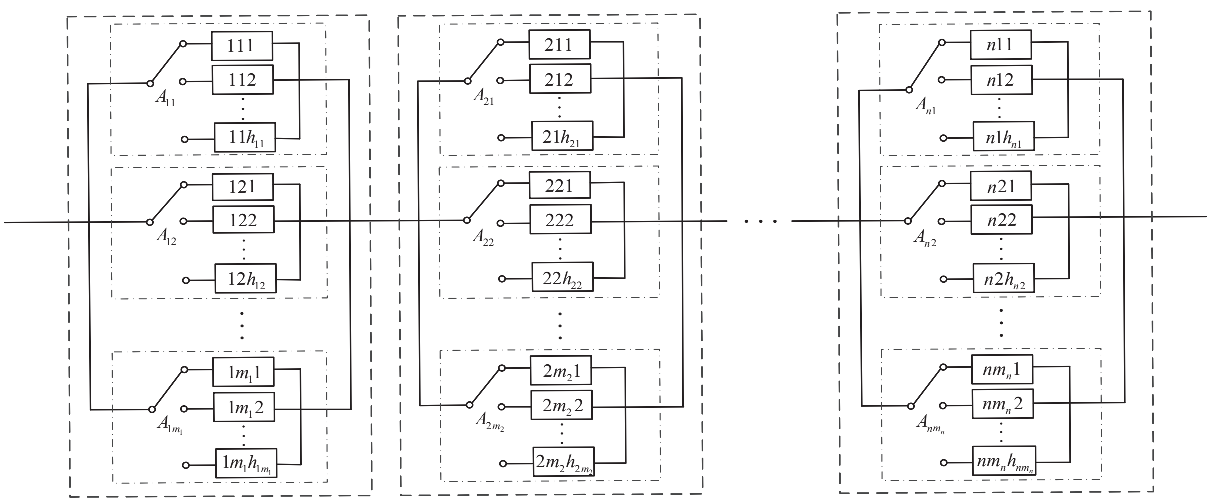


Figure 1: A warm standby series-parallel system

4 Warm standby series-parallel system

4.1 Warm standby series-parallel system with prefect conversion switches

In this subsection, we assume that the conversion switches of the warm standby system are absolutely reliable. Let ξ_{ij}^p be the lifetime of subsystem $A_{ij}(i=1,2,\cdots,n, j=1,2,\cdots,m_i)$. Obviously, the lifetime of the series-parallel system with perfect conversion switches is

$$\xi(\boldsymbol{h},\boldsymbol{\xi}) = \bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i} \xi_{ij}^p,$$

where

$$\boldsymbol{h} = (h_{11}, h_{12}, \cdots, h_{1m_1}, \cdots, h_{n1}, h_{n2}, \cdots, h_{nm_n}),$$

$$\boldsymbol{\xi} = (\xi_{111}^w, \xi_{112}^w, \cdots, \xi_{11h_{11}}^w, \cdots, \xi_{nm_n1}^w, \xi_{nm_n2}^w, \cdots, \xi_{nm_nh_{nm_n}}^w).$$

Let $\xi_{ijv}^w(v=1,2,\cdots,h_{ij})$, a bi-uncertain variable, be the working lifetime of the vth element in subsystem $A_{ij}(i=1,2,\cdots,n, j=1,2,\cdots,m_i)$. Obviously, the lifetime of subsystem A_{ij} can be expressed as

$$\xi_{ij}^p = \xi_{ij1}^w + \xi_{ij2}^w + \cdots + \xi_{ijh_{ij}}^w,$$

where h_{ij} represents the number of elements in subsystem A_{ij}. It is assumed that the degradation rate of all elements in the overall system is the same, which is denoted as λ. According to Liu [26], the working lifetime of the vth element in subsystem A_{ij} is expressed as

$$\xi_{ijv}^w = \xi_{ijv} - \lambda(\xi_{ij(v-1)}^w + \cdots + \xi_{ij2}^w + \xi_{ij1}^w),$$

where ξ_{ijv} is the lifetime of the vth element in subsystem A_{ij} at time 0, then, the working lifetime of all elements in subsystem A_{ij} can be expressed

$$\begin{aligned}
\xi_{ij1}^w &= \xi_{ij1}, \\
\xi_{ij2}^w &= \xi_{ij2} - \lambda\xi_{ij1}^w, \\
\xi_{ij3}^w &= \xi_{ij3} - \lambda(\xi_{ij2}^w + \xi_{ij1}^w), \\
&\vdots \\
\xi_{ijv}^w &= \xi_{ijv} - \lambda(\xi_{ij(v-1)}^w + \cdots + \xi_{ij2}^w + \xi_{ij1}^w).
\end{aligned}$$

Then the lifetime of subsystem A_{ij} is

$$\xi_{ij}^p = (1-\lambda)^{h_{ij}-1}\xi_{ij1} + (1-\lambda)^{h_{ij}-2}\xi_{ij2} + \cdots + \xi_{ijh_{ij}}.$$

So, the lifetime ξ of the system with prefect conversion switches is

$$\xi = \bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} \xi_{ij}^p = \bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} \sum_{1\le v\le hij} (1-\lambda)^{h_{ij}-v}\xi_{ijv}.$$

Theorem 4.1. *Let $a_{ij1}, a_{ij2}, \cdots, a_{ijk_{ij}}$ be independent uncertain parameters of the lifetime distribution of standby element in subsystem A_{ij} with regular uncertainty distributions $\Upsilon_{ij1}, \Upsilon_{ij2} \cdots, \Upsilon_{ijk_{ij}}$, respectively. If the uncertain reliability variable of the system is strictly increasing with respect to $a_{ij1}, a_{ij2}, \cdots, a_{ijg_{ij}}$, and strictly decreasing with respect to $a_{ij(g_{ij}+1)}, a_{ij(g_{ij}+2)}, \cdots, a_{ijk_{ij}}$, then the reliability function and MTTF of the warm standby series-parallel system with perfect conversion switches are*

$$R(t) = \int_0^1 \bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} \Big(1 - \Phi_{ij}\Big(\Upsilon_{ij1}^{-1}(\alpha), \cdots, \Upsilon_{ijg_{ij}}^{-1}(\alpha), \Upsilon_{ij(g_{ij}+1)}^{-1}(1-\alpha), \cdots, \Upsilon_{ijk_{ij}}^{-1}(1-\alpha); \frac{\lambda t}{1-(1-\lambda)^{h_{ij}}}\Big)\Big)\mathrm{d}\alpha \quad (4)$$

and

$$\mathrm{MTTF} = \int_0^{+\infty}\int_0^1 \bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} \Big(1 - \Phi_{ij}\Big(\Upsilon_{ij1}^{-1}(\alpha), \cdots, \Upsilon_{ijg_{ij}}^{-1}(\alpha), \Upsilon_{ij(g_{ij}+1)}^{-1}(1-\alpha), \cdots, \Upsilon_{ijk_{ij}}^{-1}(1-\alpha); \frac{\lambda t}{1-(1-\lambda)^{h_{ij}}}\Big)\Big)\mathrm{d}\alpha\mathrm{d}t, \quad (5)$$

respectively, where $\Upsilon_{ijg_{ij}}^{-1}(\alpha)$ is the inverse uncertainty distribution of uncertain variable $a_{jg_{ij}}$, k_{ij} is the number of uncertain parameters contained in subsystem A_{ij} with $1 \le g_{ij} \le k_{ij}$, $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m_i$.

Proof. According to Definition 3.3, the uncertain reliability variable $R_{ij}^{var}(\boldsymbol{a}_{ij1}, \boldsymbol{a}_{ij2}, \cdots, \boldsymbol{a}_{ijh_{ij}}; t)$ of subsystem A_{ij} is

$$\begin{aligned}
R_{ij}^{var}(\boldsymbol{a}_{ij1}, \boldsymbol{a}_{ij2}, \cdots, \boldsymbol{a}_{ijh_{ij}}; t) &= \mathcal{M}\left\{\xi_{ij1}^w + \xi_{ij2}^w + \cdots + \xi_{ijh_{ij}}^w > t\right\} \\
&= \mathcal{M}\left\{(1-\lambda)^{h_{ij}-1}\xi_{ij1} + (1-\lambda)^{h_{ij}-2}\xi_{ij2} + \cdots + \xi_{ijh_{ij}} > t\right\},
\end{aligned}$$

where $\boldsymbol{a}_{ijv} = (a_{ij1}, a_{ij2}, \ldots, a_{ijk_{ij}}), v = 1, 2, \cdots, h_{ij}$. If $\xi_{ij1}, \xi_{ij2}, \cdots, \xi_{ijh_{ij}}$ have the same uncertainty distribution $\Phi_{ij}(a_{ij1}, a_{ij2}, \cdots, a_{ijk_{ij}}; t)$, the uncertain reliability variable of subsystem A_{ij} is

$$R_{ij}^{var}(\boldsymbol{a}_{ij1}, \boldsymbol{a}_{ij2}, \cdots, \boldsymbol{a}_{ijh_{ij}}; t) = 1 - \Phi_{ij}\Big(a_{ij1}, a_{ij2}, \cdots, a_{ijk_{ij}}; \frac{\lambda t}{1-(1-\lambda)^{h_{ij}}}\Big).$$

Then, the uncertain reliability variable of the system is

$$\begin{aligned}
R^{var}(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n; t) &= \bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} R_{ij}^{var}(\boldsymbol{a}_{ij1}, \boldsymbol{a}_{ij2}, \cdots, \boldsymbol{a}_{ijh_{ij}}; t) \\
&= \bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} \Big(1 - \Phi_{ij}\Big(a_{ij1}, a_{ij2}, \cdots, a_{ijk_{ij}}; \frac{\lambda t}{1-(1-\lambda)^{h_{ij}}}\Big)\Big),
\end{aligned}$$

where $\boldsymbol{a}_i = (a_{i11}, a_{i12}, \cdots, a_{i1k_{i1}}, \cdots, a_{ij1}, a_{ij2}, \cdots, a_{ijk_{ij}}, \cdots, a_{im_i1}, a_{im_i2}, \cdots, a_{im_ik_{im_i}})$, $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m_i$. According to Definition 3.3, the reliability function of the system with perfect switches can be determined by

$$\begin{aligned} R(t) &= E\Big[R^{var}(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n; t)\Big] \\ &= E\Big[\bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} \Big(1 - \Phi_{ij}\Big(a_{ij1}, a_{ij2}, \cdots, a_{ijk_{ij}}; \frac{\lambda t}{1-(1-\lambda)^{h_{ij}}}\Big)\Big)\Big]. \end{aligned}$$

It is known that the uncertain reliability variable $R^{var}(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n; t)$ is strictly increasing with respect to $a_{ij1}, a_{ij2}, \cdots, a_{ijg_{ij}}$ and strictly decreasing with respect to $a_{ij(g_{ij}+1)}, a_{ij(g_{ij}+2)}, \cdots, a_{ijk_{ij}}$. According to Theorem 2.5, the inverse uncertainty distribution $\Psi^{-1}(\alpha)$ of the uncertain reliability variable $R^{var}(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n; t)$ is

$$\Psi^{-1}(\alpha) = \bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} \Big(1 - \Phi_{ij}\Big(\Upsilon^{-1}_{ij1}(\alpha), \cdots, \Upsilon^{-1}_{ijg_{ij}}(\alpha), \Upsilon^{-1}_{ij(g_{ij}+1)}(1-\alpha), \cdots, \Upsilon^{-1}_{ijk_{ij}}(1-\alpha); \frac{\lambda t}{1-(1-\lambda)^{h_{ij}}}\Big)\Big).$$

Then by Theorem 2.9, we can obtain the reliability function of the system which is

$$R(t) = E\left[R^{var}(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n; t)\right] = \int_0^1 \Psi^{-1}(\alpha)\mathrm{d}\alpha.$$

So, Equation (4) is obtained. And the MTTF of the system is formulated owing to Definition 3.4, that is

$$\mathrm{MTTF} = \int_0^{+\infty} R(t)\mathrm{d}t = \int_0^{+\infty} \int_0^1 \Psi^{-1}(\alpha)\mathrm{d}\alpha\mathrm{d}t.$$

Then, Equation (5) can be obtained. □

When $\lambda = 0$, the warm standby system degenerates to the cold standby system. In order to express the special case of the deterioration rate λ, the following Remark 4.2 is made.

Remark 4.2. *The reliability function and MTTF of the cold standby series-parallel system with prefect conversion switches are*

$$R(t) = \int_0^1 \bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} \Big(1 - \Phi_{ij}\Big(\Upsilon^{-1}_{ij1}(\alpha), \cdots, \Upsilon^{-1}_{ijg_{ij}}(\alpha), \Upsilon^{-1}_{ij(g_{ij}+1)}(1-\alpha), \cdots, \Upsilon^{-1}_{ijk_{ij}}(1-\alpha); \frac{t}{n}\Big)\Big)\mathrm{d}\alpha, \tag{6}$$

and

$$\mathrm{MTTF} = \int_0^{+\infty} \int_0^1 \bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} \Big(1 - \Phi_{ij}\Big(\Upsilon^{-1}_{ij1}(\alpha), \cdots, \Upsilon^{-1}_{ijg_{ij}}(\alpha), \Upsilon^{-1}_{ij(g_{ij}+1)}(1-\alpha), \cdots, \Upsilon^{-1}_{ijk_{ij}}(1-\alpha); \frac{t}{n}\Big)\Big)\mathrm{d}\alpha\mathrm{d}t, \tag{7}$$

respectively.

4.2 Warm standby series-parallel system with discrete imperfect conversion switches

In this subsection, we consider a warm standby subsystem with imperfect conversion switches. The conversion switch lifetime is a discrete uncertain variable, the uncertainty measures of operation and failure of the conversion switch are b_{ij} and $1 - b_{ij}$, respectively. There are two cases of subsystem failure, those are

1) When the functioning element fails, it is immediately replaced by a standby element. If the conversion switch fails, the subsystem fails.

2) The subsystem fails when $h_{ij} - 1$ times of using conversion switch is normal, and the h_{ij}th element fails.

Here, we define an uncertain variable

$$\eta_{ij} = \begin{cases} \mu_{ij} & \textit{the conversion switch fails for the first time when using switch for the } \mu_{ij}\text{th } \textit{times}, \\ & \mu_{ij} = 1, 2, \cdots, h_{ij} - 1, \\ h_{ij} & \textit{the conversion switch is normal when using switch for the } (h_{ij}-1)\text{th } \textit{times}. \end{cases}$$

In addition, $\mathcal{M}\{\eta_{ij} = 1\} = 1 - b_{ij}$; $\mathcal{M}\{\eta_{ij} = \mu_{ij}\} = b_{ij} \wedge (1 - b_{ij})$, $\mu_{ij} = 2, 3, \cdots, h_{ij} - 1$; $\mathcal{M}\{\eta_{ij} = h_{ij}\} = b_{ij}$. Obviously, the lifetime of subsystem A_{ij} with discrete conversion switches can be expressed as

$$\xi^d_{ij} = \xi^w_{ij1} + \xi^w_{ij2} + \cdots + \xi^w_{ij\eta_{ij}}.$$

So, the lifetime ξ of the system with discrete conversion switches is

$$\xi = \bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}\xi_{ij}^d = \bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}\sum_{1\le v\le \eta_{ij}}\xi_{ijv}^w.$$

Theorem 4.3. *Let $a_{ij1}, a_{ij2}, \cdots, a_{ijk_{ij}}$ be independent uncertain parameters of the lifetime distribution of standby element in subsystem A_{ij} with regular uncertainty distributions $\Upsilon_{ij1}, \Upsilon_{ij2}\cdots, \Upsilon_{ijk_{ij}}$, respectively. If the uncertain reliability variable of the system is strictly increasing with respect to $a_{ij1}, a_{ij2}, \cdots, a_{ijg_{ij}}$, and strictly decreasing with respect to $a_{ij(g_{ij}+1)}, a_{ij(g_{ij}+2)}, \cdots, a_{ijk_{ij}}$, then the reliability function and MTTF of the warm standby series-parallel system with discrete imperfect switches are*

$$R(t) = \int_0^1 \bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}\bigvee_{1\le \mu_{ij}\le h_{ij}}\Big\{\mathcal{M}(\eta_{ij}=\mu_{ij})\wedge\Big\{1-\Phi_{ij}(\Upsilon_{ij1}^{-1}(\alpha),\cdots,\Upsilon_{ijg_{ij}}^{-1}(\alpha), \Upsilon_{ij(g_{ij}+1)}^{-1}(1-\alpha),\cdots,\Upsilon_{ijk_{ij}}^{-1}(1-\alpha);\frac{\lambda t}{1-(1-\lambda)^{\mu_{ij}}})\Big\}\Big\}\mathrm{d}\alpha \tag{8}$$

and

$$\mathrm{MTTF} = \int_0^{+\infty}\int_0^1 \bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}\bigvee_{1\le \mu_{ij}\le h_{ij}}\Big\{\mathcal{M}(\eta_{ij}=\mu_{ij})\wedge\Big\{1-\Phi_{ij}(\Upsilon_{ij1}^{-1}(\alpha),\cdots,\Upsilon_{ijg_{ij}}^{-1}(\alpha), \Upsilon_{ij(g_{ij}+1)}^{-1}(1-\alpha),\cdots,\Upsilon_{ijk_{ij}}^{-1}(1-\alpha);\frac{\lambda t}{1-(1-\lambda)^{\mu_{ij}}})\Big\}\Big\}\mathrm{d}\alpha\mathrm{d}t, \tag{9}$$

respectively, where $\Upsilon_{ijg_{ij}}^{-1}(\alpha)$ is the inverse uncertainty distribution of uncertain variable $a_{jg_{ij}}$, k_{ij} is the number of uncertain parameters contained in subsystem A_{ij} with $1\le g_{ij}\le k_{ij}$, $i=1,2,\cdots,n$, $j=1,2,\cdots,m_i$.

Proof. According to Definition 3.3, the uncertain reliability variable $R_{ij}^{var}(\boldsymbol{a}_{ij1},\boldsymbol{a}_{ij2},\cdots,\boldsymbol{a}_{ijh_{ij}};t)$ of subsystem A_{ij} with discrete conversion switches is

$$\begin{aligned}R_{ij}^{var}(\boldsymbol{a}_{ij1},\boldsymbol{a}_{ij2},\cdots,\boldsymbol{a}_{ijh_{ij}};t) &= \mathcal{M}\Big\{\xi_{ij1}^w+\xi_{ij2}^w+\cdots+\xi_{ij\eta_{ij}}^w>t\Big\}\\ &= \mathcal{M}\Big\{\bigcup_{1\le\mu_{ij}\le h_{ij}}(\eta_{ij}=\mu_{ij})\cap\Big\{\xi_{ij1}^w+\xi_{ij2}^w+\cdots+\xi_{ij\mu_{ij}}^w>t\Big\}\Big\}\\ &= \bigvee_{1\le\mu_{ij}\le h_{ij}}\Big\{\mathcal{M}(\eta_{ij}=\mu_{ij})\wedge\mathcal{M}\Big\{\xi_{ij1}^w+\xi_{ij2}^w+\cdots+\xi_{ij\mu_{ij}}^w>t\Big\}\Big\}.\end{aligned}$$

If $\xi_{ij1},\xi_{ij2},\cdots,\xi_{ijh_{ij}}$ have the same uncertainty distribution $\Phi_{ij}(a_{ij1},a_{ij2},\cdots,a_{ijk_{ij}};t)$, the uncertain reliability variable of subsystem A_{ij} with discrete conversion switches is

$$\begin{aligned}&R_{ij}^{var}(\boldsymbol{a}_{ij1},\boldsymbol{a}_{ij2},\cdots,\boldsymbol{a}_{ijh_{ij}};t)\\ &= \bigvee_{1\le\mu_{ij}\le h_{ij}}\Big\{\mathcal{M}(\eta_{ij}=\mu_{ij})\wedge\Big\{1-\mathcal{M}\Big\{\xi_{ij1}^w+\xi_{ij2}^w+\cdots+\xi_{ij\mu_{ij}}^w\le t\Big\}\Big\}\Big\}\\ &= \bigvee_{1\le\mu_{ij}\le h_{ij}}\Big\{\mathcal{M}(\eta_{ij}=\mu_{ij})\wedge\Big\{1-\mathcal{M}\Big\{(1-\lambda)^{\mu_{ij}-1}\xi_{ij1}+(1-\lambda)^{\mu_{ij}-2}\xi_{ij2}+\cdots+\xi_{ij\mu_{ij}}\le t\Big\}\Big\}\Big\}\\ &= \bigvee_{1\le\mu_{ij}\le h_{ij}}\Big\{\mathcal{M}(\eta_{ij}=\mu_{ij})\wedge\Big\{1-\Phi_{ij}(a_{ij1},a_{ij2},\cdots,a_{ijk_{ij}};\frac{\lambda t}{1-(1-\lambda)^{\mu_{ij}}})\Big\}\Big\},\end{aligned}$$

where $\boldsymbol{a}_{ijv}=(a_{ij1},a_{ij2},\ldots,a_{ijk_{ij}}), v=1,2,\cdots,h_{ij}$. Then, the uncertain reliability variable of the system is

$$\begin{aligned}R^{var}(\boldsymbol{a}_1,\boldsymbol{a}_2,\cdots,\boldsymbol{a}_n;t) &= \bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}R_{ij}^{var}(\boldsymbol{a}_{ij1},\boldsymbol{a}_{ij2},\cdots,\boldsymbol{a}_{ijh_{ij}};t)\\ &= \bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}\bigvee_{1\le\mu_{ij}\le h_{ij}}\Big\{\mathcal{M}(\eta_{ij}=\mu_{ij})\wedge\Big\{1-\Phi_{ij}(a_{ij1},a_{ij2},\cdots,a_{ijk_{ij}};\frac{\lambda t}{1-(1-\lambda)^{\mu_{ij}}})\Big\}\Big\},\end{aligned}$$

where $\boldsymbol{a}_i = (a_{i11}, a_{i12}, \cdots, a_{i1k_{i1}}, \cdots, a_{ij1}, a_{ij2}, \cdots, a_{ijk_{ij}}, \cdots, a_{im_i1}, a_{im_i2}, \cdots, a_{im_ik_{im_i}})$, $i = 1, 2, \cdots, n$. According to Definition 3.3, the reliability function of the system with discrete imperfect switches can be determined by

$$\begin{aligned} R(t) &= E\Big[R^{var}(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n; t)\Big] \\ &= E\Big[\bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}\bigvee_{1\le \mu_{ij}\le h_{ij}}\Big\{\mathcal{M}(\eta_{ij}=\mu_{ij})\wedge\Big\{1-\Phi_{ij}(a_{ij1}, a_{ij2}, \cdots, a_{ijk_{ij}}; \frac{\lambda t}{1-(1-\lambda)^{\mu_{ij}}})\Big\}\Big\}\Big]. \end{aligned}$$

It is known that the uncertain reliability variable $R^{var}(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n; t)$ is strictly increasing with respect to $a_{ij1}, a_{ij2}, \cdots, a_{ijg_{ij}}$ and strictly decreasing with respect to $a_{ij(g_{ij}+1)}, a_{ij(g_{ij}+2)}, \cdots, a_{ijk_{ij}}$. According to Theorem 2.5, the inverse uncertainty distribution $\Psi^{-1}(\alpha)$ of the uncertain reliability variable $R^{var}(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n; t)$ is

$$\begin{aligned} \Psi^{-1}(\alpha) = &\bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}\bigvee_{1\le \mu_{ij}\le h_{ij}}\Big\{\mathcal{M}(\eta_{ij}=\mu_{ij})\wedge\Big\{1-\Phi_{ij}(\Upsilon^{-1}_{ij1}(\alpha), \cdots, \Upsilon^{-1}_{ijg_{ij}}(\alpha), \\ &\Upsilon^{-1}_{ij(g_{ij}+1)}(1-\alpha), \cdots, \Upsilon^{-1}_{ijk_{ij}}(1-\alpha); \frac{\lambda t}{1-(1-\lambda)^{\mu_{ij}}})\Big\}\Big\}. \end{aligned}$$

According to Theorem 2.9, the reliability function of the system is

$$R(t) = E\left[R^{var}(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n; t)\right] = \int_0^1 \Psi^{-1}(\alpha)\mathrm{d}\alpha.$$

So, Equation (8) is obtained. And the MTTF of the system is formulated owing to Definition 3.4, that is

$$\mathrm{MTTF} = \int_0^{+\infty} R(t)\mathrm{d}t = \int_0^{+\infty}\int_0^1 \Psi^{-1}(\alpha)\mathrm{d}\alpha\mathrm{d}t.$$

Then, Equation (9) can be obtained. □

When $\lambda = 0$, the warm standby system degenerates to the cold standby system. Considering the special case of the deterioration rate λ, the Remark 4.4 is procured.

Remark 4.4. *The reliability function and MTTF of the cold standby series-parallel system with discrete imperfect switches are*

$$\begin{aligned} R(t) = \int_0^1 &\bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}\bigvee_{1\le \mu_{ij}\le h_{ij}}\Big\{\mathcal{M}(\eta_{ij}=\mu_{ij})\wedge\Big\{1-\Phi_{ij}(\Upsilon^{-1}_{ij1}(\alpha), \cdots, \Upsilon^{-1}_{ijg_{ij}}(\alpha), \\ &\Upsilon^{-1}_{ij(g_{ij}+1)}(1-\alpha), \cdots, \Upsilon^{-1}_{ijk_{ij}}(1-\alpha); \frac{t}{\mu_{ij}})\Big\}\Big\}\mathrm{d}\alpha, \end{aligned} \tag{10}$$

and

$$\begin{aligned} \mathrm{MTTF} = \int_0^{+\infty}\int_0^1 &\bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}\bigvee_{1\le \mu_{ij}\le h_{ij}}\Big\{\mathcal{M}(\eta_{ij}=\mu_{ij})\wedge\Big\{1-\Phi_{ij}(\Upsilon^{-1}_{ij1}(\alpha), \cdots, \Upsilon^{-1}_{ijg_{ij}}(\alpha), \\ &\Upsilon^{-1}_{ij(g_{ij}+1)}(1-\alpha), \cdots, \Upsilon^{-1}_{ijk_{ij}}(1-\alpha); \frac{t}{\mu_{ij}})\Big\}\Big\}\mathrm{d}\alpha\mathrm{d}t, \end{aligned} \tag{11}$$

respectively.

4.3 Warm standby series-parallel system with continuous imperfect conversion switches

Let ξ^{η}_{ij}, a bi-uncertain variable, be the lifetime of conversion switch in subsystem A_{ij}. The lifetime ξ^{η}_{ij} is distributed with uncertainty distribution $\Phi^{\eta}_{ij}(a^{\eta}_{ij1}, a^{\eta}_{ij2}, \cdots, a^{\eta}_{ijk^{\eta}_{ij}})$, where $a^{\eta}_{ij1}, a^{\eta}_{ij2}, \cdots, a^{\eta}_{ijk^{\eta}_{ij}}$ are independent uncertain variables. Obviously, the lifetime of subsystem A_{ij} with continuous conversion switches can be expressed as

$$\xi^{c}_{ij} = (\xi^{w}_{ij1} + \xi^{w}_{ij2} + \cdots + \xi^{w}_{ijh_{ij}}) \wedge \xi^{\eta}_{ij}.$$

So, the lifetime ξ of the system with continuous conversion switches is

$$\xi = \bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}\xi^{c}_{ij} = \bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}\Big(\Big(\sum_{1\le v\le h_{ij}}\xi^{w}_{ijv}\Big)\wedge\xi^{\eta}_{ij}\Big).$$

Theorem 4.5. *If the uncertain reliability variable of the system is strictly increasing with respect to* $a_{ij1}, a_{ij2}, \cdots, a_{ijg_{ij}}$, $a^{\eta}_{ij1}, a^{\eta}_{ij2}, \cdots, a^{\eta}_{ijg^{\eta}_{ij}}$, *strictly decreasing with respect to* $a_{ij(g_{ij}+1)}, a_{ij(g_{ij}+2)}, \cdots, a_{ijk_{ij}}, a^{\eta}_{ij(g^{\eta}_{ij}+1)}, a^{\eta}_{ij(g^{\eta}_{ij}+2)}, \cdots, a^{\eta}_{ijk^{\eta}_{ij}}$, *then the reliability function and MTTF of the warm standby series-parallel system with continuous imperfect switches are*

$$\begin{aligned} R(t) =& \int_0^1 \bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} \Big(1-\Phi_{ij}\Big(\Upsilon^{-1}_{ij1}(\alpha),\cdots,\Upsilon^{-1}_{ijg_{ij}}(\alpha),\Upsilon^{-1}_{ij(g_{ij}+1)}(1-\alpha),\cdots,\Upsilon^{-1}_{ijk_{ij}}(1-\alpha);\frac{\lambda t}{1-(1-\lambda)^{h_{ij}}}\Big) \\ &\vee \Phi^{\eta}_{ij}\Big(\Upsilon^{-1(\eta)}_{ij1}(\alpha),\cdots,\Upsilon^{-1(\eta)}_{ijg^{\eta}_{ij}}(\alpha),\Upsilon^{-1(\eta)}_{ij(g^{\eta}_{ij}+1)}(1-\alpha),\cdots,\Upsilon^{-1(\eta)}_{ijk^{\eta}_{ij}}(1-\alpha);t\Big)\Big)\mathrm{d}\alpha, \end{aligned} \tag{12}$$

and

$$\begin{aligned} \mathrm{MTTF} =& \int_0^{+\infty}\int_0^1 \bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} \Big(1-\Phi_{ij}\Big(\Upsilon^{-1}_{ij1}(\alpha),\cdots,\Upsilon^{-1}_{ijg_{ij}}(\alpha),\Upsilon^{-1}_{ij(g_{ij}+1)}(1-\alpha),\cdots,\Upsilon^{-1}_{ijk_{ij}}(1-\alpha);\frac{\lambda t}{1-(1-\lambda)^{h_{ij}}}\Big) \\ &\vee \Phi^{\eta}_{ij}\Big(\Upsilon^{-1(\eta)}_{ij1}(\alpha),\cdots,\Upsilon^{-1(\eta)}_{ijg^{\eta}_{ij}}(\alpha),\Upsilon^{-1(\eta)}_{ij(g^{\eta}_{ij}+1)}(1-\alpha),\cdots,\Upsilon^{-1(\eta)}_{ijk^{\eta}_{ij}}(1-\alpha);t\Big)\Big)\mathrm{d}\alpha\mathrm{d}t, \end{aligned} \tag{13}$$

respectively, where $\Upsilon^{-1}_{ijg_{ij}}(\alpha)$ *is the inverse uncertainty distribution of uncertain variable* $a_{jg_{ij}}$, $\Upsilon^{-1(\eta)}_{ijg^{\eta}_{ij}}(\alpha)$ *is the inverse uncertainty distribution of uncertain variable* $a_{ijg^{\eta}_{ij}}$, k_{ij} *is the number of uncertain parameters contained in subsystem* A_{ij} *with* $1 \le g_{ij} \le k_{ij}$, k^{η}_{ij} *is the number of uncertain parameters contained in conversion switch of subsystem* A_{ij} *with* $1 \le g^{\eta}_{ij} \le k^{\eta}_{ij}$, $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m_i$.

Proof. Let $R^{var}_{ij\eta}(a^{\eta}_{ij1}, a^{\eta}_{ij2}, \cdots, a^{\eta}_{ijk^{\eta}_{ij}}; t)$ denote the uncertain reliability variable of conversion switch in subsystem A_{ij}, that is

$$R^{var}_{ij\eta}\Big(a^{\eta}_{ij1}, a^{\eta}_{ij2}, \cdots, a^{\eta}_{ijk^{\eta}_{ij}}; t\Big) = \mathcal{M}\left\{\xi^{\eta}_{ij} > t\right\} = 1 - \Phi^{\eta}_{ij}\Big(a^{\eta}_{ij1}, a^{\eta}_{ij2}, \cdots, a^{\eta}_{ijk^{\eta}_{ij}}; t\Big).$$

Then, the uncertain reliability variable of subsystem A_{ij} with continuous conversion switches is

$$\begin{aligned} R^{var}_{ij}\Big(\boldsymbol{a}_{ij1}, \boldsymbol{a}_{ij2}, \cdots, \boldsymbol{a}_{ijh_{ij}}, \boldsymbol{a}^{\eta}_{ij}; t\Big) &= \mathcal{M}\Big\{(\xi^{w}_{ij1} + \xi^{w}_{ij2} + \cdots + \xi^{w}_{ijh_{ij}}) \wedge \xi^{\eta}_{ij} > t\Big\} \\ &= \mathcal{M}\Big\{(\xi^{w}_{ij1} + \xi^{w}_{ij2} + \cdots + \xi^{w}_{ijh_{ij}} > t) \cap (\xi^{\eta}_{ij} > t)\Big\} \\ &= \mathcal{M}\Big\{(1-\lambda)^{h_{ij}-1}\xi_{ij1} + (1-\lambda)^{h_{ij}-2}\xi_{ij2} + \cdots + \xi_{ijh_{ij}} > t\Big\} \wedge \mathcal{M}\left\{\xi^{\eta}_{ij} > t\right\}. \end{aligned}$$

If $\xi_{ij1}, \xi_{ij2}, \cdots, \xi_{ijh_{ij}}$ have the same uncertainty distribution $\Phi_{ij}(a_{ij1}, \cdots, a_{ijk_{ij}}; t)$, the uncertain reliability variable of subsystem A_{ij} with continuous conversion switches is

$$R^{var}_{ij}\Big(\boldsymbol{a}_{ij1},\boldsymbol{a}_{ij2}, \cdots, \boldsymbol{a}_{ijh_{ij}}, \boldsymbol{a}^{\eta}_{ij}; t\Big) = 1 - \Phi_{ij}\Big(a_{ij1}, a_{ij2}, \cdots, a_{ijk_{ij}}; \frac{\lambda t}{1-(1-\lambda)^{h_{ij}}}\Big) \vee \Phi^{\eta}_{ij}\Big(a^{\eta}_{ij1}, a^{\eta}_{ij2}, \cdots, a^{\eta}_{ijk^{\eta}_{ij}}; t\Big),$$

where $\boldsymbol{a}_{ijv} = (a_{ij1}, a_{ij2}, \ldots, a_{ijk_{ij}})$, $v = 1, 2, \cdots, h_{ij}$; $\boldsymbol{a}^{\eta}_{ij} = (a^{\eta}_{ij1}, a^{\eta}_{ij2}, \cdots, a^{\eta}_{ijk^{\eta}_{ij}})$, $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m_i$. Hence, the uncertain reliability variable of the system is

$$\begin{aligned} R^{var}(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n; t) &= \bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} R^{var}_{ij}\Big(\boldsymbol{a}_{ij1}, \boldsymbol{a}_{ij2}, \cdots, \boldsymbol{a}_{ijh_{ij}}, \boldsymbol{a}^{\eta}_{ij}; t\Big) \\ &= \bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} \Big(1 - \Phi_{ij}\Big(a_{ij1}, a_{ij2}, \cdots, a_{ijk_{ij}}; \frac{\lambda t}{1-(1-\lambda)^{h_{ij}}}\Big) \vee \Phi^{\eta}_{ij}\Big(a^{\eta}_{ij1}, a^{\eta}_{ij2}, \cdots, a^{\eta}_{ijk^{\eta}_{ij}}; t\Big)\Big), \end{aligned}$$

where $\boldsymbol{a}_i = (a_{i11}, a_{i12}, \cdots, a_{i1k_{i1}}, a^{\eta}_{i11}, a^{\eta}_{i12}, \cdots, a^{\eta}_{i1k^{\eta}_{i1}}, \cdots, a_{ij1}, a_{ij2}, \cdots, a_{ijk_{ij}}, a^{\eta}_{ij1}, a^{\eta}_{ij2}, \cdots, a^{\eta}_{ijk^{\eta}_{ij}}, \cdots, a_{im_i1}, a_{im_i2}, \cdots, a_{im_ik_{im_i}}, a^{\eta}_{im_i1}, a^{\eta}_{im_i2}, \cdots, a^{\eta}_{im_ik^{\eta}_{im_i}})$, $i = 1, 2, \cdots, n$. According to Definition 3.3, the reliability function of the system with continuous imperfect switches can be determined by

$$\begin{aligned} R(t) &= E\Big[R^{var}\Big(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n; t\Big)\Big] \\ &= E\Big[\bigwedge_{1\le i\le n} \bigvee_{1\le j\le m_i} \Big(1 - \Phi_{ij}\Big(a_{ij1}, a_{ij2} \cdots, a_{ijk_{ij}}; \frac{\lambda t}{1-(1-\lambda)^{h_{ij}}}\Big) \vee \Phi^{\eta}_{ij}\Big(a^{\eta}_{ij1}, a^{\eta}_{ij2}, \cdots, a^{\eta}_{ijk^{\eta}_{ij}}; t\Big)\Big)\Big]. \end{aligned}$$

It is known that the uncertain reliability variable $R^{var}(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n; t)$ is strictly increasing with respect to $a_{ij1}, a_{ij2}, \cdots, a_{ijg_{ij}}, a^{\eta}_{ij1}, a^{\eta}_{ij2}, \cdots, a^{\eta}_{ijg^{\eta}_{ij}}$ and strictly decreasing with respect to $a_{ij(g_{ij}+1)}, a_{ij(g_{ij}+2)}, \cdots, a_{ijk_{ij}}, a^{\eta}_{ij(g^{\eta}_{ij}+1)}, a^{\eta}_{ij(g^{\eta}_{ij}+2)}, \cdots, a^{\eta}_{ijk^{\eta}_{ij}}$. According to Theorem 2.5, the inverse uncertainty distribution $\Psi^{-1}(\alpha)$ of the uncertain reliability variable $R^{var}(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n; t)$ is

$$\begin{aligned}\Psi^{-1}(\alpha) = \bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}\Big(1-\Phi_{ij}\Big(\Upsilon^{-1}_{ij1}(\alpha),\cdots,\Upsilon^{-1}_{ijg_{ij}}(\alpha),\Upsilon^{-1}_{ij(g_{ij}+1)}(1-\alpha),\cdots,\Upsilon^{-1}_{ijk_{ij}}(1-\alpha);\frac{\lambda t}{1-(1-\lambda)^{h_{ij}}}\Big)\\ \vee\,\Phi^{\eta}_{ij}\Big(\Upsilon^{-1(\eta)}_{ij1}(\alpha),\cdots,\Upsilon^{-1(\eta)}_{ijg^{\eta}_{ij}}(\alpha),\Upsilon^{-1(\eta)}_{ij(g^{\eta}_{ij}+1)}(1-\alpha),\cdots,\Upsilon^{-1(\eta)}_{ijk^{\eta}_{ij}}(1-\alpha);t\Big)\Big).\end{aligned}$$

According to Theorem 2.9, the reliability function of the system is

$$R(t) = E\left[R^{var}(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n; t)\right] = \int_0^1 \Psi^{-1}(\alpha)\mathrm{d}\alpha.$$

So, Equation (12) is obtained. And the MTTF of the system is formulated owing to Definition 3.4, that is

$$\mathrm{MTTF} = \int_0^{+\infty} R(t)\mathrm{d}t = \int_0^{+\infty}\int_0^1 \Psi^{-1}(\alpha)\mathrm{d}\alpha\mathrm{d}t.$$

Then, Equation (13) can be obtained. □

When $\lambda = 0$, the warm standby system degenerates to the cold standby system. The special case of deterioration rate λ is described in Remark 4.6.

Remark 4.6. *The reliability function and MTTF of the cold standby series-parallel system with continuous imperfect switches are*

$$\begin{aligned}R(t) = \int_0^1 \bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}\Big(1-\Phi_{ij}\Big(\Upsilon^{-1}_{ij1}(\alpha),\cdots,\Upsilon^{-1}_{ijg_{ij}}(\alpha),\Upsilon^{-1}_{ij(g_{ij}+1)}(1-\alpha),\cdots,\Upsilon^{-1}_{ijk_{ij}}(1-\alpha);\frac{t}{h_{ij}}\Big)\\ \vee\,\Phi^{\eta}_{ij}\Big(\Upsilon^{-1(\eta)}_{ij1}(\alpha),\cdots,\Upsilon^{-1(\eta)}_{ijg^{\eta}_{ij}}(\alpha),\Upsilon^{-1(\eta)}_{ij(g^{\eta}_{ij}+1)}(1-\alpha),\cdots,\Upsilon^{-1(\eta)}_{ijk^{\eta}_{ij}}(1-\alpha);t\Big)\Big)\mathrm{d}\alpha,\end{aligned} \tag{14}$$

and

$$\begin{aligned}\mathrm{MTTF} = \int_0^{+\infty}\int_0^1 \bigwedge_{1\le i\le n}\bigvee_{1\le j\le m_i}\Big(1-\Phi_{ij}\Big(\Upsilon^{-1}_{ij1}(\alpha),\cdots,\Upsilon^{-1}_{ijg_{ij}}(\alpha),\Upsilon^{-1}_{ij(g_{ij}+1)}(1-\alpha),\cdots,\Upsilon^{-1}_{ijk_{ij}}(1-\alpha);\frac{t}{h_{ij}}\Big)\\ \vee\,\Phi^{\eta}_{ij}\Big(\Upsilon^{-1(\eta)}_{ij1}(\alpha),\cdots,\Upsilon^{-1(\eta)}_{ijg^{\eta}_{ij}}(\alpha),\Upsilon^{-1(\eta)}_{ij(g^{\eta}_{ij}+1)}(1-\alpha),\cdots,\Upsilon^{-1(\eta)}_{ijk^{\eta}_{ij}}(1-\alpha);t\Big)\Big)\mathrm{d}\alpha\mathrm{d}t,\end{aligned} \tag{15}$$

respectively.

5 Numerical example

In this section, numerical examples of different indicators are given to illustrate the application of the developed models. The reliability function and MTTF of systems with perfect, discrete or continuous conversion switch are analyzed when the lifetime is considered as uncertain and bi-uncertain variable respectively.

5.1 Numerical analysis

Consider a warm standby series-parallel system consisting of subsystems $A_{ij}(i = 1, 2, 3, j = 1, \cdots, m_i,\ m_1 = m_2 = m_3 = 3)$, each subsystem consists of one original element and two warm standby elements, as show in Figure 2. Let $\lambda = 0.18$ be the deterioration rate of element in the warm standby system. The lifetime distributions of elements with uncertain parameters are given in Table 1, and the lifetime distributions of conversion switches are given in Table 2. In order to illustrate the numerical results clearly, we initially make the necessary explanations for systems description:

1) $\lambda = 0$ means the warm standby system is degenerated to the cold standby system.

2) The system is called the original system, if each subsystem of the system consists of one original element and none standby elements.

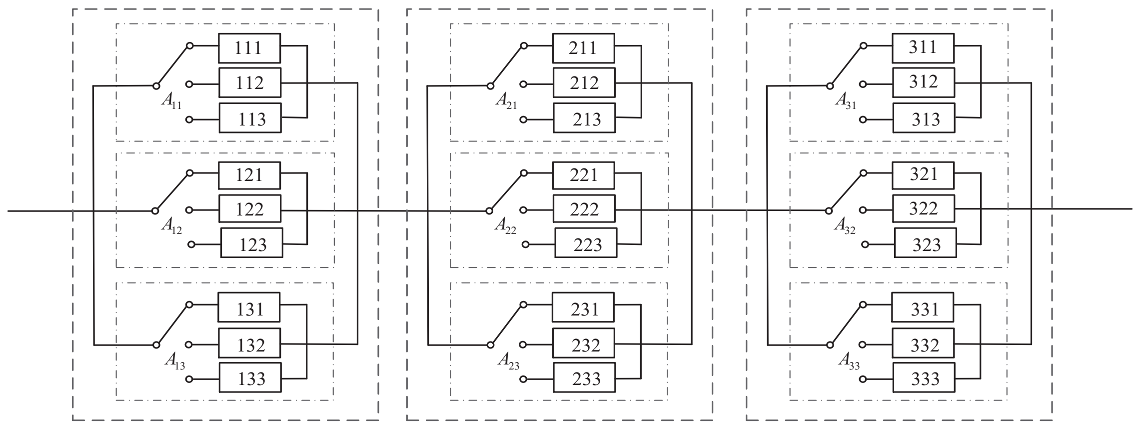


Figure 2: A warm standby series-parallel system with nine subsystems

3) A system with bi-uncertain variables means whose lifetime distribution has uncertain parameters, while a system with uncertain variables means whose lifetime distribution has constant parameters.

4) $\mathcal{L}(a,b)$ means the linear uncertain variable. $\mathcal{LOGN}(e,\sigma)$ means the lognormal uncertain variable.

The main calculate steps of the proposed numerical example are as follows:

Step 1. Calculate lifetimes of subsystem A_{ij} and system with prefect conversion switches.

$$\xi_{ij} = 0.6724\xi_{ij1} + 0.82\xi_{ij2} + \xi_{ij3}.$$

$$\xi = \bigwedge_{1\leqslant i\leqslant 3} \bigvee_{1\leqslant j\leqslant 3} (0.6724\xi_{ij1} + 0.82\xi_{ij2} + \xi_{ij3}).$$

Step 2. Calculate the uncertain reliability variables of subsystem A_{ij} and system with prefect conversion switches.

$$R_{ij}^{var}(e_{ij},\sigma_{ij};t) = 1 - (1 + \exp(\frac{\pi(e_{ij} - \ln 0.4012 - \ln t)}{\sqrt{3}\sigma_{ij}}))^{-1}.$$

$$R^{var}(e_{ij};t) = \bigwedge_{1\leqslant i\leqslant 3} \bigvee_{1\leqslant j\leqslant 3} (1 - (1 + \exp(\frac{\pi(e_{ij} - \ln 0.4012 - \ln t)}{\sqrt{3}\sigma_{ij}}))^{-1}).$$

Step 3. Calculate the inverse uncertainty distribution of the uncertain reliability variable of the system.

$$\Psi^{-1}(\alpha) = \bigwedge_{1\leqslant i\leqslant 3} \bigvee_{1\leqslant j\leqslant 3} (1 - (1 + \exp(\frac{\pi((1-\alpha)a_{ij} + \alpha b_{ij} - \ln 0.4012 - \ln t)}{\sqrt{3}\sigma_{ij}}))^{-1}).$$

Step 4. Calculate the reliability function and MTTF of the system with perfect conversion switches.

$$R(t) = \int_0^1 \bigwedge_{1\leqslant i\leqslant 3} \bigvee_{1\leqslant j\leqslant 3} (1 - (1 + \exp(\frac{\pi((1-\alpha)a_{ij} + \alpha b_{ij} - \ln 0.4012 - \ln t)}{\sqrt{3}\sigma_{ij}}))^{-1})\mathrm{d}\alpha.$$

$$\mathrm{MTTF} = \int_0^{+\infty} \int_0^1 \bigwedge_{1\leqslant i\leqslant 3} \bigvee_{1\leqslant j\leqslant 3} (1 - (1 + \exp(\frac{\pi((1-\alpha)a_{ij} + \alpha b_{ij} - \ln 0.4012 - \ln t)}{\sqrt{3}\sigma_{ij}}))^{-1})\mathrm{d}\alpha \mathrm{d}t.$$

Step 5. Substitute $a_{ij}, b_{ij}, \sigma_{ij}$ into constant values according to the Table 1. Then, the reliability and MTTF of the system with perfect conversion switches are calculated by MATLAB.

Step 6. Select the expected value of $e_{ij} \sim \mathcal{L}(a,b)$ as the constant parameter of each element lifetime distribution, the reliability and MTTF of warm standby series-parallel system with perfect conversion switches and uncertain lifetimes are obtained.

Step 7. Similarly, the system reliability and MTTF of the system with discrete imperfect switches and continuous imperfect switches are obtained according to the theorems in this article.

Table 1: The lifetime distributions of elements with uncertain parameters

Lifetime distribution			Uncertain parameter
$\xi_{11v} \sim \mathcal{LOGN}(e_{11}, 0.6)$	$\xi_{21v} \sim \mathcal{LOGN}(e_{21}, 0.9)$	$\xi_{31v} \sim \mathcal{LOGN}(e_{31}, 0.5)$	$e_{1j}(j = 1, 2, 3) \sim \mathcal{L}(1.8, 4.2)$
$\xi_{12v} \sim \mathcal{LOGN}(e_{12}, 0.7)$	$\xi_{22v} \sim \mathcal{LOGN}(e_{22}, 0.6)$	$\xi_{32v} \sim \mathcal{LOGN}(e_{32}, 0.7)$	$e_{2j}(j = 1, 2, 3) \sim \mathcal{L}(1.5, 3.8)$
$\xi_{13v} \sim \mathcal{LOGN}(e_{13}, 0.9)$	$\xi_{23v} \sim \mathcal{LOGN}(e_{23}, 0.5)$	$\xi_{33v} \sim \mathcal{LOGN}(e_{33}, 0.6)$	$e_{3j}(j = 1, 2, 3) \sim \mathcal{L}(1.2, 3.4)$

Table 2: The lifetime distributions of conversion switches

Switch	Subsystem A_{11}, A_{12}, A_{13}	Subsystem A_{21}, A_{22}, A_{23}	Subsystem A_{31}, A_{32}, A_{33}
0-1 switch lifetime	$b_{1j}(j = 1, 2, 3) = 0.998$	$b_{2j}(j = 1, 2, 3) = 0.9985$	$b_{3j}(j = 1, 2, 3) = 0.999$
Continuous switch lifetime	$\xi^{\eta}_{ij}(i = 1, 2, 3, j = 1, 2, 3) \sim \mathcal{LOGN}(e_\eta + \ln 4, 0.8)$, where $e_\eta \sim \mathcal{L}(1.25, 2.65)$		

Table 3: Results of reliability functions in the warm standby redundant systems with imperfect switches

Time t	3	5	7	9	11	13	15	17	19
Discrete imperfect switch	0.9980	0.9813	0.9455	0.8927	0.8326	0.7727	0.7163	0.6644	0.6171
Continuous imperfect switch	0.9909	0.9720	0.9429	0.9058	0.8630	0.8171	0.7701	0.7235	0.6782

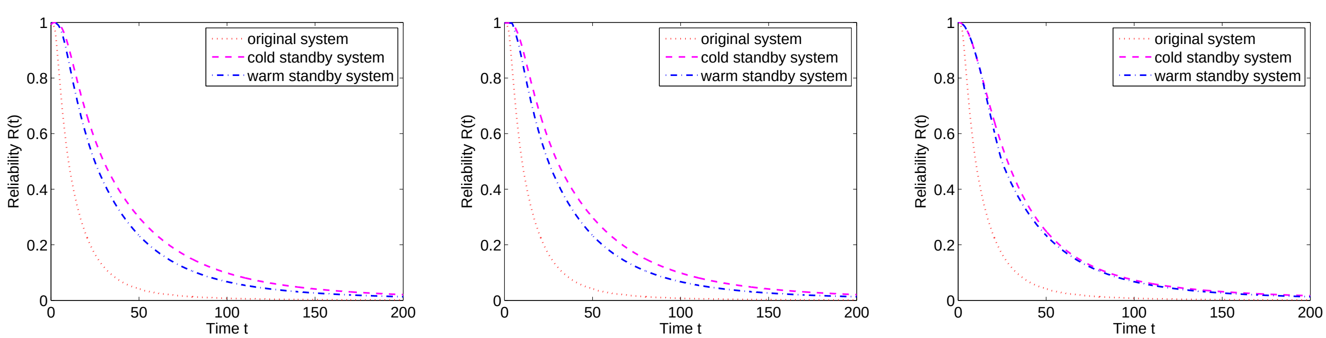
Table 4: Results of reliability functions in the cold standby redundant systems with imperfect switches

Time t	2	6	10	14	18	22	26	30	34
Discrete imperfect switch	0.9980	0.9815	0.9123	0.8134	0.7175	0.6337	0.5615	0.4990	0.4484
Continuous imperfect switch	0.9963	0.9586	0.8849	0.7936	0.7006	0.6140	0.5368	0.4694	0.4111

5.2 Comparative analysis

In this subsection, we provide a comprehensive comparison of the reliability function and MTTF of system with different conversion switches. In order to illustrate the comparison between the different parameters and the corresponding system reliability indexes, we analyze the system reliability when the lifetime is considered as bi-uncertain variable and uncertain variable, respectively.

For warm and cold standby series-parallel systems with discrete imperfect switches and continuous imperfect switches, the reliability can be calculated by Equations (8), (10), (12) and (14), as shown in Tables 3 and 4. From Tables 3 and 4, we can see that the reliability of the warm and cold standby series-parallel systems are indeed affected by the imperfect switches. In addition, The reliability of the warm and cold standby series-parallel systems with perfect switches can be obtained by Equations (4) and (6). The reliability of the original system is also obtained according to the theorems proved by Liu et al. [28]. The reliability functions of the original system, cold standby system and warm standby system under different switches are shown in Figure 3.

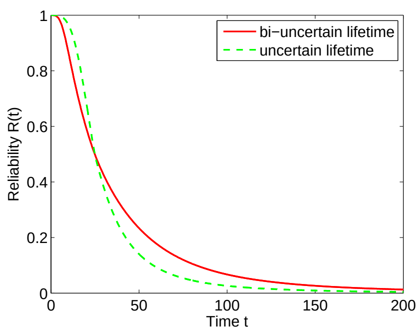


(a) Perfect switch (b) Discrete imperfect switch (c) Continuous imperfect switch

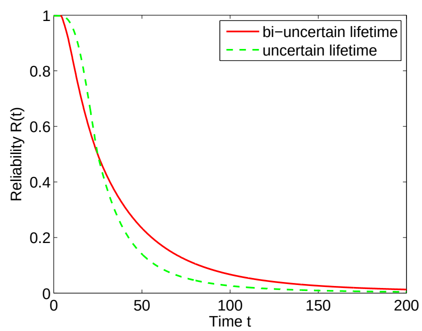
Figure 3: Reliability function of the system with different conversion switches

For the same warm standby series-parallel system with uncertain lifetimes, we select the expected value of $e_{ij} \sim$

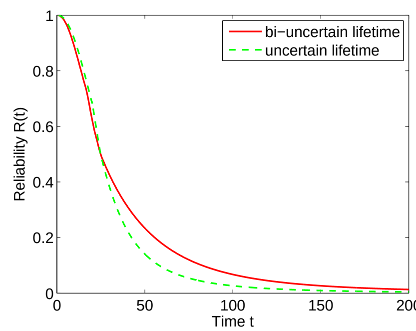
$\mathcal{L}(a, b)(i = 1, 2, 3, j = 1, 2, 3)$ as the constant parameter of each element lifetime distribution. According to the theorems derived by Liu et al. [29], the reliability of warm standby series-parallel system with uncertain lifetimes is obtained. The comparison of reliability between warm standby system with uncertain lifetimes and that with bi-uncertain lifetimes is depicted in Figure 4. From Figure 4, we can see that system reliability is sensitive to different parameters (uncertain parameters and constant parameters).



(a) Perfect switch



(b) Discrete imperfect switch



(c) Continuous imperfect switch

Figure 4: Reliability function of the warm standby system

According to Equations (5), (9) and (13) shown in this paper and the theorems derived by Liu et al. [29], the MTTF of the system with different switches is given when the lifetimes of elements are considered as bi-uncertain and uncertain variables respectively, as show in Table 5. The results in Table 5 show that the MTTF of the warm standby system with bi-uncertain lifetimes is longer than that of the system with uncertain lifetimes. For each type of switch, the MTTF of the warm standby series-parallel system is longer than that of the cold standby series-parallel system.

Table 5: MTTF of the system

System	Original	Warm standby			Cold standby		
		Perfect	Discrete	Continuous	Perfect	Discrete	Continuous
MTTF with bi-uncertain lifetime	15.5781	37.7943	37.8031	37.2913	42.8189	44.8183	39.2388
MTTF with uncertain lifetime	13.2452	32.5796	32.5726	30.9163	38.9323	38.9739	36.7961

6 Conclusions

In this paper, a warm standby series-parallel system is proposed to extend a generalization for system reliability analysis by using uncertainty theory. The lifetimes of system elements are assumed as bi-uncertain variables. Three conversion switch models of the warm standby series-parallel system are established, which are absolutely reliable mode, discrete mode and continuous mode. In addition, the expressions of reliability function and the mean time to failure for these system models are presented. Numerical example is presented to show the developed system models when the lifetimes of elements are considered as uncertain variables and bi-uncertain variables respectively.

In the warm standby series-parallel system with different switches, the lifetimes of system elements are assumed to be bi-uncertain variables. However, in some cases, human uncertainty and objective randomness may coexist in a complex system. In future research, based on the chance theory, we will devote to the warm standby series-parallel system with different switches, whose element lifetimes are uncertain random variables.

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Reliability analysis of a warm standby series-parallel system with different switches and bi-uncertain lifetimes

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تجزیه و تحلیل قابلیت اطمینان یک سیستم سری‌های – موازی آماده به کار گرم با سوئیچ‌های مختلف و طول عمر دو- احتمالی

چکیده. قابلیت اطمینان یک سیستم سری‌های – موازی آماده به کار گرم بدون نمونه‌های کافی بر اساس نظریه عدم قطعیت مطالعه شده‌است. فرض بر این است که طول عمر عناصر سیستم، از توزیع عدم قطعیت مستقل با پارامترهای نامشخص پیروی می‌کند. سه مدل سوئیچ مختلف، از جمله حالت کاملاً قابل اعتماد، حالت گسسته و حالت مداوم، برای سیستم سری‌های موازی آماده به کار گرم توسعه داده شده‌اند. علاوه براین، سیستم سری‌های موازی آماده به کار سرد به عنوان یک مورد خاص مورد بحث قرار گرفته‌است. عملکرد قابل اطمینان و میانگین زمان شکست هر مدل توسعه یافته تجزیه و تحلیل می‌شود. یک مثال عددی برای سیستم با سوئیچ‌های مختلف برای نشان دادن کاربرد و کارایی مدل‌های پیشنهادی، ارائه شده‌است.