Iranian Journal of Fuzzy Systems
Volume 18, Number 4, (2021), pp. 131-148
Original paper

University of Sistan and Baluchestan

A multi-attribute assessment of fuzzy regression models

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Abstract

Most of the fuzzy regression approaches proposed in the literature adopted a single objective function in the generation of fuzzy regression models. These approaches mostly being criticized by their weak performances analysis and their sensitivity to outliers. Therefore, this paper develops a new multi-objective two-stage optimization and decision technique for fuzzy regression modeling problems in order to handle both of the criticisms. To handle the outlier problems, in the first stage, dynamic robust to outlier objective functions is considered in the estimation problem. The estimation problem is solved by running an algorithm which generates a set of fuzzy regression models. Then, in the next stage, we design a decision schema by employing Multi-Attribute Decision Making (MADM) problem. Here, the VIKOR method is employed as a proper means to provide a design to rank the generated fuzzy regression models by the first stage to introduce the most desirable model. We include simulation numerical results and a real-world house price problem in order to highlight the advantages of the proposed method in a comparison study. The results demonstrate the effectiveness of the proposed multi-objective optimization method to handle outlier detection problem while the prediction accuracy of the model is improved.

Keywords: Fuzzy regression, robust estimation, multi-attribute decision making (MADM), VIKOR, outlier analysis.

1 Introduction

Determining and estimating the coefficients values are important to measure relationship in any algebraic expression and to build a mathematical model though it might be complex and troublesome in some environments such as fuzzy environments with imprecise data and vague information. Additionally, providing precise value for the coefficient of any model structure is an important issue which may be difficult when it deals with fuzzy information which increase the complexity of deciding the optimal coefficients as well. Seeking high goodness-of-fit criteria values by improving estimation process is an important research topic in the literature, especially in fuzzy regression analysis. However, in this context, most of the researches on fuzzy regression analysis are mainly the possibilistic regression [67, 68] which using the possibilistic concepts formulates the problem of estimating the parameters as a linear/non-linear programming problem [5, 27, 28, 51, 50, 61] and the fuzzy Least-Squares (LS) regression [6, 19] which is based on an optimization problem based on a distance on fuzzy numbers [4, 7, 10, 20, 21, 23, 30]. Several researches, however, have pointed out that both possibilistic and fuzzy LS methods are sensitive to outliers [3, 14, 35, 42, 54]. Moreover, some methods such as fuzzy least-absolutes regression as well as some heuristic approaches have been provided to overcome the outlier problems [12, 15, 45, 41, 66]. Although routine and popular in estimation, these methods have been criticized by

1. A single outlier can wrongly affect the estimated parameters in such a way that the computed model could be grossly erroneous at the presence of the outlier [7, 22, 42].

2. The outlier points could not be detected [57], while the breakdown point of the estimators is zero [34].

Recently, robust approaches to fuzzy regression have been considered as alternative approaches to overcome the difficulties mentioned above. In addition, many papers are exclusively devoted to this methodology on the topic of fuzzy

Corresponding Author: J. Chachi
Received: April 2020; Revised: July 2020; Accepted: February 2021.

regression [16]. For instance, Varga [69] presented robust estimation approaches to fuzzy and non-fuzzy regression models. Nasrabadi and Hashemi [49] suggested a robust nonlinear fuzzy regression model using multilayered feedforward neural networks. Kula and Apaydin [46] proposed a robust fuzzy regression analysis based on the ranking of fuzzy sets. With the help of hyperelliptic functions and possibility maximization, Yabuuchi and Watada [72, 73] developed some robust fuzzy regression models. Using genetic algorithms, Hu [32, 33] represented robust interval regression models. Kelkinnama and Taheri [41] proposed a fuzzy least-absolutes regression using shape preserving operations. Ferraro and Giordani [26] dealt with robustness in the field of regression analysis for imprecise information managed in terms of fuzzy sets. Yang et al. [74] present a robustified fuzzy varying coefficient model for fuzzy input-fuzzy output variables. Shakouri and Nadimi [62] investigated a method for outlier detection in fuzzy linear regression problems. Leski and Kotas [47], by introducing an objective function based on the Huber's M-estimators and the Yager's OWA operators, proposed a robust fuzzy c-regression models. D'Urso [22, 23] used the term robust parameter design in fuzzy regression modeling to address the problem of outliers with regard to robust estimation methods and detection criteria for outliers. D'Urso et al. [23] proposed a robust fuzzy linear regression model based on the least median squares-weighted least-squares estimation procedure. D'Urso and Massari [22] proposed weighted least-squares and least median squares estimation for fuzzy linear regression analysis. Chachi et al. [13], using the generalized Hausdorff-metric, presented two least-absolutes/least squares approaches to multiple regression analysis with real-world dataset in predicting imperfections of cotton yarn. Chachi and Roozbeh [8] proposed a robust least trimmed squares estimation method for fuzzy regression that helps to identify and/or ignore irregular data. Zhou et al. [78] modeled affordable levels of house prices using fuzzy linear regression analysis. Chachi [7] discussed a robustified weighted objective function for estimation of fuzzy regression model that prevents the drawbacks of ordinary least squares estimation in the presence of outliers. Arefi [3] investigated a quantile fuzzy regression based on fuzzy outputs and fuzzy parameters. Akbari and Hesamian [2] investigated a partial-robust-ridge-based regression model with fuzzy predictors-responses. Khammar et al. [42, 43] proposed general robust approaches to fuzzy regression models based on different loss functions for crisp/fuzzy input and fuzzy output. Hesamian and Akbari [31] proposed a robust varying coefficient approach to fuzzy multiple regression model. Taheri and Chachi [65] investigated a robust variable-spread fuzzy regression model. In most of the above discussed papers, the problem of outliers has been considered with regard to both robust estimation methods and detection criteria for outliers. There are some of the other fuzzy regression approaches that are also dedicated to the detection and/or influence of outliers, e.g. Colla et al. [17] investigated methods based on a fuzzy inference system and fuzzy logic that were capable of pointing out outliers in a series of data points. To address the pertinence of outlier results, Jin et al. [38] proposed an outlier detection approach which aims at finding anomalies in full dimensions that lack pertinence and comprehensibility. Sharma et al. [63, 64] proposed two outlier detection approaches, namely as high dimensional fuzzy outlier detection method and fuzzy constraint based outlier detection method, to address the pertinence of outlier results in a dataset. Zazzaro and Martone [75] focused on an approach to mine large datasets by applying the clustering algorithm, in order to detect potential outliers. By investigating weighted aggregation of individual fitness values, Chachi and Taheri [11] proposed an outlier detection method in fuzzy regression models.

However, addressing the limitations in robust techniques involves the realization of several conflicting objectives which might be complex in nature. In this regard, the aim of the present study is to propose a new technique in the estimation process to maximize efficiency of fuzzy regression models while considering outliers in the dataset, avoiding disruptive effects of them, and not scarifying the prediction accuracy. In the estimation technique the robust process is the most vital step which uses hybrid algorithm(s) with high accuracy to reduce the overall disruptive effects of outliers while improving the accuracy of prediction. To this aim, a new estimation approach will be introduced to estimate the coefficient values of fuzzy regression model with multi-objective optimization problem for which statistical data contains simultaneous fuzzy information. In ordinary fuzzy regression optimization problems, a given goodness-of-fit criterion with a single objective function is optimized to find solutions. In addition, only one solution can be obtained based on a single objective function, while, in reality, decision makers may wish to consider several optimal solutions simultaneously which are obtained under different scenarios and then decide which one is the most preferred one. In a problem of multi-objective optimization, however, more than one objective function is involved. Multi-objective optimization should often also satisfy several numbers of constraints while giving overall optimal values for the objectives. Such problems mainly have the following properties [37]:

1. They have various search spaces. The complexity of this space represents an important difference between multi-objective and single-objective optimization processes. Multi-objective optimization involves a search in an objective space which is multi-dimensional.

2. There are different goals of optimization. Also there is more than one optimal set, hence yielding Pareto fronts.

In the proposed approach, the multi-objective optimization problem on the robustified objective functions will firstly be used to develop a set of mathematical candidate solutions. An algorithm is then explained to solve the multi-objective optimization problem generating the set of candidate solutions for the fuzzy regression model. Each candidate solution is the optimum solution of one of the conflicting objective functions. Associated with each solution is an uncertainty profile consisting of the set of possible outcomes that can occur if we choose this alternative. Now, we have a collection of alternatives from which we must choose one. Secondly, we will employ a technique to rank the set of solutions, here, the set of fuzzy regression models, based on their goodness-of-fit performance analysis when applied on modeling a dataset. Concretely speaking, the problem of finding the best model is considered as an MADM problem in which the set of candidate models which are the solutions of the multi-objective problem are alternatives and the recorded dataset of individuals for input-output variables are attributes [36]. Since each data point gets different goodness-of-fit value by the models inside the aforementioned set, therefore, the weights of attributes and the attribute values, which are considered as the most important points in MADM problems, are obtained by a proper technique. The decision information matrix which generally includes the attribute weights and the attribute values provides distinct values among alternatives by using proper means [56]. In the proposed technique, the entries of the decision matrix are considered as the fitness values of the data points by the models (or alternatives). It is clear that besides giving us how well a data point is fitted by the models, the variety of matrix entries demonstrate how vast the distinction among alternatives is. The fitness values are then normalized to eliminate the units of criterion functions and transformed to vector values for the data points (or attributes) illustrating the importance of each data point in the estimation method of models. These transformed values can create a distinction among the models (or alternatives) as well as the good data set and the bad data set in the estimation process helping us to select the best model. Ultimately, the VIKOR method [48] is employed at this point to provide an algorithm to rank the models to select the best choice [39, 40, 44, 76]. The efficiency and performance of the proposed method are then compared against several common methods based on crisp input-fuzzy output data derived from simulated experiments.

Rest of the paper is organized as follows. Next section introduces an optimization method which establishes and explains the generation process of the candidate fuzzy regression models as well as selecting the best model. Based on a couple of datasets in Section 3, comparison studies are provided to demonstrate the effectiveness of the proposed model. Section 5 gives some concluding remarks.

2 The proposed hybrid method

2.1 The proposed model

The general form of a fuzzy regression model for crisp/fuzzy-input data and fuzzy-output data is as $\widetilde{y} = \widetilde{f_{\mathbf{\Lambda}}(\mathbf{x})}$ [16] where

1. $\mathbf{x} = [1, x_1, \ldots, x_p]$ (or $\mathbf{x} = [1, \widetilde{x}_1, \ldots, \widetilde{x}_p]$) is a crisp (or fuzzy) input vector.
2. $\widetilde{\mathbf{y}} = [\widetilde{y}_1, \ldots, \widetilde{y}_n]$ determines the fuzzy values of the output variable.
3. $\mathbf{\Lambda} = [\Lambda_0, \Lambda_1, \ldots, \Lambda_p]$ (or $\mathbf{\Lambda} = [\widetilde{\Lambda}_0, \widetilde{\Lambda}_1, \ldots, \widetilde{\Lambda}_p]$) is a crisp (or fuzzy) vector parameter.
4. $f_{\mathbf{\Lambda}}(\cdot)$ determines the relationship between input-output variables.

In this study, let $(\widetilde{y}_1, \mathbf{x}_1), \ldots, (\widetilde{y}_n, \mathbf{x}_n)$ consists of n observations where

1. $\widetilde{y}_j = (y_j, l_j, r_j)_{LR}$ $(j = 1, \ldots, n)$ determines the fuzzy observation of the dependent variable.
2. $\mathbf{x}_j = [x_{0j}, x_{1j}, \ldots, x_{pj}]$ $(j = 1, \ldots, n; p < n; x_{0j} = 1)$, determines the crisp observed of the independent variables.
3. $\mathbf{\Theta} = [\Theta_0, \Theta_1, \ldots, \Theta_p]$ is a vector of parameters in $\mathbb{R}^3$, i.e. $\Theta_k = (\theta_k, \alpha_k, \beta_k) \in \mathbb{R}^3$, where $\theta_k \in \mathbb{R}$, $\alpha_k \in \mathbb{R}$, $\beta_k \in \mathbb{R}$, $k = 0, \ldots, p$. The parameters are considered in $\mathbb{R}^3$, because the proposed prediction error is the g-prediction error, due to the scale transformations of the spreads by means of the invertible function $g : \mathbb{R}^+ \to \mathbb{R}$ [20, 21, 22, 24].

Consider the following relationship between $\widetilde{y}$ and $\mathbf{x}$ which is known as a g-scale transformation model of $\widetilde{y}$ on $\mathbf{x}$

$$\begin{aligned}
(y, g(l), g(r)) &= \bigoplus_{k=0}^{p} (\Theta_k \otimes x_k) \qquad (1)\\
&= \Theta_0 \oplus (\Theta_1 \otimes x_1) \oplus \ldots \oplus (\Theta_p \otimes x_p)\\
&= (\theta_0, \alpha_0, \beta_0) \oplus ((\theta_1, \alpha_1, \beta_1) \otimes x_1) \oplus ((\theta_p, \alpha_p, \beta_p) \otimes x_p)\\
&= (\theta_0 + \theta_1 x_1 + \ldots + \theta_p x_p, \alpha_0 + \alpha_1 x_1 + \ldots + \alpha_p x_p, \beta_0 + \beta_1 x_1 + \ldots + \beta_p x_p).
\end{aligned}$$

Here, the major processes of estimating the parameter $\mathbf{\Theta} = [\Theta_0, \Theta_1, \ldots, \Theta_p]$ in model (1) are described as a two-phase procedure. The first phase includes the formulation of a multi-objective optimization model, and the second phase employs an MADM problem for determining the final best solution among the set of solutions obtained in the first phase. Solving the optimization model using an employed algorithm and the determination of the final optimal solution as well as detecting outlier points, minimizing the effect of outliers, tuning the number of outliers and minimizing modeling errors will be considered by the two-phase procedure. After obtaining the final solution, the aim is to predict the value of the fuzzy dependent variable $\widetilde{y}$ as

$$\widehat{\widetilde{y}} \;=\; (\widehat{\theta}_0 + \widehat{\theta}_1 x_1 + \ldots + \widehat{\theta}_p x_p, g^{-1}(\widehat{\alpha}_0 + \widehat{\alpha}_1 x_1 + \ldots + \widehat{\alpha}_p x_p), g^{-1}(\widehat{\beta}_0 + \widehat{\beta}_1 x_1 + \ldots + \widehat{\beta}_p x_p)).$$

2.2 Multi-objective optimization problem

In ordinary fuzzy regression optimization problems, a given goodness-of-fit criterion with a single objective function is optimized to find estimations/solutions. In the problem of multi-objective optimization, however, more than one objective function is involved [37, 51, 53, 59, 60]. One important situation which needs to be considered in the formulation of objective functions occurs in determining outliers. Although, many possible choices exist for the formulation of the objective functions as well as the determination outlier values, it must be noted that the notion of an outlier is somehow fuzzy and each observation value can be considered as an outlier with some degree/notion/definition. Our proposed procedure is based on minimizing the q^{th} quantile of the ordered fitted errors between the observed values of output variable and its estimated values with respect to the following distance

$$\mathcal{D}(\widetilde{A}, \widetilde{B}) = |a - b| + c_1 |l_a - l_b| + c_2 |r_a - r_b|,$$

where $\widetilde{A} = (a, l_a, r_a)_{LR}$ and $\widetilde{B} = (b, l_b, r_b)_{LR}$ are two LR-fuzzy numbers, $c_1 = \int_0^1 L^{-1}(\alpha) d\alpha$, and $c_2 = \int_0^1 R^{-1}(\alpha) d\alpha$ [18]. To formulate this strategy, the estimation value of $\mathbf{\Theta}$ is obtained by minimizing the following objective functions

$$\min_{\mathbf{\Theta}} \left[O(\mathbf{\Theta}, [\frac{n}{2}] + 1), \ldots, O(\mathbf{\Theta}, n) \right], \qquad (2)$$

where

$$\begin{aligned} O(\mathbf{\Theta}, q) &= \frac{d_{(q)}}{\sum_{j=1}^{n} d_{(j)}}, \qquad q = [\frac{n}{2}] + 1, \ldots, n, \\ d_j &= \mathcal{D}\left((y_j, g(l_j), q(r_j)), \bigoplus_{k=0}^{p} (\Theta_k \otimes x_k) \right) \quad j = 1, \ldots, n, \end{aligned}$$

and $d_{(1)} \leq \ldots \leq d_{([\frac{n}{2}])} \leq \ldots \leq d_{(q)} \leq \ldots \leq d_{(n)}$.

Here, optimization problem (2) must be evaluated to find a group of optimal solutions by using different values of q which determine different objective functions. The optimal value of q must be determined in such a way that suitably tunes the negative effects of outliers in the estimation process. It also determines the number of good data points in the dataset. Its domain is between $\frac{n}{2}$ (at least half of the dataset are good data points) and n (there is no outlier point in the dataset). As each datum influences the estimation process as well as the regression model more or less, the outliers are compensated by data that produce high value of errors in the estimation process. Thus, the objective function of minimizing the effect of outliers is formulated as minimizing the q^{th} quantile of errors. The usual way of solving such multi-objective optimization problem is the elemental set approach which is used as one type of multi-objective evolutionary algorithms to solve this kind of problems [58, 57]. By using the elemental set approach, computing parameter estimates and selecting the ideal value of q for model (1) are explained in the algorithm below. The codes have been implemented in R Software [55].

Algorithm: estimates for fuzzy regression with crisp inputs and fuzzy output data.

Input: An $n \times 1$ fuzzy response vector $\widetilde{\boldsymbol{y}}$ and the corresponding $n \times (p+1)$ predictor matrix $\boldsymbol{X}$.

Output: The optimal estimates of coefficient $\mathbf{\Theta}$ and the corresponding outlier points in the dataset.

Step 1: Generate every elemental set for $q = [\frac{n}{2}] + 1, \ldots, n$.

Step 2: For each elemental set, compute the exactly fitting regression model (1), and get the residuals on all data.

Step 3: Calculate the objective $O(\mathbf{\Theta}, q)$ in (2).

Step 4: Choose $\widehat{\mathbf{\Theta}}_q$, which minimizes $O(\mathbf{\Theta}, q)$.

Each candidate solution $\widehat{\mathbf{\Theta}}_q$ which minimizes the objective function $O(\mathbf{\Theta}, q)$ contains the settings of estimated coefficients for model (1). After obtaining the set of the candidate solutions for all of the objective functions, it is essential to determine and select the final optimal solution to estimate model (1). The selecting process is conventionally performed based on the importance of individual objectives, individual points as well as decision makers' judgements and experience which is explained in the second phase.

2.3 Multi-attribute decision making analysis to fuzzy regression model

In the process of MADM, we generally need to compare the overall attribute values of the considered alternatives. According to the information theory, if all alternatives have similar attribute values with respect to an attribute, then a small weight should be assigned to the attribute; it is due to this fact that the attribute does not help to differentiate alternatives. As a result, in the process of ranking the alternatives, the attribute which has bigger deviations among the alternatives should be assigned larger weight. Especially, if there is no difference among the attribute values of all the alternatives with respect to the j^{th} attribute, then the j^{th} attribute will play no role in ranking the alternatives, and thus, its weight can be assigned zero [36, 70].

Therefore, the problem of finding the most desirable solution in $\mathcal{E} = \{\widehat{\mathbf{\Theta}}_q | q = [\frac{n}{2}] + 1, \ldots, n\}$ is considered as an MADM problem. Here, the set of candidate models are considered as the alternatives and the recorded dataset of individuals play the attributes. Since each data point is fitted differently by the models inside the aforementioned set, therefore, the tolerance of the fitness values helps in differentiating alternatives (the models). Notice, if a data point gets almost similar value of fitted errors by the models then it means that this data point does not help to differentiate alternatives, i.e. all of the models similarly fits to this data point. On the other hand for a data point which is fitted differently by the models we conclude that this data point gets small to high value of fitness values by the models. Therefore, the data points and their fitness values by the models help to differentiate alternatives using a decision information matrix. The decision matrix generally includes the attribute weights and the attribute values and provides distinct values between alternatives by using a proper means. Since, the central aim of multi-attribute evaluation is to obtain the best alternative among a set of evaluated alternatives, therefore, an appropriate method known as VIKOR technique [52] is employed as following steps to determine the best solution based on the decision matrix [44, 48, 56, 76].

1. Consider decision matrix $D = [d_{ij}]_{m\times n}$, $i = 1, \ldots, m = |\mathcal{E}|$ and $j = 1, \ldots, n$, where $|\mathcal{E}|$ is the cardinal of the collection $\mathcal{E} = \{\widehat{\mathbf{\Theta}}_q | q = [\frac{n}{2}] + 1, \ldots, n\}$, n is the sample size or number of data points, and for each $\widehat{\mathbf{\Theta}} \in \mathcal{E}$

$$d_{ij} = \mathcal{D}\left((y_j, g(l_j), g(r_j)), \bigoplus_{k=0}^{p}(\widehat{\Theta}_k \otimes x_k)\right).$$

2. Define ideal and nadir (virtual) model as $D^+ = (d_1^+, \ldots, d_n^+)$ and $D^- = (d_1^-, \ldots, d_n^-)$, respectively, where in which $d_j^+ = \min_{i=1,\ldots,m} d_{ij}$ and $d_j^- = \max_{i=1,\ldots,m} d_{ij}$ are the best ideal value and the nadir value for attribute j^{th}, respectively, $j = 1, \ldots, n$.

3. Let $S^+ = \min\{S_1, \ldots, S_m\}$, $S^- = \max\{S_1, \ldots, S_m\}$, $R^+ = \min\{R_1, \ldots, R_m\}$, $R^- = \max\{R_1, \ldots, R_m\}$, where for each $i = 1, \ldots, m$,

$$S_i = \textstyle\sum_{j=1}^{n} w_j \frac{d_{ij} - d_j^+}{d_j^- - d_j^+}, \qquad R_i = \max\left\{ w_j \frac{d_{ij} - d_j^+}{d_j^- - d_j^+} | j = 1, \ldots, n \right\},$$

where $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

4. Select $\nu \in [0, 1]$ as the weight of the decision making strategy "the majority of criteria" (or "the maximum group utility") and $1 - \nu$ is the weight of the individual target.

5. For each $i = 1, \ldots, m$, let

$$Q(A_i) = Q_i = \nu\left(\frac{S_i - S^+}{S^- - S^+}\right) + (1 - \nu)\left(\frac{R_i - R^+}{R^- - R^+}\right).$$

6. Order the values of Q_i, S_i and R_i as

$$\begin{array}{l} Q_{(1)} \le Q_{(2)} \le \ldots \le Q_{(i)} \le \ldots \le Q_{(m)}, \\ S_{(1)} \le S_{(2)} \le \ldots \le S_{(i)} \le \ldots \le S_{(m)}, \\ R_{(1)} \le R_{(2)} \le \ldots \le R_{(i)} \le \ldots \le R_{(m)}. \end{array}$$

Table 1: The table of errors to be aggregated by n attribute (here the data points) with respect to m alternatives (here the models in $\mathcal{E}$)

Alternatives: Estimated models in $\mathcal{E}$ i	Attributes: Data points j 1	2	$\dots$	n	S_i	R_i	Q_i	$Q_{(i)}$	$S_{(i)}$	$R_{(i)}$
Model 1: $f_{\widehat{\boldsymbol{\Theta}}_{[\frac{n}{2}]+1}}(\cdot)$	d_{11}	d_{12}	$\dots$	d_{1n}	S_1	R_1	Q_1	$Q_{(1)}$	$S_{(1)}$	$R_{(1)}$
Model 2: $f_{\widehat{\boldsymbol{\Theta}}_{[\frac{n}{2}]+2}}(\cdot)$	d_{21}	d_{22}	$\dots$	d_{2n}	S_2	R_2	Q_2	$Q_{(2)}$	$S_{(2)}$	$R_{(2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Model m: $f_{\widehat{\boldsymbol{\Theta}}_n}(\cdot)$	d_{m1}	d_{m2}	$\dots$	d_{mn}	S_m	R_m	Q_m	$Q_{(m)}$	$S_{(m)}$	$R_{(m)}$
$d_j^+ = \min_{i=1,\dots,n} d_{ij}$	d_1^+	d_2^+	$\dots$	d_n^+	$S^+ = \min_{i=1,\dots,m} S_i$					
$d_j^- = \max_{i=1,\dots,n} d_{ij}$	d_1^-	d_2^-	$\dots$	d_n^-	$S^- = \max_{i=1,\dots,m} S_i$					
					$R^+ = \min_{j=1,\dots,m} R_i$					
					$R^- = \max_{j=1,\dots,m} R_i$					
w_j	w_1	w_2	$\dots$	w_n	$\sum_{j=1}^n w_j = 1$					

7. Sort the alternatives as $A^{(1)}$, $A^{(2)}$, ..., $A^{(m)}$ with their position in the ranking list by $Q_{(i)}$, $i = 1, \dots, m$. Indeed $A^{(i)}$ for $i = 1, \dots, m$ is the alternative that has the i^{th} position in the ranking list value of Q_i. Propose as a compromise solution the alternative $A^{(1)}$, i.e. select the alternative that has the minimum value of Q_i, as the best choice if

   (a) $Q_{(2)} - Q_{(1)} \geq \frac{1}{m-1}$ and

   (b) Alternative $A^{(1)}$ also has the best rank by R_i and/or S_i.

   Note that, if $A^{(1)}$ satisfies only in the first above condition (a) and dose not satisfy in the second above condition (b) then $A^{(1)}$ and $A^{(2)}$ are selected as the best choices. Also, if $A^{(1)}$ dose not satisfy in the first above condition (a) then $A^{(1)}, A^{(2)}, \dots, A^{(K)}$ are selected as the best choices, where K is the biggest digit number obtained as follows

$$Q_{(K)} - Q_{(1)} < \frac{1}{m-1}.$$

The mathematical computations of the Algorithm is shown in Table 1.

3 Numerical example

3.1 Illustrative example

Here, we provide numerical examples to explain how the proposed method is applicable to obtain a suitable regression model for fuzzy observations. Consider the fuzzy input-output data set in Table 2 which are simulated as

$$\begin{array}{ll}
x_j \sim \mathrm{U}(2,5), & j = 1, \dots, 7, \\
x_j \sim \mathrm{U}(10,15), & j = 8, 9, 10, \\
y_j \sim 1 + 5x_j + N(0,1), & j = 1, \dots, 7, \\
y_j \sim -10 + x_j + N(0,1), & j = 8, 9, 10, \\
l_j \sim 1 + |N(0, 0.25)|, & j = 1, \dots, 10, \\
r_j \sim 1 + |N(0, 0.25)|, & j = 1, \dots, 10,
\end{array}$$

Note that $U(a, b)$ is the Uniform distribution on the interval (a, b) and $N(\mu, \sigma^2)$ is the Normal distribution with mean μ and variance σ^2. Here, we are going to estimate the following model

$$(y, \ln(l), \ln(r)) = \Theta_0 \oplus (\Theta_1 \otimes x). \tag{3}$$

To formulate the first phase of the proposed strategy, the estimated values of $\boldsymbol{\Theta} = (\Theta_0, \Theta_1)$ are obtained by minimizing the following objective functions, each at a time

$$\min_{\boldsymbol{\Theta}} \left[O(\boldsymbol{\Theta}, 6), \dots, O(\boldsymbol{\Theta}, 10) \right],$$

Table 2: Simulated crisp input- fuzzy output data

j	x_j	$\widetilde{y}_j = (y_j, l_j, r_j)_T$
1	3.2	$(19.2, 1.4, 1.0)_T$
2	3.2	$(15.6, 1.5, 1.3)_T$
3	4.7	$(25.3, 1.0, 1.4)_T$
4	2.6	$(16.1, 1.1, 1.2)_T$
5	3.1	$(17.1, 1.6, 1.4)_T$
6	3.7	$(19.4, 1.5, 1.5)_T$
7	2.2	$(11.5, 1.0, 1.0)_T$
8	10.8	$(1.4, 2.5, 1.6)_T$
9	12.4	$(4.5, 1.5, 1.6)_T$
10	14.7	$(5.3, 1.1, 1.1)_T$

Table 3: The solutions of the objective functions as the candidate parameters obtained by the Algorithm

q	$\widehat{\Theta}_0^q$	$\widehat{\Theta}_1^q$
6	(1.97, 1.360, 0.3316)	(4.9, -0.2856, 0.0044)
7	(-0.27, 0.040, -0.2794)	(5.4, -0.0058, 0.1339)
8	(20.81,-0.208, 0.0460)	(-1.8, 0.1054, 0.0410)
9	(19.21,-0.024, 0.0756)	(-1.2, 0.0360, 0.0299)
10	(-19.83, 4.470, 0.7979)	(2.0, -0.3261,-0.0283)

where

$$O(\mathbf{\Theta}, q) = \frac{d_{(q)}}{\sum_{j=1}^{10} d_{(j)}}, \qquad q = 6, \ldots, 10,$$
$$d_j = \mathcal{D}\Big((y_j, \ln(l_j), \ln(r_j)), \widehat{\Theta}_0 \oplus (\widehat{\Theta}_1 \otimes x_j)\Big) \quad j = 1, \ldots, 10,$$

By running the proposed algorithm the solutions have been achieved, and the corresponding operational parameters of these solutions are listed in Table 3. These five solutions are the elements of the collection $\mathcal{E}$. In the next phase, by the MADM problem the most desirable element will be chosen as the final estimated parameters value for the model. From the basic dataset and the elements of the collection $\mathcal{E}$ the errors and the normalized values of both beneficial and non-beneficial attributes are calculated and summarized in Table 4. The mathematical computations of VIKOR method are summarized in Table 4 and continued in Table 5. the provided results show that both estimated models $f_{\widehat{\mathbf{\Theta}}_6}$ and $f_{\widehat{\mathbf{\Theta}}_7}$ are the most desirable choices for estimating model (3). The results also show that both models fit almost the same to the data set. Notice the model $f_{\widehat{\mathbf{\Theta}}_6}$ minimizes target function $\frac{d_{(6)}}{\sum_{j=1}^{10} d_{(j)}}$ which declares that there are four potential outlier points in the dataset. While the model $f_{\widehat{\mathbf{\Theta}}_7}$ minimizes target function $\frac{d_{(7)}}{\sum_{j=1}^{10} d_{(j)}}$ which considers three data points as the potential outlier points. The reported individual deviations of model $f_{\widehat{\mathbf{\Theta}}_6}$ in Table 4 labels the observations of No. 7, No. 8, No. 9 and No. 10 as outliers. But, except for data point of No. 7, the other observations provide extraordinary amounts of fitting deviations. Therefore if we choose this model as the best model four observations including the observation of No. 7 must be labeled as outlier points. Although, the observation of No. 7 gets fitted very well by the estimated model but it is labeled as an outlier point. The reported individual deviations of the desirable model $f_{\widehat{\mathbf{\Theta}}_7}$ are shown in Table 4. Considering this model, three observations of No. 8, No. 9 and No. 10 are labeled as outliers. It is clear that these three observations provide extraordinary amounts of total fitting deviations. Also, there is a deep gap between fitted values of No. 8 and of No. 7. Therefore, it is better to choose $f_{\widehat{\mathbf{\Theta}}_7}$ as the best final model for modeling the dataset. As we expected, the best model for modeling the simulated dataset in which there are three outliers is the model which minimizes target function $\frac{d_{(7)}}{\sum_{j=1}^{10} d_{(j)}}$. The prposed method labels three observations of No. 8, No. 9 and No. 10 as outliers, properly. Finally the model is estimated as follows

$$\widehat{\widetilde{y}} = (-0.27 + 5.4x, \exp\{0.04 - 0.0058x\}, \exp\{-0.2794 + 0.1339\})_T.$$

Table 4: Summarized mathematical computations of VIKOR method

Alternatives: Estimated models in $\mathcal{E}$ i	Attributes: Data points j 1	2	3	4	5	6	7	8	9	10
Model 1: $f_{\widehat{\Theta}_6}(\cdot)$	0.0070	0.0121	0.0000	0.0071	0.0000	0.0059	0.0089	0.2850	0.3088	0.3652
Model 2: $f_{\widehat{\Theta}_7}(\cdot)$	0.0102	0.0090	0.0000	0.0101	0.0055	0.0043	0.0000	0.2834	0.3094	0.3681
Model 3: $f_{\widehat{\Theta}_8}(\cdot)$	0.0883	0.0153	0.2660	0.0000	0.0425	0.1103	0.1120	0.0000	0.1278	0.2379
Model 4: $f_{\widehat{\Theta}_9}(\cdot)$	0.1072	0.0114	0.3158	0.0000	0.0497	0.1290	0.1403	0.1414	0.0000	0.1052
Model 5: $f_{\widehat{\Theta}_{10}}(\cdot)$	0.1468	0.1312	0.1594	0.1394	0.1388	0.1423	0.1246	0.0000	0.0000	0.0176
$d_j^+ = \min_{i=1,\ldots,n} d_{ij}$	0.0070	0.0090	0.0000	0.0000	0.0000	0.0043	0.0000	0.0000	0.0000	0.0176
$d_j^- = \max_{i=1,\ldots,n} d_{ij}$	0.1468	0.1312	0.3158	0.1394	0.1388	0.1423	0.1403	0.2850	0.3094	0.3681
w_j	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Table 5: Table 4 continued

Alternatives: Estimated models in $\mathcal{E}$ i	S_i	S_i^N	R_i	R_i^N	Q_i	$Q_{(i)}$	$S_{(i)}$	$R_{(i)}$
Model 1: $f_{\widehat{\Theta}_6}(\cdot)$	0.3141	0.0036	0.1000	1.0000	0.0942	0.0909	0.3129	0.0842
Model 2: $f_{\widehat{\Theta}_7}(\cdot)$	0.3129	0.0000	0.1000	1.0000	0.0909	0.0942	0.3141	0.1000
Model 3: $f_{\widehat{\Theta}_8}(\cdot)$	0.4390	0.3862	0.0842	0.0000	0.3511	0.3511	0.4390	0.1000
Model 4: $f_{\widehat{\Theta}_9}(\cdot)$	0.4744	0.4948	0.1000	1.1000	0.5407	0.5407	0.4744	0.1000
Model 5: $f_{\widehat{\Theta}_{10}}(\cdot)$	0.6393	1.0000	0.1000	1.1000	1.0000	1.0000	0.6393	0.1000

3.2 Comparison study

In this section, we compare our proposed method with the recently robust approaches proposed by Chachi [7] and ordinary least-squares method (LS) and ordinary least-absolutes method (LA). Here, we provide a simulated numerical example to explain how the proposed method is applicable to obtain a suitable regression model for the observations. Consider the symmetric triangular fuzzy output and crisp input data set in Table 6 which are simulated as follows

$$
\begin{array}{ll}
x_{j1} \sim U(10,50), & j=1,\ldots,15,\\
x_{j1} \sim U(300,350), & j=15,\ldots,20,\\
x_{j2} \sim N(15,4), & j=1,\ldots,15,\\
x_{j2} \sim N(150,16), & j=15,\ldots,20,\\
x_{j3} \sim \chi^2_{(df=10)}, & j=1,\ldots,15,\\
x_{j3} \sim \chi^2_{(df=100)}, & j=15,\ldots,20,\\
y_j \sim 1 + x_{j1} + x_{j2} + x_{j3} + N(0,1), & j=1,\ldots,15,\\
y_j \sim -200 + x_{j1} + x_{j2} + x_{j3} + N(0,1), & j=15,\ldots,20,\\
l_j \sim 1 + \exp\{0.01 \times y_j\} + |N(0,1)|, & j=1,\ldots,15,\\
l_j \sim 1 + \exp\{0.001 \times y_j\} + |N(0,1)|, & j=15,\ldots,20,
\end{array}
$$

where $\chi^2_{(df=t)}$ is the Chi-Squared distribution with t degrees of freedom. Here, we estimate the following model

$$(y, \ln(l)) = \Theta_0 \oplus (\Theta_1 \otimes x_1) \oplus (\Theta_2 \otimes x_2) \oplus (\Theta_3 \otimes x_3),$$

by minimizing the following objective functions, each by each

$$\min_{\boldsymbol{\Theta}} \left[O(\boldsymbol{\Theta}, 11), \ldots, O(\boldsymbol{\Theta}, 20)\right]. \tag{4}$$

The candidate solutions of the optimization problem (4) are given in Table 7. Table 7 also shows the solution of the least-absolutes method and the solution of the least-squares method which are obtained by minimizing the following

Table 6: Simulated dataset in comparison study

No.	x_1	x_2	x_3	$(y,l,r)_T$
1	34.14	13.19	12.38	$(60.02, 3.67, 3.67)_T$
2	30.54	12.51	9.20	$(54.71, 4.18, 4.18)_T$
3	40.67	11.17	6.57	$(60.06, 3.34, 3.34)_T$
4	13.80	17.08	12.75	$(45.23, 3.87, 3.87)_T$
5	24.03	14.99	13.83	$(54.89, 3.26, 3.26)_T$
6	16.80	17.10	13.75	$(48.53, 2.79, 2.79)_T$
7	23.59	16.70	12.21	$(52.28, 4.30, 4.30)_T$
8	25.24	14.69	12.60	$(53.54, 3.07, 3.07)_T$
9	43.99	9.75	9.14	$(64.30, 4.57, 4.57)_T$
10	18.24	11.47	9.58	$(39.83, 3.21, 3.21)_T$
11	26.55	12.17	11.42	$(51.90, 3.79, 3.79)_T$
12	40.73	19.39	8.69	$(71.86, 3.35, 3.35)_T$
13	13.89	14.58	13.38	$(41.73, 2.90, 2.90)_T$
14	30.78	12.23	4.44	$(47.62, 3.72, 3.72)_T$
15	16.59	14.92	13.74	$(45.93, 2.66, 2.66)_T$
16	323.84	154.08	80.25	$(357.39, 3.58, 3.58)_T$
17	319.08	150.74	103.23	$(369.92, 3.35, 3.35)_T$
18	326.57	155.47	125.18	$(407.06, 3.59, 3.59)_T$
19	343.87	147.48	103.83	$(395.37, 3.46, 3.46)_T$
20	345.55	154.12	107.23	$(405.04, 4.07, 4.07)_T$

target functions, respectively,

$$\begin{aligned}
E_{LA} &= \sum_{j=1}^{20} \mathcal{D}\left((y_j, g(l_j), g(l_j)), \Theta_0 \oplus (\Theta_1 \otimes x_{j1}) \oplus (\Theta_2 \otimes x_{j2}) \oplus (\Theta_3 \otimes x_{j3})\right) \\
&= \sum_{j=1}^{20} \left[|y_j - \sum_{k=0}^{3} x_{kj}\, \theta_k| + 0.5|g(l_j) - \sum_{k=0}^{3} x_{kj}\, \alpha_k| + 0.5|g(r_j) - \sum_{k=0}^{3} x_{kj}\, \beta_k| \right], \\
E_{LS} &= \sum_{j=1}^{20} \mathcal{D}^2\left((y_j, g(l_j), g(l_j)), \Theta_0 \oplus (\Theta_1 \otimes x_{j1}) \oplus (\Theta_2 \otimes x_{j2}) \oplus (\Theta_3 \otimes x_{j3})\right) \\
&= \sum_{j=1}^{20} \left[(y_j - \sum_{k=0}^{3} x_{kj}\, \theta_k)^2 + 0.5(g(l_j) - \sum_{k=0}^{3} x_{kj}\, \alpha_k)^2 + 0.5(g(r_j) - \sum_{k=0}^{3} x_{kj}\, \beta_k)^2 \right],
\end{aligned}$$

Tables 8, 9 and 10 summarize the mathematical computations of VIKOR technique for the problem. Considering VIKOR steps in selecting the best solutions, it is clear that the alternatives $\{f_{\widehat{\Theta}_{11}}(\cdot), \ldots, f_{\widehat{\Theta}_{15}}(\cdot)\}$ are the best solutions for modeling the simulated dataset, because $Q_{(5)} - Q_{(1)} = 0.0034 < 0.1111 = \frac{1}{m-1} = \frac{1}{9}$ and $Q_{(6)} - Q_{(1)} = 0.1698 > 0.1111$.

Remark 3.1. *Each model inside the collection $\{f_{\widehat{\Theta}_{11}}(\cdot), \ldots, f_{\widehat{\Theta}_{15}}(\cdot)\}$ fits almost the same to the dataset. If someone wants to pick the final model it is better to choose the model with the highest value of q, i.e $f_{\widehat{\Theta}_{15}}(\cdot)$, because this model covers the most number of data points and labels the minimum number of data points as outliers/unusual points.*

In order to provide a comparison study, three following well-known criteria are employed to compare the performance values of the competitive models [1, 3, 4, 9, 29]

$$G_1 = \frac{1}{n}\sum_{i=1}^{n} \frac{\int \min\{\widetilde{y}_i(t), \widehat{\widetilde{y}}_i(t)\}\, dt}{\int \max\{\widetilde{y}_i(t), \widehat{\widetilde{y}}_i(t)\}\, dt}, \qquad G_2 = \frac{1}{n}\sum_{i=1}^{n} \int |\widetilde{y}_i(t) - \widehat{\widetilde{y}}_i(t)|\, dt, \qquad G_3 = \frac{1}{n}\sum_{i=1}^{n} \frac{\int |\widetilde{y}_i(t) - \widehat{\widetilde{y}}_i(t)|\, dt}{\int \widetilde{y}_i(t)\, dt}.$$

By considering the results in Table 11 it can be concluded that the models obtained as the solution of the proposed procedure produce superior results than the others. Also, it is remarkable that the other alternatives $\{f_{\widehat{\Theta}_{16}}(\cdot), \ldots, f_{\widehat{\Theta}_{20}}(\cdot)\}$ which are not selected by VIKOR technique do not work perfectly for such a simulated dataset.

Table 7: The solutions of the objective functions (4) as the candidate parameters obtained by the Algorithm

q	$\widehat{\Theta}_0^q$	$\widehat{\Theta}_1^q$	$\widehat{\Theta}_2^q$	$\widehat{\Theta}_3^q$
11	(-5.2,1.738)	(1.15,0.0062)	(0.959,-0.05226)	(1.32,0.00575)
12	(-3.7,1.115)	(1.07,0.0020)	(1.131,-0.00012)	(1.15,0.00154)
13	(-5.6,1.294)	(1.09,0.0130)	(1.205,-0.02823)	(1.13,-0.00805)
14	(-1.0,1.208)	(1.04,0.0142)	(1.088,0.02105)	(0.98,-0.05669)
15	(-6.8,2.362)	(1.11,0.0010)	(1.213,-0.03469)	(1.16,-0.06012)
16	(34.7,1.566)	(0.32,-0.0141)	(3.879,0.09440)	(-4.72,-0.12799)
17	(19.6,1.616)	(0.90,0.0100)	(-3.575,-0.02842)	(5.83,0.00658)
18	(45.5,1.662)	(8.90,0.0860)	(41.736,0.40549)	(-85.31,-0.86249)
19	(-317.9,-1.513)	(1.18,0.0029)	(1.379,0.01215)	(0.99,-0.00049)
20	(-317.9,-1.513)	(1.18,0.0029)	(1.379,0.01215)	(0.99,-0.00049)
LS	(20.91,1.263)	(0.813,0.0048)	(-0.041,-0.0096)	(0.986,-0.0010)
LA	(20.7,1.245)	(0.77,0.0116)	(0.024,-0.02418)	(1.06,0.00011)

Table 8: Decision matrix for mathematical computations of VIKOR method. Each row of the table is the vector of errors of the models on data points

Alternatives: Estimated models in $\mathcal{E}$	Attributes: Data points j									
i	1	2	3	4	5	6	7	8	9	10
Model 1: $f_{\widehat{\Theta}_{11}}(\cdot)$	0.0021	0.0007	0.0007	0.0013	0.0000	0.0000	0.0014	0.0007	0.0017	0.0005
Model 2: $f_{\widehat{\Theta}_{12}}(\cdot)$	0.0016	0.0009	0.0000	0.0003	0.0000	0.0008	0.0019	0.0007	0.0007	0.0000
Model 3: $f_{\widehat{\Theta}_{13}}(\cdot)$	0.0010	0.0013	0.0006	0.0010	0.0006	0.0003	0.0015	0.0002	0.0000	0.0007
Model 4: $f_{\widehat{\Theta}_{14}}(\cdot)$	0.0009	0.0012	0.0005	0.0010	0.0010	0.0000	0.0014	0.0001	0.0000	0.0000
Model 5: $f_{\widehat{\Theta}_{15}}(\cdot)$	0.0010	0.0013	0.0008	0.0011	0.0007	0.0001	0.0012	0.0000	0.0000	0.0011
Model 6: $f_{\widehat{\Theta}_{16}}(\cdot)$	0.0244	0.0059	0.0000	0.0000	0.0219	0.0078	0.0033	0.0147	0.0238	0.0000
Model 7: $f_{\widehat{\Theta}_{17}}(\cdot)$	0.0391	0.0037	0.0150	0.0000	0.0342	0.0138	0.0000	0.0252	0.0341	0.0285
Model 8: $f_{\widehat{\Theta}_{18}}(\cdot)$	0.0284	0.0000	0.0333	0.0331	0.0460	0.0411	0.0186	0.0322	0.0000	0.0225
Model 9: $f_{\widehat{\Theta}_{19}}(\cdot)$	0.0662	0.0669	0.0664	0.0670	0.0668	0.0666	0.0663	0.0666	0.0664	0.0670
Model 10: $f_{\widehat{\Theta}_{20}}(\cdot)$	0.0662	0.0669	0.0664	0.0670	0.0668	0.0666	0.0663	0.0666	0.0664	0.0670
$d_j^+ = \min_{i=1,\ldots,10} d_{ij}$	0.0009	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$d_j^- = \max_{i=1,\ldots,10} d_{ij}$	0.0662	0.0669	0.0664	0.0670	0.0668	0.0666	0.0663	0.0666	0.0664	0.0670
w_j	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Table 9: Table 8 continued

Alternatives: Estimated models in $\mathcal{E}$	Attributes: Data points j									
i	11	12	13	14	15	16	17	18	19	20
Model 1: $f_{\widehat{\Theta}_{11}}(\cdot)$	0.0000	0.0004	0.0004	0.0000	0.0003	0.1916	0.1979	0.2015	0.1981	0.2007
Model 2: $f_{\widehat{\Theta}_{12}}(\cdot)$	0.0003	0.0000	0.0011	0.0006	0.0008	0.1947	0.1986	0.1998	0.1972	0.1998
Model 3: $f_{\widehat{\Theta}_{13}}(\cdot)$	0.0009	0.0000	0.0004	0.0000	0.0000	0.1955	0.1986	0.1999	0.1971	0.2003
Model 4: $f_{\widehat{\Theta}_{14}}(\cdot)$	0.0009	0.0012	0.0007	0.0012	0.0000	0.1991	0.1991	0.1957	0.1970	0.1991
Model 5: $f_{\widehat{\Theta}_{15}}(\cdot)$	0.0010	0.0000	0.0000	0.0005	0.0003	0.1941	0.1982	0.2008	0.1973	0.2005
Model 6: $f_{\widehat{\Theta}_{16}}(\cdot)$	0.0174	0.0116	0.0101	0.0263	0.0142	0.0000	0.1525	0.2874	0.1897	0.1890
Model 7: $f_{\widehat{\Theta}_{17}}(\cdot)$	0.0373	0.0863	0.0414	0.0461	0.0395	0.3231	0.0000	0.2016	0.0310	0.0000
Model 8: $f_{\widehat{\Theta}_{18}}(\cdot)$	0.0311	0.0531	0.0534	0.0531	0.0529	0.2836	0.0000	0.2166	0.0010	0.0000
Model 9: $f_{\widehat{\Theta}_{19}}(\cdot)$	0.0669	0.0660	0.0668	0.0664	0.0668	0.0000	0.0000	0.0000	0.0000	0.0011
Model 10: $f_{\widehat{\Theta}_{20}}(\cdot)$	0.0669	0.0660	0.0668	0.0664	0.0668	0.0000	0.0000	0.0000	0.0000	0.0011
$d_j^+ = \min_{i=1,\ldots,10} d_{ij}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$d_j^- = \max_{i=1,\ldots,10} d_{ij}$	0.0669	0.0863	0.0668	0.0664	0.0668	0.3231	0.1991	0.2874	0.1981	0.2007
w_j	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Table 10: Summarized mathematical computations of VIKOR method ($Q = 0.9 * S^N + 0.1 * R^N$)

Alternatives: Estimated models in $\mathcal{E}$ i	S_i	S_i^N	R_i	R_i^N	Q_i	$Q_{(i)}$	$S_{(i)}$	$R_{(i)}$
Model 1: $f_{\widehat{\mathbf{\Theta}}_{11}}(\cdot)$	0.4426	0.0019	0.1000	1.0000	0.1017	0.0983	0.4406	0.0878
Model 2: $f_{\widehat{\mathbf{\Theta}}_{12}}(\cdot)$	0.4421	0.0014	0.0998	0.9830	0.0996	0.0996	0.4414	0.0998
Model 3: $f_{\widehat{\mathbf{\Theta}}_{13}}(\cdot)$	0.4406	0.0000	0.0998	0.9830	0.0983	0.0999	0.4417	0.0998
Model 4: $f_{\widehat{\mathbf{\Theta}}_{14}}(\cdot)$	0.4417	0.0011	0.1000	1.0000	0.1010	0.1010	0.4421	0.0999
Model 5: $f_{\widehat{\mathbf{\Theta}}_{15}}(\cdot)$	0.4414	0.0007	0.0999	0.9926	0.0999	0.1017	0.4426	0.1000
Model 6: $f_{\widehat{\mathbf{\Theta}}_{16}}(\cdot)$	0.6342	0.1868	0.1000	1.0000	0.2681	0.2681	0.6342	0.1000
Model 7: $f_{\widehat{\mathbf{\Theta}}_{17}}(\cdot)$	0.8224	0.3684	0.1000	1.0000	0.4316	0.3930	0.8224	0.1000
Model 8: $f_{\widehat{\mathbf{\Theta}}_{18}}(\cdot)$	0.8931	0.4366	0.0877	0.0000	0.3930	0.4316	0.8931	0.1000
Model 9: $f_{\widehat{\mathbf{\Theta}}_{19}}(\cdot)$	1.4770	1.0000	0.1000	1.0000	1.0000	1.0000	1.4770	0.1000
Model 10: $f_{\widehat{\mathbf{\Theta}}_{20}}(\cdot)$	1.4770	1.0000	0.1000	1.0000	1.0000	1.0000	1.4770	0.1000

Table 11: Comparison between different fuzzy regression models

Methods	G_1	G_2	G_3
Ordinary least-absolutes method	0.5526	2.5266	0.7536
Ordinary least-squares method	0.4596	2.9909	0.8730
Model proposed by Chachi [7]	0.4753	2.9255	0.8583
Model 1: $f_{\widehat{\mathbf{\Theta}}_{11}}(\cdot)$	0.5251	2.0123	0.5474
Model 2: $f_{\widehat{\mathbf{\Theta}}_{12}}(\cdot)$	0.5254	3.6339	1.0157
Model 3: $f_{\widehat{\mathbf{\Theta}}_{13}}(\cdot)$	0.5406	2.2344	0.6159
Model 4: $f_{\widehat{\mathbf{\Theta}}_{14}}(\cdot)$	0.5414	10.4728	2.9074
Model 5: $f_{\widehat{\mathbf{\Theta}}_{15}}(\cdot)$	0.5203	2.0029	0.5513
Model 6: $f_{\widehat{\mathbf{\Theta}}_{16}}(\cdot)$	0.2151	4.5643	1.3149
Model 7: $f_{\widehat{\mathbf{\Theta}}_{17}}(\cdot)$	0.2356	6.0410	1.7773
Model 8: $f_{\widehat{\mathbf{\Theta}}_{18}}(\cdot)$	0.0200	3.958785e+08	1.106131e+08
Model 9: $f_{\widehat{\mathbf{\Theta}}_{19}}(\cdot)$	0.2039	3.1798	0.8937
Model 10: $f_{\widehat{\mathbf{\Theta}}_{20}}(\cdot)$	0.2039	3.1798	0.8937

4 Real world experiments

To investigate the robust performance of our proposed model in the presence of anomalous data the following numerical experiment will be performed. The dataset are information about the Prediction of Affordable Levels of House Prices [78]. A survey for the city of Shanghai was conducted with 147 individual questionnaires to collect the observation of the following variables:

- $\widetilde{y}$: acceptable purchase price (fuzzy dependent variable),
- x_1: housing size,
- x_2: mortgage interest rate,
- x_3: real estate tax,
- x_4: down payment ratio,
- x_5: annual household income,
- x_6: family population.

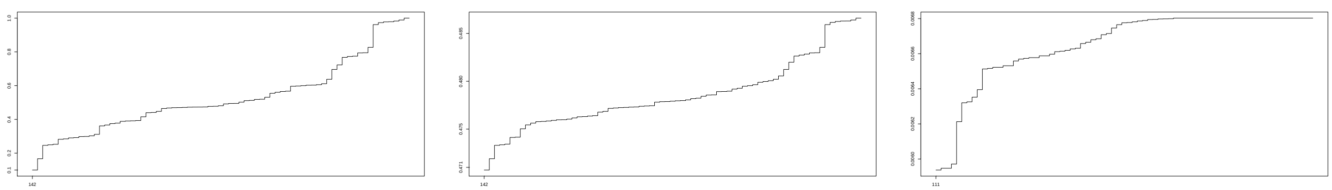
4.1 Model estimation

To analyse such a dataset, first, the following objective functions are minimized running the proposed algorithm

$$\min_{\mathbf{\Theta}} [O(\mathbf{\Theta}, 73), \ldots, O(\mathbf{\Theta}, 147)]. \quad (5)$$

Table 12: Summarized results of VIKOR method employed on the house price dataset. Staircase plots of values of Q, S and R are plotted in Figure 1a, Figure 1b and Figure 1c, respectively

Rank (i)	q	$Q_{(i)} = Q_q$	q'	$S_{(i)} = S_{q'}$	q''	$R_{(i)} = R_{q''}$
1	142	0.100	142	0.471	111	0.006
2	132	0.167	132	0.472	113	0.006
3	140	0.246	140	0.473	112	0.006
4	143	0.250	143	0.473	110	0.006
5	144	0.253	144	0.473	80	0.006
6	93	0.282	93	0.474	116	0.006
7	92	0.285	92	0.474	114	0.006
8	110	0.290	118	0.475	118	0.006
9	118	0.292	116	0.475	115	0.006
10	111	0.298	131	0.476	99	0.007
⋮	⋮	⋮	⋮	⋮	⋮	⋮



(a) Staircase plot of the vector values of $Q_{(\cdot)}$ in Table 12 (b) Staircase plot of the vector values of $S_{(\cdot)}$ in Table 12 (c) Staircase plot of the vector values of $R_{(\cdot)}$ in Table 12

Figure 1: Full results of VIKOR method in Table 12

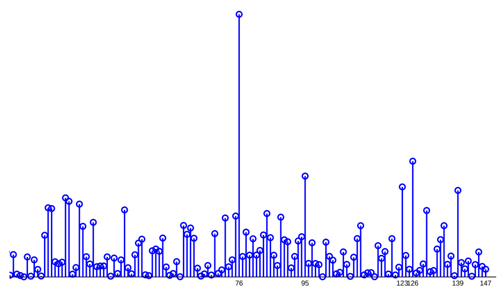
Table 13: Coefficients' estimates of $f_{\boldsymbol{\Theta}_{142}}(\cdot)$ in modeling house price dataset ($\widehat{\boldsymbol{\Theta}}_{142} = (\widehat{\Theta}_0, \widehat{\Theta}_1, \ldots, \widehat{\Theta}_6)$)

j	Var.	$\widehat{\Theta}_j$
0	Int.	(-562.9086, 2.9000, 2.8607)
1	x_1	(7.6605, 0.0190, 0.0124)
2	x_2	(-6.4251,-0.0019, 0.0093)
3	x_3	(-69.2543,-0.0595,-0.0222)
4	x_4	(-1.0295,-0.0051,-0.0061)
5	x_5	(14.0904, 0.0180, 0.0262)
6	x_6	(-1.3635,-0.0052,-0.0252)

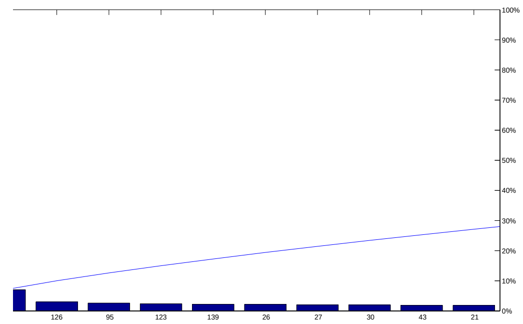
Then, the best ranked solutions of the optimization problem (5) is obtained by VIKOR technique. A summarized mathematical computations of VIKOR method is given in Table 12 showing the first ten best ranked alternatives. The sorted vector values of Q, S and R are plotted in Figure 1a, Figure 1b and Figure 1c as staircase plots, respectively. Considering VIKOR's steps in selecting the best solution, it can be concluded the alternative $f_{\widehat{\boldsymbol{\Theta}}_{142}}(\cdot)$ is selected as the best solution for modeling such a dataset. The estimated parameters values of $\boldsymbol{\Theta}_{142}$ are given in Table 13.

4.2 Outlier detection analysis

In order to investigate that the outlier points can be detected by the proposed method, the individual deviations of the optimal model (5) are shown in Figure 2a and Figure 2b. The estimated model labels 142 data points as good data points and the remaining 5 data points as outlier points. In order to detect the outlier points the Pareto Graph of the error values is plotted in Figure 2b. The Pareto Graph displays the values of the errors in descending order which have the most influence on the total sum of errors. Now, by analyzing the results depicted in Figure 2a and Figure 2b, it is clear that the observations of No. $\{76, 95, 123, 126, 139\}$ are labeled as the outlier/unusual points by the proposed method. Here, $e_{(147)} = e_{76}$, $e_{(146)} = e_{126}$, $e_{(145)} = e_{95}$, $e_{(144)} = e_{123}$, $e_{(143)} = e_{139}$ are the five biggest values of the errors when the estimated model $f_{\widehat{\boldsymbol{\Theta}}_{142}}(\cdot)$ is fitted to the dataset.



(a) Individual errors of estimated model $f_{\widehat{\Theta}_{142}}(\cdot)$



(b) The Pareto Graph which displays the first ten biggest values in the vector of errors as bars drawn in descending order. These values provide 28% of the total sum of errors. The first five biggest values provide 17.5% of the total sum of errors

Figure 2: Analysis of errors of estimated model $f_{\widehat{\Theta}_{142}}(\cdot)$

Table 14: Comparison between different fuzzy regression models

Methods	G_1	G_2	G_3
Arefi [3]	0.745	37.02	0.30
Khammare et al. [42]	0.730	38.95	0.32
Taheri and Chachi [65]	0.742	39.36	0.33
Chachi [7]	0.753	34.05	0.31
Xu and Li [71]	0.739	37.87	0.32
Ferraro et al. [25]	0.732	38.58	0.33
Zeng et al. [77]	0.724	39.72	0.36
$f_{\widehat{\Theta}_{142}}(\cdot)$	0.755	36.41	0.29

4.3 Comparison study

Here, in order to provide a comparison study the performance analysis of several fuzzy regression models in the literature will be examined and compared. To do so, the goodness-of-fit performance analysis of the methods proposed by Chachi [7], Arefi [3], Khammar et al. [42], Xu and Li [71], Taheri and Chachi [65], Ferraro et al. [25] and Zeng et al. [77] will be compared together. The competitive fuzzy regression models are fitted to the dataset and their goodness-of-fit values are obtained in Table 14. Although the results given in Table 14 are in favour of the proposed method but it is important to consider the following items.

Remark 4.1. *The obtained goodness-of-fit values in Table 14 are more or less the same because there are not effective outlier points in such a dataset. Notice for such a datset with six independent variables and 147 individuals the existence of such a low amount of potential outliers does not provide enough effective strength. In order to provide a big difference between robust and non-robust methods at least* 10% *to* 25% *of the dataset must have the potentialities of being outlier points. But first note that, the notion of an outlier is somewhat fuzzy, i.e, all data can be qualified as outliers to some degree, based on how well the model fits each datum. Then, by down-weighting poorly fitted data between 0 and 1, which are considered outliers, we can limit their effect on the model.*

5 Concluding remarks

In this paper a new technique was developed to address the problem of selecting the most desirable fuzzy regression model based on MADM method. In this regard, we discussed multi-attribute decision-making method in a fuzzy environment to solve the fuzzy regression problem. The proposed solution method emphasizes two important points. First, the associated alternatives are determined by a multi-objective optimization procedure for fuzzy regression model, where the outlier points observed in the dataset are considered during the evaluation. Second, we provided a multi-attribute evaluation scheme to obtain the total evaluation and rank the alternatives for multi-attribute decision making.

The analysis also provides a complete rank order of the alternatives, an ordered list of best alternatives, and the most appropriate alternative. Meanwhile, the most appropriate alternative also determines and labels the set of outlier points. Goodness-of-fit criteria have been used to evaluate the performance of alternatives while compared to several other known methods in fuzzy regression optimization techniques.

Here, we discuss some limitations of the proposed approach and give in-depth directions for future researches.

1. This paper proposes a multi-objective optimization approach to fuzzy regression modeling. Although the multi-objective optimization approach can be regarded as a generalized version for a single optimization method, what remains unsolved as the biggest issue is the derivation of appropriate solution that plays the key role of determining the final performance. Two methods are involved in explaining multi-objective optimization problems:

   (a) In the first method, all the objectives are aggregated into one function, or all but one objective is moved into the sets of constraint sets.

   (b) In the latter case, the aim is to find a group of Pareto-optimal solutions (optimal solutions) instead of one best solution, upon which the final decision can be made by the designer.

   Methods like utility theory and weighted sum technique are used to deal with the first approach. The problem with this approach is that it is not easy to select the weights and utility functions. One commonly used technique to aggregate the information in the multi-objective optimization problem is to obtain the representative values that can be used to compare the alternatives.

2. Some of above proposed methods have their own complexity in their context. We can propose several methods to reduce the complexity of such methods. Fast algorithm solving methods can be investigated. In this regard, in order to reduce the complexity of the method, we proposed the elemental set approach as one of the approaches to reduce the complexity of computations for larger datasets. Some other techniques can be formalized to facilitate the problem of large scale dataset. Many methods which propose fast algorithm solving such a problem, e.g., methods for Least Trimmed Squares (LTS).

3. One could think about whether it is possible to combine the parameters associated with the model and/or to combine the objective functions associated with the model using weighted operators into one error function that can then be simultaneously optimized. Such a technique can also be used to answer the limitations in fuzzy regression estimation problems at the presence of outlier points [7]. Using weighted operators, robust to outlier operators on ordered errors can be investigated as objective functions. Also some interesting aspects of (ordered) weighted operators (e.g. non-trimmed weighting methods) can be really investigated.

4. A systematical approach of deriving the breakdown point q (or cutoff point for outliers) or even providing a guideline as to optimizing this breakdown point, which is paramount considering the optimization of model parameters can be conducted, even in a data-driven manner. From the theoretical point of view, it can be investigated how to determine the optimal value of q while giving us the highest breakdown point, limits the convergence and coverage of the method. Presumably, here some contributions can be investigated to look at the rolls of the cutoff point for outliers in a data-driven manner.

5. The proposed approach is practical to be applied in some case studies even for very large-sized datasets.

6. Other proper MADM techniques can be investigated similarly to determine the final solution.

7. Other studies can be proposed in the framework of multi-objective optimization approach to fuzzy regression where any combination of the inputs, coefficients and outputs could be "fuzzy", or another MADM technique is used.

Acknowledgement

The Authors would like to thank the reviewers for the thoughtful comments and the constructive suggestions which helped to improve the quality of this manuscript. They also appreciate the Editor's warm work earnestly, which helped to make the manuscript suitable for possible publication in the eminent journal.

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Iranian Journal of Fuzzy Systems
Volume 18, Number 4 (2021), pp. 211
University of Sistan and Baluchestan

A multi-attribute assessment of fuzzy regression models

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رویکرد چند شاخصه در مدل‌های رگرسیون فازی

چکیده. اکثر مدل‌های رگرسیون فازی که تاکنون در ادبیات تحقیق مربوطه پیشنهاد شده‌است، از یک تابع هدف منفرد به منظور برآورد مدل‌های رگرسیون فازی استفاده می‌کنند. این رویکردها اغلب به خاطر عملکرد ناکافی در تحلیل نتایج و یا حساسیت نسبت به مشاهدات پرت، مورد انتقاد واقع می‌شوند. از این رو، به منظور برطرف کردن انتقادات پیش گفته شده، این مقاله یک رویکرد بهینه‌سازی چند هدفه-دو مرحله‌ای را در مسائل مدل‌های رگرسیون فازی معرفی می‌کند. به منظور حل و فصل مسائل مربوط به مشاهدات پرت، در مرحله اول یک تابع هدف پویا نسبت به مشاهدات پرت در فرآیند برآوردیابی در نظر گرفته شده‌است. مسأله برآوردیابی با اجرای الگوریتمی که مجموعه‌ای از مدل‌های رگرسیون فازی را تولید می‌کند، حل می‌شود. در مرحله بعد، یک طرح تصمیم‌گیری چند معیاره پیشنهاد می‌شود. بدین منظور، جهت فراهم نمودن فرآیند رتبه‌بندی مدل‌های رگرسیون فازی تولید شده در مرحله اول، روش ویکور VIKOR، به عنوان یک روش مناسب به کار برده می‌شود تا بر این اساس مطلوب‌ترین مدل انتخاب شود. به منظور تأکید بر امتیازات رویکرد پیشنهادی، نتایج مربوط به مثال‌های شبیه‌سازی شده و داده‌های واقعی مربوط به قیمت منازل مسکونی آورده شده‌است. این نتایج مؤید مؤثر بودن روش بهینه‌سازی چند هدفه پیشنهادی در حل و فصل مسأله مشاهدات پرت با بهبود دقت پیش‌بینی مدل است.