Iranian Journal of Fuzzy Systems
Volume 18, Number 4, (2021), pp. 95-112
Original paper

University of Sistan and Baluchestan

An interactive fuzzy programming approach for a new multi-objective multi-product oil pipeline scheduling problem

F. Khalili Goudarzi[1], H. R. Maleki[2] and S. Niroomand[3]

[1,2]Department of Mathematics, Faculty of Applied Mathematics, Shiraz University of Technology, Shiraz, Iran
[3]Department of Industrial Engineering, Firouzabad Institute of Higher Education, Firouzabad, Iran

f.khalili@sutech.ac.ir, maleki@sutech.ac.ir, sadegh.niroomand@yahoo.com

Abstract

In this paper, a new fuzzy multi-objective multi-product pipeline scheduling problem is introduced. The system consists of a single refinery, a unique distribution center, and a multi-product pipeline. Restrictions such as batch sizing, discharging rate, forbidden sequences, batch volumetric, etc. are considered. Due to the uncertain nature of real-world problems, some parameters of the system are considered as fuzzy values. Tardiness and earliness penalties are considered with a time dependent non-linear function. The basic aim of this scheduling problem is to achieve the optimal sequence for pumping batches of oil products to maximize the financial benefit and simultaneously satisfies the customers with on-time delivery as a multi-objective problem. To tackle this problem, a two-stage methodology is proposed. In the first stage, the fuzzy formulation is converted to its equivalent crisp form by a credibility-based chance-constrained programming approach. In the second stage, the multi-objective crisp formulation is solved by some well-known approaches in the literature. Some test problems are generated and solved by the proposed approaches and the obtained Pareto-optimal solutions are analyzed and compared using some distance-based comparison metrics.

Keywords: Credibility-based chance-constrained modeling, fuzzy number, multi-objective optimization, multi-product pipeline scheduling.

1 Introduction

Pipeline as the best way to transport petroleum products plays a major role in the oil industry. This role is more intensive when large volumes of products are shipped to far distances over short periods. It is the most economical and convenient method of transportation for petroleum products on both land and sea. A multi-product pipeline is a typical pipeline that transports different refined petroleum products from refineries to distribution centers and depots. In this type of pipeline, several types of oil products such as gasoline, kerosene, liquid gas, etc. are transported. These products are pumped into the pipeline consecutively without any physical separator. The drawback of consecutive pumping is the creation of an interference area known as interface between two consecutive products in the pumping sequence. The interface occurs usually at the beginning or end of a product shipment because products are shipped in sequence to keep the pipeline full and flowing. These volumes must be split into the original products, which will be relatively costly.

Pipeline scheduling problems have received a growing cause of debate in the past decades due to its importance and numerous studies have focused on solving these kinds of problems. Linear programming models are of the basic mathematical models used to formulate the pipeline scheduling problem and have a brilliant background in the oil industry in most developed countries. Some of the basic key-decisions in this field consists of some questions such as, when and which kind of products should be transferred; how to manage the operating costs associated with these transfers; how much liquid should be transferred in each pumping and how to satisfy all demands.

Corresponding Author: H. R. Maleki
Received: April 2020; Revised: October 2020; Accepted: February 2021.

Many researchers have made significant efforts in the use of mathematical models for transferring oil products. They mainly formulated the problem as a mixed integer linear programming model and attempted to solve them by exact or heuristic algorithms. The researchers of this field mainly have investigated both kinds of pipeline systems such as straight and branch pipeline systems. Straight pipeline as the basic system plays a crucial role in transferring petroleum products and is divided into three groups. The first group consists of a straight system connecting a single refinery to a single distribution center. Many studies are focused on this kind of system (Relvas et al. [46] applied a reactive framework; Cafaro and Cerda [10] considered daily customer product demands; MirHassani and BeheshtiAsl [26] determined batch sequencing and monitoring batch location in pipeline and inventory at a distribution center (DC); Relvas et al. [45] presented two mathematical models to attain a set of planning objectives such as fulfilling costumers demands while minimizing the medium flow rate; Moradi and MirHassani [33] considered continuous and discrete flow rate and high peak electricity periods; Zhang et al. [56] applied ant colony algorithm and simplex method as solution approaches). The second group is single-refinery to multiple DCs pipeline which can be seen in many studies in this field (Rejowski and Pinto [44] presented a discrete-time programming model; Cafaro and Cerdá [9] presented a non-discrete mixed integer linear formulation; Cafaro and Cerdá [11] considered continuous-time programming model to reduce pumping and transmix reprocessing costs; MirHassani et al. [29] considered daily demands, forcing settling periods of quality control, predetermined shutdown periods; Moradi et al. [35] used an efficient decomposition-based algorithm for long-term pipeline scheduling problem). The tree pipeline structure as the last group has a more complicated system than other groups and it can be seen in the works of some researchers (MirHassani and Ghorbanalizadeh [28] considered discrete-time and a division of the pipeline by equal size segments; MirHassani and Fani Jahromi [27] proposed an algorithm for long-term scheduling problem; Mostafaei et al. [37] considered simultaneous deliveries from a unique refinery to multiple downstream terminals; Castro and Mostafaei [14] proposed a new monolithic formulation coping with a weekly demand plan; Liao et al. [22] used a heuristic method for detailed scheduling branched multi-product pipeline networks; Losenkov et al. [25] investigated a new approach to oil-flow scheduling for a certain period). Non-Accessing periods of pipelines (predetermined intervals for repairing, maintenance, line inspections, and so on), tracing batches constrains, settling period, customers demands limitations are considered in the work of Cafaro et al. [10]. Cafaro et al. [12] considered pipeline networks with simultaneous injections and consequently led to better utilization of transportation capacities and reduced total time to meet the demands of depots. The reversible flow was considered in Cafaro and Cerdá [9]. Castro and Mostafaei [13] presented a product-centric continuous-time formulation.

However certain parameters are taken to consideration in many studies, but usually, real-world situations are faced by imprecision rather than exactness (Nasseri et al. [39]). Data uncertainty is a prominent aspect of many real-world problems and ignoring uncertain data may lead to unacceptable solutions. Many researchers have considered this issue in their researches. A review of fuzzy mathematical programming models according to fuzzy components was investigated by Baykasoğlu and Gocken [5]. They introduced fifteen types of these models and the existing solution methodologies were also reviewed in their paper. Goudarzi et al. [18] proposed a new fuzzy approach to solve a new open shop scheduling problem by considering setup times, sequence-dependent removal times, and inaccessible times for machines. Baykasoglu and Subulan [6] presented a detailed review and analysis of the solution methodologies for solving fully fuzzy linear programming problems. They proposed a novel parametric method that incorporated the decision-maker(s)s attitude toward risk. They also developed a fuzzy-stochastic optimization model for an intermodal fleet management system of a large international transportation company. They applied a hybrid chance-constrained programming and fuzzy interactive resolution based approach in their work (Baykasolu and Subulan [7]). Mozaffari et al. [38] proposed a bi-level hybrid intelligent tool for the identification and optimization of PHEV fuel economy. To cope with the uncertainty associated with the collected database, they adopted a fuzzy polynomial system which leads to model the fuel consumption rate, by using the values of motor and engine powers. Salehi et al. [47] proposed an assembly line balancing problem with workers skills. This problem was formulated in an uncertain environment with fuzzy parameters. Shen and Zhu [49] proposed a parallel-machine scheduling problem with periodic maintenance under an uncertain environment. In this study, processing and maintenance times are considered as uncertain parameters. Rahbari et al. [43] investigated the perishability of products in vehicle routing and scheduling in combination with the cross-docking problems under uncertainty in the travel times of the outbound vehicles and freshness-life products. He et al. [19] introduced a yard crane scheduling problem with the uncertainty of the task groups arriving times and handling volumes. More applications of these issues can be seen in ([3],[16],[17],[36]).

In the pipeline scheduling problem, the values of some parameters may be unknown or uncertain. These parameters can be due dates of delivery, customer demands, flow rate, interface volumes, shutdown periods, etc. and a small change in these data may lead to infeasible solutions. Although data uncertainty is an unavoidable aspect of many real-world problems especially in many real scheduling problems, this issue has been rarely addressed in the literature of pipeline scheduling (only a few exceptions). Moradi and MirHassani [34] provided a robust scheduling formulation for the multi-product pipeline problem under uncertainty. Delivery time is uncertain in this work. They presented a robust planning approach and showed that a robust optimization approach can be used when product demands are uncertain. In another study, BeheshtiAsl and MirHassani [8] provided a new scenario-based MILP two-stage stochastic model for scheduling and managing pipeline with flow rate uncertainty.

Unlike other studies of this field, many aspects of the problem such as the uncertainty of some parameters like interfaces, demands, due dates, the costs of tardiness, earliness, and flow rate are simultaneously considered. For the first time in the pipeline scheduling problems, tardiness and earliness penalty functions are determined which made the problem more realistic. Although multi-objective optimization is an effective and complete search procedure in order to let decision makers carry out the best choice and investigate many aspects of a problem, it has been rarely applied in pipeline scheduling problem by decision makers who are the planning managers of the oil industry in this study. Optimization of the operational costs and on-time deliveries are two significant issues that have always faced the energy and oil industries. This has a significant impact on the profitability of oil companies, the oil industry, and consequently the governments. To cope with the uncertainty nature and achieve the above cases, an interactive fuzzy programming approach is applied for a new multi-objective multi-product oil pipeline scheduling problem. The main contributions of this research are as follows,

1) Proposing a new multi-objective MILP formulation for multi-product pipeline scheduling problems.

2) Optimizing two objective functions such as minimizing all operational costs and simultaneously minimizing the tardiness and earliness penalties of each product under fuzzy type uncertainty of some parameters like due date and demand for each product, interface cost between two products, and flow rate cost.

3) For the first time, applying a credibility-based chance-constrained programming approach to convert the fuzzy formulation into a crisp form in pipeline scheduling problem.

4) For the first time, employing interactive approaches to interact with planning managers during the optimization process.

5) Considering some parameters as a fuzzy number due to their uncertain nature for the first time.

6) Finding the best approach among other approaches in the literature by comparing them.

The rest of this article is organized as follows: problem assumptions and its fuzzy formulation are described in Section 2. A two-phase solution approach is proposed in Section 3 to solve the proposed fuzzy formulation. Computational experiments are provided in Section 4. Finally, conclusions and future studies are given in Section 5.

2 Problem description and formulation

This paper addresses the scheduling problem of a straight multi-product pipeline with a single refinery and a unique distribution center (DC). Various refined petroleum products such as gas oil, gasoline, kerosene, etc. are pumped into the pipeline consecutively and without any physical separation. These products are transported as batches through the pipeline and after a long-distance are discharged at the DC. This process leads to contamination areas at the interface portions of the successive products which are known as the interfaces. Figure 1 depicts a schematic overview of this system showing the interfaces.

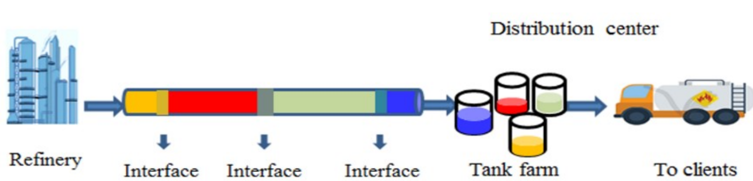

Figure 1: Schematic overview of a multi-product pipeline.

The main aim of planning this problem is to obtain the optimal sequence of the products for pumping them to achieve some financial benefits and satisfying the customers with on-time delivery while observing many operational restrictions. Delay in any system, not only effects clients but also declines the reliability of companies or governments. On the other hand, earliness may cause maintenance, warehousing, deteriorate costs, etc. In general, in the pipeline system, proper and accurate scheduling can greatly reduce the contamination of products, reduce its associated costs as well as reduce lateness and earliness damages, and thereby can increase credit. In this study, some restrictions such as the size of the batches, discharging rate, forbidden sequence of the products, batch volumetric, demand satisfaction, delay, and early delivery restrictions, etc. are considered. The following assumptions are met in this system,

• The pipeline is always full and the only way to discharge a volume of a product is to pump the same quantity into the pipeline from the refinery.
• Only one batch can be discharged at any time.
• At the beginning of the planning time horizon, the pipeline is filled with products.
• The interface cost between any pair of successive batches is a fuzzy type uncertain parameter.
• A set of pumping flow rates is considered and only one of the flow rates can be used for each batch.
• Forbidden sequences of some products are considered.
• Demand for each product is a fuzzy type uncertain parameter and must be satisfied at the end of the planning horizon.
• The volume of each batch is not known but it is limited to a predetermined interval for each product type.
• Due dates are not considered as exact parameters for each product.
• Late and early delivery penalties depending on the type of product are considered.
• The capacity for production in the refinery is unlimited.
• The discharging flow rate cost for each product is not uncertain and depends on the value of discharge flow rates.
• The penalty cost of tardiness or earliness for a product is a fuzzy type uncertain parameter. Under the above-mentioned conditions and assumptions, the notations of Tables 1-3 including indices, parameters, and decision variables are considered to formulate the problem.

Before formulating the problem the penalty cost function of a product (say product p) is explained here. As mentioned earlier, either tardiness or earliness of a product has penalty costs. In order to determine tardiness and earliness penalty functions, instead of the unique due date of dd_p, four due dates of $dd_{1,p}$,$dd_{2,p}$,$dd_{3,p}$,$dd_{4,p}$ are considered for product p as a fuzzy due date where $dd_{1,p} \leq dd_{2,p} \leq dd_{3,p} \leq dd \leq 4, p$. Then the normalized tardiness value and earliness values of product p based on its completion time are obtained by equations (1) and (2) where their functions are depicted by Figure 2. This type of due date has been considered by Ishii et al. [20] as fuzzy due dates to scheduling problems, fuzzy due date scheduling problems have been investigated by many researchers (see Molla-Alizadeh-Zavardehi et al. [31, 32]).

$$\mu_{td,p}(C_p) = \begin{cases} 1, & C_p > dd_{4,p} \\ \frac{C_p - dd_{3,p}}{dd_{4,p} - dd_{3,p}}, & dd_{3,p} \leq C_p \leq dd_{4,p} \\ 0, & C_p < dd_{3,p} \end{cases} \tag{1}$$

$$\mu_{e,p}(C_p) = \begin{cases} 1, & C_p < dd_{1,p} \\ \frac{dd_{2,p} - C_p}{dd_{2,p} - dd_{1,p}}, & dd_{1,p} \leq C_p \leq dd_{2,p} \\ 0, & C_p > dd_{2,p} \end{cases} \tag{2}$$

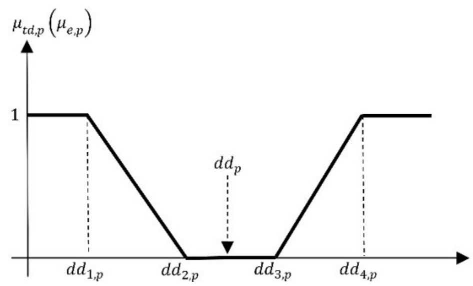

Figure 2: The penalty function of product p proposed for the problem.

Table 1: The sets used for formulating the problem

Sets	Index	Description
I	i	Set of batches
K	k	Set of flow rate options
P	p, q	Set of products

Table 2: The parameters used for the formulation the problem

Parameter	Description
initial	Initial volume in the pipeline from the previous planning horizon
$seq_{p,q}$	Forbidden sequence
$maxlot_p$	The maximum volume of batches containing product p
$minlot_p$	The minimum size of the batches containing product p
vb_k	k^{th} rate option
dd_p	Due date of product p
$\widetilde{dem}_p$	Fuzzy demand for product p
$\widetilde{cin}_{p,q}$	Fuzzy interface cost between products p and q
$\widetilde{cost}_k$	Fuzzy cost of k^{th} flow rate
ρ	Staff salaries per unit of time
$\widetilde{A}_p$	Fuzzy tardiness penalty
$\widetilde{B}_p$	Fuzzy earliness penalty
M	A large number at least as big as the planning horizon

Conceptually, for the completion time of C_p, either the earliness penalty of $\widetilde{B}_p\mu_{e,p}$ or the tardiness penalty of $\widetilde{A}_p\mu_{td,p}$ is considered. Based on the notations introduced by Tables 1 to 3, and the penalty functions (1) and (2), the multi-objective mixed integer formulation for the proposed pipeline scheduling problem can be written as below.

$$minz_1 = \sum_{i\in I}\sum_{k\in k}\frac{\widetilde{cost}_k Qu_{i,k}}{vb_k} + \sum_{i\in I} W_i + \rho C_{max} \tag{3}$$

$$minz_2 = \sum_{p\in P}\tilde{A}_p\mu_{td,p} + \sum_{p\in P}\tilde{B}_p\mu_{e,p} \tag{4}$$

$s.t.$

$$\sum_{p\in P} y_{i,p} \leqslant 1 \qquad \forall i\in I \tag{5}$$

$$\sum_{p\in P} y_{i,p} \leqslant \sum_{p\in P} y_{i-1,p} \qquad \forall i\in I,\quad i>1 \tag{6}$$

$$y_{i-1,p} + y_{i,q} \leqslant 1 + seq_{p,q} \qquad \forall p,q\in P,\quad \forall i>1 \tag{7}$$

$$minlot_p y_{i,p} \leqslant Qsize_{i,p} \leqslant maxlot_p y_{i,p} \qquad \forall p\in P,\quad \forall i>1 \tag{8}$$

$$\sum_{p\in P} Qsize_{i,p} - M(1-fr_{i,k}) \leqslant Qu_{i,k} \leqslant Mfr_{i,k} \qquad \forall i\in I,\quad \forall k\in K \tag{9}$$

$$\sum_{p\in P} Qsize_{i,p} = \sum_{k\in K} Qu_{i,k} \qquad \forall i\in I \tag{10}$$

$$\sum_{k\in K} fr_{i,k} = \sum_{p\in P} y_{i,p} \qquad \forall i\in I \tag{11}$$

$$Tdis_{i,p} = \sum_{k\in K}\frac{Qu_{i,k}}{vb_k} \qquad i=1,\ p=1 \tag{12}$$

$$\frac{initial}{max_k\{vb_k\}} y_{i,p} \leqslant Tdis_{i,p} \leqslant My_{i,p} \qquad \forall p\in P,\quad \forall i\in I \tag{13}$$

$$T_i \geqslant T_{i-1,p} + \sum_{k\in K}\frac{Qu_{i,k}}{vb_k} \qquad \forall i\in I \tag{14}$$

$$T_i = \sum_{p\in P} Tdis_{i,p} \qquad \forall i\in I \tag{15}$$

$$C_p \geqslant Tdis_{i,p} \qquad \forall p\in P,\quad \forall i\in I \tag{16}$$

$$W_i \geqslant \widetilde{cin}_{p,q}(y_{i-1,p} + y_{i,q} - 1) \qquad \forall i\in I,\quad i>1 \tag{17}$$

$$\sum_{i\in I} Qsize_{i,p} \geqslant \widetilde{dem}_p \qquad \forall p \in P \qquad (18)$$

$$y_{i,p}, fr_{i,k} \in \{0,1\} \qquad \forall i \in I,\ \forall p \in P \qquad (19)$$

$$Qsize_{i,p}, Qu_{i,k}, Tdis_{i,p}, T_i, W_i \geqslant 0 \qquad \forall i \in I,\ \forall p \in P\ \forall k \in K \qquad (20)$$

Table 3: The variables used for the formulation of the problem

Variables	Description
$y_{i,p}$	Binary variable, equal to 1 if batch i contains product p; else 0
$Qsize_{i,p}$	The volume of batch i containing product p
$Qu_{i,k}$	Size of batch i discharged by flow rate vb_k
$fr_{i,k}$	Binary variable, equal to 1 if batch i is discharged by the rate vb_k; else 0
$Tdis_{i,p}$	Discharging time of batch i containing product p
T_i	Discharging time of batch i
C_p	Completion time of product p
W_i	Interface cost between batches i and i+1
Td_p	Tardiness of product p
E_p	Earliness of product p
$\mu_{td,p}$	The normalized tardiness of product p
$\mu_{e,p}$	The normalized earliness of product p
$Td_{1,p}$	Binary variables for calculating tardiness of product p
$Td_{2,p}$	Non-negative variable for calculating tardiness of product p
$E_{1,p}$	Binary variables for calculating earliness of product$p$
$E_{2,p}$	Non-negative variable for calculating earliness of product p
C_{max}	Discharging time of the last batch

In the proposed formulation, the objective function (3) minimizes all operational costs involving pumping costs, interfacing costs, and staff salaries according to the total hours of operation. The objective function (4) minimizes the total tardiness and earliness penalties of discharging the products. This objective is important because delays lead to reduce the credit of oil companies and earliness increases the costs such as warehousing cost, staff costs, etc. Sets of constraints (5) and (6) allocate the products to the batches. Due to these constraints, each batch contains at most one product and the empty batches are located at the end of the sequence. In constraints set (7), some sequences in the pipeline are forbidden. These are due to product contamination. Constraint (8), determines the volume of each batch which must be in a specified range of its product. Discharging flow rate is considered in constraints sets (9)-(11). There are some rate options for the discharging flow where each batch must be pumped by one of these options. Constraints sets (12)-(15) determine the discharging time of each batch. All batches are discharged successively. Therefore, the discharging time of each batch is equal to its discharging length plus the discharging time of the previous discharged batch. The discharging completion time of each product is calculated in constraints set (16) which is equal to the last batch containing the intended product. Interfacing costs are described in the constraints set (**??**). In order to satisfy all demands, constraints set (17) is applied. Constraints sets (19) and (20) describe the sign of all variables of the model.

The model (3)-(20) is a non-linear formulation due to the non-linear objective function (4). In order to linearize this objective function, using the equations (1) and (2) it is formulated as the following model,

$$minz_2 = \sum_{p\in P} \tilde{A}_p \mu_{td,p} + \sum_{p\in P} \tilde{B}_p \mu_{e,p} \qquad (21)$$

$$s.t.$$

$$\mu_{td,p} = 1 \qquad \text{if} \qquad C_p > dd_{4,p} \qquad (22)$$

$$\mu_{td,p} = \frac{C_p - dd_{3,p}}{dd_{4,p} - dd_{3,p}} \qquad \text{if} \qquad dd_{3,p} \leq C_p \leq dd_{4,p} \qquad (23)$$

$$\mu_{e,p} = 1 \qquad \text{if} \qquad C_p < dd_{1,p} \qquad (24)$$

$$\mu_{e,p} = \frac{dd_{2,p} - C_p}{dd_{2,p} - dd_{1,p}} \qquad \text{if} \qquad dd_{1,p} \leq C_p \leq dd_{2,p} \qquad (25)$$

The model (21)-(25) is converted to the following non-linear objective function for calculating the total degree of dissatisfaction.

$$minz_2 = \sum_{p\in P}\tilde{A}_p[(\frac{max\{0, C_p - dd_{3,p}\}}{C_p - dd_{3,p}})(\frac{max\{0, dd_{4,p} - C_p\}}{dd_{4,p} - C_p})(\frac{C_p - dd_{3,p}}{dd_{4,p} - dd_{3,p}}) + \frac{max\{0, C_p - dd_{4,p}\}}{C_p - dd_{4,p}}] \\ + \sum_{p\in P}\tilde{B}_p[(\frac{max\{0, dd_{2,p} - C_p\}}{dd_{2,p} - C_p})(\frac{max\{0, C_p - dd_{1,p}\}}{C_p - dd_{1,p}})(\frac{dd_{2,p} - C_p}{dd_{2,p} - dd_{1,p}}) + \frac{max\{0, dd_{1,p} - C_p\}}{dd_{1,p} - C_p}] \tag{26}$$

The proposed fuzzy objective function (26) is non-linear due to its multiplication of variables and max function and also the conditional expressions in the constraint sets (21)- (25). For linearizing this objective function, binary variables $Td_{1,p}$ and $E_{1,p}$, and continuous non-negative variables $Td_{2,p}$ and $E_{2,p}$ are introduced and applied (see Table 3 for the definitions). Now, the following mixed integer linear mathematical model is used instead of the non-linear model (21)-(25) or the objective function (26).

$$minz_2 = \sum_{p\in P}\tilde{A}_p(Td_{1,p} + Td_{2,p}) + \sum_{p\in P}\tilde{B}_p(E_{1,p} + E_{2,p}) \tag{27}$$

$$s.t.$$

$$C_p - dd_{4,p} \leqslant M(Td_{1,p}) \qquad \forall p \in P \tag{28}$$

$$Td_{2,p} \geqslant \frac{C_p - dd_{3,p}}{dd_{4,p} - dd_{3,p}} - M(Td_{1,p}) \qquad \forall p \in P \tag{29}$$

$$dd_{1,p} - C_p \leqslant M(E_{1,p}) \qquad \forall p \in P \tag{30}$$

$$E_{2,p} \geqslant \frac{dd_{2,p} - C_p}{dd_{2,p} - dd_{1,p}} - M(E_{1,p}) \qquad \forall p \in P \tag{31}$$

$$Td_{1,p}, E_{1,p} \in \{0,1\} \qquad \forall p \in P \tag{32}$$

$$Td_{2,p}, E_{2,p} \geqslant 0 \qquad \forall p \in P \tag{33}$$

Finally, the mathematical models (3)-(20) and (27)-(33) are combined in order to construct the proposed multi-objective mixed integer linear formulation of the problem of this section. Therefore, the following multi-objective formulation is obtained.

$$minz_1 = \sum_{i\in I}\sum_{k\in k}\frac{\widetilde{cost}_k Qu_{i,k}}{vb_k} + \sum_{i\in I} W_i + \rho C_{max} \tag{34}$$

$$minz_2 = \sum_{p\in P}\tilde{A}_p(Td_{1,p} + Td_{2,p}) + \sum_{p\in P}\tilde{B}_p(E_{1,p} + E_{2,p}) \tag{35}$$

$$s.t.$$

$$Constraints\ (5) - (17) \tag{36}$$

$$Constraints (28) - (31) \tag{37}$$

$$y_{i,p}, fr_{i,k}, Td_{1,p}, E_{1,p} \in \{0,1\} \qquad \forall i \in I,\ \forall p \in P \tag{38}$$

$$Qsize_{i,p}, Qu_{i,k}, Tdis_{i,p}, T_i, W_i, C_p, Td_{2,p}, E_{2,p} \geqslant 0 \qquad \forall i \in I,\ \forall p \in P,\ \forall k \in K \tag{39}$$

3 Solution methodology

In this section, a two-phase solution methodology is proposed to solve the fuzzy multi-objective formulation of previous section. In the first phase, a method is used to convert the fuzzy multi-objective formulation to a crisp multi-objective formulation. In the second phase, some multi-objective solution approaches of the literature are used to obtain Pareto-optimal (efficient) solutions of the crisp multi-objective formulation. In order to compare the results of different multi-objective solution approaches, some distance metrics are used to obtain the distances of the obtained efficient solutions from the ideal solutions of the objective functions. The proposed two-phase solution methodology and the distance metrics are detailed in the rest of this section.

3.1 Phase 1: Equivalent crisp multi-objective formulation

Uncertainty theory is a branch of mathematics based on normality, monotonicity, self-duality, and countable subadditivity axioms. In order to solve the above fuzzy formulation, the first step is to convert it to a crisp form. For this aim, three measures have efficient roles in dealing with fuzzy parameters such as possibility, necessity, and credibility measures. The possibility measure was suggested by Zadeh [54] for the first time. In this method, a fuzzy event may be interpreted as a possibility distribution where its membership function plays the role of a possibility distribution function. A fuzzy variable as a random variable also is associated with a probability distribution. This measure considers an optimistic point of view for the occurrence of a fuzzy event. The necessity measure as a dual part of possibility measure was proposed by Zadeh [55]. This measure considers a pessimistic point of view for the occurrence of a fuzzy event. Both of these measures do not have self-duality property. This means that a fuzzy event may not occur even if its possibility is equal to one, and may occur even if its necessity is equal to zero. To overcome this issue, the credibility measure as the average of possibility and necessity measures was presented by Liu and Liu [24]. It provides the commonness of probability theory, credibility theory, and chance theory. Credibility measures in vague environments are to quantify the approximate chance of occurrence of fuzzy events. These three measures have been applied to the fuzzy problems of literature from different fields such as decision making, portfolio optimization, supply chain problems, etc. (see [41], [50], [57]). The possibility, necessity, and credibility measures of fuzzy event $\{\widetilde{\xi} \leq a\}$ are respectively defined as follow,

$$Pos\{\widetilde{\xi} \leq a\} = sup_{x \leq a}\mu(x); \tag{40}$$

$$Nec\{\widetilde{\xi} \leq a\} = 1 - Pos\{\widetilde{\xi} > a\} = 1 - sup_{x>a}\mu(x) \tag{41}$$

$$Cr\{\widetilde{\xi} \leq a\} = \frac{1}{2}(Pos\{\widetilde{\xi} \leq a\} + Nec\{\widetilde{\xi} \leq a\}) \tag{42}$$

where $\widetilde{\xi}$ is a fuzzy variable, $\mu(x)$ represents its membership function, x and a denote real numbers. More details about these measures can be found in[4],[23],[53].

Considering $\widetilde{\xi} = (\xi_1, \xi_2, \xi_3, \xi_4)$ as a trapezoidal fuzzy number (event), Figure 3 demonstrates its membership function of this fuzzy number and the credibility measures of this number are determined as follows [47],

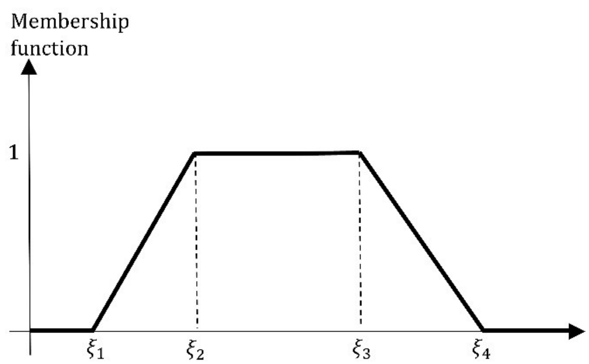

Figure 3: The membership function of the fuzzy trapezoidal number $\widetilde{\xi}$

$$Cr\{\widetilde{\xi} \leq a\} = \begin{cases} 0, & a \leq \xi_1; \\ \frac{a-\xi_1}{2(\xi_2-\xi_1)}, & \xi_1 \leq a \leq \xi_2; \\ \frac{1}{2}, & \xi_2 \leq a \leq \xi_3; \\ \frac{a-2\xi_3+\xi_4}{2(\xi_4-\xi_3)}, & \xi_3 \leq a \leq \xi_4; \\ 1, & \xi_4 \leq a. \end{cases} \tag{43}$$

$$Cr\{\widetilde{\xi} \geq a\} = \begin{cases} 1, & a \leq \xi_1; \\ \frac{2\xi_2-\xi_1-a}{2(\xi_2-\xi_1)}, & \xi_1 \leq a \leq \xi_2; \\ \frac{1}{2}, & \xi_2 \leq a \leq \xi_3; \\ \frac{\xi_4-a}{2(\xi_4-\xi_3)}, & \xi_3 \leq a \leq \xi_4; \\ 0, & \xi_4 \leq a. \end{cases} \tag{44}$$

The above formulations at the confidence level of $\lambda > 0.5$ are converted to the following crisp formulations [58],

$$Cr\{\widetilde{\xi} \leq a\} \geq \lambda \Leftrightarrow a \geq (2-2\lambda)\xi_3 + (2\lambda-1)\xi_4; \quad (45)$$

$$Cr\{\widetilde{\xi} \geq a\} \geq \lambda \Leftrightarrow a \leq (2\lambda-1)\xi_1 + (2-2\lambda)\xi_2. \quad (46)$$

The formula (45) and (46) are the summary of credibility fuzzy chance constrained modeling to be applied for converting the proposed fuzzy multi-objective formulation to its crisp form. It is notable to mention that in this conversion procedure the fuzzy objective functions first are defined as fuzzy constraints and then are converted to crisp form. Therefore, the following form for the fuzzy multi-objective formulation is considered.

$$min \ : \ z_1 \quad (47)$$

$$min \ : \ z_2 \quad (48)$$

$$Cr\{\sum_{i\in I}\sum_{k\in k}\frac{\widetilde{cost_k}Qu_{i,k}}{vb_k} + \sum_{i\in I}W_i + \rho C_{max} + \sum_{i\in I}W_i + \rho C_{max} \leqslant z_1\} \geqslant \alpha_1 \quad (49)$$

$$Cr\{\sum_{p\in P}\tilde{A}_p(Td_{1,p}+Td_{2,p}) + \sum_{p\in P}\tilde{B}_p(E_{1,p}+E_{2,p})\} \leqslant z_2\} \geqslant \alpha_2 \quad (50)$$

$$Cr\{W_i + \widetilde{cin}_{p,q}(y_{i-1,p}+y_{i,q}-1) \leqslant 0\} \geqslant \alpha_3 \qquad \forall p,q \in P, \forall i > 1 \quad (51)$$

$$Cr\{\sum_{i\in I}Qsize_{i,p} \geqslant \widetilde{dem}_p\} \geqslant \alpha_4 \qquad \forall p \in P \quad (52)$$

$$Constraints(5)-(16) \quad (53)$$

$$Constraints(28)-(31) and (36) \quad (54)$$

$$Qu_{i,k}, Tdis_{i,p}, T_i, W_i, \mu_{td,p}, \mu_{e,p}, C_p, Td_{2,p}, E_{2,p}, C_p, z_1, z_2 \geqslant 0 \qquad \forall i \in I, p \in P, k \in K \quad (55)$$

As mentioned earlier, the value of confidence levels must be $\alpha_i > 0.5$ $(i = 1,..,4)$. Then, applying equations (45) and (46) the constraints (49)-(52) are crisped and the following equivalent crisp model is obtained.

$$min \ : \ z_1 \quad (56)$$

$$min \ : \ z_2 \quad (57)$$

$s.t.$

$$\sum_{i\in I,k\in K}Qu_{i,k}(\frac{cost_{k,3}(2-2\alpha_1)+cost_{k,4}(2\alpha_1-1)}{vb_k}) + \sum_{i\in I}W_i + \rho C_{max} \leqslant z_1 \quad (58)$$

$$\sum_{p\in P}(A_{p,3}(Td_{1,p}+Td_{2,p}) + B_{p,3}(E_{1,p}+E_{2,p}))(2-2\alpha_2) + (A_{p,4}(Td_{1,p}+Td_{2,p}) + B_{p,4}(E_{1,p}+E_{2,p}))(2\alpha_2-1) \leqslant z_2 \quad (59)$$

$$-W_i + ((2-2\alpha_3)cin_{p,q,3} + (2\alpha_3-1)cin_{p,q,4}) \times (y_{i-1,p}+y_{i,q}-1) \leqslant 0 \qquad \forall i \in I, \quad i > 1 \quad (60)$$

$$\sum_{i\in I}Qsize_{i,p} \geqslant dem_{p,1} \qquad \forall p \in P \quad (61)$$

$$Constraints\ (5)-(16) \quad (62)$$

$$Constraints\ (28)-(31) \quad (63)$$

$$Constraints\ (36)-(37) \quad (64)$$

Therefore, the (56)-(64) formulation is the equivalent crisp formulations of the fuzzy multi-objective formulation (34)-(37). In the next sub-section, the formulation (56)-(64) is solved by some multi-objective solution approaches.

3.2 Phase 2: Multi-objective solution approaches

Dealing with multi-objective models is one of the most important difficulties of combinatorial optimization. In the literature numerous approaches have been applied for solving multi-objective problems in order to obtain good efficient solutions. Among these approaches, fuzzy programming approach is an effective approach due to its capability of measuring the satisfaction degree of each objective function explicitly [51]. This ability leads to choose an acceptable solution according to both satisfaction degree and relative importance of each objective function. In this issue, Zimmermann [59] defined the most common approach of this area called the max- min approach. But this method is not always capable to find efficient solutions [42]. In order to deal with this difficulty, many studies have focused on this shortcoming and developed modifications of fuzzy programming approach. For instance, Lai and Hwang [21] presented a new method called the LH method, Torabi and Hasssini [51] introduced the TH method, Demirli and Yimer [15] introduced the DY method, Selim and Ozkarahan [48] proposed the SO method, Mosallaeipour et al. [38] introduced the MMNV method. These modified methods follow mainly the same structure. In this study, some of these approaches are used to obtain efficient solutions of the crisp multi-objective formulation (56)-(64). The structures of these multi-objective approaches for the formulation (56)-(64) are summarized in the following steps.

Step 1: The values of parameters of the problem and the confidence levels of the chance constraints are determined.

Step 2: To obtain a satisfactory feasible solution, the positive ideal solution (PIS) and the negative ideal solution (NIS) are determined for each objective function of the problem. The PIS value for each objective is obtained as follows:

$$\begin{aligned} &z_i^{PIS} = min\ z_k \qquad\qquad i = 1,2 \\ &Constraints\ (56) - (64) \end{aligned} \tag{65}$$

Let x_h^* denote the vector of decision variable values obtained for the PIS of objective function h. The NIS value for each objective function is obtained as follow,

$$z_i^{NIS} = max_{\{h=1,2\}}\ \{z_i(x_h^*)\} \qquad i = 1,2 \tag{66}$$

Step 3: Determine the linear membership function for each objective function according to the achieved values of Step 2. The membership function is obtained as follow,

$$\mu_h(z_h) = \begin{cases} 1, & \text{if} \quad z_h \le z_h^{PIS}; \\ \frac{z_h^{NIS} - z_h}{z_h^{NIS} - z_h^{PIS}}, & \text{if} \quad z_h^{PIS} \le z_h \le z_h^{NIS}; \qquad h = 1,2 \\ 0, & \text{if} \quad z_h \ge z_h^{NIS}. \end{cases} \tag{67}$$

Step 4: Convert the multi-objective formulation (56)-(64) to an equivalent single objective formulation using the following methods,

LH method (Lai and Hwang[21])

$$\begin{aligned} max: \quad & \lambda(x) = \lambda_0 + \delta \sum_{i=1}^{2} \theta_i \mu_i(z_i), \\ s.t. \quad & \\ & \lambda_0 \le \mu_i(z_i), \qquad\qquad i = 1,2 \\ & Constraints\ (58) - (64), \\ & \lambda_0 \in [0,1]. \end{aligned} \tag{68}$$

where λ_0 and $\mu_i(z_i)$ denote the minimum satisfaction degree of both objectives and the membership function of each objective, respectively. θ_i denotes the weight for the importance of ith objective, where $\sum_{i=1}^{2} \theta_i = 1$. The weight values are determined by decision makers. δ is a small value as it is usually set to 0.01.

TH method (Torabi and Hassini[51])

$$\begin{aligned} max: \quad & \lambda(x) = \gamma\lambda_0 + (1-\gamma) \sum_{i=1}^{2} \theta_i \mu_i(z_i), \\ s.t. \quad & \\ & \lambda_0 \le \mu_i(x), \qquad\qquad i = 1,2 \\ & Constraints\ (58) - (64), \\ & \gamma, \lambda_0 \in [0,1], \qquad\qquad i = 1,2 \end{aligned} \tag{69}$$

where γ is a weight value determined by decision makers to balance the objective function. In the literature, it is usually set to 0.2.

MW method (Werners [52], Selim and Ozkarahan[48])

$$\begin{aligned}
&max: \quad \lambda = \gamma\lambda_0 + (1-\gamma)\sum_{i=1}^{2}\theta_i\lambda_i, \\
&s.t. \\
&\qquad \lambda_0 + \lambda_i \leq \mu_i(z_i), \qquad\qquad i = 1, 2, \\
&\qquad Constraints\ (58) - (64), \\
&\qquad \gamma, \lambda_0, \lambda_i \in [0, 1], \qquad\qquad i = 1, 2.
\end{aligned} \tag{70}$$

In the literature, the value of γ is usually set to 0.4.

DY method (Demirli and Yimer [15])

$$\begin{aligned}
&max: \quad \lambda, \\
&s.t. \\
&\qquad \theta_i\lambda \leq \mu_i(z_i), \qquad\qquad i = 1, 2, \\
&\qquad Constraints\ (58) - (64), \\
&\qquad \lambda \in [0, 1], \qquad\qquad i = 1, 2.
\end{aligned} \tag{71}$$

Niroomand method (Niroomand[40])

$$\begin{aligned}
&max: \quad \lambda(x) = \gamma\lambda_0 + (1-\gamma)\sum_{k=1}^{i}\lambda_i, \\
&s.t. \\
&\qquad \lambda_0 + \lambda_i \leq \mu_i(z_i), \qquad\qquad i = 1, 2, \\
&\qquad \lambda_0 \leq \mu_i(z_i) \\
&\qquad Constraints\ (58) - (64), \\
&\qquad \gamma, \lambda_0, \lambda_i \in [0, 1], \qquad\qquad i = 1, 2.
\end{aligned} \tag{72}$$

Solving any of these single objective methods gives a Pareto-optimal solution for the proposed formulation.

3.3 Overall procedure of the proposed solution methodology

The proposed solution methodology solving the fuzzy multi-objective formulation (34)-(39) is summarized in the flowchart of Figure 4.

3.4 Comparison metrics

In order to compare the performance of the multi-objective solution approaches mentioned in Section 3.2, some distance metrics have been used in the literature. These distances calculate the differences between the obtained objective function values and the ideal solutions. For this aim, a general formulation has been proposed in the literature as follow (see [1],[2],[47]),

$$D_p(\theta, i) = [\Sigma_{i=1}^{K}\theta_i^p(1-\mu_i(\mathbf{x}))^p]^{\frac{1}{p}}, \qquad p \in \mathbb{N} \tag{73}$$

where $\sum_{i=1}^{k}\theta_i = 1$ and $\theta_i > 0$ denotes the importance weight of objective function i. Parameter k denotes the number of objective functions. The power p is the index of distance type. The most important values of this parameter, which

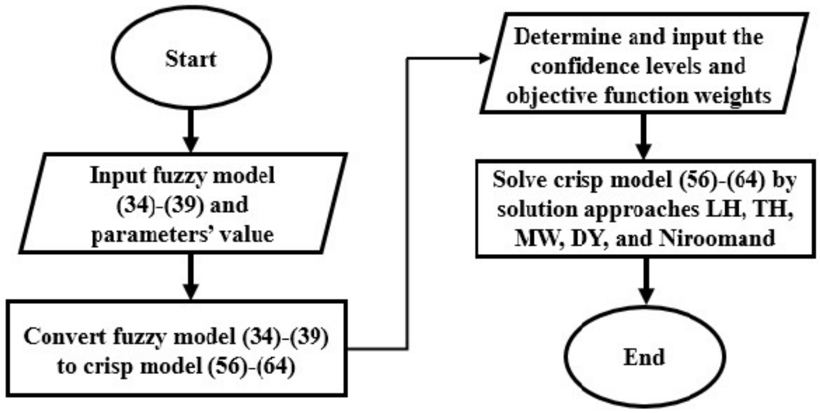

Figure 4: The procedure of the proposed solution methodology.

are usually used in the literature, are 1, 2, and ∞. These measures are known as Manhattan distance (D_1), Euclidean distance (D_2) and Tchebycheff distance (D_∞), respectively. These distance measures are defined as follow,

$$D_1(\theta, i) = 1 - \Sigma_{i=1}^{K} \theta_i \mu_i(\mathbf{x}) \tag{74}$$

$$D_2(\theta, i) = [\Sigma_{i=1}^{K} \theta_i^2 (1 - \mu_i(\mathbf{x}))^2]^{\frac{1}{2}} \tag{75}$$

$$D_\infty(\theta, i) = \max_i [\theta_i (1 - \mu_i(\mathbf{x}))] \tag{76}$$

Obviously, obtaining an efficient solution with fewer distances values is favored.

4 Computational experiments

In order to compare and analyze the performance of the multi-objective solution approaches of this study, two test problems are considered. These test problems are generated randomly with the following characterizations,
1. Test problem 1 has 5 products and 2 flow rate options.
2. Test problem 2 has 6 products and 3 flow rate options.

In order to perform the experiments, two weight combinations of $(\theta_1,\ \theta_2) = (0.3, 0.7)$ and $(\theta_1,\ \theta_2) = (0.7, 0.3)$ are considered. Furthermore, three values for the confidence levels are considered as $\{0.55,\ 0.75,\ 0.95\}$.. In order to apply the confidence level values, when considering each of the values, all required confidence levels of the constraints are set to that value. The solution approaches and mathematical formulations are coded in AIMMS 3.12 software with CPLEX 12.4 solver and run on a computer with Intel® Core$^{\text{TM}}$ i7, 2.80 GHz processor.

4.1 Results of the test problems

The results obtained for the test problems including the membership function values and the distance metrics of the obtained Pareto-optimal solutions are reported by Table 4 and Table 5. The average CPU time for the first case problem was 12.64 sec. and 258.31 sec. for the second case. This value has been the same in almost all methods. Focusing on the results of test problem 1 which is summarized in Table 4, it is concluded that the multi-objective solution methods are sensitive to the changes in the confidence level values. This sensitivity can be seen in the confidence level of 0.95 comparing to other confidence level values. Among the multi-objective solution methods, the DY method has higher values of distance measures compared to the other methods in all confidence levels and all objective function weight values, therefore, it has less performance in comparison to other proposed methods. Although the LH and TH methods approximately have the same performance, in some cases of this table the TH method performs better than the LH method. Results obtained by the Niroomand method show the same performance comparing to the TH and LH methods, but this method is less sensitive to changes in the objective function weight values than other methods. The MW method has similar performance to the TH and LH methods. In conclusion, for test problem 1, the results obtained by the LH, MW, and TH methods have more balance in satisfaction degree of objectives and have less distance to the ideal values.

Table 4: Results of the test problem 1.

Confidence level ($\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4$)	Objective function weights (θ_1, θ_2)	Solution approach	μ_i		Distance		
			μ_1	μ_2	D_1	D_2	D_∞
		DY	0.43	1	0.171	0.171	0.171
		LH	0.94	0.96	0.046	0.0333	0.028
0.55	$(\theta_1, \theta_2) = (0.3, 0.7)$	MW	0.94	0.96	0.046	0.0333	0.028
		TH	0.94	0.96	0.046	0.0333	0.028
		Niroomand	0.94	0.96	0.046	0.0333	0.028
		DY	1	0.43	0.171	0.171	0.171
		LH	0.94	0.96	0.054	0.04368	0.042
0.55	$(\theta_1, \theta_2) = (0.7, 0.3)$	MW	0.94	0.96	0.054	0.04368	0.042
		TH	0.94	0.96	0.054	0.04368	0.042
		Niroomand	0.94	0.96	0.05	0.03606	0.03
		DY	0.43	1	0.171	0.171	0.171
		LH	0.94	0.96	0.046	0.0333	0.028
0.75	$(\theta_1, \theta_2) = (0.3, 0.7)$	MW	0.94	0.96	0.046	0.0333	0.028
		TH	0.94	0.96	0.046	0.0333	0.028
		Niroomand	0.94	0.96	0.05	0.03606	0.03
		DY	1	0.43	0.171	0.171	0.171
		LH	0.94	0.96	0.054	0.04368	0.042
0.75	$(\theta_1, \theta_2) = (0.7, 0.3)$	MW	0.94	0.96	0.054	0.04368	0.042
		TH	0.94	0.96	0.054	0.04368	0.042
		Niroomand	0.94	0.96	0.05	0.03606	0.03
		DY	0.43	0.99	0.178	0.17114	0.171
		LH	0.89	0.96	0.061	0.04328	0.033
0.95	$(\theta_1, \theta_2) = (0.3, 0.7)$	MW	0.89	0.96	0.061	0.04328	0.033
		TH	0.89	0.96	0.061	0.04328	0.033
		Niroomand	0.89	0.96	0.075	0.05852	0.055
		DY	1	0.43	0.171	0.171	0.171
		LH	0.89	0.96	0.089	0.07793	0.077
0.95	$(\theta_1, \theta_2) = (0.7, 0.3)$	MW	1	0.81	0.057	0.057	0.057
		TH	1	0.81	0.057	0.057	0.057
		Niroomand	0.89	0.96	0.075	0.05852	0.055

Table 5: Results of the test problem 2.

Confidence level ($\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4$)	Objective function weights (θ_1, θ_2)	Solution approach	μ_i		Distance		
			μ_1	μ_2	D_1	D_2	D_∞
		DY	0.42	0.98	0.188	0.17456	0.035
		LH	0.97	0.96	0.037	0.02941	0.028
0.55	$(\theta_1, \theta_2) = (0.3, 0.7)$	MW	0.95	0.98	0.029	0.02052	0.015
		TH	0.98	0.95	0.041	0.03551	0.035
		Niroomand	0.98	0.95	0.035	0.02693	0.025
		DY	1	0.43	0.171	0.171	0.171
		LH	0.97	0.95	0.036	0.02581	0.021
0.55	$(\theta_1, \theta_2) = (0.7, 0.3)$	MW	0.98	0.95	0.029	0.02052	0.015
		TH	0.98	0.95	0.029	0.02052	0.015
		Niroomand	0.98	0.95	0.035	0.02693	0.025
		DY	0.42	0.98	0.188	0.17456	0.035
		LH	0.97	0.95	0.044	0.03614	0.035
0.75	$(\theta_1, \theta_2) = (0.3, 0.7)$	MW	0.94	0.97	0.039	0.02766	0.021
		TH	0.94	0.97	0.039	0.02766	0.021
		Niroomand	0.98	0.95	0.032	0.02332	0.02
		DY	1	0.43	0.171	0.171	0.171
		LH	0.97	0.95	0.036	0.02581	0.021
0.75	$(\theta_1, \theta_2) = (0.7, 0.3)$	MW	0.98	0.95	0.029	0.02052	0.015
		TH	0.98	0.95	0.029	0.02052	0.015
		Niroomand	0.98	0.95	0.032	0.02332	0.02
		DY	0.42	0.98	0.188	0.17456	0.174
		LH	0.98	0.98	0.02	0.01523	0.014
0.795	$(\theta_1, \theta_2) = (0.3, 0.7)$	MW	0.98	0.98	0.02	0.01523	0.014
		TH	0.98	0.98	0.02	0.01523	0.014
		Niroomand	0.98	0.98	0.02	0.01414	0.01
		DY	1	0.43	0.171	0.171	0.171
		LH	0.98	0.98	0.02	0.01523	0.014
0.95	$(\theta_1, \theta_2) = (0.7, 0.3)$	MW	0.99	0.97	0.016	0.0114	0.009
		TH	0.98	0.98	0.02	0.01523	0.014
		Niroomand	0.98	0.98	0.02	0.01414	0.01

Table 6: The output of the ANOVA analysis

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	0.205	4	0.051	46.725	0.000
Within Groups	0.060	55	0.001		
Total	0.265	59			

Table 7: Multiple Comparisons by using Tukey test

		Mean			95%Confidence Interval	
(I) type	(J) type	Difference (I-J)	Std. Error	Sig.	Lower Bound	Upper Bound
DY	LH	0.1429166667*	0.0135105832	0.000	0.104812397	0.181020936
	MW	0.1482500000*	0.0135105832	0.000	0.110145730	0.186354270
	TH	0.1461666667*	0.0135105832	0.000	0.108062397	0.184270936
	Niroomand	0.1465000000*	0.0135105832	0.000	0.108395730	0.184604270
LH	DY	−0.142916667*	0.0135105832	0.000	-0.181020936	-0.104812397
	MW	0.0533333333	0.0135105832	0.995	-0.032770936	0.043437603
	TH	0.0032500000	0.0135105832	0.999	-0.034854270	0.041354270
	Niroomand	0.0035833333	0.0135105832	0.999	-0.034520936	0.041687603
MW	DY	−0.1482500000*	0.0135105832	0.000	-0.186354270	-0.110145730
	LH	-0.0053333333	0.0135105832	0.995	-0.043437603	0.032770936
	TH	−0.0020833333	0.0135105832	1.000	-0.040187603	0.036020936
	Niroomand	−0.0017500000	0.0135105832	1.000	-0.039854270	0.036354270
TH	DY	−0.1461666667*	0.0135105832	0.000	-0.184270936	-0.108062397
	LH	-0.0032500000	0.0135105832	0.999	-0.041354270	0.034854270
	MW	0.0020833333	0.0135105832	1.000	-0.036020936	0.040187603
	Niroomand	0.0003333333	0.0135105832	1.000	-0.037770936	.038437603
Niroomand	DY	−0.1465000000*	0.0135105832	0.000	-0.184604270	-0.108395730
	LH	-0.0035833333	0.0135105832	0.999	-0.041687603	0.034520936
	MW	0.0017500000	0.0135105832	1.000	-0.036354270	0.039854270
	TH	−0.0003333333	0.0135105832	1.000	-0.038437603	0.037770936

*. The mean difference is significant at the 0.05 level.

According to the results of test problem 2 which is given by Tables 5, it is concluded that the multi-objective solution methods are sensitive to the changes in the confidence level values. This sensitivity can be seen in all confidence level values where the satisfaction degree values are changed by changing in the confidence levels. Among the multi-objective solution methods, the DY method has higher values of distance measures compared to the other methods in all confidence levels and all objective function weight values, therefore, it has less performance in comparison to other proposed methods. The solution approaches LH, MW, TH, and Niroomand in all cases approximately perform similarly. In experiments 1 and 2 the Niroomand method performs better than other methods in terms of satisfaction degree levels.

4.2 Statistical analysis

For more investigation and accurate comparison, statistical analysis is applied in this subsection. ANOVA provides a statistical test for investigating the equality of the means of different groups. Tables 6-8 are extracted from IBM SPSS software. It can be seen in Table 6 that the significance value (sig) is 0.000 which is below 0.05. Therefore, there are statistically significant differences between the groups as a whole.

In order to show which groups differ from others, the Tukey post hoc test is employed. This test is generally the preferred test for conducting post hoc tests on a one-way ANOVA. In this research, this test is applied for multiple comparisons of all five approaches. Tables 7 and 8 provide some very useful descriptive statistics including mean difference, standard error, significant differences, 95% confidence intervals for multiple approaches, and homogeneous subsets. It can be seen from these tables that there is a statistically significant difference between DY approaches to all others ($sig < 0.05$). This approach has less performance in comparison to other proposed methods. The solution approaches LH, MW, TH, and Niroomand approximately perform similarly ($sig > 0.05$). Among these four methods, Niroomand and MW methods have the best performances due to the results in Tables 7 and 8. The superiority of the Niroomand method over other methods is due to the lower sensitivity of this method to changes in the objective function weight values compared to other methods.

Table 8: Homogeneous subsets

Type	Subset for Sig.=0.05	
	1	2
MW	0.026583333	
Niroomand	0.028333333	
TH	0.028666667	
LH	0.031916667	
DY		0.174833333
Sig.	0.995	1.000

4.3 Managerial insight

From the managerial point of view, this problem and the proposed procedure have the following implications:
1. The model and solution approaches can be applied to obtain financial benefits in the oil industry and oil companies.
2. The uncertainty and its solution methodology can be employed for other supply chain network design problems easily.
3. The uncertainty type of this study is an efficient and functional tool to apply the idea of experts and decision makers, especially in a lack of data or when the data of the problem do not exist historically.
4. On-time deliveries have significant effects on the profitability of oil companies, the oil industry, and consequently the governments. The on-time delivery penalty function applied in this study can easily be used in other optimization problems.

5 Conclusions and future studies

In this study, a multi-objective mathematical formulation for a new multi-product pipeline scheduling problem was developed in an uncertain environment. Batch sizing, discharging rate, forbidden sequence, batch volumetric restrictions were considered in the problem. Objectives such as all operational costs involving pumping costs, interfacing costs and staff salaries according to the total hours of operation and also the tardiness and earliness of discharging each product were to be minimized simultaneously. A penalty function was introduced for tardiness and earliness calculations. Some parameters of the model were considered as trapezoidal fuzzy values due to the uncertain nature of the problem. In order to tackle such a multi-objective fuzzy environment, five two-phase fuzzy interactive methods were applied. In the first phase, the fuzzy formulation was converted to its equivalent crisp form using a credibility-based chance-constrained programming approach. In the second phase, the multi-objective formulation was converted to a single objective formulation using fuzzy interactive programming approaches. In order to investigate the efficiency of these five methods, two problem instances were applied to demonstrate the application of the proposed methodology as well as to compare the performances of the solution approaches in terms of three performance measures.

As future studies, there are several recommendations as follows
1) Improving the proposed model by considering the multi-DCs pipeline.
2) Applying meta-heuristic algorithms for solving the problem.
3) Applying other objectives of the problem.
4) Considering other restrictions such as settling and shutdown periods.
5) Applying other methods to tackle uncertainty.

References

[1] W. F. Abd El-Wahed, S. M. Lee, *Interactive fuzzy goal programming for multi-objective transportation problems*, Omega, **34**(2) (2006), 158-166.

[2] M. H. Alavidoost, H. Babazadeh, S. T. Sayyari, *An interactive fuzzy programming approach for bi-objective straight and U-shaped assembly line balancing problem*, Applied Soft Computing, **40** (2016), 221-235.

[3] Z. Alizadeh, S. H. Nasseri, I. Mahdavi, M. M. Paydar, *A fuzzy stochastic multi-objective optimization model to configure a supply chain considering new product development*, Applied Mathematical Modelling, **40**(17-18) (2016), 7545-7570.

[4] O. A. Ark, *Credibility based chance constrained programming for project scheduling with fuzzy activity durations*, An International Journal of Optimization and Control: Theories and Applications, **9**(2) (2019), 208-215.

[5] A. Baykasoğlu, T. Gocken, *A review and classification of fuzzy mathematical programs*, Journal of Intelligent and Fuzzy Systems, **19**(3) (2008), 205-229.

[6] A. Baykasoğlu, K. Subulan, *An analysis of fully fuzzy linear programming with fuzzy decision variables through logistics network design problem*, Knowledge-Based Systems, **90** (2015), 165-184.

[7] A. Baykasoğlu, K. Subulan, *A fuzzy-stochastic optimization model for the intermodal fleet management problem of an international transportation company*, Transportation Planning and Technology, **42**(8) (2019), 777-824.

[8] N. Beheshti Asl, S. A. Mir Hassani, *Benders decomposition with integer sub-problem applied to pipeline scheduling problem under flow rate uncertainty*, Computers and Chemical Engineering, **123** (2019), 222-235.

[9] D. C. Cafaro, J. Cerdá, *Optimal scheduling of multiproduct pipeline systems using a non-discrete MILP formulation*, Computers and Chemical Engineering, **28**(10) (2004), 2053-2068.

[10] D. C. Cafaro, J. Cerdá, *Efficient tool for the scheduling of multiproduct pipelines and terminal operations*, Industrial and Engineering Chemistry Research, **47**(24) (2008), 9941-9956.

[11] D. C. Cafaro, J. Cerdá, *Optimal scheduling of refined products pipelines with multiple sources*, Industrial and Engineering Chemistry Research, **48**(14) (2009), 6675-6689.

[12] D. C. Cafaro, J. Cerdá, *Operational scheduling of refined products pipeline networks with simultaneous batch injections*, Computers and Chemical Engineering, **34**(10) (2010), 1687-1704.

[13] M. Castro, H. Mostafaei, *Product-centric continuous-time formulation for pipeline scheduling*, Computers and Chemical Engineering, **104** (2017), 283-295.

[14] M. Castro, H. Mostafaei, *New continuous-time scheduling formulation for multilevel treelike pipeline systems*, Computer Aided Chemical Engineering, **43** (2018), 973-978.

[15] K. Demirli, A. D. Yimer, *Fuzzy scheduling of a build-to-order supply chain*, International Journal of Production Research, **46**(14) (2008), 3931-3958.

[16] F. K. Goudarzi, A. Ghaffari, *Fuzzy interactive approach for a multi-objective supplier selection problem under robust uncertainty*, Journal of Modeling and Optimization, **12**(2) (2020), 71-83.

[17] F. K. Goudarzi, S. H. Nasseri, N. A. Taghinezhad, *A new interactive approach for solving fully fuzzy mixed integer linear programming*, Yugoslav Journal of Operations Research, **30**(1) (2020), 71-89.

[18] F. K. Goudarzi, N. A. Taghinezhad, S. H. Nasseri, *A new fuzzy approach to solve a novel model of open shop scheduling problem*, Scientific Bulletin: Series A, **76**(3) (2014), 199-210.

[19] J. He, C. Tan, Y. Zhang, *Yard crane scheduling problem in a container terminal considering risk caused by uncertainty*, Advanced Engineering Informatics, **39** (2019), 14-24.

[20] H. Ishii, M. Tada, T. Masuda, *Two scheduling problems with fuzzy due-dates*, Fuzzy Sets and Systems, **46**(3) (1992), 229-347.

[21] Y. J. Lai, C. L. Hwang, *A new approach to some possibilistic linear programming problems*, Fuzzy Sets and Systems, **49**(2) (1992), 121-133.

[22] Q. Liao, H. Zhang, Y. Wang, W. Zhang, Y. Liang, *Heuristic method for detailed scheduling of branched multiproduct pipeline networks*, Chemical Engineering Research and Design, **140**(2) (2018), 82-101.

[23] B. Liu, *Uncertainty theory: An introduction to its axiomatic foundations*, Springer-Verlag Berlin Heidelberg, 2004.

[24] B. Liu, Y. K. Liu, *Expected value of fuzzy variable and fuzzy expected value models*, IEEE Transactions on Fuzzy Systems, **10**(4) (2002), 445-450.

[25] A. S. Losenkov, T. S. Yushchenko, S. A. Strelnikova, D. E. Michkova, *Optimization of oil-flow scheduling in branched pipeline system*, Journal of Pipeline Systems Engineering and Practice, **10**(3) (2019), doi:10.1061/(ASCE)PS.1949-1204.0000378.

[26] S. A. Mir Hassani, N. BeheshtiAsl, *A heuristic batch sequencing for multiproduct pipelines*, Computers and Chemical Engineering, **56** (2013), 58-67.

[27] S. A. Mir Hassani, H. Fani Jahromi, *Scheduling multi-product tree-structure pipelines*, Computers and Chemical Engineering, **35**(1) (2011), 165-176.

[28] S. A. Mir Hassani, M. Ghorbanalizadeh, *The multi-product pipeline scheduling system*, Computers and Mathematics with Applications, **56**(4) (2008), 891-897.

[29] S. A. Mir Hassani, S. Moradi, N. Taghinezhad, *Algorithm for long-term scheduling of multiproduct pipelines*, Industrial and Engineering Chemistry Research, **50**(24) (2011), 13899-13910.

[30] S. Molla Alizadeh Zavardehi, R. Tavakkoli Moghaddam, F. H. Lotfi, *Hybrid metaheuristics for solving a fuzzy single batch-processing machine scheduling problem*, The Scientific World Journal, (2014), 1-10.

[31] S. Molla Alizadeh Zavardehi, R. Tavakkoli Moghaddam, F. H. Lotfi, *Electromagnetism-like algorithm for fuzzy flow shop batch processing machines scheduling to minimize total weighted earliness and tardiness*, Industrial Journal of Industrial Mathematics, **7**(1) (2015), 13-24.

[32] S. Molla Alizadeh Zavardehi, R. Tavakkoli Moghaddam, F. H. Lotfi, *A modified imperialist competitive algorithm for scheduling single batch-processing machine with fuzzy due date*, The International Journal of Advanced Manufacturing Technology, **85**(9-12) (2016), 2439-2458.

[33] S. Moradi, S. A. Mir Hassani, *Transportation planning for petroleum products and integrated inventory management*, Applied Mathematical Modelling, **39**(23-24) (2015), 7630-7642.

[34] S. Moradi, S. A. Mir Hassani, *Robust scheduling for multi-product pipelines under demand uncertainty*, The International Journal of Advanced Manufacturing Technology, **87**(9-12) (2016), 2541-2549.

[35] S. Moradi, S. A. Mir Hassani, F. Hooshmand, *Efficient decomposition-based algorithm to solve long-term pipeline scheduling problem*, Petroleum Science, **16**(5) (2019), 1159-1175.

[36] S. Mosallaeipour, A. Mahmoodirad, S. Niroomand, B. Vizvari, *Simultaneous selection of material and supplier under uncertainty in carton box industries: A fuzzy possibilistic multicriteria approach*, Soft Computing, **22**(9) (2018), 2891-2905.

[37] H. Mostafaei, Y. Alipouri, J. Shokri, *A mixed-integer linear programming for scheduling a multi-product pipeline with dual-purpose terminals*, Computational and Applied Mathematics, **34**(3) (2014), 979-1007.

[38] A. Mozaffari, N. L. Azad, A. Fathi, *Simultaneous knowledge-based identification and optimization of PHEV fuel economy using hyper-level Pareto-based chaotic Lamarckian immune algorithm*, International Journal of Intelligent Computing and Cybernetics, **8**(1) (2015), 1-27.

[39] S. H. Nasseri, A. Ebrahimnejad, B. Y. Cao, *Fuzzy linear programming: Solution techniques and applications (Studies in Fuzziness and Soft Computing)*, Springer International Publishing, **379** (2019), doi:10.1007/978-3-030-17421-7.

[40] S. Niroomand, A. Mahmoodirad, S. Mosallaeipour, *A hybrid solution approach for fuzzy multi objective dual supplier and material selection problem of carton box production systems*, Expert Systems, **36**(1) (2019), 1-17.

[41] G. Pasi, M. Viviani, A. Carton, *A multi-criteria decision making approach based on the Choquet integral for assessing the credibility of user-generated content*, Information Sciences, **503** (2019), 574-588.

[42] M. S. Pishvaee, S. A. Torabi, *A possibilistic programming approach for closed-loop supply chain network design under uncertainty*, Fuzzy Sets and Systems, **161**(20) (2010), 2668-2683.

[43] A. Rahbari, M. M. Nasiri, F. Werner, M. Musavi, F. Jolai, *The vehicle routing and scheduling problem with cross-docking for perishable products under uncertainty: Two robust bi-objective models*, Applied Mathematical Modelling, **70** (2019), 605-625.

[44] R. Rejowski, J. M. Pinto, *Scheduling of a multiproduct pipeline system*, Computers and Chemical Engineering, **27**(8-9) (2003), 1229-1246.

[45] S. Relvas, S. N. Boschetto Magatáo, A. P. F. D. Barbosa-Póvoa, F. Neves, *Integrated scheduling and inventory management of an oil products distribution system*, Omega, **41**(6) (2013), 955-968.

[46] S. Relvas, H. A. Matos, A. P. F. D. Barbosa-Póvoa, J. Fialho, *Reactive scheduling framework for a multiproduct pipeline with inventory management*, Industrial and Engineering Chemistry Research, **46**(17) (2007), 5659-5672.

[47] M. Salehi, H. R. Maleki, S. Niroomand, *A multi-objective assembly line balancing problem with workerś skill and qualification considerations in fuzzy environment*, Applied Intelligence, **48**(8) (2017), 2137-2156.

[48] H. Selim, I. Ozkarahan, *A supply chain distribution network design model: An interactive fuzzy goal programming-based solution approach*, The International Journal of Advanced Manufacturing Technology, **36**(3-4) (2008), 401-418.

[49] J. Shen, Y. Zhu, *A parallel-machine scheduling problem with periodic maintenance under uncertainty*, Journal of Ambient Intelligence and Humanized Computing, **10**(8) (2018), 3171-3179.

[50] H. Taghizadeh, E. Hafezi, *The investigation of supply chainś reliability measure: A case study*, Journal of Industrial Engineering International, **8**(1) (2012), doi:10.1186/2251-712X-8-22.

[51] S. A. Torabi, E. Hassini, *An interactive possibilistic programming approach for multiple objective supply chain master planning*, Fuzzy Sets and Systems, **159**(2) (2008), 193-214.

[52] B. M. Werners, *Aggregation models in mathematical programming*, Mathematical Models for Decision Support, (1988), 295-305.

[53] D. Yu-Zhen, C. Huan, *Credibility measure theory analysis*, Procedia Engineering, **15** (2011), 1722-1726.

[54] L. A. Zadeh, *Fuzzy sets as a basis for a theory of possibility*, Fuzzy Sets and Systems, **1**(1) (1978), 3-28.

[55] L. A. Zadeh, *A theory of approximate reasoning*, In: J. Hayes (ed), Mathematical Frontiers of the Social and Policy Sciences. Westview Press, Boulder, (1979), 69-129.

[56] H. R. Zhang, Y. T. Liang, Q. Xiao, M. Y. Wu, Q. Shao, *Supply-based optimal scheduling of oil product pipelines*, Petroleum Science, **13**(2) (2016), 355-367.

[57] X. Zhang, W. G. Zhang, R. Cai, *Portfolio adjusting optimization under credibility measures*, Journal of Computational and Applied Mathematics, **234**(5) (2010), 1458-1465.

[58] H. Zhu, J. Zhang, *A credibility-based fuzzy programming model for APP problem*, International Conference on Artificial Intelligence and Computational Intelligence, **1** (2009), 455-459.

[59] H. J. Zimmermann, *Fuzzy programming and linear programming with several objective functions*, Fuzzy Sets and Systems, **234**(1) (1978), 45-55.

Iranian Journal of Fuzzy Systems
Volume 18, Number 4 (2021), pp. 209
University of Sistan and Baluchestan

An interactive fuzzy programming approach for a new multi-objective multi-product oil pipeline scheduling problem

F. Khalili Goudarzi, H. R. Maleki and S. Niroomand

یک رویکرد برنامه‌ریزی فازی تعاملی برای یک مسأله چند هدفی زمان‌بندی خط لوله چند محصولی

چکیده. در این مقاله، یک مسأله چند هدفی زمان‌بندی خط لوله چند محصولی فازی جدید معرفی شده‌است. این سیستم متشکل از یک واحد پالایشگاهی، یک مرکز توزیع منحصر به فرد و یک خط لوله چند محصولی است. محدودیت‌هایی مانند اندازه محموله‌ها، سرعت تخلیه، توالی‌های ممنوعه، حجم محموله‌ها و ... در نظر گرفته شده است. به دلیل ماهیت ناقطعی مسائل دنیای واقعی، برخی از پارامترهای سیستم به عنوان مقادیر فازی در نظر گرفته شده‌اند. جریمه تأخیر و زودکرد با یک تابع غیرخطی وابسته به زمان معرفی شده‌است. هدف اصلی این مسأله‌ی برنامه‌ریزی، دستیابی به توالی بهینه برای پمپاژ محموله‌های فرآورده‌های نفتی در جهت به حداکثر رساندن سود مالی و نیز به طور همزمان برآورده کردن نیاز مشتریان با تحویل به موقع محصولات در قالب یک مسأله چند هدفه است. برای حل این مسأله، یک روش دو مرحله‌ای پیشنهاد شده‌است. در مرحله اول، فرمول‌بندی فازی توسط یک رویکرد برنامه‌ریزی مبتنی بر اعتبار محدودیت شانسی به صورت معادل قطعی آن تبدیل می‌شود. در مرحله دوم، فرمول قطعی چند هدفه توسط برخی رویکردهای شناخته شده در ادبیات موضوع حل می‌شود. برخی نمونه‌ها برای مسأله تولید و توسط روش‌های پیشنهادی حل می‌شوند و جواب‌های بهینه پارتو به دست آمده با استفاده از برخی معیارهای مقایسه مبتنی بر فاصله تجزیه و تحلیل و مقایسه می‌شوند.