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Original paper

Continuous probability-interval valued fuzzy preference relations and its application in group decision making

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Abstract

Probabilistic hesitant fuzzy set represents the occurrence probabilities of elements. The probabilistic hesitant fuzzy preference relations can more effectively express the hesitant preference information of decision makers. But in the existing research, all of them are based on discrete probability distribution. In order to give decision maker more evaluation space, continuous probability distribution is necessary to be considered. Therefore, in this paper, the continuous probability-interval valued fuzzy set is defined and its probability is represented by a probability density function. A method of converting probabilistic hesitant fuzzy set into continuous probability-interval valued fuzzy set is developed to transform discrete data into continuous data. Then, the continuous probability-interval valued fuzzy preference relations is presented. In order to consider the consistency of continuous probability-interval valued fuzzy preference relations, the multiplication consistent expected preference relations is proposed. The individual consistency index and group consensus index are also presented to determine the consistency level. And then, an algorithm is introduced for checking and improving the individual consistency level and group consensus level. Finally, a numerical example is shown to the effectiveness of proposed algorithm, the comparative analysis is given with the existing methods to show the superiority of this algorithm.

Keywords: Continuous probability-interval valued fuzzy set, continuous probability-interval valued fuzzy preference relations, consistency, group decision making.

1 Introduction

In 1965, Zadeh [35] first proposed the concept of fuzzy set (FS), which describe and model a large number of fuzzy concepts and phenomena in the real world with precise mathematical means. It is widely applied in decision making [3, 5, 14, 29], clustering analysis [19, 33], pattern recognition [15, 16], fuzzy control [10, 31] and so on. With the development of FS, Gorzaczany [7] extended FS to interval valued fuzzy set (IVFS). But FS only consider membership function and has one-sidedness, such as voting system. In order to improve this shortcoming, the non-membership function and hesitant function were added in FS. Based on it, intuitionistic fuzzy set (IFS) was introduced by Atanassov [1] in 1986. To consider more information, Atanassov and Gargov [2] extended IFS to interval-valued intuitionistic fuzzy set (IVIFS). Further, Torra [20] introduced the concept of hesitant fuzzy set (HFS), which express membership function as a set of possible values. It shows the hesitation of decision makers in a better way. After, Xia et al. [28] expressed the definition of HFS in a mathematical manner. However, the probabilities of elements in the HFS are all equal. In order to provide better decision space for decision makers, Zhu [39] proposed the probabilistic hesitant fuzzy set (PHFS), which assign each element to the corresponding probability. Combing with IVFS and PHFS, Zhang et al. [36] developed probability-interval valued hesitant fuzzy set (P-IVHFS).

For fuzzy decision making problem, decision making with fuzzy preference relations is an important method. Fuzzy preference relations [4, 8, 9], interval valued fuzzy preference relations [21, 24], intuitionistic fuzzy preference relations [6, 32] and interval valued intuitionistic fuzzy preference relations [18, 22, 23] are widely used in decision making.

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Because of the probabilistic hesitant fuzzy preference relations (PHFPRs) [40] can better reflect the wishes of decision makers. It has attracted the attention and research of scholars in recent years. For preference relations, their transitivity should be satisfied, that is, if X is preferred to Y, Y is preferred to Z, then X is preferred to Z. However, most of the preference information provided by decision makers is inconsistent in decision making process. Thus, consistency test is an important topic. The consistency test is mainly divided into multiplicative consisitency and additive consistecy. Some consensus optimization models [17, 25, 27] has been proposed. In PHFPRs, for multiplicative consistecy, Zhou and Xu [38] developed the probability calculation method for the PHFPRs based on the expected multiplicative consistency. Jian and Wang [11] introduced a programming model to meet the individual consistency level and group consensus level simultaneously in group decision making (GDM) with PHFPRs. Li and Wang [13] constructed the expected multiplicative consistency index to determine the consistency level for PHFPRs. For additive consistecy, Zhou and Xu [37] introduced a probability calculation method based on expected additive consistency and then developed a consistency-improving iterative algorithm for GDM with PHFPRs. Li and Wang [12] proposed expected additive consistency PHFPRs and then to make the GDM. Wu et al. [26] developed a novel consensus reaching process with PHFPRs and application in local feedback strategy. There are also some researchers in probability-interval valued hesitant fuzzy preference relations (P-IVHFPRs). Zhang et al. [36] proposed a fusion method in GDM with P-IVHFPRs. Xu et al. [34] introduced a decomposition method to deal with the consistency of P-IVHFPRs.

Although there are many studies on PHFPRs and P-IVHFPRs, and the probability distribution is discrete. The advantage of continuous data is that it is applicable to the situation where all possible values of variables cannot be listed one by one, but only any point in a certain interval on the number axis, such as the life of electronic components, measurement error, height change and so on. Therefor, comparing with discrete data, continuous data can better express uncertain information and give decision makers more decision space. Therefore, we introduce the concept of continuous probability-interval valued fuzzy set (CP-IVFS) and its preference relations. Considering the consistency of preference relations, we develop a method to address GDM problem under continuous probability-interval valued fuzzy preference relations (CP-IVFPRs). The main contributions of this paper are as follows:

(1) Inspired by continuous random variable, the interval valued corresponding probability is expressed as a probability density function. Constructing a method for transforming discrete fuzzy data into continuous fuzzy data.

(2) Consistency test is an important research issue in the preference relations. Thus, individual consistency and group consensus are considered base on CP-IVFPRs in this paper. And the algorithm is proposed to improve the individual consistency and group consensus level.

(3) Zhang et al. and Xu et al. [34, 36] considers the discrete probability interval valued hesitant fuzzy preference information for GDM. Xu et al. [30] considers the interval valued hesitant fuzzy preference information in GDM. However, the methods[30, 36] do not consider the consistency of their preference information. Therefore, comparing with the existing methods [30, 34, 36], our method not only expresses the probability with continuous distribution function, but also considers individual consistency and group consensus. From the example in this paper, the results obtained by our method are more effective and reasonable.

The rest structure of this paper is organized as follows: In section 2, some basic concepts are reviewed. In section 3, the concept of CP-IVFS and normalization CP-IVFS are developed. The ranking method of CP-IVFS is also proposed. Further, based on CP-IVFS, the CP-IVFPRs is introduce. In section 4, multiplication consistent expected preference relations of CP-IVFPRs is defined. The CP-IVFPRs consistency index and a programming model to improve the consistency level are also presented. In section 5, group consensus index is introduced. A model is developed to meet individual consistency and group consensus level. An algorithm for GDM with CP-IVFPRs is proposed. In section 6, an example for overseas high-level talent introduction program is solved, and the comparative analysis shows the advantages of our method. In section 7, the conclusions are presented.

2 Preliminaries

In 1987, Gorzałczany [7] first introduced interval valued fuzzy set (IVFS) to deal with more uncertain information.

Definition 2.1. [7] *Let X be a reference set, A IVFS is definition as $A = \{x, \mu_A(x)|x \in X\}$, where $\mu_A(x) = [\mu_A^L(x), \mu_A^U(x)]$, $\mu_A^L, \mu_A^U : X \to [0,1], \forall x \in X$, $\mu_A^L \leq \mu_A^U$. μ_A^L is the low limit of membership degree and μ_A^U is the upper limit of membership degree.*

Torra [20] proposed the concept of hesitant fuzzy set (HFS) by expressing membership degree with several possible values.

Definition 2.2. [20] *Let X be a reference set, then a HFS associated with X in term of a function to X return a subset of $[0,1]$.*

Xia et al. [28] expressed the definition of HFS in a mathematical manner as follows:

$$A = \{< x, h_A(x) > | x \in X\},$$

where function $h_A(x)$ is a set of different values in [0,1] representing the possible membership degrees of element x in X to A.

In order to give the decision makers more space for hesitant, Zhu [39] introduced the concept of probabilistic hesitant fuzzy set (PHFS) as an extension of HFS, which membership degrees are given corresponding probability.

Definition 2.3. [39] *Let X be a universal set, then a PHFS P on X can be defined as:*

$$P = \{< x, h_x(p_x) > | x \in X\},$$

where the function h_x is a set of different valued in [0,1], which is described by the probability distribution p_x; where h_x denotes the possible membership degree of element x in X to P. For convenience, $h_x(p_x)$ is named probabilistic hesitant fuzzy element(PHFE) and denoted as $h(p)$. It is indicated by:

$$h(p) = \{\gamma_i(p_i) | i = 1, 2, ..., \#h\},$$

where p_i satisfying $\Sigma_{i=1}^{\#h} p_i = 1$, is the probability of the possible valued γ_i, and $\#h$ is the number of all $\gamma_i(p_i)$ in $h(p)$.

3 Continuous probability-interval valued fuzzy set

According to the current literature, the existing studies on PHFS and P-IVHFS are aimed at discrete data and interval data, continuous data can express more information than discrete data. Therefore, in this section, we propose the continuous probability-interval valued fuzzy set (CP-IVFS) which membership degrees is interval fuzzy set and probability is a probability density function. Further, we also introduce a method for transforming PHFS into CP-IVFS.

3.1 Continuous probability-interval valued fuzzy set

Following, we introduce CP-IVFS, normalized continuous probability-interval valued fuzzy set (NCP-IVFS) and its ranking method based on score function and deviation function.

Definition 3.1. *Let X be a universal set, and $D[0,1]$ be the set of all closed subintervals of [0,1]. A CP-IVFS on X is*

$$A = \{< x, h_x(f(x)) > | x \in X\},$$

where $h_x : X \to D[0,1]$ denotes interval value membership degrees of the element $x \in X$ to the set A, $f(x)$ is a probability density function, which is used to describe the probability distribution of h_x. For convenience, $h_x(f(x))$ is named continuous probability-interval valued fuzzy element (CP-IVFE).

In Definition 3.1, $h_x = [h^L, h^U]$ is an interval number, $h^L = inf h_x$ is the lower limit of h_x, and $h^U = sup h_x$ is the upper limit of h_x.

The commonly used probability distribution functions are as follows:

(1) Uniform distribution: $U_{[a,b]}(x) = \begin{cases} \frac{1}{b-a}, & a < x < b; \\ 0, & else. \end{cases}$ is its density distribution function. Where a and b are two boundaries. The uniform distribution is applicable to the case where the distribution probability of the same length interval is equal.

(2) Exponential distribution: $g_\lambda(x) = \begin{cases} \lambda e^{\lambda x}, & x > 0; \\ 0, & x \leq 0. \end{cases}$ is its density distribution function. Where $\lambda > 0$ is a parameter of distribution, often referred to as rate parameter. The exponential distribution is applicable to the case where events occur continuously and independently at a constant average rate.

(3) Normal distribution: $N_{(\mu,\sigma^2)}(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ is its density distribution function, denotes the random variable X obey a normal distribution with mathematical expectation of μ and variance of σ^2. Normal distribution is applicable to a case where a variable is affected by a large number of small, independent random factors.

(4) F distribution: $f_{(m,n)}(x) = \begin{cases} \frac{\Gamma(\frac{m+n}{2})}{\Gamma(\frac{n}{2})\Gamma(\frac{m}{2})} m^{\frac{m}{2}} n^{\frac{n}{2}} x^{\frac{m}{2}-1} (n + mx)^{-\frac{m+n}{2}}, & x > 0; \\ 0, & x \leq 0. \end{cases}$ is its density distribution function, denotes the random variable X obeys the F distribution with degrees of freedom n and m, where $\Gamma(x) = \int_0^{+\infty} t^{x-1} e^{-t}\, dt$. F distribution is suitable for sampling distribution of two normal population.

(5) t distribution: $t_n(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\sqrt{n\pi}}\left(1+\frac{x^2}{n}\right)^{\frac{n+1}{2}}$ is its density distribution function, denotes the random variable X obeys the t distribution with degree of freedom n. t distribution is suitable for estimating normal distribution and unknown variance based on small samples.

Example 3.2. *Let $X = \{x_1, x_2, x_3\}$ be a universal set, and the three CP-IVFEs*

$$h_{x_1}(f(x_1)) = [0.6, 0.9](N_{(0.75,0.9)}(x)), \quad h_{x_2}(f(x_2)) = [0.4, 0.7](U_{[0.5,1]}(x)), \quad h_{x_3}(f(x_3)) = [0.1, 0.4](g_2(x)),$$

denotes the membership degrees of $x_i (i = 1, 2, 3)$ to the set A, A is a CP-IVFS, where

$$A = \{< x_1, [0.3, 0.5](f_{(1,10)}(x)) >, < x_2, [0.4, 0.7](U_{[0.5,1]}(x)) >, < x_3, [0.1, 0.4](g_2(x)) >\}.$$

Following, we develop NCP-IVFS.

Definition 3.3. *Let X be a universal set, $A = \{< x, h_x(f(x)) > | x \in X\}$ is a CP-IVFS. Then the NCP-IVFS is defined as*

$$A' = \left\{\left\langle x, h_x\left(\frac{f(x)}{\int_{infh_x}^{suph_x} f(x)\,dx}\right)\right\rangle | x \in X\right\}.$$

Example 3.4. *For Example 3.2, the NCP-IVFS is*

$$A = \left\{\left\langle x_1, [0.6, 0.9]\left(\frac{N_{(0.75,0.9)}(x)}{\int_{0.6}^{0.9} N_{(0.75,0.9)}(x)\,dx}\right)\right\rangle, \left\langle x_2, [0.4, 0.7]\left(\frac{U_{[0.5,1]}(x)}{\int_{0.4}^{0.7} U_{[0.5,1]}(x)\,dx}\right)\right\rangle, \left\langle x_3, [0.1, 0.4]\left(\frac{g_2(x)}{\int_{0.1}^{0.4} g_2(x)\,dx}\right)\right\rangle\right\}.$$

In order to rank the CP-IVFE, we propose the following comparison method of CP-IVFE.

Definition 3.5. *Let $h(f(x)) =< [a, b](f(x)) | [a, b] \in [0, 1]) >$ be a CP-IVFE. Its score function is defined as follow:*

$$E(h(f(x))) = \frac{\int_a^b x f(x)\,dx}{\int_a^b f(x)\,dx}.$$

Definition 3.6. *Let $h(f(x)) =< [a, b](f(x)) | [a, b] \in [0, 1]) >$ be a CP-IVFE. Its deviation function is defined as follow:*

$$V(h(f(x))) = \int_a^b (x - E(h(f(x))))^2 f(x)\,dx.$$

Based on the score function and deviation function, the ranking method for two CP-IVFEs is as follows.

Definition 3.7. *Let $h_1(f_1(x)) =< [a, b](f_1(x)) | [a, b] \in [0, 1]) >$ and $h_2(f_2(x)) =< [c, d](f_2(x)) | [c, d] \in [0, 1]) >$ be two CP-IVFEs. Their ranking law is defined as follows:*
(1) If $E(h_1(f_1(x))) < E(h_2(f_2(x)))$, then $h_1(f_1(x)) \prec h_2(f_2(x))$;
(2) If $E(h_1(f_1(x))) = E(h_2(f_2(x)))$, then
(a) If $V(h_1(f_1(x))) < V(h_2(f_2(x)))$, then $h_1(f_1(x)) \succ h_2(f_2(x))$;
(b) If $V(h_1(f_1(x))) = V(h_2(f_2(x)))$, then $h_1(f_1(x)) \sim h_2(f_2(x))$.

Example 3.8. *For Example 3.2, the score function of $x_i (i = 1, 2, 3)$ is*

$$E(h_{x_1}(f(x_1))) = 0.75, E(h_{x_2}(f(x_2))) = 0.75, E(h_{x_3}(f(x_3))) = 0.26.$$

Because $E(h_{x_1}(f(x_1))) = E(h_{x_2}(f(x_2)))$, then according to deviation function, we can get

$$V(h_{x_1}(f(x_1))) = 0.00094, V(h_{x_1}(f(x_1))) = 0.0475.$$

Therefore, the ranking result is $x_1 \succ x_2 \succ x_3$.

3.2 Data processing

In the existing researchs on PHFS, the data of membership degree and probability distribution given in the literatures are discrete. In order to construct a continuous probability distribution function based on existing discrete data to better describe hesitation information, we use the following method to transform probabilistic hesitation fuzzy information into continuous probabilistic-interval valued fuzzy information:

(1) Ranking probabilistic hesitant fuzzy information according to membership degree from small to large.

(2) An interval value membership degree $[a, b]$ is obtained by taking the minimum membership degree as the lower bound and the maximum membership degree as the upper bound.

(3) Observing the probability distribution of probabilistic hesitant fuzzy information, we select a known continuous probability density function which is very similar to this set of data (the parameters of this function are undetermined), the parameters are obtained by fitting. Then, the continuous probability density function $f(x)$ that best represents the characteristics of the data set is obtained.

(4) Finally, continuous probabilistic-interval valued fuzzy information $[a, b](f(x))$ is got.

Example 3.9. *Let $h(x_1) = \{0.2(0.4), 0.5(0.35), 0.8(0.25)\}$ and $h(x_2) = \{0.2(0.2), 0.4(0.4), 0.5(0.3), 0.7(0.1)\}$ be two PHFEs, according to the above description and MATLAB fitting toolbox, we can get that the corresponding CP-IVHFE are*

$$h(f(x_1)) = [0.2, 0.8](0.401e^{-(\frac{x-0.1526}{0.9418})^2}),$$

and

$$h(f(x_2)) = [0.2, 0.7](0.3831e^{-(\frac{x-0.3951}{0.248})^2}).$$

4 Continuous probability-interval valued fuzzy preference relations

In decision-making, decision makers provide evaluation information in different forms, in which preference relations can better express the preferences of decision makers. Therefore, in this section, we propose continuous probability-interval valued fuzzy preference relations (CP-IVFPRs) to describe continuous probability-interval valued fuzzy preference information. We also introduce the expected preference relations matrix and its distance measurement.

Definition 4.1. *Let $X = \{x_1, x_2, ..., x_n\}$ be a universal set. A CP-IVFPRs matrix R on X is represented by $R = (h_{ij}(f_{ij}(x)))_{n\times n}$, where $h_{ij}(f_{ij}(x))$ is a CP-IVFE, indicating preference degree of x_i to x_j. Moreover, $h_{ij}(f_{ij}(x))$ should satisfy*

$$\begin{cases} infh_{ij} + suph_{ji} = suph_{ij} + infh_{ji} = 1 \\ f_{ij}(x) = f_{ji}(1-x) \\ h_{ii}(f_{ii}(x)) = 0.5(1), \qquad i, j = 1, 2, ..., n. \end{cases}$$

where $\inf h_{ij}$ and $\sup h_{ij}$ denote the lower and upper limits of h_{ij} respectively.

To understand the meaning of each element in a CP-IVFPRs as a determined valued, we define the concept of expected preference relations.

Definition 4.2. *Let $R = (h_{ij}(f_{ij}(x)))_{n\times n}$ be a CP-IVFPRs matrix. Expected preference relations matrix corresponding to R is denoted as $E_R = (e_{h_{ij}})_{n\times n}$, where $e_{h_{ij}} = \frac{\int_{infh_{ij}}^{suph_{ij}} xf_{ij}(x)\,dx}{\int_{infh_{ij}}^{suph_{ij}} f_{ij}(x)\,dx}$.*

Theorem 4.3. *Let $R = (h_{ij}(f_{ij}(x)))_{n\times n}$ be a CP-IVFPRs matrix. The expected preference relations matrix is $E_R = (e_{h_{ij}})_{n\times n}$. Then, E_R is a fuzzy preference relations matrix.*

Proof. By $x \in [0, 1]$ and $f_{ij}(x)$ is a probability density function, then $0 \le xf_{ij}(x) \le f_{ij}(x)$. Thus, $0 \le \int_{infh_{ij}}^{suph_{ij}} xf_{ij}(x)\,dx \le \int_{infh_{ij}}^{suph_{ij}} f_{ij}(x)\,dx$, that is, $0 \le \frac{\int_{infh_{ij}}^{suph_{ij}} xf_{ij}(x)\,dx}{\int_{infh_{ij}}^{suph_{ij}} f_{ij}(x)\,dx} \le 1$. So, E_R is a fuzzy matrix.

By $e_{h_{ij}} = \frac{\int_{infh_{ij}}^{suph_{ij}} xf_{ij}(x)\,dx}{\int_{infh_{ij}}^{suph_{ij}} f_{ij}(x)\,dx}$ and $e_{h_{ji}} = \frac{\int_{1-suph_{ij}}^{1-infh_{ij}} xf_{ji}(x)\,dx}{\int_{1-suph_{ij}}^{1-infh_{ij}} f_{ji}(x)\,dx}$. Let $x = 1 - t$, then

$$e_{h_{ji}} = \frac{\int_{suph_{ij}}^{infh_{ij}} (1-t)f_{ji}(1-t)\,dt}{\int_{suph_{ij}}^{infh_{ij}} f_{ji}(1-t)\,dt} = \frac{\int_{infh_{ij}}^{suph_{ij}} (1-t)f_{ji}(1-t)\,dt}{\int_{infh_{ij}}^{suph_{ij}} f_{ji}(1-t)\,dt}.$$

According to $f_{ij}(x) = f_{ji}(1-x)$, therefore,

$$e_{h_{ij}} + e_{h_{ji}} = \frac{\int_{infh_{ij}}^{suph_{ij}} x f_{ij}(x)\,dx}{\int_{infh_{ij}}^{suph_{ij}} f_{ij}(x)\,dx} + \frac{\int_{infh_{ij}}^{suph_{ij}} (1-x) f_{ji}(1-x)\,dx}{\int_{infh_{ij}}^{suph_{ij}} f_{ji}(1-x)\,dx} = \frac{\int_{infh_{ij}}^{suph_{ij}} x f_{ij}(x)\,dx}{\int_{infh_{ij}}^{suph_{ij}} f_{ij}(x)\,dx} + \frac{\int_{infh_{ij}}^{suph_{ij}} (1-x) f_{ij}(x)\,dx}{\int_{infh_{ij}}^{suph_{ij}} f_{ij}(x)\,dx} = 1.$$

Thus, E_R is a fuzzy preference relations matrix. □

Example 4.4. *Suppose a decision maker provides a CP-IVFPRs for four alternatives $\{x_1, x_2, x_3, x_4\}$ as follows:*

$$R = \begin{pmatrix} 0.5(1) & [0.1, 0.5](t_{10}(x)) & [0.7, 0.9](t_5(x)) & [0.3, 0.4](t_7(x)) \\ [0.5, 0.9](t_{10}(1-x)) & 0.5(1) & [0.3, 0.7](t_1(x)) & [0.2, 0.5](t_3(x)) \\ [0.1, 0.3](t_5(1-x)) & [0.3, 0.7](t_1(1-x)) & 0.5(1) & [0.4, 0.8](t_5(x)) \\ [0.6, 0.7](t_7(1-x)) & [0.5, 0.8](t_3(1-x)) & [0.2, 0.6](t_5(1-x)) & 0.5(1) \end{pmatrix}.$$

The consistency level is set to $CI_0 = 0.94$. Based on Definition 4.2, the expected fuzzy preference relations matrix is

$$E_R = \begin{pmatrix} 0.5 & 0.3 & 0.8 & 0.35 \\ 0.7 & 0.5 & 0.5 & 0.35 \\ 0.2 & 0.5 & 0.5 & 0.6 \\ 0.65 & 0.65 & 0.4 & 0.5 \end{pmatrix}.$$

Following, we propose the distance measurement of fuzzy preference relations.

Definition 4.5. *Let $R_1 = (h^1_{ij}(f^1_{ij}(x)))_{n\times n}$ and $R_2 = (h^2_{ij}(f^2_{ij}(x)))_{n\times n}$ be two CP-IVFPRs matrices, $E_{R_1} = (e_{h^1_{ij}})_{n\times n}$ and $E_{R_2} = (e_{h^2_{ij}})_{n\times n}$ be their corresponding expected preference relations matrices. The distance between two expected preference relations matrix is defined as follows:*

$$d(E_{R_1}, E_{R_2}) = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} |e_{h^1_{ij}} - e_{h^2_{ij}}|. \quad (1)$$

5 The consistency of CP-IVFPRs

In fuzzy preference relations, consistency is an important topic. However, the preference relations given by the decision makers in the decision process is not necessarily coordinated. Thus, in the process of dealing with fuzzy preference relations, the first problem to be solved is their consistency. In this section, we first propose the concept of multiplication consistency expected preference relations of CP-IVFPRs. To check the CP-IVFPRs consistency level, we develop CP-IVFPRs consistency index. If CP-IVFPRs is unacceptable consistency, then we also construct a programming model to improve the consistency level for CP-IVFPRs.

5.1 Consistency index of CP-IVFPRs

Based on the expected preference relations and multiplication transitivity, we introduce the definition of multiplicative consistency expected preference relations with CP-IVFPRs.

Definition 5.1. *Let $R = (h_{ij}(f_{ij}(x)))_{n\times n}$ be a CP-IVFPRs matrix. The expected preference relations matrix of R is $E_R = (e_{h_{ij}})_{n\times n}$. If*

$$e_{h_{ij}} e_{h_{jk}} e_{h_{ki}} = e_{h_{ji}} e_{h_{kj}} e_{h_{ik}},$$

then E_R is a multiplicative consistency expected preference relations matrix.

In order to measure the consistency level, the concept of consistency index for CP-IVFPRs matrix can be defined as follows.

Definition 5.2. *Let $R = (h_{ij}(f_{ij}(x)))_{n\times n}$ be a CP-IVFPRs matrix, its expected preference relations matrix is $E_R = (e_{h_{ij}})_{n\times n}$. The consistency index of R is defined as follows:*

$$CI(R) = 1 - \frac{6}{n(n-1)(n-2)} \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} |e_{h_{ij}} e_{h_{jk}} e_{h_{ki}} - e_{h_{ji}} e_{h_{kj}} e_{h_{ik}}|. \quad (2)$$

The lager the valued of $CI(R)$, the higher the consistency level of R. A CP-IVFPRs matrix is multiplicative consistency if and only if its consistency index $CI(R) = 1$. However, the given preference relations matrix may be inconsistent. Therefore, consistency threshold CI_0 is developed in practical application. If $CI(R) \geq CI_0$, then R is acceptable consistency; otherwise, R is unacceptable consistency.

5.2 Consistency improvement process

Given CP-IVFPRs matrix $R = (h_{ij}(f_{ij}(x)))_{n\times n}$, then we can get its expected preference relations matrix E_R. According to Eq.2, we can obtain the consistency index of R. If R is unacceptable consistency, then we construct a programming model to improve the consistency level.

To improve the consistency level of R, the basic idea is to find a consistent expected fuzzy preference relations matrix $C_R = (c_{h_{ij}})_{n\times n}$. The distance between C_R and E_R should be minimum for better preserve the original information. We have the following optimization model to obtain acceptably consistency expected fuzzy preference relations matrix C_R.

$$(Model1)\min\ \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}|e_{h_{ij}} - c_{h_{ij}}| \tag{3}$$

$$s.t.\ \begin{cases} 1 - \frac{6}{n(n-1)(n-2)}\sum\limits_{i=1}^{n-2}\sum\limits_{j=i+1}^{n-1}\sum\limits_{k=j+1}^{n}|c_{h_{ij}}c_{h_{jk}}c_{h_{ki}} - c_{h_{ji}}c_{h_{kj}}c_{h_{ik}}| \geq CI_0 \\ c_{h_{ij}} + c_{h_{ji}} = 1(i=1,2,...,n-1; i<j) \\ c_{h_{ii}} = 0.5(i=1,2,...,n) \\ 0 \leq c_{h_{ij}} \leq 1(i=1,2,...,n-1; i<j) \end{cases} \tag{4}$$

According to Model 1, we develop the following programming model to obtain an optimal solution.

Let $d_{ij} = e_{h_{ij}} - c_{h_{ij}}$, $d_{ij}^{+} = \frac{|d_{ij}|+d_{ij}}{2}$ and $d_{ij}^{-} = \frac{|d_{ij}|-d_{ij}}{2}$, then $|d_{ij}| = d_{ij}^{+} + d_{ij}^{-}$. Similarity, let $\delta_{ijk} = c_{h_{ij}}c_{h_{jk}}c_{h_{ki}} - c_{h_{ji}}c_{h_{kj}}c_{h_{ik}}$, $\delta_{ijk}^{+} = \frac{|\delta_{ijk}|+\delta_{ijk}}{2}$ and $\delta_{ijk}^{-} = \frac{|\delta_{ijk}|-\delta_{ijk}}{2}$, then $|\delta_{ijk}| = \delta_{ijk}^{+} + \delta_{ijk}^{-}$. In this way, Model 1 is equivalent to Model 2:

$$(Model2)\min\ \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}(d_{ij}^{+} + d_{ij}^{-}) \tag{5}$$

$$s.t.\ \begin{cases} 1 - \frac{6}{n(n-1)(n-2)}\sum\limits_{i=1}^{n-2}\sum\limits_{j=i+1}^{n-1}\sum\limits_{k=j+1}^{n}(\delta_{ijk}^{+} + \delta_{ijk}^{-}) \geq CI_0 \\ e_{h_{ij}} - c_{h_{ij}} + d_{ij}^{-} - d_{ij}^{+} = 0(i=1,2,...,n-1; i<j) \\ c_{h_{ij}}c_{h_{jk}}c_{h_{ki}} - c_{h_{ji}}c_{h_{kj}}c_{h_{ik}} + \delta_{ijk}^{-} - \delta_{ijk}^{+} = 0(i=1,2,...,n-2; i<j<k) \\ c_{h_{ij}} + c_{h_{ji}} = 1(i=1,2,...,n-1; i<j) \\ c_{h_{ii}} = 0.5(i=1,2,...,n) \\ 0 \leq c_{h_{ij}} \leq 1(i=1,2,...,n-1; i<j) \\ d_{ij}^{+}, d_{ij}^{-} > 0(i=1,2,...,n-1; i<j) \\ \delta_{ijk}^{+}, \delta_{ijk}^{-} > 0(i=1,2,...,n-2; i<j<k) \end{cases} \tag{6}$$

Example 5.3. *We consider the consistency problem of Example 4.4. According to Eq.2, $CI(R) = 0.8125$. Since $CI(R) < CI_0$, then the consistency expected fuzzy preference relations matrix C_R is obtained based on Model 2.*

$$C_R = \begin{pmatrix} 0.5 & 0.3 & 0.3 & 0.35 \\ 0.7 & 0.5 & 0.5 & 0.35 \\ 0.7 & 0.5 & 0.5 & 0.6 \\ 0.65 & 0.65 & 0.4 & 0.5 \end{pmatrix},$$

where $CI_{C_R} = 0.9413$.

Based on the above analysis, the algorithm for improving the consistency level of R is given in the following.

Algorithm 1 An algorithm for improving the individual consistency and group consensus

Require: An initial CP-IVFPRs matrix $R = (h_{ij}(f_{ij}(x)))_{n\times n}$, consistency threshold CI_0.
Ensure: Acceptably consistency expected fuzzy preference relations matrix C_R.

```
Calculate expected fuzzy preference relations matrix E_p = (e_{h^p_{ij}})_{n×n} according to definition 4.2;
for each E_p = (e_{h^p_{ij}})_{n×n} do
    Compute the individual consistency of E_p = (e_{h_{ij}})_{n×n} according to Eq.2 and Eq.7;
    if  CI(E_p) ≥ CI_0(p = 1, 2, ..., m); then
        Let C_p = E_p;
    end if
end for
while CI(E_p) < CI_0(p = 1, 2, ..., m)  do
    Calculate C_p according to Model 2;
end while
Return C_p.
```

6 An algorithm for group decision making with CP-IVFPRs

In decision making process, many decision making problems are group decision making (GDM) problems. Thus, in this section, we propose an algorithm for GDM with CP-IVFPRs. First, we develop a group consensus index to measure the group consensus level. If the group is unacceptable consistency, we also develop a programming model to improve the group consensus level. Finally, we introduce an algorithm for GDM with CP-IVFPRs.

Let $A = \{A_1, A_2, ..., A_n\}$ be a set of alternatives, $D = \{D_1, D_2, ..., D_m\}$ be a set of decision makers. Assume $\omega = \{\omega_1, \omega_2, ..., \omega_m\}$ is the weight vector of D, where $\sum_{p=1}^{m} \omega_p = 1$ and $\omega_p \in [0, 1]$. Let $R^p = (h^p_{ij}(f_{ij}))_{n\times n}(p = 1, 2, ..., m)$ be m CP-IVFPRs matrices over a set of alternative $A = \{A_1, A_2, ..., A_n\}$, where $h^p_{ij}(f_{ij})$ is a CP-IVFE. Based on Definition 4.2, the corresponding expected fuzzy preference relations matrix $E^p = (e_{h^p_{ij}})_{n\times n}$ can obtained.

6.1 Group consensus index with CP-IVFPRs

For GDM problem, different decision makers give different continuous probability-interval valued fuzzy preference information. Therefore, we need to consider its consistency level. Following, we can get a group consensus index to measure the consistency level among decision makers $D_1, D_2, ..., D_m$.

The smaller the distance between E^p and E^q, the higher their similarity degree. Therefore, we can obtain the following similarity degree between E^p and E^q based on Eq.1:

$$S(E^p, E^q) = 1 - d(E^p, E^q) = 1 - \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} |e_{h^p_{ij}} - e_{h^q_{ij}}|.$$

The similarity degree between E^p and other expected fuzzy preference relation matrices $E^1, ..., E^{p-1}, E^{p+1}, ..., E^m$ can be obtained as follows:

$$D(E^P) = \frac{1}{m-1}\sum_{q=1,q\neq p}^{m} S(E^p, E^q) = \frac{1}{m-1}\sum_{q=1,q\neq p}^{m}\left(1 - \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} |e_{h^p_{ij}} - e_{h^q_{ij}}|\right).$$

The consensus among all decision makers can obtain as follows:

$$Con(E) = \frac{1}{m}\sum_{p=1}^{m} T(E^p) = \frac{1}{m(m-1)}\sum_{p=1}^{m}\sum_{q=1,q\neq p}^{m}\left(1 - \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} |e_{h^p_{ij}} - e_{h^q_{ij}}|\right).$$

Based on $S(E^p, E^q) = S(E^q, E^p)$, then

$$Con(E) = \frac{2}{m(m-1)}\sum_{p=1}^{m-1}\sum_{q=p+1}^{m}\left(1 - \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} |e_{h^p_{ij}} - e_{h^q_{ij}}|\right). \quad (7)$$

Let GCI_0 be a group consistency index, if $Con(E) \geq GCI_0$, then the group is an acceptable consistency. Otherwise, the group is an unacceptable consistency. Next, we propose a programming model to improve its consistency level when the group is unacceptable consistency.

6.2 Improving the individual consistency and group consensus index with CP-IVFPRs

Based on group consensus index, if the group is unacceptable consistency, then the following programming model can be improved the group consensus level.

Let $E^p = (e_{h^p_{ij}})_{n\times n}$ be m expected preference relations matrices. If at least one expected preference relations matrix is unacceptably individual consistency or unacceptably group consensus, then the consistency expected preference relations matrix $C^p = (c_{h^p_{ij}})_{n\times n}$ can be obtained according to the following programming model to satisfy both individual consistency level and group consensus level.

$$(Model3)\min\ \frac{1}{mn^2}\sum_{p=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n}|e_{h^p_{ij}} - c_{h^p_{ij}}| \tag{8}$$

$$s.t.\ \begin{cases} 1-\frac{6}{n(n-1)(n-2)}\sum\limits_{i=1}^{n-2}\sum\limits_{j=i+1}^{n-1}\sum\limits_{k=j+1}^{n}|c_{h^p_{ij}}c_{h^p_{jk}}c_{h^p_{ki}} - c_{h^p_{ji}}c_{h^p_{kj}}c_{h^p_{ik}}| \geq CI_0 (p=1,2,...,m) \\ \frac{2}{m(m-1)}\sum\limits_{p=1}^{m-1}\sum\limits_{q=p+1}^{m}\left(1-\frac{1}{n^2}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}|c_{h^p_{ij}} - c_{h^q_{ij}}|\right) \geq GCI_0 \\ c_{h^p_{ij}} + c_{h^p_{ji}} = 1 (i=1,2,...,n-1; i<j; p=1,2,...,m) \\ c_{h^p_{ii}} = 0.5 (i=1,2,...,n; p=1,2,...,m) \\ 0 \leq c_{h^p_{ij}} \leq 1 (i=1,2,...,n-1; i<j; p=1,2,...,m) \end{cases} \tag{9}$$

Model 3 is a multi-objective programming problem and cannot be directly resolved. Thus, we introduce the following optimization model to get an optimal solution.

Let $d^p_{ij} = e_{h^p_{ij}} - c_{h^p_{ij}}$, $d^{p+}_{ij} = \frac{|d^p_{ij}|+d^p_{ij}}{2}$ and $d^{p-}_{ij} = \frac{|d^p_{ij}|-d^p_{ij}}{2}$, then $|d^p_{ij}| = d^{p+}_{ij} + d^{p-}_{ij}$. Similarity, let $\delta^p_{ijk} = c_{h^p_{ij}}c_{h^p_{jk}}c_{h^p_{ki}} - c_{h^p_{ji}}c_{h^p_{kj}}c_{h^p_{ik}}$, $\delta^{p+}_{ijk} = \frac{|\delta^p_{ijk}|+\delta^p_{ijk}}{2}$ and $\delta^{p-}_{ijk} = \frac{|\delta^p_{ijk}|-\delta^p_{ijk}}{2}$, then $|\delta^p_{ijk}| = \delta^{p+}_{ijk} + \delta^{p-}_{ijk}$. $\xi^{pq}_{ij} = c_{h^p_{ij}} - c_{h^q_{ij}}$, $\xi^{pq+}_{ij} = \frac{|\xi^{pq}_{ij}|+\xi^{pq}_{ij}}{2}$ and $\xi^{pq-}_{ij} = \frac{|\xi^{pq}_{ij}|-\xi^{pq}_{ij}}{2}$, then $|\xi^{pq}_{ij}| = \xi^{pq+}_{ij} + \xi^{pq-}_{ij}$ In this way, Model 3 can be transformed to Model 4.

$$(Model4)\min\ \frac{1}{mn^2}\sum_{p=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n}(d^{p+}_{ij} + d^{p-}_{ij}) \tag{10}$$

$$s.t.\ \begin{cases} 1-\frac{6}{n(n-1)(n-2)}\sum\limits_{i=1}^{n-2}\sum\limits_{j=i+1}^{n-1}\sum\limits_{k=j+1}^{n}(\delta^{p+}_{ijk} + \delta^{p-}_{ijk}) \geq CI_0 (p=1,2,...,m) \\ \frac{2}{m(m-1)}\sum\limits_{p=1}^{m-1}\sum\limits_{q=p+1}^{m}\left(1-\frac{1}{n^2}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}(\xi^{pq+}_{ij} + \xi^{pq-}_{ij})\right) \geq GCI_0 \\ e_{h^p_{ij}} - c_{h^p_{ij}} + d^{p-}_{ij} - d^{p+}_{ij} = 0 (i=1,2,...,n-1; i<j; p=1,2,...,m) \\ c_{h^p_{ij}}c_{h^p_{jk}}c_{h^p_{ki}} - c_{h^p_{ji}}c_{h^p_{kj}}c_{h^p_{ik}} + \delta^{p-}_{ijk} - \delta^{p+}_{ijk} = 0 (i=1,2,...,n-2; i<j<k; p=1,2,...,m) \\ c_{h^p_{ij}} - c_{h^q_{ij}} + \xi^{pq-}_{ij} - \xi^{pq+}_{ij} = 0 (i=1,2,...,n-1; i<j; p=1,2,...,m) \\ c_{h^p_{ij}} + c_{h^p_{ji}} = 1 (i=1,2,...,n-1; i<j; p=1,2,...,m) \\ c_{h^p_{ii}} = 0.5 (i=1,2,...,n; p=1,2,...,m) \\ 0 \leq c_{h^p_{ij}} \leq 1 (i=1,2,...,n-1; i<j; p=1,2,...,m) \\ d^{p-}_{ij}, d^{p+}_{ij}, \xi^{pq-}_{ij}, \xi^{pq+}_{ij} > 0 (i=1,2,...,n-1; i<j; p=1,2,...,m) \\ \delta^{p-}_{ijk}, \delta^{p+}_{ijk} > 0 (i=1,2,...,n-2; i<j<k; p=1,2,...,m) \end{cases} \tag{11}$$

Based on the above analysis, an algorithm for improving the individual consistency and group consensus of $R^p (p = 1,2,...,m)$ is given in the following.

Algorithm 2 An algorithm for improving the individual consistency and group consensus

Require: An initial CP-IVFPRs matrices $R^p = (h_{ij}^p(f_{ij}(x)))_{n\times n}(p = 1, 2, ..., m)$, individual consistency threshold CI_0, group consensus threshold GCI_0.
Ensure: Expected fuzzy preference relation matrix C_p is acceptably consistent.

```
Calculate expected fuzzy preference relations matrix E_p = (e_{h_{ij}^p})_{n×n} according to definition 4.2;
for each E_p = (e_{h_{ij}^p})_{n×n} do
    Compute the individual consistency and group consensus of E_p = (e_{h_{ij}})_{n×n} according to Eq.2 and Eq.7;
    if CI(E_p) ≥ CI_0(p = 1, 2, ..., m), Con(E) ≥ GCI_0; then
        Let C_p = E_p;
    end if
end for
while CI(E_p) < CI_0(p = 1, 2, ..., m) or Con(E) < GCI_0 do
    Calculate C_p according to Model 4;
end while
Return C_p.
```

6.3 An algorithm for group decision making with CP-IVFPRs

To solve GDM problem with CP-IVFPRs, an algorithm is proposed as follows.

Step 1 Decision makers provide decision making information with CP-IVFPRs matrices.

Each expert provides CP-IVPRs matrix is $R_p = (h_{ij}^p(f_{ij}(x)))_{n\times n}$, where $i, j = 1, 2, ..., n; p = 1, 2, ..., m$.

Step 2 Calculate expected preference relations matrix.

We can get the expected preference relations matrices $E_p = (e_{h_{ij}^p})_{n\times n}$ according to Definition 4.2.

Step 3 Check individual consistency and group consensus.

Individual consistency is judged by Eq.1 and group consensus is judged by Eq. 7. If all given CP-IVFPRs matrices satisfy their individual consistency level and the group consensus level reaches its given consistency threshold valued, then proceed to Step 5. If at least one expected preference relations matrix is unacceptably individual consistency or unacceptably group consensus, then proceed to next step.

Step 4 Obtaining a group of consistency expected preference relations matrices.

According to the Model 4, the consistency expected preference relations matrix $C_p = (c_{h_{ij}^p})_{n\times n}$ can be obtained, where $C_p = (c_{h_{ij}^p})_{n\times n}$ not only satisfy the individual consistency level, but also reaches its given group consensus threshold valued.

Step 5 Calculate the individual priority weights of the alternatives.

According to the priority formula (Ref. [29]) and consistency expected preference relations matrix $C_p = (c_{h_{ij}^p})_{n\times n}$, the individual priority of alternatives can obtained as follows.

$$w_i^p = \frac{1}{n(n-1)}\left(\sum_{j=1}^{n} c_{h_{ij}^p} + \frac{n}{2} - 1\right), \tag{12}$$

where $i, j = 1, 2, ..., n; p = 1, 2, ..., m$.

Step 6 Calculate overall priority weights of the alternatives.

We can get the overall priority weights of alternatives as follows:

$$w = (\omega_1, \omega_2, ..., \omega_m) \times \begin{pmatrix} w_1^1 & w_2^1 & ... & w_n^1 \\ w_1^2 & w_2^2 & ... & w_n^2 \\ ... & ... & ... & ... \\ w_1^m & w_2^m & ... & w_n^m \end{pmatrix}. \tag{13}$$

where $\omega = (\omega_1, \omega_2, ..., \omega_m)$ is the weight vector of decision makers.

Step 7 Ranking.

According to w_i, ranking from biggest to smallest can get its ranking result. The largest valued is the best alternative.

7 Numerical example and comparative analysis

In this section, we validate the effectiveness of our method based on the example of an overseas high-level talent introduction programs and reflect the superiority of our method by comparing with existing methods.

7.1 Numerical example

In China, the overseas high-level talent introduction programs are mainly focused on the strategic objectives of national development. At the same time, the provinces have also introduced a number of overseas high-level talents in accordance with the needs of economic and social development and industrial restructuring in the region. Since 2008, about 2000 talents have been introduced. Focus on supporting a group of strategic scientists and leaders to enter China's innovation and entrepreneurship, who can break through key technologies, develop high-tech industries and drive emerging disciplines. In 2012, the program has introduced 4180 high-end talents in various fields. In 2016, the government again stressed the need to intensify efforts to promote the program.

The talents introduced by the program need to provide relevant personal proof materials, mainly scientific research achievements or entrepreneurship achievements, ideas for work after returning to China or coming to China, including the content, objectives, plans, existing foundations, teams and so on. Special requirements for working conditions can also be put forward. According to these conditions, relevant departments organize experts for evaluation and comprehensive selection. In order to evaluate the four talents A_1, A_2, A_3, A_4, three domestic and foreign peer experts provide evaluation information according to the prescribed conditions, the experts weight vector is $\omega = (0.3, 0.5, 0.2)^T$. In order to better express the preferences of evaluation experts, the information provided by experts is CP-IVFPRs matrix.

In order to evaluate and decision making for four talents, we use the method proposed in this paper to decision making. The steps are as follows.

Step 1 Decision makers provide decision making information with CP-IVFPRs matrix.

The CP-IVFPRs matrices given by the three experts are as follows.

$$R_1 = \begin{pmatrix} 0.5(1) & [0.3,0.7](N_{(0.5,0.7)}(x)) & [0.4,1](N_{(0.7,1)}(x)) & [0.4,0.6](N_{(0.5,0.6)}(x)) \\ [0.3,0.7](N_{(0.5,0.7)}(1-x)) & 0.5(1) & [0.6,0.9](N_{(0.75,0.9)}(x)) & [0.4,0.8](N_{(0.6,0.8)}(x)) \\ [0,0.6](N_{(0.7,1)}(1-x)) & [0.1,0.4](N_{(0.75,0.9)}(1-x)) & 0.5(1) & [0.5,0.9](N_{(0.7,0.9)}(x)) \\ [0.4,0.6](N_{(0.5,0.6)}(1-x)) & [0.2,0.6](N_{(0.6,0.8)}(1-x)) & [0.1,0.5](N_{(0.7,0.9)}(1-x)) & 0.5(1) \end{pmatrix},$$

$$R_2 = \begin{pmatrix} 0.5(1) & [0.2,0.4](N_{(0.3,0.4)}(x)) & [0.5,0.7](N_{(0.6,0.7)}(x)) & [0.2,0.5](N_{(0.35,0.5)}(x)) \\ [0.6,0.8](N_{(0.3,0.4)}(1-x)) & 0.5(1) & [0.9,1](N_{(0.95,1)}(x)) & [0.5,0.8](N_{(0.65,0.8)}(x)) \\ [0.3,0.5](N_{(0.6,0.7)}(1-x)) & [0,0.1](N_{(0.95,1)}(1-x)) & 0.5(1) & [0.3,0.4](N_{(0.35,0.4)}(x)) \\ [0.5,0.8](N_{(0.35,0.5)}(1-x)) & [0.2,0.5](N_{(0.65,0.8)}(1-x)) & [0.6,0.7](N_{(0.35,0.4)}(1-x)) & 0.5(1) \end{pmatrix},$$

$$R_3 = \begin{pmatrix} 0.5(1) & [0.4,0.7](N_{(0.55,0.7)}(x)) & [0.2,0.5](N_{(0.35,0.5)}(x)) & [0.3,0.5](N_{(0.4,0.5)}(x)) \\ [0.3,0.6](N_{(0.55,0.7)}(1-x)) & 0.5(1) & [0.6,1](N_{(0.8,1)}(x)) & [0.8,1](N_{(0.9,1)}(x)) \\ [0.5,0.8](N_{(0.35,0.5)}(1-x)) & [0,0.4](N_{(0.8,1)}(1-x)) & 0.5(1) & [0.5,0.8](N_{(0.65,0.8)}(x)) \\ [0.5,0.7](N_{(0.4,0.5)}(1-x)) & [0,0.2](N_{(0.9,1)}(1-x)) & [0.2,0.5](N_{(0.65,0.8)}(1-x)) & 0.5(1) \end{pmatrix}.$$

Step 2 Calculate expected preference relations matrix.

According to Definition 4.2, we can get the following expected preference relations matrices.

$$E_1 = \begin{pmatrix} 0.5 & 0.5 & 0.7 & 0.5 \\ 0.5 & 0.5 & 0.75 & 0.6 \\ 0.3 & 0.25 & 0.5 & 0.7 \\ 0.5 & 0.4 & 0.3 & 0.5 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 0.5 & 0.3 & 0.6 & 0.35 \\ 0.7 & 0.5 & 0.95 & 0.65 \\ 0.4 & 0.05 & 0.5 & 0.35 \\ 0.65 & 0.35 & 0.65 & 0.5 \end{pmatrix}, \quad E_3 = \begin{pmatrix} 0.5 & 0.55 & 0.35 & 0.4 \\ 0.45 & 0.5 & 0.8 & 0.9 \\ 0.65 & 0.2 & 0.5 & 0.65 \\ 0.6 & 0.1 & 0.35 & 0.5 \end{pmatrix}.$$

Step 3 Check individual consistency and group consensus.

By Eq. 1, we can get individual consistency index: $CI(E_1) = 0.89, CI(R_2) = 0.9313, CI(E_3) = 0.8525$. By Eq. 7, we can get group consistency index is $GCI(E) = 0.8667$. Assume $CI_0 = 0.92$ and $GCI_0 = 0.9$, then R_2 is acceptably consistency, R_1 and R_3 are unacceptably consistency. $R = \{R_1, R_2, R_3\}$ is unacceptably group consensus.

Step 4 Derive a group of expected preference relations matrices which satisfy individual consistency and group consensus.

According to Model 4, we get a group of expected preference relations matrix are shown as follows.

$$C_1 = \begin{pmatrix} 0.5 & 0.3 & 0.6 & 0.4 \\ 0.7 & 0.5 & 0.78 & 0.65 \\ 0.4 & 0.2 & 0.5 & 0.64 \\ 0.6 & 0.35 & 0.36 & 0.5 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 0.5 & 0.3 & 0.6 & 0.35 \\ 0.7 & 0.5 & 0.95 & 0.65 \\ 0.4 & 0.05 & 0.5 & 0.35 \\ 0.65 & 0.35 & 0.65 & 0.5 \end{pmatrix}, \quad C_3 = \begin{pmatrix} 0.5 & 0.3 & 0.5 & 0.4 \\ 0.7 & 0.5 & 0.8 & 0.65 \\ 0.5 & 0.2 & 0.5 & 0.64 \\ 0.6 & 0.35 & 0.36 & 0.5 \end{pmatrix}.$$

By Eq. 1, we can get individual consistency index: $CI(C_1) = 0.92, CI(C_2) = 0.9313, CI(C_3) = 0.92$. By Eq. 7, we can get group consensus index is $GCI(R) = 0.95$. Therefore, the expected preference relations matrices C_1, C_2, C_3 satisfies individual consistency and group consensus.

Step 5 Calculate the individual priority weights of $A_i (i = 1, 2, 3, 4)$.

According to Eq. 12, we can get the following individual priority weights of $A_i (i = 1, 2, 3, 4)$.

$$w_1^1 = 0.2333, w_2^1 = 0.3030, w_3^1 = 0.2294, w_4^1 = 0.2343,$$
$$w_1^2 = 0.2292, w_2^2 = 0.3167, w_3^2 = 0.1917, w_4^2 = 0.2625,$$
$$w_1^3 = 0.2252, w_2^3 = 0.3042, w_3^3 = 0.2363, w_4^3 = 0.2343.$$

Step 6 Calculate overall priority weights of $A_i (i = 1, 2, 3, 4)$.

According to Eq. 13, we can obtain the following overall priority weights of $A_i (i = 1, 2, 3, 4)$.

$$w = (0.3, 0.5, 0.2) \times \begin{pmatrix} 0.2333 & 0.3030 & 0.2294 & 0.2343 \\ 0.2292 & 0.3167 & 0.1917 & 0.2625 \\ 0.2252 & 0.3042 & 0.2363 & 0.2343 \end{pmatrix} = (0.2296, 0.3101, 0.2119, 0.2484).$$

Step 7 Rank $A_i (i = 1, 2, 3, 4)$.

By $w = (0.2370, 0.3035, 0.2053, 0.2541)$, then $w_2 > w_4 > w_1 > w_3$, that is, $A_2 \succ A_4 \succ A_1 \succ A_3$ Therefore, the best one is A_2.

7.2 Comparative analysis

For this GDM problem, we compare our method with existing methods based on probability-interval valued hesitant fuzzy preference relations (P-IVHFPRs) [34, 36] and interval fuzzy preference relations (IFPRs) [30].

Xu et al. proposed a decision making method based on P-IVHFPRs (Ref. [34]). The probability distribution of P-IVHFPRs is discrete. Therefore, we take the probability corresponding to the average of the interval valued from CP-IVFPRs in this example as the discrete probability of the interval valued. Combining the values of the corresponding positions of R_1, R_2 and R_3 into one matrix. Because there will be overlapping interval when combine R_1, R_2 and R_3 into one matrix, according to the method of dealing with overlapping intervals (Ref. [34]). R can be obtain as follows:

$$R = \begin{pmatrix} 0.5 & \{[0.2,0.3](0.175),[0.3,0.4](0.255),[0.4,0.7](0.57)\} & \{[0.2,0.4](0.3),[0.4,0.5](0.18),[0.5,0.7](0.42),[0.7,1](0.1)\} & \{[0.2,0.3](0.1),[0.3,0.4](0.3),[0.4,0.5](0.45),[0.5,0.6](0.15)\} \\ \{[0.7,0.8](0.175),[0.6,0.7](0.255),[0.3,0.6](0.57)\} & 0.5 & \{[0.6,0.9](0.6),[0.9,1](0.4)\} & \{[0.4,0.5](0.075),[0.5,0.8](0.625),[0.8,1](0.3)\} \\ \{[0.6,0.8](0.3),[0.5,0.6](0.18),[0.3,0.5](0.42),[0,0.3](0.1)\} & \{[0.1,0.4](0.6),[0,0.1](0.4)\} & 0.5 & \{[0.3,0.4](0.4),[0.5,0.8](0.525),[0.8,0.9](0.075)\} \\ \{[0.7,0.8](0.1),[0.6,0.7](0.3),[0.5,0.6](0.45),[0.4,0.5](0.15)\} & \{[0.5,0.6](0.075),[0.2,0.5](0.625),[0,0.2](0.3)\} & \{[0.6,0.7](0.4),[0.2,0.5](0.525),[0.1,0.2](0.075)\} & 0.5 \end{pmatrix} \quad (14)$$

According to a decomposition method proposed by Xu et al. (Ref. [34]) to deal with the consistency of P-IVHFPRs, we obtain the following consistent P-IVHFPR $\hat{R}$.

$$
\hat{R} = \left(\begin{array}{c}
0.5 \\
\{[0.7, 0.8](0.175), [0.6, 0.7](0.255), [0.3, 0.6](0.57)\} \\
\{[0.3051, 0.7507](0.175), [0.2437, 0.6737](0.125), [0.222, 0.6061](0.13), [0.1057, 0.5304](0.05), [0.0839, 0.5](0.12), [0, 0.2221](0.3), \\
.1813 \\
(0.1)\} \\
\{[0.69730.8596](0.075), [0.5839, 0.8374](0.025), [0.5514, 0.8142](0.075), [0.5181, 0.7741](0.125), [0.4857, 0.7276](0.1), \\
,0.6353 \\
(0.03), \\
[0.2274, 0.6](0.05), [0.1732, 0.5652](0.22), [0.1106, 0.4477](0.15), [0.0992, 0.4178](0.05), [0, 0.3455](0.0.025), [0, 0.2614](0.075)\} \\
\{[0.2, 0.3](0.175), [0.3, 0.4](0.255), [0.4, 0.7](0.57)\} \\
0.5 \\
\{[0.1, 0.4](0.6), [0, 0.1](0.4)\} \\
\{[0.3939, 0.7779](0.075), [0.3748, 0.8415](0.325), [0.2748, 0.9491](0.2), [0.1768, 1](0.1), [0.3894, 1](0.225), [0.2071, 1](0.075)\} \\
\{[0.2493, 0.6949](0.175), [0.3263, 0.7563](0.125), [0.3939, 0.778](0.13), [0.4696, 0.8943](0.05), [0.5, 0.9161](0.12), [0.7779, 1](0.3), [0.8187, 1](0.1)\} \\
\{[0.6, 0.9](0.6), [0.9, 1](0.4)\} \\
0.5 \\
\{[0.3, 0.4](0.4), [0.5, 0.8](0.525), [0.8, 0.9](0.075)\} \\
\{[0.14040.3027](0.075), [0.1626, 0.4161](0.025), [0.1858, 0.4486](0.075), [0.2259, 0.4819](0.125), [0.2724, 0.5143](0.1), [0.3647, 0.6944](0.03), \\
[0.4, 0.7726](0.05), [0.4348, 0.8268](0.22), [0.5523, 0.8894](0.15), [0.5822, 0.9008](0.05), [0.6545, 1](0.0.025), [0.7386, 1](0.075)\} \\
\{[0.2221, 0.6061](0.075), [0.1585, 0.6252](0.325), [0.0509, 0.7252](0.2), [0, 0.8232](0.1), [0, 0.6106](0.225), [0, 0.7929](0.075)\} \\
\{[0.6, 0.7](0.4), [0.2, 0.5](0.525), [0.1, 0.2](0.075)\} \\
0.5
\end{array} \right) \tag{15}
$$

where we take the acceptable multiplicative consistent level is 0.1, and $\lambda = 0.7$ (Refs. [34]). Finally, we can obtain the ranking as follows:

$$A_2 \succ A_1 \succ A_3 \succ A_4.$$

Zhang et al. proposed generalized probabilistic interval-valued hesitant fuzzy weighted averaging (GPIVHFWA) operator for P-IVHFPRs (Ref. [36]). The P-IVHFPRs matrix R is shown as Eq.14. The method is to fusion preference relations by using aggregation operators to decision making. Therefore, we can get the following result for this example:

$$A_2 \succ A_1 \succ A_3 \succ A_4.$$

The IFPRs without probabilities has been extracted from the CP-IVFPRs as the initial judgment matrices. It shown as follows:

$$
R_1 = \begin{pmatrix} [0.5, 0.5] & [0.3, 0.7] & [0.4, 1] & [0.4, 0.6] \\ [0.3, 0.7] & [0.5, 0.5] & [0.6, 0.9] & [0.4, 0.8] \\ [0, 0.6] & [0.1, 0.4] & [0.5, 0.5] & [0.5, 0.9] \\ [0.4, 0.6] & [0.2, 0.6] & [0.1, 0.5] & [0.5, 0.5] \end{pmatrix}, \quad R_2 = \begin{pmatrix} [0.5, 0.5] & [0.2, 0.4] & [0.5, 0.7] & [0.2, 0.5] \\ [0.6, 0.8] & [0.5, 0.5] & [0.9, 1] & [0.5, 0.8] \\ [0.3, 0.5] & [0, 0.1] & [0.5, 0.5] & [0.3, 0.4] \\ [0.5, 0.8] & [0.2, 0.5] & [0.6, 0.7] & [0.5, 0.5] \end{pmatrix},
$$

$$
R_3 = \begin{pmatrix} [0.5, 0.5] & [0.4, 0.7] & [0.2, 0.5] & [0.3, 0.5] \\ [0.3, 0.6] & [0.5, 0.5] & [0.6, 1] & [0.8, 1] \\ [0.5, 0.8] & [0, 0.4] & [0.5, 0.5] & [0.5, 0.8] \\ [0.5, 0.7] & [0, 0.2] & [0.2, 0.5] & [0.5, 0.5] \end{pmatrix}.
$$

According to uncertain power weighted average(UPWA) operator for group decision making with IFPRs (Ref. [30]), we can get the following result:

$$A_2 \succ A_1 \succ A_4 \succ A_3.$$

For convenient, we put results of four kinds of decision making method into Table 1 for comparison.

Based on the above results, we can know that the best alternative is A_2, whether it is our method or other existing methods (Refs. [36, 34, 30]). But the ranking results of our method are difference from GPIVHFWA operator decision method (Ref. [36]), decomposition method of P-IVHFPRs (Ref. [34]) and UPWA operator decision method (Ref. [30]), the ranking results of GPIVHFWA operator decision method and decomposition method of P-IVHFPRs are the same. The main reason for the different results are as follow.

(1) Both discrete probabilistic interval hesitant fuzzy preference information and interval fuzzy preference information without probabilities will result in the loss of original continuous probability-interval valued fuzzy preference information.

Table 1: Ranking result of four kinds of decision making method

method	ranking results
Our method	$A_2 \succ A_4 \succ A_1 \succ A_3$
GPIVHFWA operator [36]	$A_2 \succ A_1 \succ A_3 \succ A_4$
Decomposition method of P-IVHFPRs[34]	$A_2 \succ A_1 \succ A_3 \succ A_4$
UPWA operator [30]	$A_2 \succ A_1 \succ A_4 \succ A_3$

(2) The consistent matrix obtained for decomposition method of P-IVHFPRs is a P-IVHFPR matrix which elements are complex and tedious, and our method obtains a consistent expected matrix, which is more simple and straightforward in form.

(3) Consistency is an important topic in preference relations. However, GPIVHFWA operator and UPWA operator directly aggregation preference information without considering the consistency of preference relations, while our method takes into account both individual consistency and group consensus.

(4) Although the ranking results of GPIVHFWA operator decision method and decomposition method of P-IVHFPRs are the same, but we can not ignore the disadvantage of GPIVHFWA operator decision method that does not consider the consistency of preference relations, which will result in unreasonable ranking result.

Compared with P-IVHFS, the implementation of our method in CP-IVFPRs continuous probability-interval valued fuzzy environment is more complex. This method involves the calculation of integral and requires higher calculation. Therefore, more simple calculation methods can be considered to reduce the complexity of the calculation for future research.

In spite of what has been mentioned above, compared with PHFSs and P-IVHFSs, CP-IVFS can describe the actual preferences of decision makers and better reflect their uncertainty, hesitancy, and inconsistency. Although the representation of CP-IVFS looks complex, they can depict fuzzy information clearly and retain the completeness of original data, and ensure the accuracy of the final results. Moreover, the complexity and amount of computation can be clearly reduced with the assistance of programming software.

8 Conclusions

In this paper, the concept of CP-IVFS and CP-IVFPRs has been proposed. As an important tool in group decision making, CP-IVFS can describe the actual preferences of decision makers and better reflect their uncertainty, hesitancy, and inconsistency. Based on related research, a method has been developed to deal with the consistency of CP-IVFPRs. A number example has also been provided to illustrate the use of the proposed method. The main contributions of this paper are summarized as follows.

(1) The concept of CP-IVFS and CP-IVFPRs has been defined.

(2) Expected multiplicative consistency with CP-IVFPRs has been developed, using the multiplicative transitivity to verify the consistency of a CP-IVFPRs. Based on the consistence, the individual consistency index and group consensus index has been developed. Moreover, a method has been proposed to deal with the consistency of CP-IVFPRs.

(3) Based on the multiplicative consistency of hesitant fuzzy preference relations, an algorithms has been designed to obtain acceptable individual consistency and group consensus and improve the consistency level to meet the set threshold.

In future research, other group decision making methods can be considered in continuous probability-interval valued fuzzy environment, such as the multi-attribute decision making problem based on aggregative operators, distance and preference relations.

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Continuous probability-interval valued fuzzy preference relations and its application in group decision making

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روابط ترجیحی فازی با ارزش بازه احتمال پیوسته و کاربرد آن در تصمیم‌گیری گروهی

چکیده. مجموعه فازی مردد احتمالی، احتمال وقوع عناصر را نشان می‌دهد. روابط ترجیحی فازی مردد احتمالی می‌تواند به طور مؤثرتری اطلاعات ترجیح مردد تصمیم‌گیرندگان را بیان کند. اما در تحقیقات موجود، همه آن‌ها بر اساس توزیع احتمال گسسته است. برای ایجاد فضای ارزیابی بیشتر برای تصمیم‌گیرنده، لازم است توزیع احتمال پیوسته در نظر گرفته شود. بنابراین، در این مقاله، مجموعه فازی با ارزش بازه پیوسته تعریف شده و احتمال آن با یک تابع تراکم احتمال نشان داده می‌شود. برای تبدیل مجموعه فازی مردد احتمالی به مجموعه فازی با ارزش بازه احتمال پیوسته روشی تعمیم داده شده تا داده‌های گسسته به داده‌های پیوسته تبدیل شوند. سپس، روابط ترجیحی فازی با ارزش بازه احتمال پیوسته ارائه می‌شود. به منظور درنظر گرفتن سازگاری روابط ترجیحی فازی با ارزش بازه پیوسته، روابط ترجیحی مورد انتظار سازگار ضربی پیشنهاد شده‌است. شاخص سازگاری فردی و شاخص اجماع گروه نیز برای تعیین سطح سازگاری ارائه شده‌است و سپس، الگوریتمی برای بررسی و بهبود سطح سازگاری فردی و سطح اجماع گروه معرفی شده‌است. سرانجام، یک مثال عددی برای اثر بخشی الگوریتم پیشنهادی نشان داده شده‌است، تحلیل مقایسه‌ای با روش‌های موجود، برای نشان دادن برتری این الگوریتم ارائه شده‌است.