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Original paper

Adaptive control design for fixed-time synchronization of fuzzy stochastic cellular neural networks with discrete and distributed delay

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Abstract

This paper studies the fixed-time synchronization problem of fuzzy stochastic cellular neural networks (FSCNNs) with discrete and distributed delay. Compared with the finite-time synchronization in the existing literature, the fixed-time synchronization of FSCNNs is studied for the first time, and the convergence time obtained does not depend on the upper bound of the initial value of the system. In addition, two kinds of control are designed, one is feedback control and the other is adaptive control. Besides, it is the first time to achieve fixed-time synchronization of FSCNNs via adaptive control. Finally, two numerical examples are also proposed to illustrate the practicability and validity of the results we proposed.

Keywords: Fixed-time synchronization, adaptive control, stochastic perturbations, fuzzy cellular neural networks.

1 Introduction

Chua and Yang [4] first proposed cellular neural networks (CNNs) in 1988. CNNs are a parallel computing networks similar to biological neural networks, the difference from other neural networks are that they only allow communication between adjacent units. CNNs have been widely used in various fields such as pattern recognition [3] and associative memory [7]. In 1996, fuzzy cell neural networks (FCNNs) are proposed by Yang et al.[24], in which fuzzy logic is added to traditional CNNs, which takes into account the uncertainty and ambiguity in real-world applications. Unlike traditional CNNs, FCNNs also have fuzzy logic between its template input and/or output besides the "sum of product" operation. FCNNs widely used in pattern recognition [12], image encryption [14] and other fields. However, no matter in nature or human society, neural networks are inevitably interfered by stochastic perturbations. Therefore, it is more practical to study fuzzy stochastic cellular neural networks (FSCNNs).

As an important dynamic behavior of FSCNNs, many scholars have studied synchronization [13], such as exponential synchronization [5] and asymptotic synchronization[2]. However, the above synchronization can only be achieved when the time tends to infinity. In real life, people often hope to achieve synchronization goals as soon as possible. In recent years, in order to further satisfy scientific researches and actual needs, people proposed the concept of finite-time synchronization [1, 19, 26]. Finite-time synchronization has the best convergence speed, and synchronization can be achieved in a finite time. In addition, the finite time synchronization also shows good robustness and anti-interference characteristics. However, the convergence time of finite time synchronization largely depends on the initial value of the system, and different initial values have different convergence time.

In practical applications, not all initial values in the system are known. Therefore, in order to overcome the shortcoming that the synchronization time varies with the initial value, a special case of finite time synchronization is proposed, that is, fixed-time synchronization. The fixed-time synchronization can achieve synchronization in a finite time, and the convergence time does not depend on the upper bound of the initial value, which satisfies some production

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and life realities, and broadens the application range of synchronization. In recent years, some scholars have studied fixed-time synchronization [17, 20, 21, 22].

Adaptive control can continuously modify its own characteristics in accordance with adaptive rules, so that the entire network reaches the desired synchronization target. The adaptive control method is robust against uncertain factors such as interference and unknowns. In addition, adaptive control can identify unknown parameters in the model based on input and output data, and can be automatically adjusted according to different update rules. In [10], the author studied the finite-time and fixed-time robust synchronization of fuzzy Cohen-Grossberg neural networks with discontinuous activation by designing a simple switch adaptive controller.

The author of [15] designed a nonlinear delay state feedback control scheme and a novel adaptive control to study the fixed-time synchronization of stochastic memristor-based neural networks. The author mainly studies the global fixed-time synchronization of chaotic systems with different dimensions through adaptive control in [6]. However, the results of using adaptive control to study the fixed-time synchronization of FSCNNs have not yet been obtained.

In [23], the author has proposed a model of multi-coupling fuzzy cellular neural networks with stochastic perturbations and mixed delays. Nowadays, fuzzy cellular neural networks have been studied in depth, such as the author of [8] investigate the general decay synchronization analysis of discontinuous fuzzy neutral-type neural networks. In [9], the author aims to provide a new framework to study the general decay synchronization analysis of a class of fuzzy neural networks with discontinuous activations, discrete and distributed time-delays. In addition, the author of [11] aims to investigate the fixed-time synchronization analysis for discontinuous fuzzy inertial neural networks in the presence of parameter uncertainties.

However, in literature [23], the author studied the finite-time synchronization of FSCNNs by constructing feedback control, but this paper studies the fixed-time synchronization of FSCNNs, which can make FSCNNs synchronized in a finite time, and the convergence time does not depend on the system Initial value upper bound. In addition, this paper designed the feedback control to make FSCNNs synchronized in a fixed time, but because the feedback gain of feedback control is difficult to determine, an adaptive control is further designed, which can also achieve fixed-time synchronization of FSCNNs.

Motivated by the above discussions, we investigate the fixed-time synchronization problem of FSCNNs with discrete and distributed delay. By utilizing Lyapunov method, some sufficient conditions guaranteeing fixed-time synchronization of FSCNNs are obtained. Then, two numerical examples illustrate the practicality and effectiveness of our theoretical results. In comparison with the relevant results, the main contributions of this paper are listed as follows.

- The first attempt at investigating fixed-time synchronization of FSCNNs with discrete and distributed delays rather than exponential synchronization, asymptotic synchronization and finite-time synchronization. In addition, the convergence time does not depend on the system initial value upper bound.

- This article designs two kinds of controls to realize the fixed-time synchronization of FSCNNs, one is feedback control, the other is adaptive control. In addition, the first attempt to design adaptive control to achieve fixed-time synchronization of FSCNNs.

The remaining of this paper is organized in following parts. In Section 2, some preliminaries and model descriptions are introduced. Moreover, main results are presented in Section 3, and we show two numerical examples in Section 4. Finally, Section 5 ends this study and concisely summarizes main conclusions of this paper.

2 Preliminaries and model descriptions

Throughout this paper, let $\mathbb{N} = \{1, 2, \cdots, n\}$, $\mathbb{Z}^+ = \{1, 2, \cdots\}$ $\mathbb{R}^+ = [0, +\infty)$, $\mathbb{R}^n$ denotes the n dimensional Euclidean space respectively. The superscript "T" stands for the transpose of a vector or a matrix. Write $\|x\|$ for the Euclidean norm of vectors. Set $\text{sign}(\cdot)$ is a sign function and $\text{Tr}[A]$ denotes trace of matrix A. $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ is a complete probability of the sample space with a filtration $\mathbb{F} = \{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). And $B(t)$ is a one-dimensional Brownian motion defined on the probability space.

The mathematical expectation with respect to the given probability measure $\mathbb{P}$ is denoted by $\mathbb{E}(\cdot)$. And $C^{2,1}(\mathbb{R}^n \times \mathbb{R}^+, \mathbb{R}^+)$ represents the family of all nonnegative functions $V(x, t)$ on $\mathbb{R}^n \times \mathbb{R}^+$ which are continuously twice differentiable in x and once in t.

In this paper, based on previous works, we consider a class of fuzzy stochastic cellular neural networks with discrete and distributed delay:

$$
\begin{aligned}
\mathrm{d}x_i(t) =\Bigg[& -\alpha_i x_i(t) + \sum_{j=1}^{n} k_{ij} f(x_j(t)) + \sum_{j=1}^{n} h_{ij} f(x_j(t-\tau)) + \sum_{j=1}^{n} w_{ij} \int_{t-\tau}^{t} f(x_j(s))\mathrm{d}s \\
& + \bigwedge_{j=1}^{n} m_{ij} g(x_j(t-\tau)) + \bigvee_{j=1}^{n} \iota_{ij} g(x_j(t-\tau)) + \bigwedge_{j=1}^{n} s_{ij} \int_{t-\tau}^{t} g(x_j(s))\mathrm{d}s \\
& + \bigvee_{j=1}^{n} v_{ij} \int_{t-\tau}^{t} g(x_j(s))\mathrm{d}s + \bigwedge_{j=1}^{n} G_{ij} p_j + \bigvee_{j=1}^{n} J_{ij} p_j + A_i \Bigg]\mathrm{d}t + \sigma_i(x_i(t), t)\mathrm{d}B(t),
\end{aligned}
\tag{1}
$$

where $x_i(t)$ denotes the ith neurons state at time t; α_i denotes the passive decay rate to the state of ith unit; the parameters k_{ij}, h_{ij} and w_{ij} denote the elements of feedback templates; $f(\cdot) : \mathbb{R} \to \mathbb{R}$ and $g(\cdot) : \mathbb{R} \to \mathbb{R}$ are the activation function of the jth neuron;$\sigma_i : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$ is the noise intensity function; p_i and A_i represent input and bias of the ith neuron. m_{ij}, s_{ij} are elements of fuzzy feedback MIN templates, and ι_{ij}, v_{ij} are fuzzy feedback MAX templates, respectively; G_{ij} and J_{ij} are fuzzy feed-forward MIN template and fuzzy feed-forward MAX template, respectively.

Throughout the paper, we consider system (1) as drive system and response system can be described as follows:

$$
\begin{aligned}
\mathrm{d}y_i(t) =\Bigg[& -\alpha_i y_i(t) + \sum_{j=1}^{n} k_{ij} f(y_j(t)) + \sum_{j=1}^{n} h_{ij} f(y_j(t-\tau)) + \sum_{j=1}^{n} w_{ij} \int_{t-\tau}^{t} f(y_j(s))\mathrm{d}s \\
& + \bigwedge_{j=1}^{n} m_{ij} g(y_j(t-\tau)) + \bigvee_{j=1}^{n} \iota_{ij} g(y_j(t-\tau)) + \bigwedge_{j=1}^{n} s_{ij} \int_{t-\tau}^{t} g(y_j(s))\mathrm{d}s \\
& + \bigvee_{j=1}^{n} v_{ij} \int_{t-\tau}^{t} g(y_j(s))\mathrm{d}s + \bigwedge_{j=1}^{n} G_{ij} p_j + \bigvee_{j=1}^{n} J_{ij} p_j + A_i + u_i(t) \Bigg]\mathrm{d}t + \sigma_i(y_i(t), t)\mathrm{d}B(t),
\end{aligned}
\tag{2}
$$

where $y_i(t)$ denotes the ith neurons state at time t that is the solutions of response system and $u_i(t)$ is an appropriate control designed to ensure that drive-response systems (1) and (2) can achieve synchronization in fixed time.

We define the fixed-time synchronization error as $e_i(t) = y_i(t) - x_i(t)$. Then the error system can be expressed as follows:

$$
\begin{aligned}
\mathrm{d}e_i(t) =\Bigg[& -\alpha_i e_i(t) + \sum_{j=1}^{n} k_{ij} F(e_j(t)) + \sum_{j=1}^{n} h_{ij} F(e_j(t-\tau)) + \sum_{j=1}^{n} w_{ij} \int_{t-\tau}^{t} F(e_j(s))\mathrm{d}s \\
& + \bigwedge_{j=1}^{n} m_{ij} G(e_j(t-\tau)) + \bigvee_{j=1}^{n} \iota_{ij} G(e_j(t-\tau)) + \bigwedge_{j=1}^{n} s_{ij} \int_{t-\tau}^{t} G(e_j(s))\mathrm{d}s \\
& + \bigvee_{j=1}^{n} v_{ij} \int_{t-\tau}^{t} G(e_j(s))\mathrm{d}s + u_i(t) \Bigg]\mathrm{d}t + \sigma_i(e_i(t), t)\mathrm{d}B(t),
\end{aligned}
\tag{3}
$$

where

$$
\begin{aligned}
F(e_j(\cdot)) &= f(y_j(\cdot)) - f(x_j(\cdot)), \\
\bigwedge_{j=1}^{n} m_{ij} G(e_j(\cdot)) &= \bigwedge_{j=1}^{n} m_{ij} g(e_j(\cdot)) - \bigwedge_{j=1}^{n} m_{ij} g(e_j(\cdot)), \\
\bigvee_{j=1}^{n} \iota_{ij} G(e_j(\cdot)) &= \bigvee_{j=1}^{n} \iota_{ij} g(e_j(\cdot)) - \bigvee_{j=1}^{n} \iota_{ij} g(e_j(\cdot)), \\
\sigma_i(e_i(t), t) &= \sigma_i(y_i(t), t) - \sigma_i(x_i(t), t).
\end{aligned}
$$

The initial conditions of system (1) and system (2) are $\phi(s) = (\phi_1(s), \phi_2(s), \cdots, \phi_n(s)) \in \mathcal{L}^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$, $\varphi(s) = (\varphi_1(s), \varphi_2(s), \cdots, \varphi_n(s)) \in \mathcal{L}^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$. In this paper, we always assume that the functions $f, g (i, j \in \mathbb{N})$ satisfy all necessary conditions to ensure that systems (1) and (2) exist solutions with corresponding initial conditions. Then, the initial condition of error system (3) is denoted by $\psi(t) \in \mathcal{L}^2_{\mathcal{F}_0}([-\delta, 0]; \mathbb{R}^n)$.

Consider the following stochastic nonlinear system:

$$
\mathrm{d}x(t) = f(t, x(t))\mathrm{d}(t) + g(t, x(t))\mathrm{d}B(t), \tag{4}
$$

where $x(t_0) = x_0$, $x(t) \in \mathbb{R}^n$ denotes the state of system, $B(t)$ is an 1-dimensional Brownian motion defined on the complete probability space. The first hitting time is denoted as $T(x_0, B) = \inf\{T \geq 0 | x_0 = 0, t \geq T\}$, which is a settling time function. For the convenience of the presentation of the main results, some definitions, assumptions and lemmas follow.

Definition 2.1. [15] *For any initial state $x_0(t) \in \mathbb{R}^n$, the given positive constants T_ϵ, M exist such that the following statements hold:*

(1) *The origin is globally stochastic finite-time stable in probability.*

(2) *The mathematical expectation of settling time function $T(x_0, B)$ is bounded by a positive constant M, which do not depend on the initial condition, i.e.,*

$$T_\epsilon = \mathbb{E}(T(x_0, B)) \leq M, \forall x_0 \in \mathbb{R}^n,$$

then the trivial solution of system (4) is said to be stochastic fixed-time stable in probability.

Remark 2.2. *Nowadays, finite-time synchronization has been intensively studied by many scholars. Although finite-time synchronization enables the system to achieve synchronization in finite time, the convergence time depends heavily on the upper bound of the initial value of the system. However, the initial value of the system is difficult to obtain in many practical applications. Therefore, this paper investigates fixed-time synchronization, where the convergence time obtained is only related to the control parameters and not to the initial value of the system. This kind of synchronization does not need to calculate the initial value of the system in the process of application, which reduces the difficulty of practical application and widens the scope of application.*

Assumption 2.3. *For the nonnegative constants $\omega_i (i = 1, 2, \ldots, n)$, the noise intensity function $\sigma_i(t, e_i(t))$ satisfies the Lipschitz condition, and*

$$\sigma_i^T(t, e_i(t))\sigma_i(t, e_i(t)) \leq \omega_i e_i^T(t) e_i(t).$$

Assumption 2.4. *The functions $f(\cdot)$ and $g(\cdot)$ satisfy bounded and Lipschitz conditions, for all $s_1, s_2 \in \mathbb{R}$, if there exist positive constants L_1 and L_2, such that*

$$|f(s_1) - f(s_2)| \leq L_1 |s_1 - s_2|,$$
$$|g(s_1) - g(s_2)| \leq L_2 |s_1 - s_2|.$$

Lemma 2.5. [24] *Suppose x_1 and x_2 are two states of system (1), then we have*

$$\left| \bigwedge_{j=1}^{n} m_{ij} g(x_1) - \bigwedge_{j=1}^{n} m_{ij} g(x_2) \right| \leq \sum_{j=1}^{n} |m_{ij}||g(x_1) - g(x_2)|,$$
$$\left| \bigvee_{j=1}^{n} \iota_{ij} g(x_1) - \bigvee_{j=1}^{n} \iota_{ij} g(x_2) \right| \leq \sum_{j=1}^{n} |\iota_{ij}||g(x_1) - g(x_2)|.$$

Lemma 2.6. [27] *If $a_1, a_2, \ldots, a_n \geq 0$ and $0 < q < 1$, $p > 1$, the following inequality group is satisfied:*

$$\sum_{i=1}^{n} a_i^q \geq \left(\sum_{i=1}^{n} a_i \right)^q,$$
$$\sum_{i=1}^{n} a_i^p \geq n^{1-p} \left(\sum_{i=1}^{n} a_i \right)^p.$$

Lemma 2.7. [15] *For system (4), if there exist Lyapunov function $V(t, x(t)) \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}^+, \mathbb{R}^+)$, which is a positive definite and radially unbounded function and the real numbers $\xi_1 > 0$, $\xi_2 > 0$, $\alpha > 1$ and $0 < \beta < 1$ such that for the function $V(t, x(t))$, there have*

$$\mathcal{L}V(x) \leq -\xi_1 V^\alpha(x) - \xi_2 V^\beta(x),$$

then the origin of system (4) is stochastic fixed-time stable in probability, and the settling time

$$T_\epsilon = \mathbb{E}(T(x_0, B)) \leq \frac{1}{\xi_1} \frac{1}{\alpha - 1} + \frac{1}{\xi_2} \frac{1}{1 - \beta}.$$

3 Main results

In this Section, based on the Lyapunov method, general criterion for fixed-time synchronization of FSCNNs will be obtained.

3.1 Feedback control

First, a feedback control (5) is designed to make the FSCNNs reach synchronization in fixed time. The feedback control $u(t) = (u_1(t), u_2(t), \cdots, u_n(t))^T$ is given as

$$u_i(t) = \begin{cases} -\kappa_i e_i(t) - \sum_{j=1}^{n} \rho_{ij} |e_j(t-\tau)| \mathrm{sign}(e_i(t)) - \mu_1 |e_i(t)|^q \mathrm{sign}(e_i(t)) & \\ -\eta_1 |e_i(t)|^p \mathrm{sign}(e_i(t)) - \frac{1}{2}\sum_{j=1}^{n} \chi_{ij} \int_{t-\tau}^{t} |e_i(s)|^2 \mathrm{d}s \frac{e_i(t)}{|e_i(t)|^2}, & \text{if } e_i(t) \neq 0, \\ 0, & \text{if } e_i(t) = 0. \end{cases} \tag{5}$$

where $i \in \mathbb{N}$, the positive constants κ_i, ρ_{ij}, μ_1, η_1, χ_{ij} are the gain coefficients to be determined, and the real number q satisfies $0 < q < 1$, p satisfies $p > 1$.

Theorem 3.1. *Suppose that Assumptions2.3 and 2.4 hold, then the FSCNNs* (1) *and* (2) *are stochastic fixed-time synchronization in probability under the control* (5), *if the following three inequalities hold*

$$\begin{aligned} &-2\alpha_i - 2\kappa_i + \omega_i + \sum_{j=1}^{n} \{L_1 k_{ij} + L_1 k_{ji} + w_{ij} + s_{ij} + v_{ij}\} \leq 0, \\ &2\sum_{j=1}^{n} (L_1 h_{ij} + |m_{ij}| L_2 + |\iota_{ij}| L_2) - 2\sum_{j=1}^{n} \rho_{ij} \leq 0, \\ &\sum_{j=1}^{n} \left(\tau w_{ji} (L_1)^2 + \tau s_{ji} (L_2)^2 + \tau v_{ji} (L_2)^2\right) - \sum_{j=1}^{n} \chi_{ij} \leq 0. \end{aligned}$$

Moreover, the settling time is given by

$$T_\epsilon = \frac{1}{\mu_1}\frac{1}{1-q} + \frac{1}{\hat{\eta}_1}\frac{1}{p-1},$$

where $\hat{\eta}_1 = \eta_1 n^{1-p}$, $0 < q < 1$, $p > 1$.

Proof. First, we construct the Lyapunov function $V(t, e_i(t)) = \sum_{i=1}^{n} e_i^T(t) e_i(t)$, by the Itô formula, the infinitesimal operator of $V(t, e_i(t))$ is given by

$$\begin{aligned} \mathcal{L}V(t, e_i(t)) =& 2\sum_{i=1}^{n} e_i^T(t) \Bigg[-\alpha_i e_i(t) + \sum_{j=1}^{n} k_{ij} F(e_j(t)) + \sum_{j=1}^{n} h_{ij} F(e_j(t-\tau)) + \sum_{j=1}^{n} w_{ij} \int_{t-\tau}^{t} F(e_j(s)) \mathrm{d}s \\ &+ \bigwedge_{j=1}^{n} m_{ij} G(e_j(t-\tau)) + \bigvee_{j=1}^{n} \iota_{ij} G(e_j(t-\tau)) + \bigwedge_{j=1}^{n} s_{ij} \int_{t-\tau}^{t} G(e_j(s)) \mathrm{d}s \\ &+ \bigvee_{j=1}^{n} v_{ij} \int_{t-\tau}^{t} G(e_j(s)) \mathrm{d}s + u_i(t) \Bigg] + \sum_{i=1}^{n} \mathrm{Tr}(\sigma_i^T(t, e_i(t)) \sigma_i(t, e_i(t))). \end{aligned} \tag{6}$$

According to Assumption 2.4, we have

$$\begin{aligned} 2\sum_{i=1}^{n} e_i^T(t) \sum_{j=1}^{n} k_{ij} F(e_j(t)) &\leq 2\sum_{i=1}^{n} |e_i(t)| \sum_{j=1}^{n} k_{ij} L_1 |e_j(t)| \\ &\leq 2\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij} L_1 |e_i(t)||e_j(t)| \end{aligned}$$

$$
\begin{aligned}
&\le \sum_{i=1}^{n}\sum_{j=1}^{n}\left(k_{ij}L_1|e_i(t)|^2 + k_{ij}L_1|e_j(t)|^2\right)\\
&\le \sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}L_1|e_i(t)|^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} k_{ji}L_1|e_i(t)|^2\\
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\left(k_{ij}L_1 + k_{ji}L_1\right)|e_i(t)|^2,
\end{aligned}
\tag{7}
$$

and

$$
2\sum_{i=1}^{n} e_i^T(t)\sum_{j=1}^{n} h_{ij}F(e_j(t-\tau)) \le 2\sum_{i=1}^{n}\sum_{j=1}^{n} h_{ij}L_1|e_i(t)||e_j(t-\tau)|. \tag{8}
$$

By the Lemma 2.5 and Assumption 2.4, we can get

$$
\begin{aligned}
2\sum_{i=1}^{n} e_i^T(t)\bigwedge_{j=1}^{n} m_{ij}G(e_j(t-\tau)) &\le 2\sum_{i=1}^{n}|e_i(t)|\left|\bigwedge_{j=1}^{n} m_{ij}g(y_j(t-\tau)) - \bigwedge_{j=1}^{n} m_{ij}g(x_j(t-\tau))\right|\\
&\le 2\sum_{i=1}^{n}|e_i(t)|\sum_{j=1}^{n}|m_{ij}||G(e_j(t-\tau))|\\
&\le 2\sum_{i=1}^{n}\sum_{j=1}^{n}|m_{ij}|L_2|e_i(t)||(e_j(t-\tau))|,
\end{aligned}
\tag{9}
$$

and

$$
\begin{aligned}
2\sum_{i=1}^{n} e_i^T(t)\bigvee_{j=1}^{n} \iota_{ij}G(e_j(t-\tau)) &\le 2\sum_{i=1}^{n}|e_i(t)|\left|\bigvee_{j=1}^{n} \iota_{ij}g(y_j(t-\tau)) - \bigvee_{j=1}^{n} \iota_{ij}g(x_j(t-\tau))\right|\\
&\le 2\sum_{i=1}^{n}|e_i(t)|\sum_{j=1}^{n}|\iota_{ij}||G(e_j(t-\tau))|\\
&\le 2\sum_{i=1}^{n}\sum_{j=1}^{n}|\iota_{ij}|L_2|e_i(t)||(e_j(t-\tau))|.
\end{aligned}
\tag{10}
$$

Similarly, according to Cauchy-Schwarz inequality, we can get

$$
\begin{aligned}
2\sum_{i=1}^{n} e_i^T(t)\sum_{j=1}^{n} w_{ij}\int_{t-\tau}^{t} F(e_j(s))\mathrm{d}s &\le 2\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\,|e_i(t)|\int_{t-\tau}^{t} F(e_j(s))\mathrm{d}s\\
&\le \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\left(|e_i(t)|^2 + \left(\int_{t-\tau}^{t} F(e_j(s))\mathrm{d}s\right)^2\right)\\
&\le \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\,|e_i(t)|^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\tau\int_{t-\tau}^{t}(F(e_j(s)))^2\mathrm{d}s\\
&\le \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\,|e_i(t)|^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\tau(L_1)^2\int_{t-\tau}^{t}|e_j(s)|^2\,\mathrm{d}s,
\end{aligned}
\tag{11}
$$

and

$$
\begin{aligned}
&2\sum_{i=1}^{n} e_i^T(t)\left(\bigwedge_{j=1}^{n} s_{ij}\int_{t-\tau}^{t} G(e_j(s))\mathrm{d}s + \bigvee_{j=1}^{n} v_{ij}\int_{t-\tau}^{t} G(e_j(s))\mathrm{d}s\right)\\
\le&2\sum_{i=1}^{n} e_i^T(t)\left(\sum_{j=1}^{n} s_{ij}\int_{t-\tau}^{t} G(e_j(s))\mathrm{d}s + \sum_{j=1}^{n} v_{ij}\int_{t-\tau}^{t} G(e_j(s))\mathrm{d}s\right)
\end{aligned}
$$

$$
\begin{aligned}
&\leq 2\sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij}\left|e_i(t)\right|\int_{t-\tau}^{t} G(e_j(s))\mathrm{d}s + 2\sum_{i=1}^{n}\sum_{j=1}^{n} v_{ij}\left|e_i(t)\right|\int_{t-\tau}^{t} G(e_j(s))\mathrm{d}s \\
&\leq \sum_{i=1}^{n}\sum_{j=1}^{n}\left(s_{ij}+v_{ij}\right)\left|e_i(t)\right|^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij}\tau(L_2)^2\int_{t-\tau}^{t}\left|e_j(s)\right|^2\mathrm{d}s + \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\tau(L_2)^2\int_{t-\tau}^{t}\left|e_j(s)\right|^2\mathrm{d}s.
\end{aligned}
\tag{12}
$$

Moreover, according to Lemma 2.6, we can get

$$
\begin{aligned}
2\sum_{i=1}^{n} e_i(t)u_i(t) \leq &- 2\sum_{i=1}^{n}\kappa_i|e_i(t)|^2 - 2\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}|e_i(t)||e_j(t-\tau)| - 2\mu_1\left(\sum_{i=1}^{n} e_i^T(t)e_i(t)\right)^{\frac{1+q}{2}} \\
&- 2\eta_1 n^{1-p}\left(\sum_{i=1}^{n} e_i^T(t)e_i(t)\right)^{\frac{1+p}{2}} - \sum_{i=1}^{n}\sum_{j=1}^{n}\chi_{ij}\int_{t-\tau}^{t}|e_i(s)|^2\mathrm{d}s.
\end{aligned}
\tag{13}
$$

Substituting (7)-(13) into (6) and Assumption 2.3 holds, we can obtain

$$
\begin{aligned}
\mathcal{L}V(t,e_i(t)) \leq &\sum_{i=1}^{n}\left(-2\alpha_i - 2\kappa_i + 2\sum_{j=1}^{n}(k_{ij}L_1 + k_{ji}L_1) + \sum_{j=1}^{n}(w_{ij}+s_{ij}+v_{ij}) + \omega_i\right)|e_i(t)|^2 \\
&+ \sum_{i=1}^{n}\left(2\sum_{j=1}^{n} h_{ij}L_1 + 2\sum_{j=1}^{n}|m_{ij}|L_2 + 2\sum_{j=1}^{n}|\iota_{ij}|L_2 - 2\sum_{j=1}^{n}\rho_{ij}\right)|e_i(t)||e_j(t-\tau)| \\
&+ \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\tau w_{ji}\left(L_1\right)^2 + \sum_{j=1}^{n}\tau s_{ji}\left(L_2\right)^2 + \sum_{j=1}^{n}\tau v_{ji}\left(L_2\right)^2 - \sum_{j=1}^{n}\chi_{ij}\right)\int_{t-\tau}^{t}|e_i(s)|^2\mathrm{d}s \\
&- 2\mu_1\left(\sum_{i=1}^{n} e_i^T(t)e_i(t)\right)^{\frac{1+q}{2}} - 2\eta_1 n^{1-p}\left(\sum_{i=1}^{n} e_i^T(t)e_i(t)\right)^{\frac{1+p}{2}}.
\end{aligned}
\tag{14}
$$

Assuming that all the inequalities in Theorem 3.1 are true, we can get

$$
\begin{aligned}
\mathcal{L}V(t,e_i(t)) &\leq -2\mu_1\left(\sum_{i=1}^{n} e_i^T(t)e_i(t)\right)^{\frac{1+q}{2}} - 2\eta_1 n^{1-p}\left(\sum_{i=1}^{n} e_i^T(t)e_i(t)\right)^{\frac{1+p}{2}} \\
&\leq -2\mu_1\left(\sum_{i=1}^{n} e_i^T(t)e_i(t)\right)^{\frac{1+q}{2}} - 2\hat{\eta}_1\left(\sum_{i=1}^{n} e_i^T(t)e_i(t)\right)^{\frac{1+p}{2}} \\
&\leq -2\mu_1\left(V(t,e_i(t))\right)^{\frac{1+q}{2}} - 2\hat{\eta}_1\left(V(t,e_i(t))\right)^{\frac{1+p}{2}},
\end{aligned}
\tag{15}
$$

where $\hat{\eta}_1 = \eta_1 n^{1-p}$. Then, according to Lemma 2.7, we can calculate the settling time

$$
T_\epsilon = \frac{1}{\mu_1}\frac{1}{1-q} + \frac{1}{\hat{\eta}_1}\frac{1}{p-1}. \tag{16}
$$

This completes the proof. □

3.2 Adaptive control

Since the control gain κ_i of feedback control is difficult to determine, the next step is to design an adaptive control to make FSCNNs synchronized in a fixed time. Then, an adaptive control is designed as follows

$$
u_i(t) = \begin{cases}
-\kappa_i(t)e_i(t) - \sum_{j=1}^{n}\rho_{ij}|e_j(t-\tau)|\mathrm{sign}(e_i(t)) - \mu_2|e_i(t)|^q\mathrm{sign}(e_i(t)) \\
-\eta_2|e_i(t)|^p\mathrm{sign}(e_i(t)) - \frac{1}{2}\sum_{j=1}^{n}\chi_{ij}\int_{t-\tau}^{t}|e_i(s)|^2\mathrm{d}s\frac{e_i(t)}{|e_i(t)|^2}, & \text{if } e_i(t)\neq 0, \\
0, & \text{if } e_i(t) = 0.
\end{cases}
\tag{17}
$$

where $\kappa_i(t)$ is the adaptive regulated feedback gain, and we design adaptation rate

$$\dot{\kappa}_i(t) = e_i^T(t)e_i(t) - \mu_2 \mathrm{sign}(\kappa_i(t) - \kappa_1)(\kappa_i(t) - \kappa_1)^q - \eta_2 \mathrm{sign}(\kappa_i(t) - \kappa_1)(\kappa_i(t) - \kappa_1)^p, \tag{18}$$

where κ_1 is a constant to be determined. Then, we can get the criterion of fixed-time synchronization of FSCNNs under adaptive control (17).

Theorem 3.2. *Suppose that Assumptions2.3 and 2.4 hold, then the FsCNNs* (1) *and* (2) *are stochastic fixed-time synchronization in probability under the adaptive control* (17)*, if the following three inequalities hold*

$$\begin{aligned}
&-2\alpha_i - 2\kappa_1 + \omega_i + \sum_{j=1}^{n} \{L_1 k_{ij} + L_1 k_{ji} + w_{ij} + s_{ij} + v_{ij}\} \le 0,\\
&2\sum_{j=1}^{n} (|L_1|\, h_{ij} + |m_{ij}|L_2 + |\iota_{ij}|L_2) - 2\sum_{j=1}^{n} \rho_{ij} \le 0,\\
&\sum_{j=1}^{n} \left(\tau w_{ji} (L_1)^2 + \tau s_{ji} (L_2)^2 + \tau v_{ji} (L_2)^2\right) - \sum_{j=1}^{n} \chi_{ij} \le 0.
\end{aligned}$$

Moreover, the settling time is given by

$$T_\epsilon = \frac{1}{\mu_2}\frac{1}{1-q} + \frac{1}{\hat{\eta}_2}\frac{1}{p-1},$$

where $\hat{\eta}_2 = \eta_2 n^{1-p}$*,* $0 < q < 1$*,* $p > 1$*.*

Proof. When proving this theorem, we construct the Lyapunov function $V(t, e_i(t)) = \sum_{i=1}^{n} e_i^T(t)e_i(t) + \sum_{i=1}^{n}(\kappa_i(t) - \kappa_1)^2$. Then the infinitesimal operator of $V(t, e_i(t))$ is given by

$$\begin{aligned}
\mathcal{L}V(t, e_i(t)) =& 2\sum_{i=1}^{n} e_i^T(t)\Bigg[-\alpha_i e_i(t) + \sum_{j=1}^{n} k_{ij}F(e_j(t)) + \sum_{j=1}^{n} h_{ij}F(e_j(t-\tau)) + \sum_{j=1}^{n} w_{ij}\int_{t-\tau}^{t} F(e_j(s))\mathrm{d}s\\
&+ u_i(t) + \bigwedge_{j=1}^{n} m_{ij}G(e_j(t-\tau)) + \bigvee_{j=1}^{n} \iota_{ij}G(e_j(t-\tau)) + \bigwedge_{j=1}^{n} s_{ij}\int_{t-\tau}^{t} G(e_j(s))\mathrm{d}s\\
&+ \bigvee_{j=1}^{n} v_{ij}\int_{t-\tau}^{t} G(e_j(s))\mathrm{d}s\Bigg] + \sum_{i=1}^{n} \mathrm{Tr}(\sigma_i^T(t, e_i(t))\sigma_i(t, e_i(t))) + 2\sum_{i=1}^{n} (\kappa_i(t) - \kappa_1)\cdot \dot{\kappa}_i(t).
\end{aligned} \tag{19}$$

According to (18), we can get

$$\begin{aligned}
&2\sum_{i=1}^{n} e_i^T(t)u_i(t) + 2\sum_{i=1}^{n}(\kappa_i(t) - \kappa_1)\cdot\dot{\kappa}_i(t)\\
=&2\sum_{i=1}^{n} e_i^T(t)\Bigg(-\kappa_i(t)e_i(t) - \sum_{j=1}^{n} \rho_{ij}|e_j(t-\tau)|\mathrm{sign}(e_j(t)) - \mu_2|e_i(t)|^q\mathrm{sign}(e_i(t))\\
&-\eta_2|e_i(t)|^p\mathrm{sign}(e_i(t)) - \frac{1}{2}\sum_{j=1}^{n}\chi_{ij}\int_{t-\tau}^{t}|e_i(s)|^2\mathrm{d}s\frac{e_i(t)}{|e_i(t)|^2}\Bigg) + 2\sum_{i=1}^{n}(\kappa_i(t) - \kappa_1)\cdot\Bigg(e_i^T(t)e_i(t)\\
&-\mu_2\mathrm{sign}(\kappa_i(t) - \kappa_1)(\kappa_i(t) - \kappa_1)^q - \eta_2\mathrm{sign}(\kappa_i(t) - \kappa_1)(\kappa_i(t) - \kappa_1)^p\Bigg)\\
=&2\sum_{i=1}^{n} e_i^T(t)\Bigg(-(\kappa_i(t) - \kappa_1)\, e_i(t) - \sum_{j=1}^{n} \rho_{ij}|e_j(t-\tau)|\mathrm{sign}(e_i(t)) - \mu_2|e_i(t)|^q\mathrm{sign}(e_i(t))\\
&-\eta_2|e_i(t)|^p\mathrm{sign}(e_i(t)) - \frac{1}{2}\sum_{j=1}^{n}\chi_{ij}\int_{t-\tau}^{t}|e_i(s)|^2\mathrm{d}s\frac{e_i(t)}{|e_i(t)|^2}\Bigg) + 2\sum_{i=1}^{n}(\kappa_i(t) - \kappa_1)\cdot\Bigg(e_i^T(t)e_i(t)\\
&-\mu_2\mathrm{sign}(\kappa_i(t) - \kappa_1)(\kappa_i(t) - \kappa_1)^q - \eta_2\mathrm{sign}(\kappa_i(t) - \kappa_1)(\kappa_i(t) - \kappa_1)^p - 2\sum_{i=1}^{n}\kappa_1 e_i^T(t)e_i(t)\Bigg)
\end{aligned}$$

$$
\begin{aligned}
&\leq -2\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}|e_i(t)||e_j(t-\tau)| - 2\mu_2\left(\sum_{i=1}^{n}e_i^T(t)e_i(t)\right)^{\frac{1+q}{2}} - 2\eta_2 n^{1-p}\left(\sum_{i=1}^{n}e_i^T(t)e_i(t)\right)^{\frac{1+p}{2}}\\
&\quad -\sum_{i=1}^{n}\sum_{j=1}^{n}\chi_{ij}\int_{t-\tau}^{t}|e_i(s)|^2\mathrm{d}s - 2\mu_2\sum_{i=1}^{n}\left(\kappa_i(t)-\kappa_1\right)^{1+q} - 2\eta_2 n^{1-p}\sum_{i=1}^{n}\left(\kappa_i(t)-\kappa_1\right)^{1+p} - 2\sum_{i=1}^{n}\kappa_1 e_i^T(t)e_i(t).
\end{aligned}
\tag{20}
$$

On the basis of (7)-(11) and (20), we can obtain

$$
\begin{aligned}
\mathcal{L}V(t,e_i(t)) \leq &\sum_{i=1}^{n}\left(-2\alpha_i - 2\kappa_1 + 2\sum_{j=1}^{n}(k_{ij}L_1 + k_{ji}L_1) + \sum_{j=1}^{n}(w_{ij}+s_{ij}+v_{ij}) + \sum_{j=1}^{n}\omega_{ij}\right)|e_i(t)|^2\\
&+\sum_{i=1}^{n}\left(2\sum_{j=1}^{n}h_{ij}L_1 + 2\sum_{j=1}^{n}|m_{ij}|L_2 + 2\sum_{j=1}^{n}|\iota_{ij}|L_2 - 2\sum_{j=1}^{n}\rho_{ij}\right)|e_i(t)||e_j(t-\tau)|\\
&+\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\tau w_{ji}\left(L_1\right)^2 + \sum_{j=1}^{n}\tau s_{ji}\left(L_2\right)^2 + \sum_{j=1}^{n}\tau v_{ji}\left(L_2\right)^2 - \sum_{j=1}^{n}\chi_{ij}\right)\int_{t-\tau}^{t}|e_i(s)|^2\mathrm{d}s\\
&-2\mu_2\left(\sum_{i=1}^{n}e_i^T(t)e_i(t)\right)^{\frac{1+q}{2}} - 2\eta_2 n^{1-p}\left(\sum_{i=1}^{n}e_i^T(t)e_i(t)\right)^{\frac{1+p}{2}} - 2\mu_2\sum_{i=1}^{n}(\kappa_i(t)-\kappa_1)^{1+q}\\
&-2\eta_2 n^{1-p}\sum_{i=1}^{n}\left(\kappa_i(t)-\kappa_1\right)^{1+p}.
\end{aligned}
\tag{21}
$$

If the conditions in Theorem 3.2 hold, we can get

$$
\begin{aligned}
\mathcal{L}V(t,e_i(t)) \leq &-2\mu_2\left(\sum_{i=1}^{n}e_i^T(t)e_i(t)\right)^{\frac{1+q}{2}} - 2\eta_2 n^{1-p}\left(\sum_{i=1}^{n}e_i^T(t)e_i(t)\right)^{\frac{1+p}{2}} - 2\mu_2\sum_{i=1}^{n}\left(\kappa_i(t)-\kappa_1\right)^{1+q}\\
&-2\eta_2 n^{1-p}\sum_{i=1}^{n}\left(\kappa_i(t)-\kappa_1\right)^{1+p}\\
\leq &-2\mu_2\left(\sum_{i=1}^{n}e_i^T(t)e_i(t)\right)^{\frac{1+q}{2}} - 2\eta_2 n^{1-p}\left(\sum_{i=1}^{n}e_i^T(t)e_i(t)\right)^{\frac{1+p}{2}} - 2\mu_2\left(\sum_{i=1}^{n}\left(\kappa_i(t)-\kappa_1\right)^2\right)^{\frac{1+q}{2}}\\
&-2\eta_2 n^{1-p}\left(\sum_{i=1}^{n}\left(\kappa_i(t)-\kappa_1\right)^2\right)^{\frac{1+p}{2}}\\
\leq &-2\mu_2\left(\sum_{i=1}^{n}e_i^T(t)e_i(t) + \sum_{i=1}^{n}\left(\kappa_i(t)-\kappa_1\right)^2\right)^{\frac{1+q}{2}} - 2\eta_2 n^{1-p}\left(\sum_{i=1}^{n}e_i^T(t)e_i(t) + \sum_{i=1}^{n}\left(\kappa_i(t)-\kappa_1\right)^2\right)^{\frac{1+p}{2}}\\
\leq &-2\mu_2\left(V(t,e_i(t))\right)^{\frac{1+q}{2}} - 2\eta_2 n^{1-p}\left(V(t,e_i(t))\right)^{\frac{1+p}{2}}\\
\leq &-2\mu_2\left(V(t,e_i(t))\right)^{\frac{1+q}{2}} - 2\hat{\eta}_2\left(V(t,e_i(t))\right)^{\frac{1+p}{2}}
\end{aligned}
\tag{22}
$$

where $\hat{\eta}_2 = \eta_2 n^{1-p}$. Then, according to Lemma 2.7, we can calculate the settling time

$$
T_\epsilon = \frac{1}{\mu_2}\frac{1}{1-q} + \frac{1}{\hat{\eta}_2}\frac{1}{p-1},
\tag{23}
$$

This completes the proof. □

4 Numerical example

In this section, two numerical examples are given to illustrate the effectiveness of our results. Let $\mathbb{N} = \{1, 2, \ldots, 12\}$, drive system is given as follows:

$$\begin{aligned} \mathrm{d}x_i(t) = \Bigg[& -\alpha_i x_i(t) + \sum_{j=1}^{12} k_{ij} f(x_j(t)) + \sum_{j=1}^{12} h_{ij} f(x_j(t-\tau)) + \sum_{j=1}^{12} w_{ij} \int_{t-\tau}^{t} f(x_j(s))\mathrm{d}s \\ & + \bigwedge_{j=1}^{12} m_{ij} g(x_j(t-\tau)) + \bigvee_{j=1}^{12} \iota_{ij} g(x_j(t-\tau)) + \bigwedge_{j=1}^{12} s_{ij} \int_{t-\tau}^{t} g(x_j(s))\mathrm{d}s \\ & + \bigvee_{j=1}^{12} v_{ij} \int_{t-\tau}^{t} g(x_j(s))\mathrm{d}s + \bigwedge_{j=1}^{12} G_{ij} p_j + \bigvee_{j=1}^{12} J_{ij} p_j + A_i \Bigg] \mathrm{d}t + \sigma_i(x_i(t), t)\mathrm{d}B(t), \end{aligned} \tag{24}$$

where $\alpha_i = 1.7$, $\tau = 1$, $A_i = 0$, $i \in \mathbb{N}$, $L_1 = L_2 = 1$, $f(\cdot) = g(\cdot) = \tan(\cdot)$, $p_j = 0.1$, $G_{ij} = J_{ij} = 1$, $i, j \in \mathbb{N}$, $\sigma_1(x_1(t), t) = 3x_1(t)$, $\sigma_2(x_2(t), t) = 3.5x_2(t)$, $\sigma_3(x_3(t), t) = 3x_3(t)$, $\sigma_4(x_4(t), t) = 4.5x_4(t)$, $\sigma_5(x_5(t), t) = -3x_5(t)$, $\sigma_6(x_6(t), t) = -3.5x_6(t)$, $\sigma_7(x_7(t), t) = -4x_7(t)$, $\sigma_8(x_8(t), t) = -4.5x_8(t)$, $\sigma_9(x_9(t), t) = 3.6x_9(t)$, $\sigma_{10}(x_{10}(t), t) = -3.2x_{10}(t)$, $\sigma_{11}(x_{11}(t), t) = 2.8x_{11}(t)$, $\sigma_{12}(x_{12}(t), t) = -2.4x_{12}(t)$ and $w_{1,2} = w_{4,3} = w_{7,6} = 0.36$, $w_{2,3} = w_{3,2} = w_{6,8} = 0.3$, $w_{3,7} = w_{5,3} = w_{5,9} = w_{6,2} = w_{7,5} = 0.42$, $w_{8,2} = w_{8,8} = w_{9,8} = w_{10,8} = w_{10,11} = w_{11,7} = w_{12,8} = 0.35$, $w_{24} = 0.45$, $w_{46} = 0.49$, and others not mentioned are all zero.

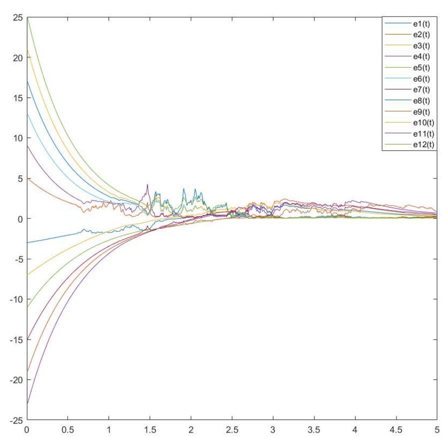

Figure 1: The trajectories of error system without controller.

Some values of k_{ij}, h_{ij}, m_{ij}, ι_{ij}, s_{ij}, v_{ij} are listed in Table 1-Table 4, s_{ij} and v_{ij} takes the same value and others not mentioned are all zero.

Table 1: The value of k_{ij} and h_{ij}.

value	The elements in the matrix									
0.6	$k_{1,3}$	$k_{1,10}$	$k_{1,12}$	$k_{2,6}$	$k_{2,9}$	$k_{3,2}$	$k_{3,5}$	$k_{3,11}$	$k_{4,2}$	$k_{4,9}$
	$k_{5,7}$	$k_{5,11}$	$k_{6,7}$	$k_{7,8}$	$k_{8,6}$	$k_{8,11}$	$k_{9,4}$	$k_{9,9}$	$k_{10,5}$	$k_{10,8}$
	$k_{11,6}$	$k_{11,2}$	$k_{12,7}$	$h_{1,3}$	$h_{1,10}$	$h_{2,1}$	$h_{2,6}$	$h_{2,11}$	$h_{3,5}$	$h_{3,12}$
	$h_{5,7}$	$h_{5,12}$	$h_{6,7}$	$h_{7,8}$	$h_{7,10}$	$h_{8,6}$	$h_{8,10}$	$h_{9,5}$	$h_{9,9}$	$h_{10,6}$
	$h_{10,9}$	$h_{11,7}$	$h_{11,3}$	$h_{11,12}$	$h_{12,8}$					
0.7	$k_{4,2}$	$k_{4,9}$	$k_{5,7}$	$k_{5,11}$	$k_{6,4}$	$k_{7,3}$	$h_{4,2}$	$h_{4,8}$	$h_{6,4}$	$h_{7,3}$
0.5	$k_{2,1}$	$h_{3,2}$								

Table 2: The value of m_{ij}.

value	The elements in the matrix									
0.024	$m_{1,3}$	$m_{1,12}$	$m_{2,1}$	$m_{2,12}$	$m_{3,9}$	$m_{4,12}$	$m_{5,7}$	$m_{5,12}$	$m_{6,4}$	$m_{6,12}$
	$m_{7,8}$	$m_{7,11}$	$m_{8,6}$	$m_{9,7}$	$m_{10,4}$	$m_{10,10}$	$m_{11,4}$	$m_{11,10}$	$m_{12,4}$	$m_{12,10}$
0.02	$m_{1,5}$	$m_{1,19}$	$m_{2,6}$	$m_{2,10}$	$m_{3,5}$	$m_{3,12}$	$m_{4,2}$	$m_{4,10}$	$m_{5,9}$	$m_{6,7}$
	$m_{6,11}$	$m_{7,3}$	$m_{7,9}$	$m_{8,11}$	$m_{9,1}$	$m_{9,5}$	$m_{9,7}$	$m_{9,12}$	$m_{10,1}$	$m_{10,7}$
	$m_{10,12}$	$m_{11,1}$	$m_{11,7}$	$m_{11,12}$	$m_{12,1}$	$m_{12,7}$	$m_{12,12}$			
0.012	$m_{4,9}$	$m_{9,9}$	$m_{10,9}$	$m_{11,9}$	$m_{12,9}$					

Table 3: The value of ι_{ij}.

value	The elements in the matrix									
0.012	$\iota_{1,3}$	$\iota_{1,12}$	$\iota_{2,1}$	$\iota_{2,12}$	$\iota_{3,9}$	$\iota_{4,12}$	$\iota_{5,7}$	$\iota_{5,12}$	$\iota_{6,4}$	$\iota_{6,12}$
	$\iota_{7,8}$	$\iota_{7,11}$	$\iota_{8,6}$	$\iota_{9,10}$	$\iota_{10,4}$	$\iota_{10,10}$	$\iota_{11,4}$	$\iota_{11,10}$	$\iota_{12,4}$	$\iota_{12,10}$
0.01	$\iota_{1,5}$	$\iota_{1,19}$	$\iota_{2,6}$	$\iota_{2,10}$	$\iota_{3,5}$	$\iota_{3,12}$	$\iota_{4,2}$	$\iota_{4,10}$	$\iota_{5,5}$	$\iota_{6,7}$
	$\iota_{6,11}$	$\iota_{7,3}$	$\iota_{7,9}$	$\iota_{8,11}$	$\iota_{9,1}$	$\iota_{5,11}$	$\iota_{9,12}$	$\iota_{9,12}$	$\iota_{10,1}$	$\iota_{10,12}$
	$\iota_{11,1}$	$\iota_{11,7}$	$\iota_{11,12}$	$\iota_{12,1}$	$\iota_{12,7}$	$\iota_{12,12}$				
0.008	$\iota_{4,9}$	$\iota_{9,9}$	$\iota_{10,9}$	$\iota_{11,9}$	$\iota_{12,9}$					
0.014	$\iota_{3,2}$									

Then, response system is given as

$$\begin{aligned}\mathrm{d}y_i(t) =\bigg[&- \alpha_i y_i(t) + \sum_{j=1}^{12} k_{ij} f(y_j(t)) + \sum_{j=1}^{12} h_{ij} f(y_j(t-\tau)) + \sum_{j=1}^{12} w_{ij} \int_{t-\tau}^{t} f(y_j(s))\mathrm{d}s \\ &+ \bigwedge_{j=1}^{12} m_{ij} g(y_j(t-\tau)) + \bigvee_{j=1}^{12} \iota_{ij} g(y_j(t-\tau)) + \bigwedge_{j=1}^{12} s_{ij} \int_{t-\tau}^{t} g(y_j(s))\mathrm{d}s \\ &+ \bigvee_{j=1}^{12} v_{ij} \int_{t-\tau}^{t} g(y_j(s))\mathrm{d}s + \bigwedge_{j=1}^{12} G_{ij} p_j + \bigvee_{j=1}^{12} J_{ij} p_j + A_i + u_i(t)\bigg]\mathrm{d}t + \sigma_i(y_i(t),t)\mathrm{d}B(t),\end{aligned} \tag{25}$$

where $\sigma_1(y_1(t),t) = 3y_1(t)$, $\sigma_2(y_2(t),t) = 3.5y_2(t)$, $\sigma_3(y_3(t),t) = 3y_3(t)$, $\sigma_4(y_4(t),t) = 4.5y_4(t)$, $\sigma_5(y_5(t),t) = -3y_5(t)$, $\sigma_6(y_6(t),t) = -3.5y_6(t)$, $\sigma_7(y_7(t),t) = -4y_7(t)$, $\sigma_8(y_8(t),t) = -4.5y_8(t)$, $\sigma_9(y_9(t),t) = 3.6y_9(t)$, $\sigma_{10}(y_{10}(t),t) = -3.2y_{10}(t)$, $\sigma_{11}(y_{11}(t),t) = 2.8y_{11}(t)$, $\sigma_{12}(y_{12}(t),t) = -2.4y_{12}(t)$.

Table 4: The value of s_{ij}.

value	The elements in the matrix									
0.22	$s_{1,3}$	$s_{1,5}$	$s_{1,9}$	$s_{2,1}$	$s_{2,3}$	$s_{2,10}$	$s_{2,12}$	$s_{3,5}$	$s_{4,2}$	$s_{4,9}$
	$s_{4,10}$	$s_{5,8}$	$s_{5,10}$	$s_{6,4}$	$s_{6,12}$	$s_{7,9}$	$s_{7,11}$	$s_{8,6}$	$s_{9,7}$	$s_{10,4}$
	$s_{10,9}$	$s_{10,12}$	$s_{11,1}$	$s_{11,4}$	$s_{11,10}$	$s_{12,9}$				
0.11	$s_{1,12}$	$s_{2,6}$	$s_{3,2}$	$s_{3,12}$	$s_{6,7}$	$s_{6,11}$	$s_{7,3}$	$s_{7,8}$	$s_{8,11}$	$s_{9,1}$
	$s_{9,7}$	$s_{9,10}$	$s_{10,7}$	$s_{11,7}$	$s_{11,9}$	$s_{11,12}$	$s_{12,4}$	$s_{12,7}$	$s_{12,12}$	$s_{1,2}$
0.33	$s_{3,9}$									

Moreover, we can get the error system

$$\begin{aligned}\mathrm{d}e_i(t) =\bigg[&- \alpha_i e_i(t) + \sum_{j=1}^{12} k_{ij} F(e_j(t)) + \sum_{j=1}^{12} h_{ij} F(e_j(t-\tau)) + \sum_{j=1}^{12} w_{ij} \int_{t-\tau}^{t} F(e_j(s))\mathrm{d}s \\ &+ \bigwedge_{j=1}^{12} m_{ij} G(e_j(t-\tau)) + \bigvee_{j=1}^{12} n_{ij} G(e_j(t-\tau)) + \bigwedge_{j=1}^{12} s_{ij} \int_{t-\tau}^{t} G(e_j(s))\mathrm{d}s \\ &+ \bigvee_{j=1}^{12} v_{ij} \int_{t-\tau}^{t} G(e_j(s))\mathrm{d}s + u_i(t)\bigg]\mathrm{d}t + \sigma_i(e_i(t),t)\mathrm{d}B(t),\end{aligned} \tag{26}$$

Example 4.1. *This numerical example is provided to verify the effectiveness of Theorem 3.1. That is, drive system* (24) *and response system* (25) *can achieve fixed-time synchronization under the effect of feedback control.*

The feedback control is designed as

$$u_i(t) = \begin{cases} -\kappa_i e_i(t) - \sum_{j=1}^{12} \rho_{ij}|e_j(t-\tau)|\mathrm{sign}(e_i(t)) - \mu_1|e_i(t)|^q \mathrm{sign}(e_i(t)) \\ -\eta_1|e_i(t)|^p \mathrm{sign}(e_i(t)) - \frac{1}{2}\sum_{j=1}^{12}\chi_{ij}\int_{t-\tau}^{t}|e_i(s)|^2 \mathrm{d}s \frac{e_i(t)}{|e_i(t)|^2}, & \text{if } e_i(t) \neq 0, \\ 0, & \text{if } e_i(t) = 0. \end{cases} \tag{27}$$

where $\alpha_i = 50$, $\mu_1 = 0.5$, $\eta_1 = 0.7$, $i \in \mathbb{N}$, $\chi_{11} = \chi_{22} = \chi_{33} = \chi_{44} = \chi_{55} = \chi_{66} = \chi_{77} = \chi_{88} = 4$, $q = 0.2$, $p = 1.2$, s_{ij} and v_{ij} takes the same value and others not mentioned are all zero. By calculation, we can verify that the condition

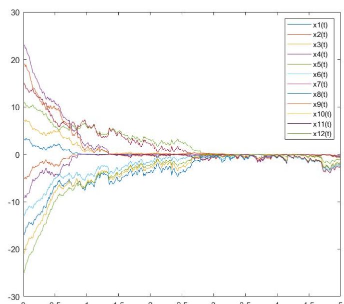

Figure 2: The trajectories of drive system (24).

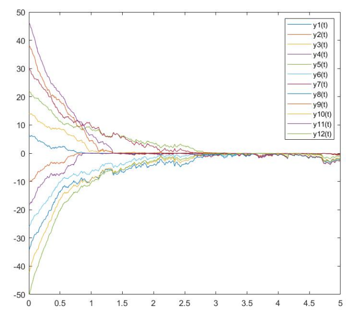

Figure 3: The trajectories of response system (25) under controller (27).

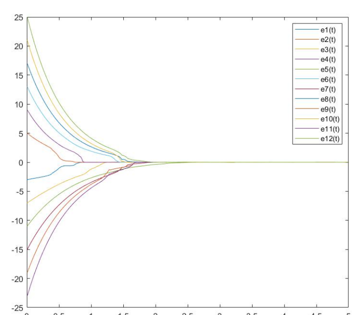

Figure 4: The trajectories of error system (26) under feedback controller (27).

in Theorem 3.1 is true, that is,

$$-2\alpha_i - 2\kappa_i + \sum_{j=1}^{12}\{L_1 k_{ij} + L_1 k_{ji} + w_{ij} + s_{ij} + v_{ij} + \omega_{ij}\} \le 0,$$

$$2\sum_{j=1}^{12}(L_1 h_{ij} + |m_{ij}|L_2 + |\iota_{ij}|L_2) - 2\sum_{j=1}^{12}\rho_{ij} \le 0,$$

$$\sum_{j=1}^{12}\left(\tau w_{ij}\left(L_1\right)^2+\tau s_{ij}\left(L_2\right)^2+\tau v_{ij}\left(L_2\right)^2\right)-\sum_{j=1}^{12}\chi_{ij}\leq 0.$$

Figure 1 shows the trajectories of the uncontrolled error system. The error system does not reach synchronization in a fixed time, which indicates that it is impossible for the system to reach synchronization without applying control. Therefore, we applied feedback control on the response system, and with the feedback control (27), the trajectories of the driven system and the controlled response system are shown in Figure 2 and Figure 3, and the trajectories of the error system is shown in Figure 4. In Figure 4, it is easy to see that the trajectories of the error system tend to zero in a fixed time, and according to the definition of the error system, it is known that the drive system and the response system reach synchronization. It can be seen that Theorem 3.1 is valid.

Example 4.2. *This numerical example is provided to verify the effectiveness of Theorem 3.2. That is, drive system* (24) *and response system* (25) *can achieve fixed-time synchronization under the effect of adaptive control.*

The adaptive control is given as

$$u_i(t)=\begin{cases} -\kappa_i(t)e_i(t)-\displaystyle\sum_{j=1}^{12}\rho_{ij}|e_j(t-\tau)|\mathrm{sign}(e_i(t))-\mu_2|e_i(t)|^q\mathrm{sign}(e_i(t)) & \\ -\eta_2|e_i(t)|^p\mathrm{sign}(e_i(t))-\dfrac{1}{2}\displaystyle\sum_{j=1}^{12}\chi_{ij}\int_{t-\tau}^{t}|e_i(s)|^2\mathrm{d}s\frac{e_i(t)}{|e_i(t)|^2}, & \text{if } e_i(t)\neq 0,\\ 0, & \text{if } e_i(t)=0.\end{cases}\tag{28}$$

where $\kappa_i(t)$ is the adaptive regulated feedback gain, and we design adaptation rate

$$\dot{\kappa}_i(t)=e_i^T(t)e_i(t)-\mu_2\mathrm{sign}(\kappa_i(t)-\kappa_1)(\kappa_i(t)-\kappa_1)^q-\eta_2\mathrm{sign}(\kappa_i(t)-\kappa_1)(\kappa_i(t)-\kappa_1)^p,\tag{29}$$

where $\kappa_1=90$, $\mu_2=0.5$, $\eta_2=0.7$, $i\in\mathbb{N}$,$\chi_{11}=\chi_{22}=\chi_{33}=\chi_{44}=\chi_{55}=\chi_{66}=\chi_{77}=\chi_{88}=4$, $q=0.2$, $p=1.2$, s_{ij} and v_{ij} takes the same value and others not mentioned are all zero. We can verify that the condition of Theorem 3.2

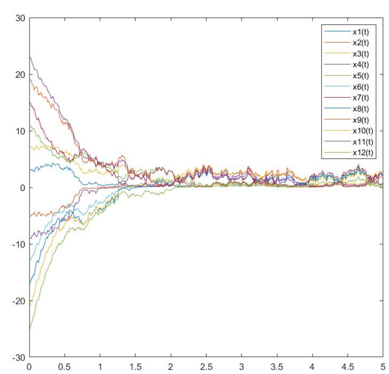

Figure 5: The trajectories of drive system (24).

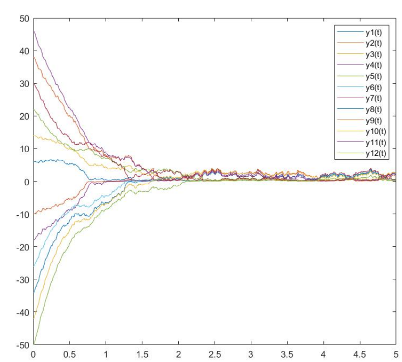

Figure 6: The trajectories of response system (25) under controller (28).

is true, that is,

$$-2\alpha_i-2\kappa_1+\sum_{j=1}^{12}\{L_1k_{ij}+L_1k_{ji}+w_{ij}+s_{ij}+v_{ij}+\omega_{ij}\}\leq 0,$$

$$2\sum_{j=1}^{12}(L_1h_{ij}+|m_{ij}|L_2+|\iota_{ij}|L_2)-2\sum_{j=1}^{12}\rho_{ij}\leq 0,$$

$$\sum_{j=1}^{12}\left(\tau w_{ij}\left(L_1\right)^2+\tau s_{ij}\left(L_2\right)^2+\tau v_{ij}\left(L_2\right)^2\right)-\sum_{j=1}^{12}\chi_{ij}\leq 0.$$

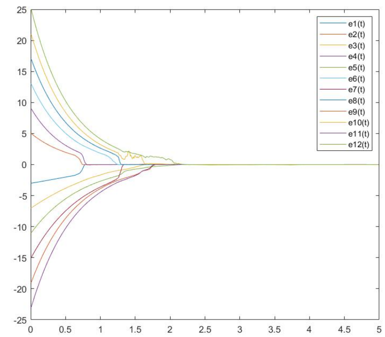

Figure 7: The trajectories of error system (26) under adaptive controller (28).

The trajectories of the response system (25) under adaptive control (28) is shown in Figure 6. In addition, Figure 7 shows the trajectories of the error system (26) under adaptive control (28). It can be seen from the figure that the error system (26) eventually converges to zero as time t changes, which shows that the drive system (24) and the response system (25) achieve fixed-time synchronization under the effect of adaptive control (28). Therefore, the validity of Theorem 3.2 is explained.

By observing Figure 4 and Figure 7, we can know that both the designed feedback control and adaptive control can make the FSCNNs reach to achieve fixed time synchronization. However, in comparison, the convergence speed of Figure 7 is faster and its error system converges to zero earlier than that of Figure 4. Therefore, the adaptive control designed in this paper is more effective in the same situation.

Remark 4.3. *The numerical simulation shows that both the feedback control and the adaptive control proposed in this paper can make the FCSNNs synchronize in fixed time and the simulation is good, which shows that our proposed controller is feasible. In addition, adaptive control has not been used for the study of fixed-time synchronization of fuzzy cellular neural networks so far, and the designed adaptive controller is more general and robust in handling fixed-time synchronization of fuzzy cellular neural networks compared with the existing results. The results in this study extend the results of previous studies and have better applicability.*

5 Conclusions

In this paper, the fixed-time synchronization of fuzzy stochastic cellular neural networks with discrete and distributed delay was studied. By utilizing Lyapunov method, some sufficient conditions guaranteeing fixed-time synchronization of FSCNNs were obtained. And two numerical examples were carried out to verify the validity of the theoretical results. In the future, the following interesting topics will be considered:1. The synchronization of neural networks with Markov switching has been widely studied recently [16, 18], so we will consider the fixed-time synchronization of FSCNNs with Markov switching in the future; 2. Because sliding mode control can overcome the uncertainty of the system, it has strong robustness to disturbances and unmodeled dynamics [21, 25], in the future work, the fixed-time synchronization of FSCNNs via sliding mode control is also one of the topic we are interested in.

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Adaptive control design for fixed-time synchronization of fuzzy stochastic cellular neural networks with discrete and distributed delay

Y. Liu, M. Liu and X. Xu

طراحی کنترل تطبیقی برای همگام سازی زمان ثابت شبکه‌های عصبی سلولی تصادفی فازی با تأخیر گسسته و توزیع شده

چکیده. این مقاله مسأله همگام سازی زمان ثابت شبکه‌های عصبی سلولی تصادفی فازی (FSCNNs) با تأخیر گسسته و توزیع شده را مطالعه می‌کند. در مقایسه با همگام سازی زمان محدود در ادبیات موجود، همگام سازی زمان ثابت FSCNN ها برای اولین بار مورد مطالعه قرار می‌گیرد، و زمان همگرایی بدست آمده به کران بالای مقدار اولیه سیستم بستگی ندارد. علاوه‌براین، دو نوع کنترل طراحی می‌شوند، یکی کنترل بازخورد و دیگری کنترل تطبیقی. همچنین، لازم به ذکر است که در این مقاله، برای اولین بار از طریق کنترل تطبیقی به همگام سازی FSCNNها با زمان ثابت می‌رسیم. سرانجام، دو مثال عددی نیز برای نشان دادن عملی بودن و اعتبار نتایج پیشنهادی ما ارائه شده است.