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Modeling and Forecasting Iranian Inflation with Time Varying BVAR Models

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Abstract¹

This paper investigates the forecasting performance of different time-varying BVAR models for Iranian inflation. Forecast accuracy of a BVAR model with Litterman's prior compared with a time-varying BVAR model (a version introduced by Doan et al., 1984); and a modified time-varying BVAR model, where the autoregressive coefficients are held constant and only the deterministic components are allowed to vary over time. Application using quarterly data of the Iranian economy from 1981:Q2 to 2006:Q1 shows that the performance of different specifications of time-varying BVAR models for forecasting inflation depends on the number of lags, hyper parameter that controls time variation, and forecast horizons. Our results, however, show that the modified time-varying BVAR model performs much better than other models regardless of the factors above.

JEL Classification: C53, C5

Key Words: Inflation, BVAR Model, Iranian economy

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Introduction

This paper examines the forecast accuracy of different Bayesian Vector Autoregressive (BVAR) models with different sources of time variation for forecasting Iranian inflation. There are a number of approaches to forecasting inflation used in academic literature and in policy making institutions. Moshiri (2001) uses a structural (an augmented Phillips Curve), a time series (an AR(1) model), and an Artificial Neural Networks (ANN) models to forecast Iranian inflation¹. Although each of these approaches has particular advantages, they are not free of limitations. This paper uses atheoretical Vector Autoregressive (VAR) models. These models are superior to the class of ARMA models (used in Moshiri, 2001) in various respects and have not yet been used in the literature to model and forecast Iranian inflation. [Moshiri and Cameron have also applied BVAR model along with ARMA and ANN models to forecast Canadian Inflation rate, See Moshiri Saeed, and Norman Cameron (2000), ANN versus Econometric Models in Forecasting Inflation, *Journal of Forecasting*, 19, Feb.]

The VAR, however, have the disadvantage of having a large number of parameters to estimate, which can be alleviated by using Bayesian methods (see, e.g., Doan, et al., 1984; Litterman, 1986; Todd; 1984). The literature suggests that BVAR, which reduces the parameter space by incorporating extraneous information, improves forecast accuracy of Unrestricted VAR (UVAR) models (see e.g., Artis and Zhang, 1990; Ballabriga et al., 1999 and 2000; Doan et al., 1984; Felix and Nunes, 2003; Kadiyala and Karlsson, 1993 and 1997; Kenny et al., 1998; Litterman, 1984 and 1986; McNees, 1986; Robertson and Tallman, 1999; Sims, 1993; Sims and Zha, 1998; Todd, 1984). The performance of the BVAR models in forecasting inflation however, has been somewhat less impressive (see e.g., Kenny et al., 1998; Litterman, 1986; McNees, 1986; Robertson and Tallman, 1999; Webb, 1995; Zarnowitz and Braun, 1992).

There are some possible explanations for the poor performance of the Traditional BVAR models for inflation forecasts. One of the most important explanations is possible regime changes (see, e.g., Boschen and Talbot, 1991; Cecchetti, 1995; and Webb, 1995). BVAR models (VAR models in general) have the disadvantage of a lack of robustness to deterministic shifts, exacerbated by the ill-determination of the intercept (see, e.g., Hendry and Clements, 2001 and 2003; Bewley, 2001, among others). The literature offers approaches to overcome this poor forecasting problem. For example, Hendry and

1. Structural and ANN models are out of the scope of this paper.

Clements (2001) have suggested intercept correction with the vector error correction model (VECM). They believe intercept corrections acts as differencing mechanisms and improve forecasts without altering policy conclusion.

Time Varying Parameter (TVP) models based on Kalman filtering could be a substitute for the methods of robustifying forecasts to structural change. Canova and Gambetti (2004) find that there are significant time variations in the coefficients of the inflation and the money equations in the reduced VAR model with US data. There is also some evidence in the literature that adding time variation coefficients to the BVAR model significantly improves the forecast of macroeconomic time series in comparison to the fixed coefficient BVAR model (see, e.g., Canova, 2002; Canova and Ciccarelli, 2004; Cogley and Sargent, 2001). In contrast, some researchers believe that there is little evidence in the literature to support the assumption of drifting autoregressive coefficients (see, e.g., Bernanke and Mihov, 1998a and 1998b; Sims 1980a, 1980b, 1999 and 2001; Stock 2001).

This inconsistency of the results on the constancy of autoregressive coefficients, and the emphasis on BVAR models' lack of robustness to deterministic (mean/drift) shifts (see, e.g., Hendry and Clements, 2001 and 2003; Bewley, 2001; among others), along with the presence of structural breaks in the mean of Iranian inflation, allows us to postulate that the autoregressive coefficients are constant and only the deterministic components are varying over time.

There are two ways in the Bayesian literature to deal with time varying coefficients (in general): one with fully hierarchical prior (pure Bayesian method) and one with Minnesota-type prior (Quasi-Bayesian method). In the first method, a Markov Chain Monte Carlo (the Gibbs sampler or other sampling methods) needs to be employed to calculate posterior distributions. This method exploits the recursive features of the posterior distribution. Given the computational complexity involved in calculating posterior Gibbs sampling estimates (especially in a simulation exercise), the paper uses the second method, which does not require iterative procedures. In the Minnesota-type prior many parameters are fixed (such as variance-covariance of innovations), and as such is referred to as Quasi-Bayesian method in comparison with the pure Bayesian method, which estimates all of them.

To study whether the presence of time variations in the deterministic components improves the quality of the forecasts of inflation in the BVAR models, the present paper estimates three models: a Traditional BVAR with Litterman's prior (BVAR), a Traditional BVAR with TVP (TVP-BVAR), and a modified TVP-BVAR model where only deterministic components evolve over time (MTV-BVAR). We compare the

performance of these models using Root Mean Square Error (RMSE) as a measure of forecast accuracy.

The results confirm that when the hyperparameters are set at the same values as Doan, et al., (1984), allowing for parameter drift in the Traditional BVAR model improves the forecast accuracy very little. The interesting result, however, is that even with this value of hyperparameters, a modified time varying BVAR model performs much better than the Traditional BVAR model for forecasts of Iranian inflation.

The rest of this paper is organized as follows. Section 2 presents a literature review of attempts to alleviate poor performance of the Traditional BVAR in forecasting inflation due to structural breaks. This section describes the alternative models, which were applied in the next section to improve the forecast accuracy of the BVAR models. Section 3 describes the data and the variables which are included in the employed model. There is also an investigation of structural breaks in the mean of Iranian inflation in this section. Section 4 presents the results and compares the forecasting performance of the various specifications with Iranian economic data. Finally, conclusions are drawn in Section 5.

1. Model specification

2.1. Literature review

A VAR model can be represented as follow:

$$y_t = a + A_1 y_{t-1} + \dots + A_p y_{t-p} + u_t \quad (1)$$

$$t = 1, 2, \dots, T$$

$$u_t \sim N(0, R)$$

where y_t is an $n \times 1$ vector of the endogenous variables. The subscript t denotes time, a is an $n \times 1$ vector of deterministic variables, and u is an $n \times 1$ vector of error terms. The parameters which describe this model are a , A_l , for $l = 1, \dots, p$, the variance-covariance matrix, R , and the lag length, p .

One of the most successful applications of the VAR models in macroeconomics has been forecasting of macroeconomic variables. Litterman (1980, 1984, and 1986)

demonstrated that the imposition of a prior (random walk) in Bayesian framework on the UVAR model improves forecast accuracy. As Litterman (1980, 1986) and Doan et al. (1984) explain, the BVAR model restricts a UVAR model by incorporating the available information about the coefficients of the model into the estimation procedure. The prior information takes the form of stochastic constraints on the coefficient parameters.

Empirical evidence suggests that the BVAR models with Litterman priors, produce forecasts that exhibit a high degree of accuracy when compared with UVAR and other alternative methods such as large scale macro-models (see, e.g., Artis and Zhang, 1990; Ballabriga et al., 1999 and 2000; Doan et al., 1984; Felix and Nunes, 2003; Kadiyala and Karlsson, 1993 and 1997; Kenny et al., 1998; Litterman, 1984 and 1986; McNees, 1986; Robertson and Tallman, 1999; Sims, 1993; Sims and Zha, 1998; Todd, 1984). Although traditional BVAR models can improve UVAR model forecasts through the use of extra information as priors, their forecasts of inflation performance has been somewhat less impressive (see, e.g., Kenny et al., 1998; Litterman, 1986; McNees, 1986; Robertson and Tallman, 1999; Webb, 1995; Zarnowitz and Braun, 1992).

One of the likely reasons for the poor performance of Traditional BVAR models in forecasting inflation can be contributed to structural breaks. In a case where any structural break is present in indicators of inflation, the correlation between inflation and candidate indicators will experience structural breaks. Moreover, the inflation process changes over time. Most importantly, the mean of inflation itself may shift at some point in time for example in response to monetary policy changes such as central bank's changes of inflation targets.

Boschen and Talbot (1991) have found evidence of unstable coefficients in the regressions of inflation on growth of the monetary base, growth of real GNP, and differenced three-month Treasury bill rate. Cecchetti (1995) also found that when the sample is divided into two sub-samples, variables that were significant in earlier periods lost their significance in subsequent periods. This might be due to several factors. One of the reasons is that this might reflect changes in the extent to which the monetary authority reacts to new information and/or structural breaks in the economic regime.¹

In the case of structural breaks, VAR models have a disadvantage of lack of robustness to deterministic shifts, exacerbated by the poor-determination of the estimated intercept. There are several options outlined in literature when forecasting with VAR models under policy-regime shifts.

1. Monetary policy changes can be responsible for large, persistent shifts in the inflation process (see, e.g., Balke and Fomby, 1991). Webb (1995) also documented that changes in the monetary policy regime give rise to inaccurate inflation forecasts for VAR models with constant coefficients.

The conventional and most simplistic approach to modelling with level shifts is to use a few dummy variables to allow some coefficients to shift and test these parameters for statistical significance. Webb (1995) used dummy variables to represent two monetary policy regime changes in the 1960s and 1980s in his model and found the most accurate inflation forecasts of the models examined. In this approach the forecaster needs to know the break points, which in practice are rarely known. This lack of information complicates the testing problem with sudden level shifts. Another disadvantage of this approach is that it is retrospective, which means that in order to adapt to future breaks in this framework the forecaster needs to re-estimate the model when new data arrives. Hence, these forecasts react to the breaks slowly.

In order to overcome poor forecasting due to structural breaks, Clements and Hendry (1996, 1998a, and 1998b) and Hendry and Clements (2001, and 2003) suggested intercept correction with VECM. They argued that intercept corrections act like differencing and improve forecasts without altering policy conclusions (Hendry and Clements, 2001, P.16). In fact, they consider that breaks in the long-run means of equilibrium error correction terms are a more important source of forecast error than breaks in drift parameters. This view is supported by Eitrheim, et al. (1999) in experimenting with the macroeconomic model of the Norwegian Central Bank.

Hendry and Clements (2001), show that the VAR in second difference (DDV) is robust to deterministic breaks that have occurred before forecasting. They show that the effects of structural breaks and policy-regime shifts on the DDV have a smaller forecast bias than the open vector error correction model (VECM). The reason is that the DDV is robust to forecasting after the equilibrium-mean shift, though it always has a large forecast-error variance.

Time varying parameter VAR models based on Kalman filtering are another method of robustifying forecasts to structural changes. The Kalman filter is the easiest to understand where the prior is normal with a fixed covariance matrix and the equation disturbance term has a known variance¹. One of the characteristics of the Kalman filter is that it shows the researcher how a rational economic agent would revise his estimates of the coefficient when new information arrives in a world of uncertainty. This can be a very important figure under a changing policy regime (see, e.g., Kim and Nelson, 1999; and also Hamilton, 1994; for more details).

1. In practice, of course, we do not know the equation disturbance variances a priori. It can be estimated by multiplication of a constant and the vector of estimated variances of residuals from least square estimates of linear univariate autoregressions, as if it were exactly the vector of variances of equation disturbances for the multivariate system. These constant terms could be fixed or estimated.

The important point about the Traditional BVAR is that coefficients are constant but unknown. Cooley and Prescott (1973) have shown that time-varying methods have good short-term forecasting properties when compared to relatively sophisticated versions of the fixed parameter method. Canova and Gambetti (2004) find that there are significant time variations in the coefficients of the inflation and the money equations in the reduced VAR model with US data.

Following Cooley and Prescott (1973), and describing the time varying coefficients, Doan et al. (1984), suggest the use of a time varying version of the BVAR with Litterman's prior¹. In general, a VAR (p) with time-varying coefficients may have the following form:

$$y_t = a_t + A_{1t}y_{t-1} + \dots + A_{pt}y_{t-p} + u_t \quad t = 1, 2, \dots, T \quad (2)$$

$$u_t \sim N(0, R)$$

where a_t denotes an $n \times 1$ vector of intercepts, and A_{lt} is an $n \times n$ time-varying regressor coefficients matrix for $l = 1, 2, \dots, p$. We can cast the model (2) in state-space form by defining a measurement and a state equation (see, e.g., Hamilton, 1994):

$$Y_t = X_t \theta_t + u_t \quad t = 1, 2, \dots, T. \quad (3)$$

where

$$X_t = \begin{bmatrix} X'_{1t} & & & & \\ & \dots & & & \\ & & \dots & & \\ & & & \dots & \\ & & & & X'_{nt} \end{bmatrix}$$

1. There are many reasons in the literature for using random coefficients VAR models: the first and most important one is the Lucas (1976) critique. As we know, VAR models are reduced form relations of structural equations and their parameters shift in response to changes in government and private decision rules. Background changes in decision rules induce random coefficients in reduced form relations. Hence random coefficient model helps account for possible structural shifts within the sample period, providing a mechanism to protect the implications of the model from the Lucas critique without having to explicitly model expectations. The economy's law of motion also varies in response to changes in fiscal policy rules (tax laws), monetary policy rules, and so on, which arguably shifted during the sample.

$$X'_{it} = (Y'_{t-1} \dots Y'_{t-m} D'_t) \quad i = 1, \dots, n$$

$$\theta'_t = (\theta'_{1t} \dots \theta'_{nt})$$

where D_t is a d - dimensional vector of deterministic variables, so X_{it} and θ_{it} are k - dimensional vectors, $k = mn + d$. The measurement matrix, $X_t = (I_n \otimes x'_t)$, is a regressors matrix, including lagged dependent variables and one predetermined variable, $x_t = (y'_{t-1}, y'_{t-2}, \dots, y'_{t-p}, 1)'$. The constant term in each equation is characterized with a k dimension vector, $k = np + 1$.

The dynamics of time-varying parameters θ_t are defined by a Markov process in which the mean reverting behaviour of θ_t is regulated by the value of the coefficients in the transition matrix,

P :

$$\theta_t - \bar{\theta}_0 = P(\theta_{t-1} - \bar{\theta}_0) + \varepsilon_t, \quad \varepsilon_t \sim N(0, \Omega),$$

or

$$\theta_t = P\theta_{t-1} + (1 - P)\bar{\theta}_0 + \varepsilon_t,$$

where P is a $k \times k$ matrix and we assume that ε_t is a $(k \times 1)$ vector of transition equation errors with $\varepsilon_t \sim N(0, \Omega)$, and $E(\varepsilon_t u'_t) = 0$. Doan et al. (1984) also choose to attribute a common behaviour for the parameters in θ_t by imposing a diagonal structure for the matrix P , as $P = \rho I_K$, in their models.

$$\theta_t - \bar{\theta}_0 = \rho(\theta_{t-1} - \bar{\theta}_0) + \varepsilon_t,$$

They further reduce the parameter space by considering the coefficient ρ in the transition equation fixed and known. It is set equal to one, forcing the parameter θ_t to evolve over time as driftless random walks:

$$\theta_t = \theta_{t-1} + \varepsilon_t, \quad (4)$$

Assuming the errors vector $\eta_t = (u_t', \varepsilon_t')'$, which have an m -variable normal distribution with zero mean vector and variance-covariance matrix $M = E(\eta_t \eta_t')$:

$$E_t(u_t \varepsilon_t') = 0 \quad \text{and} \quad M = E(\eta_t \eta_t') = E_t \begin{bmatrix} u_t & \varepsilon_t \end{bmatrix} \begin{bmatrix} u_t' \\ \varepsilon_t' \end{bmatrix} = \begin{pmatrix} R & 0' \\ 0 & \Omega \end{pmatrix}$$

In summary, state space representation form of the Doan et al. (1984) model is as follows:

$$\begin{aligned} y_t &= X_t \theta_t + u_t & t = 1, 2, \dots, T. \\ \theta_t &= \theta_{t-1} + \varepsilon_t, \end{aligned} \quad (5)$$

$$\begin{bmatrix} u_t \\ \varepsilon_t \end{bmatrix} | X_t \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} R & 0' \\ 0 & \Omega \end{bmatrix} \right)$$

where R is an $n \times n$ variance-covariance matrix for innovations in the measurement equation, and Ω is the $k \times k$ variance-covariance matrix for innovations in the state equation.

The results reported in Doan et al. (1984) using US data suggest that parameter drift is not too serious. They conclude that "allowing for parameter drift improves forecasts very little; since doing so is expensive, in many applications it will be reasonable to use fixed-coefficient models" (Doan et al., 1984, p.18).

2.1.1. Discussion

2.1.2.

One of the important reasons for moving from constant coefficient models to TVP models is the Lucas Critique (see, e.g., Doan et al., 1984; Cogley and Sargent, 2001 among others). Although TVP models can take into account past policy shifts, they are still subject to the Lucas Critique in that they cannot be used for policy evaluations.

Moreover, there are disagreements on TVP models among economists (see, e.g., Cogley and Sargent, 2001 and 2005; Sims, 2001; and Stock, 2001 among others). Lack of concurrences about drifting autoregressive coefficients in VAR models has been an interesting topic in applied macroeconometrics. While Canova (1993 and 2002) Cogley (2005) Cogley and Sargent (2001) and Primiceri (2005) posit time-varying autoregressive coefficients in their studies, Sims (1999) presents a VAR with time-invariant

autoregressive coefficients with a covariance matrix of innovations that varies over time. Moreover, in two separate comments on Cogley and Sargent (2001), Sims (2001) and Stock (2001) believe that there is little evidence in the literature to support the assumption of drifting autoregressive coefficients in VAR. Bernanke and Mihov (1998a, and 1998b) also could not reject time invariance in coefficients.

This inconsistency of applied studies about time variation of coefficients along with the emphasis on importance of mean/drift shift in forecasting (see, e.g., Clements and Hendry, 1996, 1998a, 1998b, and Hendry and Clements, 2001 and 2003) suggest that it might be feasible to assume that autoregressive coefficients are constant and only the deterministic components are varying over time. Moreover, as it is shown in 3.1, the mean of the Iranian inflation has significant variation in the period of study. So starting from a general representation, this paper introduces some alternatives to the model specification given in 5 and will be mainly executed by imposing prior restrictions on the parameter space.

2.1. Traditional BVAR Model

This model follows the standard specification of the Litterman (1986) prior¹. In this prior, the prior mean of the VAR coefficients on the first own lag is set equal to one and the mean of the remaining coefficients is equal to zero. The constant term, other deterministic and exogenous variables have diffuse prior. The prior variance covariance matrix of the coefficients (R) is diagonal and the elements are specified in a way that coefficients of higher order lags are more likely to be close to zero (the prior variance decreases when the lag length increases). Coefficients of variables other than the dependent one are assigned a smaller relative variance. In summary, the structure of R depends on some hyperparameters and assumptions: parameter controls the general tightness of the specification, one controls the decay of the prior variance as the lag length increases, and one weights the relative contribution of lags of other variables in each equation. The prior variance for the exogenous variables is diffuse. Finally, the variance covariance matrix of the error term is assumed to be fixed and known.

With respect to expenses of extension of investigation to pick up all hyperparameters, and also because the degree of parameterization of an equation is an important determination of forecast accuracy (see, e.g., Doan, et al., 1984), this study fixes all these hyperparameters, except the hyperparameter that controls overall

1. This model is known as BVAR with Litterman prior or BVAR with Minnesota prior. In recent literature it is referred as Traditional BVAR.

tightness. This hyperparameter is picked by maximizing the marginal likelihood function in an empirical Bayesian approach.

This model does not apply a fully Bayesian approach. In the fully (pure) Bayesian method, a Markov Chain Monte Carlo (any sampling method) is employed to calculate posterior distribution. In this method all parameters are estimated and forecasts are conditioned on expected values. In the method used in Litterman (1986), many parameters such as variance-covariance of innovations are fixed and Theil's (1963) mixed estimation technique are used to drive the mean of the posterior distribution/the coefficient estimator (see, e.g., Lutkepohl, 1993). Hence this method is referred to as Quasi-Bayesian method¹ in comparison to fully Bayesian approach.

2.1. BVAR Model with TVP

This model follows Doan, et al. (1984), who applied a TVP-BVAR model to a set of U.S macroeconomic data. Following Doan, et al. (1984), let the i^{th} equation in a VAR model - equation (2) - have the following representation:

$$y_t^i = c_t^i + a_{11,t}^i y_{t-1}^1 + \dots + a_{1p,t}^i y_{t-p}^1 \quad (6)$$

$$+ a_{21,t}^i y_{t-1}^2 + \dots + a_{2p,t}^i y_{t-p}^2$$

$$+ a_{n1,t}^i y_{t-1}^n + \dots + a_{np,t}^i y_{t-p}^n + \varepsilon_t^i$$

In this equation, the superscript i denotes the equation, and the subscripts denote the variable and the lag. For example a_{2p}^i is the coefficient on the p^{th} lag (subscript p) of the second variable (subscript 2) in the i^{th} equation. Suppose that equation (6) describes the first equation of the VAR model. In this case, the dependent variable is y_t^1 and the $(k \times 1)$ vector of explanatory variable is $x_t = (1, y_{t-1}^1, y_{t-1}^2, \dots, y_{t-p}^1)'$ where $k = np + 1$ and $y_t = (y_t^1, y_t^2, \dots, y_t^n)'$. Doan et al. (1984) specified the prior distribution for the initial value of the coefficient vector at time one as follows:

$$A_1 \sim N(\bar{A}, P_{10})$$

1. This method is known as Minnesota-type or Litterman-type prior in literature.

where \bar{A} is the same as the Traditional BVAR (a constant-coefficient BVAR with Litterman prior) and as the prior distribution is independent across coefficients the $P_{1|0}$ is a diagonal matrix:

$$\bar{A} = (0, 1, 0, 0, \dots, 0)'$$

Let the parameter λ_1 be overall tightness (or weight) parameter which reflects how closely the random walk approximation is to be imposed, then:

$$a_{11,1}^1 \sim N(0, \lambda_1^2)$$

The coefficient $a_{l,1}^1$ relates the value of variable one at date 1 to its own value l periods earlier. Doan et. al. (1984) had more confidence in the prior conviction that $a_{l,1}^1$ is zero the greater the lag (larger value of l). This can be represented with a harmonic series for the variance:

$$a_{l,1}^1 \sim N(0, \lambda_1^2/l) \quad \text{for } l = 2, 3, \dots, p$$

The prior distribution for the coefficient relating variable 1 to lags of other variables was taken to be:

$$a_{l,1}^1 \sim N(0, \hat{\sigma}_1^2 \lambda_1^2 \lambda_2^2 / \hat{\sigma}_j^2 l) \quad \begin{matrix} j = 2, 3, \dots, p \\ l = 1, 2, \dots, p \end{matrix}$$

where $\hat{\sigma}_j^2$ is the estimated variance of the residuals for a univariate fixed-coefficients $AR(p)$ process fitted to series j . The ratio $\hat{\sigma}_1^2 / \hat{\sigma}_j^2$ is included in the prior variance to account for the differences in the units of measurement of variables. The variance also includes λ_2 that controls the cross variable relationship. Lowering λ_2 toward zero shrinks the off-diagonal elements of A_{it} toward zero. Setting $\lambda_2 = 1$ means that there is no distinction between the lags of the dependent variable and the lags of other variables.

For the constant term, Doan et al. (1984) specified the following term:

$$c_1^1 \sim N(0, g \cdot \hat{\sigma}_1^2)$$

where g is the variance of the prior distribution for the constant and $\hat{\sigma}_1^2$ is the estimated variance of the residuals for a univariate fixed-coefficient AR(p) process fitted to series 1.

In summary, the P_{10} matrix is

$$P_{10} = \begin{bmatrix} g \cdot \hat{\sigma}_1^2 & \mathbf{0}' \\ \mathbf{0} & (B \otimes C) \end{bmatrix}$$

where

$$B = \begin{bmatrix} \lambda_1^\gamma & \cdot & \cdot & \cdots & \cdot \\ \cdot & \lambda_1^\gamma / \gamma & \cdot & \cdots & \cdot \\ \cdot & \cdot & \lambda_1^\gamma / \gamma^2 & \cdots & \cdot \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \cdot & \cdot & \cdot & \cdots & \lambda_1^\gamma / p \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2^2 \hat{\sigma}_1^2 / \hat{\sigma}_2^2 & 0 & \cdots & 0 \\ 0 & 0 & \lambda_2^2 \hat{\sigma}_1^2 / \hat{\sigma}_3^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_2^2 \hat{\sigma}_1^2 / \hat{\sigma}_n^2 \end{bmatrix}$$

Doan et al. (1984) used $\lambda_1^2 = 0.07$, $\lambda_2^2 = 1.74$ and $g = 630$. For the state equation they supposed that each of the coefficients in the measurement equation evolves over time according to the AR(1) process:

$$A_t - \bar{A} = \pi_8 (A_{t-1} - \bar{A}) + v_t$$

or

$$A_t = \pi_8 \cdot A_{t-1} + (1 - \pi_8) \cdot \bar{A} + v_t$$

where the parameter π_8 controls the rate of decay toward the prior mean. When it is set to one, the coefficient variation is treated as random walk. However, Doan et al. (1984) recommended a value of $\pi_8 = 0.999$. The disturbance v_t is assumed to be drawing from a distribution with a zero mean and has a diagonal variance-covariance matrix Q :

$$E(v_t v_t') = Q$$

where each element of Q is proportional to P_{10} (except the constant term). The proportionality factor, π_7 , determines the amount of time variation allowed in the parameter vector.

Finally, for the variance of the residuals, R , in the VAR model, Doan et al. (1984) specified it as proportional to the estimated variance of the residuals for the univariate fixed-coefficients $AR(p)$ process: $R = 0.9 \times \hat{\sigma}_1^2$.

Once the probability model is specified, they applied the Kalman filter to each equation of the VAR model to get recursively posterior of \hat{A}_t for A_t based on data up to $t - 1$. For the hyperparameters values, they fixed most of the priors and focused on two dimensions of the priors: the overall tightness and the degree of time variation of the parameters. Taking the hyperparameters values as given in Doan, et al. (1984), this paper focuses on the degree to which forecasting would be improved by searching along these two dimension priors.

2.1. Modified Time Varying BVAR Model

In this model a modified version of time varying BVAR specification is used. The model that is presented in this part has the same structure as Doan, et al. (1984) model, with a different source of variation. While Doan et al. (1984) consider a BVAR with time varying autoregressive coefficients; this model uses the information coming from variation in intercept only, rather than autoregressive coefficients. In this case, model (2) would have the following form:

$$y_t = a_t + A_1 y_{t-1} + \dots + A_p y_{t-p} + u_t \quad t = 1, 2, \dots, T \quad (7)$$

$$u_t \sim N(0, R)$$

in matrix notation

$$y_t = [1][a_t] + [A_1 \quad A_2 \quad \dots \quad A_p] \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ \vdots \\ y_{t-p} \end{bmatrix} + u_t$$

or

$$y_t = \theta_t + \theta.X_t + u_t$$

where $\theta_t = a_t$, $\theta = (A_1 \quad A_2 \quad \dots \quad A_p)$, and $X_t = (y_{t-1} \quad y_{t-2} \quad \dots \quad y_{t-p})'$.

Hence equation (5), in the case of time varying deterministic components would have the following form:

$$y_t = \theta_t + \theta.X_t + u_t \quad t = 1, 2, \dots, T.$$

$$\theta_t = \theta_{t-1} + \varepsilon_t, \tag{8}$$

$$\begin{bmatrix} u_t \\ \varepsilon_t \end{bmatrix} | X_t' \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} R & 0' \\ 0 & \Omega \end{bmatrix} \right)$$

where all the variables have been previously defined. The major difference between (5) and (8) is that in (8) only the deterministic component is allowed to evolve over time in a random walk fashion. Moreover, in (8) the variance of all autoregressive coefficients is imposed to zero except the variance of deterministic component. Hence, in this model $\Omega = \pi_\gamma * \Omega_0$ where the elements of the main diagonal of Ω_0 are zero except one, which is contributed to the constant. Similar to the previous case, π_γ controls how much time variation there is in the evolution of the law of motion of the mean.

1. Application to Iranian Data

The model estimated in this paper is the same as that presented in Heidari (2004), in terms of the number of included variables in the model, lag length, and the process of choosing hyperparameters.

It includes the logarithm of the real GDP as a measure of real output, y , the first difference of the logarithm of the GDP deflator as a measure of inflation¹, Inf , the logarithm of $M2$ as a monetary variable, $M2$, and the first difference of the logarithm of the black market exchange rate, Exe . According to Schwartz Bayesian Criterion (SBC), and the Hannan and Quinn Criterion (HQC) the lag length in this four variable VAR model is one, while Final Prediction Error (FPE) and Akiake Information Criterion (AIC) select lag length of three. We report results of different BVAR specifications for lag length one, and three later on. Moreover, the same as in Heidari (2004), an empirical Bayesian method, which maximizes the marginal likelihood function, is applied to pick hyperparameters. Quarterly data of the Iranian economy from 1981:Q2 to 2006:Q1 are used. All of the data are seasonally adjusted except for the exchange rate.

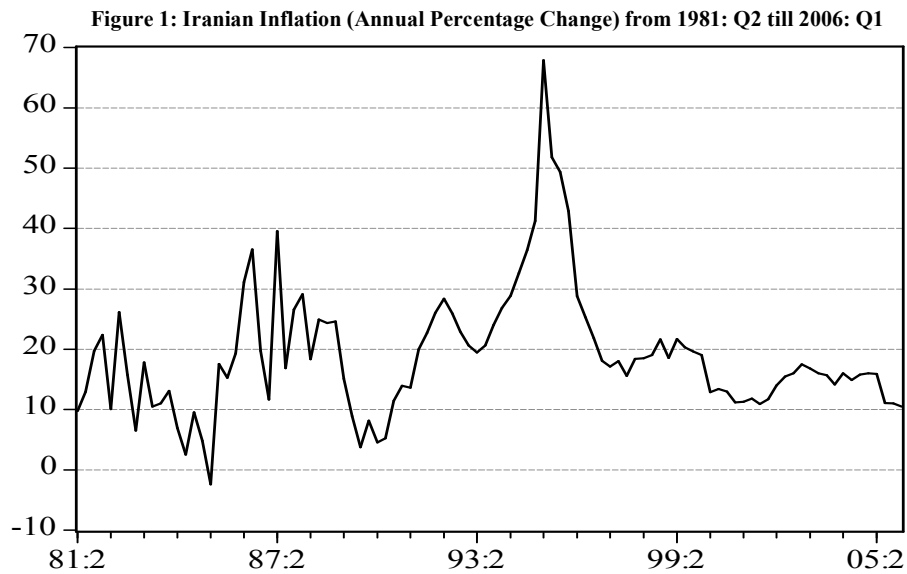
3.1. Test of Structural Breaks in the Mean of Iranian Inflation

The Iranian economy has been subject to numerous shocks and regime shifts such as the 1973-1975 oil shock, the upheavals consequential to the 1979 Islamic Revolution, the destructive eight-year war (1980-1988) with Iraq, the freezing of the country's foreign assets, a volatile international oil market, economic sanctions, and international economic isolation. In March 1993, the Iranian government embarked upon the exchange rate unification policy with consultation of the International Monetary Fund. The major objective of this policy was to unify the multiple exchange rate regimes into a single equilibrium rate by the massive intervention of the Iranian Central Bank. In other words, almost every year there has been an unusual policy change and/or external shocks to the economy resulting in the occurrence of multitude of structural breaks in macroeconomic variables. These structural breaks are of paramount importance in any forecasting exercise.

Figure 1 presents a graph of the inflation of the Iranian economy. Inflation is multiplied by 400 to express the change in price as an annual percentage change. As can

1. The most important measures to examine inflation are Consumer Price Index (CPI) and GDP deflator. As there were extensive government subsidies on consumer goods such as fuel, foods... over the period of study, CPI can not reflect the true inflation rate and, therefore, the GDP deflator is used in this paper.

be seen from this figure, there are some possible episodes during which the mean of inflation is somewhat different.



To carry out a test of no structural break against an unknown number of breaks in the Iranian inflation, this paper uses the endogenously determined multiple break test developed by Bai and Perron (1998)¹. This method tests for the presence of breaks when neither the number nor the timing of breaks is known aprior. This approach allows us to test for the presence of m breaks in the mean of inflation rate at unknown times using the following model:

$$\Delta p = \mu_j + \eta_t \quad t = T_{j-1} + 1, \dots, T_j$$

$$j = 1, 2, \dots, (m + 1)$$

1. A GAUSS algorithm to carry out these tests can be downloaded freely from Pierre Perron's homepage at <http://econ.bu.edu/perron>.

where Δp is the inflation, μ_j is the regime-specific mean inflation rate, and η_t is an error term, and $T_0 = 0$ and $T_{m+1} = T$.

Bai and Perron (1998) introduced two tests of the null hypothesis of no structural break against an unknown number of breaks given some upper bound (for most empirical applications this bound is 5, see, e.g., Bai and Perron, 2003). These tests are called Double Maximum tests ($Dmax$). The first is an equal weighted (we set all weights equal to unity) labeled by $UDmax$. The second test, $WDmax$, applies weights to the individual tests such that the marginal p -values are equal across the values of breaks. In both of these tests, break points are estimated by using the global minimization of the sum of squared residuals (for more details see, Bai and Perron, 1998 and 2003).

Table 1 presents results of $Dmax$ tests. Both $UDmax$ and $WDmax$ tests are significant. These tests show that we have at least one break in the mean of the Iranian inflation. These results are strongly supported by the $SupF_T(m)$ test introduced by Andrews (1993).

Table 1. D max Tests

Tests	UD max	WD max
Values	11.27*	19.37*

Note: * denotes significance at the 5 % level.

Moreover, Bayesian Information Criterion (BIC) suggested by Yao (1988) identifies four breaks for Iranian inflation¹. Table 2 shows the estimated means of inflation over each segment ($\hat{\mu}_i$ where $i = 1,2,3,4,5$).

Table 2. Estimated Means Over Each Segment with Four Break Points

Variables	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$	$\hat{\mu}_4$	$\hat{\mu}_5$
Estimates	12.1	25.16	15.19	32.62	15.98
	(1.96)	(1.48)	(4.0)	(4.25)	(1.05)

Note: the values in parentheses are the standard errors (robust to serial correlation) and the 95 % confidence intervals for break points.

1. There are some other methods to identify the number and date of breaks. However, it is documented in literature that in presence of breaks, BIC performs better than e.g. Schwarz Criterion (see, e.g., Perron, 1997)

It is clear from Table 2 that the differences in the estimated means over each sub-period are significant.

These results lead us to conclude that there are structural breaks in the mean of Iranian inflation. Hence, it is expected that the Traditional BVAR models give poor forecasts of inflation, and any model with time varying deterministic component might be able to improve forecasts of inflation.

1. Results of Empirical Application

This section reports the results of comparing forecast accuracy of alternative time varying BVAR models with Traditional BVAR model for forecasts of Iranian inflation. The reported results are only for the first quarter ahead and the first and second years ahead, which are of interest to policy makers, over period 2001:Q2 to 2006:Q1. The alternative specifications that were explained in section 2 are as follows:

- A BVAR model with Litterman's prior as described in part 2.1. In this model, we searched for the hyperparameter that controls the tightness of the prior distribution, and automatically picked the values that maximize the log of the marginal likelihood function. The hyperparameter that controls relative tightness on lags of other variables is fixed at 0.2. This value is the same value that Sims and Zha (1998) used for quarterly data. The original Litterman's equation by equation is used for estimation. This model is denoted as BVAR.
- A BVAR model with time-varying parameters. Like the Traditional BVAR model, we fixed the hyperparameter that controls the relative tightness on lags of other variables at 0.2 and searched for the hyperparameters that control the tightness of the prior distribution and time variation. For the hyperparameter that controls the tightness of the prior distribution, we set the value of this hyperparameter to the value that maximizes the marginal likelihood function. For the hyperparameter that controls the time variation, we searched for the value that gives accurate forecasts. We get the most accurate forecasts when we set it at 0.00001. This model denoted as TVP-BVAR.
- A modified TVP-BVAR model, where all parameters are constant and only deterministic components evolve over time. In this model all hyperparameters are treated the same as TVP-BVAR model. This specification is denoted as MTV-BVAR.

In all of these three models, the sample period is divided into two sub-samples. First we estimated the model by using data from 1981:Q2 to 2001:Q1. We then added the last 5 years of data, from 2001:Q2 to 2006:Q1, one observation at a time. We re-estimated the model and picked hyperparameters which maximize the marginal likelihood function in each re-estimation when new data added to the sample. With new optimal hyperparameters at hand, we re-estimated the model, and forecast for different horizons when new data arrived. This process continued until all the data had been used.

The forecasts of inflation in each of these models, for the current quarter as well as forecasts for the current and the subsequent calendar years, are compared with the actual values.

In the process of doing this forecasting exercise, when the hyperparameters are set at the same number as Doan, et al. (1984), our results confirm their results that “allowing for parameter drift [in Traditional BVAR] improves forecasts very little”¹. The interesting result, however, is that even with these hyperparameter values, the MTV-BVAR model performs much better than Traditional BVAR model for forecasts of inflation.

Table 3 shows the performance of forecast accuracy of the aforementioned models in forecasting Iranian inflation at the first quarter ahead, and also the following two years ahead. In this Table, the ratio of Root Mean Square Error (RMSE) of the Traditional BVAR to the RMSE of the associated model at each horizon are presented. A value greater than one means that the given model’s forecasts are more accurate than the Traditional BVAR forecasts.

Table 3 Ratios of RMSE of Traditional BVAR to the Associated Models: 2001:Q2 - 2006:Q1

Models specification	First Quarter	First Year	Second Year
<u>Lag=1</u>			
TVP-BVAR	1.073	1.459	1.779
MTV-BVAR	1.061	1.431	1.737
<u>Lags=3</u>			
TVP-BVAR	1.018	1.038	1.584
MTV-BVAR	1.130	1.711	2.102

Note: the numbers are the ratio of the RMSE of the BVAR model to the RMSE of the associated model at each horizon. A value greater than one, means that the given model’s forecasts are more accurate than the Traditional BVAR model’s forecasts.

1. As their hyperparameters that controls tightness on time variation were very small ($\pi_{\gamma} = 0.23 \times 10^{-7}$), this result could be expected before running the program. This is not reported.

Table 3 shows that the time-varying parameter version of the Traditional BVAR models improves forecasts of inflation. It also shows that the modified time-varying BVAR model, where all parameters are constant and only deterministic components are varying, improve forecast accuracy of inflation. The most interesting result is in relation to the comparison of accuracy of the Traditional BVAR model with a modified time-varying BVAR model. Our results show that a modified time-varying BVAR model performs much better than the Traditional BVAR model. The accuracy in forecasts of inflation in MTV-BVAR increases in long horizons, when the number of lags is three. In practice, a BVAR model with four variables and three lags is more common than a BVAR model with four variables and one lag. From this, we may conclude that a modified time-varying BVAR model performs much better than the Traditional BVAR, and the Traditional BVAR models with time-varying parameters.

1. Conclusion

This paper focuses on the comparison of forecast accuracy of different BVAR models. The paper discusses methods that attempt to improve forecast accuracy of the Traditional BVAR models by introducing time variation in the deterministic components. The results show that the performance of a BVAR model for forecasts of inflation depends on the number of lags, hyperparameter that controls time variation and also forecast horizons. Our results, however, show that a modified time-varying BVAR model performs much better than the Traditional BVAR model and the accuracy in forecasts of inflation in MTV-BVAR models increases in long horizons, when the number of lags is three. As a BVAR model with four variables and three lags is more common than a BVAR model with four variables and one lag, we may conclude that a modified time-varying BVAR model performs much better than the Traditional BVAR, and the Traditional BVAR models with time-varying parameters.¹

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