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A Multi-Objective Hierarchical Production Planning Model Under Stochastic Demand

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In this paper, a multi-objective Hierarchical Production Planning (HPP) model is presented for a single stage problem with stochastic demands. The HPP model is assumed to have two levels: Product type and product family, where a type is a set of families with similar costs per unit of production time. A chance-constrained goal programming approach was suggested for the first level of the HPP model, namely, Aggregate Production Plan (APP). For the second level, Disaggregate Production Plan (DPP), a cost modified function was proposed based on set-up cost and expected shortage cost within the families. The problem is also formulated as a Mixed-Integer Programming (MIP) model. Considering three alternative formulations for APP, the results of the models are compared in three directions: The versions of APP formulation, HPP versus MIP and adjusted DPP versus unadjusted DPP. The computational results demonstrate the effectiveness of the proposed approach.

INTRODUCTION

Production Planning (PP) can be defined as the process of establishing strategies for converting raw materials into finished products, so that manufacturing resources are used efficiently [1-3]. In this process, a number of factors such as: Workforce level (hiring/firing), inventory level (surplus/shortage), production rate and capacity (fixed/variable), demand forecasting (deterministic/stochastic), planning horizon (long/medium/short), organizational planning (strategic/tactical/operational) and manufacturing environment (line/batch/job shop) are involved.

Different models for PP problems can broadly be classified in two distinct categories: Monolithic Production Planning (MPP) and Hierarchical Production Planning (HPP) models [4-6].

The first category tries to consider all detailed decision problems over the entire planning horizon [7]. These approaches require data such as the forecasted demand of every item for a complete seasonal cycle, usually a full year. The second category has top-down features and decisions are made in sequence. Aggregate decisions are made first and impose constraints within which more detailed decisions are made. In turn, detailed decisions provide feedback to evaluate the quality of aggregate decision-making.

The major difference between the hierarchical and monolithic models is the existence of structural levels in the HPP models that reduce the variance of data, complexity of production planning problem and split it up into more or less independent subproblems integrated by several interfaces. Also, this approach is consistent with organizational level and decisions that lead to better performance of this type of model compared to others [6,8-10]. Also, some of the researchers refer to the HPP approach as a way for bridging the gap between theory and practice in the production planning field [11-12].

The early motivation for the HPP approach was noted by some of the developers of monolithic models, however, the first HPP model was presented by Hax and Meal [4] who considered a three-level HPP for product type, family and item. They applied Linear Programming (LP) for Aggregate Production Plan (APP) of types, the standard inventory control model for Disaggregation Production Plan (DPP) of families and, Equalization of Run Out Time (EROT) for disaggregation of items. Bitran and Hax [13], who presented two algorithms applying the knapsack method to DPP of family and item, have extended their work. Bitran et al. [5] modified the knapsack method for the case of high set up costs. Mohanty and Krishnaswamy [14] and Mohanty and Kulkarni [6]

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presented some modifications to minimize the number of backorders. Other extensions of HPP models are: Including machine groups in the HPP model [15,16], considering the shop floor problem with APP [17,18], allocating feedback mechanism [9,19], considering discrete part manufacturing, assembly [20] and extension for consumer goods [21]. Also, the issues of feasibility and consistency of DPP, which affects the result of the HPP solution, have been studied by some researchers [22-24].

All of the previously mentioned models deal with deterministic data. Because of the complexity of dealing with more realistic data, the developed stochastic models are few in number. Bitran et al. [25] developed a two-level HPP for a multi-period production planning problem with stochastic demand encountered in the manufacturing of goods, which are limited to having exactly one set-up for each family per year. Matsuo [26] extended this work and proposed a stochastic sequencing problem which simultaneously determines the production sequence and volume of style goods. The issue of consistency, in the case of independent stochastic demand, is studied by Ari and Axsater [27] and Ari [28]. Lassere and Merce [29] provided necessary and sufficient conditions for robustness of an aggregate plan.

Also, several approaches have been developed to solve the probabilistic problem for single stage production planning problem in a monolithic manner. Silver [30] suggested a procedure for derermination of the timing and the sizes of the replenishment of product with probabilistic time-varying demand. As an alternative, Askin [31] developed a model for production lot sizing with probabilistic dynamic demand, based on least cost per unit time. Some other approaches are: Considering the stockout cost as a criterion [32], deterministic approximation [33] and studying rolling horizon problem with probabilistic time-varying demand [34].

Most of the models for production planning described above are either dealing with deterministic demand or involve a single objective. To find out the most appropriate solution and trade off between conflicting goals, a multi-objective approach should be considered. In this paper, a multi-objective single-stage production planning model is considered with stochastic demand. The problem is characterized by a two-level hierarchical structure, including product types and families. A type is a set of families which have similar costs per unit of production time. The families in each type may have different set-up costs or priority of importance indicated with shortage costs. For each family it is assumed that the demand in each period is normally distributed. The forecast for each family is revised at the beginning of each period. Also, the demands for product types are stochastic independent variables with normal distribution, with their mean and variance given at the beginning for the whole of the planning horizon.

A multi-objective formulation is considered for the APP and it is solved using Preemptive Goal Programming (PGP). Then, for the DPP, the result of each period is disaggregated according to the latest information about demand data and real initial inventory. For this purpose, a non-linear programming model is developed, in which the objective function is to minimize modified set-up cost. The overall problem is also formulated as a (monolithic) mixed-integer programming model and its solution is compared with the proposed HPP model.

In the next section, the formulation for APP is presented, followed by the formulation for the DPP of families. Then, the solution of the model for an example is illustrated, finishing with some concluding remarks on the approach.

AGGREGATE PRODUCTION PLANNING

The first level of the HPP model is Aggregate Production Planning (APP), which addresses the strategic issue of selecting factors such as workforce levels, production rate and inventory for each product type. Objectives such as minimization of production and inventory cost, maximization of product profit and workforce utilization are frequently mentioned as important objectives of management [35]. In view of the multiplicity of competing objectives, Goal Programming (GP) is used as a means of considering all of them.

The product type demands are assumed to be time-varying stochastic in nature with known mean and variance, which is the summation of families demands within the type. The aggregate forecast is generated for each product type first and these are then disaggregated into product family by forecasting the proportion of the type demand corresponding to each family.

The characteristics of the problem in hand lead to the use of chance-constrained goal programming. The Chance-Constrained Goal Programming (CCGP) model can be expressed as follows:

$$\text{lex } \min\{(d_1^- + d_1^+), ..., (d_I^- + d_I^+)\}, \tag{1}$$

st:

$$\sum_{j=1}^{n} a_{ij}.x_j + (d_i^- - d_i^+) = b_i \qquad i = 1, ..., I, \tag{2}$$

$$\Pr\{d_i^- \text{ and/or } d_i^+ = 0\} \geq \alpha_i$$

$$X_j, d_i^-, d_i^+ \geq 0, \tag{3}$$

where d_i^- and d_i^+ are the negative and positive deviational variables about constraint i, the random variable a_{ij} is the technological coefficient associated with the jth decision variable in constraint i, x_j is the jth decision variable, b_i is the resource level and α_i is the service level or minimum desired probability of realizing goal $i(0 < \alpha_i < 1)$. Lex means lexicographic (or preemptive) goal requirements i.e., the goals are grouped according to priorities and the goals at a higher level are considered to be infinitely more important than goals at the lower level.

The deterministic equivalent constraints of the original CCGP constraints give the following model:

$$\text{lex } \min\{(d_1^- + d_1^+), ..., (d_I^- + d_I^+)\}$$

st:

$$F_{y(x)}^{-1}(\alpha_i) + d_i^- - d_i^+ = F_{b_i}^{-1}(\alpha_i),$$

$$x_j, d_i^-, d_i^+ \geq 0, \tag{4}$$

and:

$$F_{y(x)}^{-1}(\alpha_i) = \sum_j E[a_{ij}].x_j - Z_{\alpha_i}\sqrt{\sum \text{Var}[a_{ij}].x_j^2}, \tag{5}$$

$$F_{b_i}^{-1}(\alpha_i) = E[b_i]. + Z_{\alpha_i}\sqrt{\sum \text{Var}[b_{ij}]}, \tag{6}$$

where F^{-1} is the inverse probability distribution function and Z_α is the standard normal variation for the α fractile. When the original constraints are a less-than or equal sign, then the sign of Z_α in Equations 5 and 6 is reversed. Also, if none of the technological coefficients are stochastic, the right-hand side of Equation 4 reverts to the original one (Equation 2).

The APP model can now be formulated. It is assumed that the goal with the highest priority for management is to limit total shortage and overage inventory at the end of the planning horizon. To avoid potential opportunity cost of lost sales, it is desirable to provide a 95% service level for meeting the planning horizon demands. For overage, it is aimed to limit total production so that the extra inventory at the end of the planning horizon does not go beyond the necessary safety stock to provide the 95% service level. The goal constraint for each production type is as follows:

$$\Pr\{TI_{iT}^- + TI_{iT}^+ = 0\} \geq 0.95, \tag{7}$$

$$AI_{i0} + \sum_t x_{it} + (TI_{iT}^- - TI_{iT}^+) = D_i, \tag{8}$$

where:

i = product type number ($i = 1, ..., I$),
t = period number ($t = 1, ..., T$),
x_{it} = decision variable indicating production volume of product type i in period j,
AI_{i0} = available inventory of product type i at the beginning of the planning horizon,
D_i = the overall demand which is the sum of the demands of each product type i in the planning horizon (i.e., $D_i = \sum_t d_{it}$) with mean D_i and variance V_i,
d_{it} = the demand of product type i in period t with mean d_{it} and variance $v_{it}(D_i = \sum_t d_{it}$ and $V_i = \sum_t v_{it}$,
$TI_{iT}^-, (TI_{iT}^+)$ = under (over) achievement of goal constraint associated with total inventory for service level 95%.

The second goal is concerned with minimizing the use of overtime for production. It is assumed that there is a fixed number of regular time and a limited number of overtime in each period. Also the required time to produce each product is assumed to be deterministic. Therefore, the capacity constraints may be formulated as follows:

$$\min \sum_t W_t^+$$

$$\sum_t m_i.X_{it} + W_t^- - W_t^+ = R_t \tag{9}$$

where:

R_t = total available regular and overtime capacity in period t,

m_i = required time to produce one unit product type i,

W_t^-(W_t^+) = under (over) achievement of capacity goal constraint in period t,

The last set of goal constraints establishes the desire that overage and shortage production of each product type per period should not go beyond the amount necessary to provide the predetermined service level. It is similar to the first goal but it deals with the production level for each period rather than the total production plan of the planning horizon. Suppose a probability of 95% is also assigned for achieving these goal constraints. The appropriate set of chance-constrained goal constraints can be formulated as follows:

$$\Pr\{TI_{it}^- + TI_{it}^+ = 0\} \geq 0.95, \tag{10a}$$

$$AI_{i0} + \sum_{\tau=1}^{t} x_{i\tau} + (TI_{it}^- - TI_{it}^+)$$
$$= \sum_{\tau=1}^{t} d_{i\tau}(i = 1, ..., I)(t = 1, ..., T-1), \tag{11a}$$

where:

$TI^-_{it}(TI^+_{it})$ = under (over) achievement of goal constraint associated with ending inventory of product type i from period 1 to t.

It is possible to formulate these goal constraints in three ways, according to how the demand is represented. In the first case (APP-a) which is formulated above, the constraint in each period is concerned with the demand from the first period to the current period.

In the second case (APP-b), each goal constraint is associated with the shortage (or overage) inventory for just one period. Therefore, it may be written as:

$$\Pr\{I^-_{it} + I^+_{it} = 0\} \geq 0.95, \tag{10b}$$

$$(EI^+_{it-1} - EI^-_{it-1}) + x_{it} + (I^-_{it} - I^+_{it}) = d_{it}. \tag{11b}$$

where:

$I^-_{it}(I^+_{it})$ = under (over) achievement of the goal constraint associated with inventory of product type i in period t,

$EI^+_{it}(EI^-_{it})$ = expected ending inventory of product type i at the end of period t.

As these goal constraints deal only with individual period demand, all of the Equations 7, 8, 10a and 11a will be omitted and Equations 10b and 11b will become the high priority goal constraints of APP-b.

The last case (APP-c) is a combination of the first and second cases i.e., the first goal is held for the sum of demands in the planning horizon, but the individual demand for goal constraint is used for each period.

Since the standard deviation of the sum of some independent random variable (r.v.) is less than the sum of their standard deviations, the solution results for these cases are not similar. The results of these formulations are compared according to their effects on DPP solutions.

The whole proposed chance-constrained goal programming model for the APP-a (the first level of HPP) may now be written as follows:

$$\text{lex min } f_1 = \sum_i (TI^-_{iT} + TI^+_{iT}) \quad (i = 1, ..., I),$$

$$f_2 = \sum_t W^+_t,$$

$$f_3 = \sum_i \sum_t TI^-_{it} + TI^+_{it} \quad (i = 1, ..., I),\ (t = 1, ..., T-1)$$

$$f_4 = TC^+$$

st:

$$AI_{i0} + \sum_t x_{it} + (TI^-_{iT} - TI^+_{iT}) = F^{-1}_{D_i}(\alpha_i) \quad (i = 1, ..., I), \tag{12}$$

$$\sum_i m_{it}.x_{it} + (W^-_t - W^+_t) = R_t \quad (t = 1, ..., T), \tag{13}$$

$$AI_{i0} + \sum_\tau x_{i\tau} + (I^-_{it} - I^+_{it}) = F^-_{\sum d_{i\tau}}(\alpha_{it}) \quad (i = 1, ..., I),\ (t = 1, ..., T-1), \tag{14}$$

$$(EI^+_{it-1} - EI^+_{it-1}) = (EI^+_{it} - EI^+_{it}) + x_{it} - d_{it} \quad (i = 1, ..., I),\ (t = 1, ..., T), \tag{15}$$

$$\sum_i \sum_t (c_{it}.x_{it} + h^-_{it}.EI^-_{it} + h^+_{it}.EI^+_{it}) + \sum_t (w_t.W_t + w^+_t.W^+_t) + (TC^- - TC^+) = 0. \tag{16}$$

Equation 15 shows the inventory relationship between periods. The expected ending inventory in this equation is calculated according to the mean demand in each period. Equation 16 calculates the minimum total cost for the APP problem. The parameters in this equation are as follows:

c_{it} = unit production cost (excluding labor) for type i in period t,

$h^-_{it}(h^+_{it})$ = inventory backorder (carrying) cost for type i in period t,

$w_t(w^+_t)$ = cost per man-hour of regular (overtime) labor in period t,

TC^+ = total cost of APP model.

An alternative formulation for APP was presented by Ghazanfari et al. [36,37]. The set-up cost has been eliminated from the APP model and will be considered within the DPP model, which deals with families belonging to each type. The proposed preemptive goal programming model was solved using the GAMS/MIONS package [38,39]. The solution was then disaggregated for each period, in turn, by applying the DPP procedure. This will be discussed in the next section.

DISAGGREGATE PRODUCTION PLANNING

The second level of HPP is DPP, which can be defined as the total process of going from an aggregated plan to a feasible and consistent, detailed plan. It translates the strategic decisions into operational assignments, thus, the effectiveness of the APP approach and, consequently, the performance of HPP depends, to a great extent, on this second process.

The disaggregation process is done period by period. The results of the first period of the planning horizon are disaggregated, then, at the beginning of the second period, the available inventory and demand forecasts are updated and the APP results for this period are disaggregated and so on. The main constraint for a coherent disaggregation is that the summation of production quantities for each family must be less than or equal to the amount dictated by the higher level for this type.

Bitran and Hax [13] and also Bitran et al. [5] used an objective function, including the original set-up cost to disaggregate the APP results for the case of a deterministic problem. Here, their work has been extended and a new objective function proposed to minimize total adjusted set-up costs among families. The adjusted set-up cost for a family has been defined as the summation of its original set-up cost and the expected shortage cost. For families with similar original set-up cost, this modification prioritizes families with high expected shortage cost to assign the limited available production capacity. Also, for families with great demand variance, it causes the model to allocate more safety stock.

Since the family demand is assumed to be normally distributed with mean μ_D and variance σ_D^2, the expected shortage units for a product family j may be computed as follows:

$$E[I_j^-]=\int_{AI_j+Y_j}^{\infty}(D_j-(AI_j+Y_j)).f(D_j).dD_j$$

$$=\sigma_{D_j}\int_{k_j}^{\infty}(u_0-k_j).f_u(u_0).du_0$$

$$=\sigma_{D_j}.G_u(k_j), \quad (17)$$

$$G_u(k_k)=f_u(k_k)-k_j.P_{u\geq}(k_j), \quad (18)$$

$$P_{u\geq}(k_j)=\int_{k_j}^{\infty}f_u(u_0).du_0, \quad (19)$$

$$f_u(k_j)=\frac{1}{\sqrt{2\pi}}exp(-k_j^2/2), \quad (20)$$

$$k_j=\frac{Y_j+AI_j-\mu_{D_j}}{\sigma_{D_j}}, \quad (21)$$

where:

AI_j = available initial inventory of family j,
$E[I_j^-]$ = expected shortage of family j on the condition that demand r.v. is D_j and the production, plus inventory level, is AI_j+Y_j,
μ_{D_j},σ_{D_j} = mean and standard deviation of demand of family j,
Y_j = production level for family j,
$f(D_j)$ = probability density function of demand of family j,
u_0 = standard normal r.v.,
$f_u(u_0)$ = probability density function of r.v. u_0,
$G_u(k_j)$ = unit normal linear loss integral,
$P_{u\geq}(k_j)$ = probability that u is at least as large as a certain value k_j,
k_j = equivalent standardized normal r.v. of r.v. $D_j/\ (Y_j+AI_j)$.

The following proposed DPP model has to be solved for every product type i;

DPP_i

$$\min\sum_j\{S_j+h_j^-.E[I_j^-]\}\frac{\mu_{D_j}}{AIj+Y_j}, \quad (22)$$

st:

$$\sum_j Y_j=X_i, \quad (23)$$

$$E[I_j^-]=\sigma_{D_j}.G(k_j), \quad (24)$$

$$k_j=(Y_j+AI_j-\mu_{D_j})/\sigma_{D_j}, \quad (j=1,...,J), \quad (25)$$

$$AI_j+Y_j\geq\mu_{D_j}, \quad (j=1,...,J), \quad (26)$$

$$Y_j\geq 0, \quad (27)$$

where:

S_j = set-up cost of family j,
h_j^- = shortage cost per unit of backorder of family j.

The proposed model generates a solution reflecting the competition between families of each type for limited capacity, according to their set-up costs, shortage costs and demand means and variances. The GAMS/MINOS package has been used to solve this non-linear programming model. To show the effect of the adjusted set-up cost, its result will be compared to the DPP solution without this modification.

NUMERICAL APPLICATION

The authors conducted a series of experiments to examine the performance of the HPP model. Also, for comparison with an alternative solution, a (monolithic) MIP model has been solved using the constraints of APP (a, b and c).

Description of the Experiments

The assumed product structure used for the tests is given in Figure 1. Product types 1 and 2 have two and three families, respectively. The planning horizon consists of four periods of equal length.

Table 1 shows the demand mean and variance for each product type. The demand mean of each family is based on its proportion of the total type demand. Also, the variance of each family has been calculated according to the assumption of equality between the summation of variances of families demands within each type with variance of that type demand. For more details about mean and variance of family demand see Appendix A.

The only difference between the data for the HPP and MIP experiments is the demand variances. It is assumed that the family demand standard deviations in the HPP experiments are 80% of those of MIP. This is due to the fact that in the HPP approach the demand forecasted for family level is revised at the beginning of each period, which makes it more accurate; an advantage which monolithic approaches do not have.

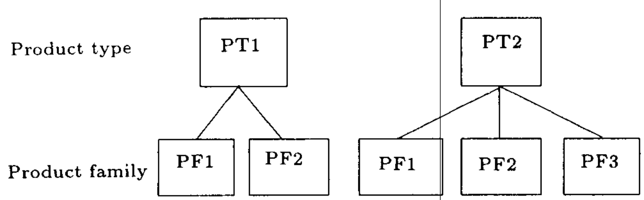

Figure 1. Product structure.

The information about demand proportions, holding costs and shortage costs for all families are shown in Table 2. Also, the cost data, labor capacity and other type relevant information is given in Table 3.

The APP model has been run using preemptive goal programming for its a, b and c versions. Then, for each APP result, the DPP has been solved periodically using a normal random number generator, to produce demand data for each period based on the demand means and variances of families. These numbers operate as real demand data and help to specify the real ending inventory, which is necessary to disaggregate the next period solution.

This simulation is repeated 100 times. At the end of the simulation, total set-up costs, shortage and overage costs, regular and overtime labor costs and the number of inventory and backorder were accounted for. The same procedure is applied to the MIP models to obtain comparable results.

Results of the Experiments

The initial results of the APP and DPP level are shown in Tables B1 to B4 of Appendix B. These are a small

Table 1. Forecast demand.

Period	Type TP1		Type TP2	
	Demand Mean	Demand ST. Dev.	Demand Mean	Demand St. Dev.
1	5000	214.29	6000	257.14
2	4000	228.57	6000	285.71
3	6000	428.57	6000	321.43
4	4000	342.86	6000	342.86

Table 2. Data related to family.

Family Name	Proportion of Total Type Demand	Holding Cost ($/Unit/Period)	Shortage Cost ($/Unit/Period)	Set-up Cost
PT1-PF1	0.6	0.2	0.6	90
PT1-PT2	0.4	0.4	0.3	90
PT1-PF1	0.2	0.3	0.4	120
PT2-PF2	0.3	0.4	0.3	120
PT2-PF3	0.5	0.5	0.2	120

Table 3. Cost data of produce type.

	PT1	PT2
Type holding cost	0.30 ($/unit/period)	0.40 ($/unit/period)
Type holding cost	0.45 ($/unit/period)	0.30 ($/unit/period)
Type backorder cost	4 ($/hour)	4 ($/hour)
Regular time cost	10 ($/hour)	10 ($/hour)
Overtime cost	0.10 (hour/unit)	0.05 (hour/unit)
Total regular workforce capacity		700 (hour/period)
Total overtime workforce capacity		200 (hour/period)

sample of a large number of experiments carried out to compare the alternative models. Analysis of the results is given in four section:

1. Comparison between alternative versions (a, b and c) for APP in HPP,
2. Comparison between the same alternative versions for MIP,
3. Comparing the HPP approach with MIP as the alternative method to solve the production planning problems,
4. Considering the effect of proposed cost modification for DPP.

Comparison 1

Figure 2a depicts the total shortage level for 3 formulations of APP. As can be seen, APP-a generally gives the better performance. When the number of overage inventory is an important criterion, the APP-b results in a better solution than the others (Figure 2b).

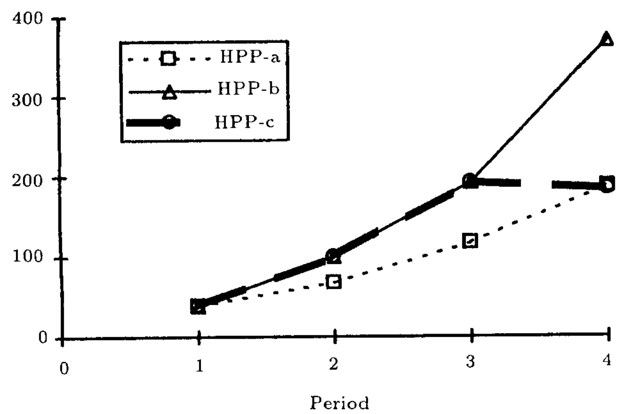

Figure 2a. Total no. of shortage (Comparison 1).

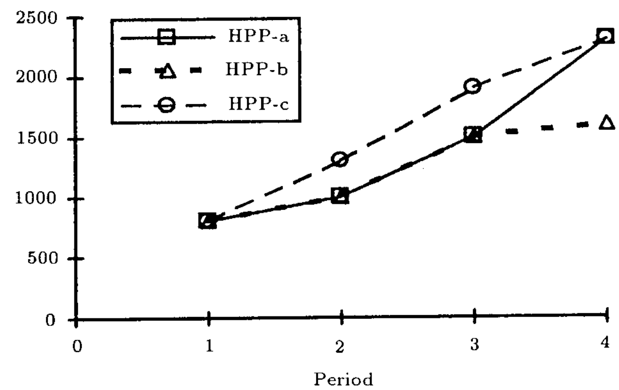

Figure 2b. Total no. of overage (Comparison 1).

Comparison 2

The results from the MIP models (shown in Figures 3a and 3b) are a little different. Dealing with preemptive goal programming and lowering the set-up cost from a high level to a lower level than the APP model in the HPP approach, are the main reasons for these differences. The MIP-c gives a superior shortage performace but highest overage inventory level. Compared to others, an MIP-b with a low level of overage inventory causes the most shortage.

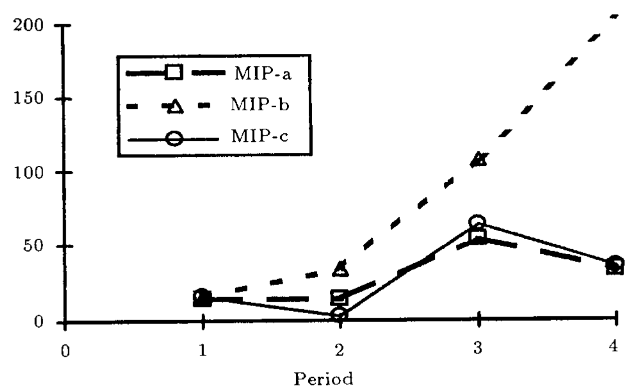

Figure 3a. Total no. of shortage (Comparison 2).

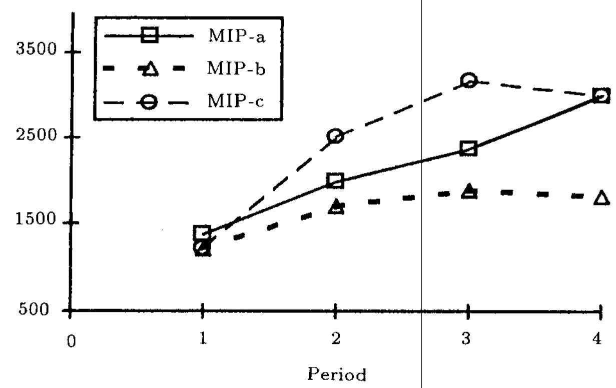

Figure 3b. Total no. of overage (Comparison 2).

Comparison 3

Figures 4a and 4b depict a comparison between HPP and MIP approaches in terms of total ending inventory. Both of them result in shortage because of the uncertainty situation involved in the models. The MIP approach causes more extra inventory in each period and reduces the probability of facing shortage.

Also, in term of cost, the HPP approach yields a solution with less labor cost, higher inventory cost and equal set-up cost. Therefore, considering the well-known limitations of MIP algorithms for solving problems with great size, it could be concluded

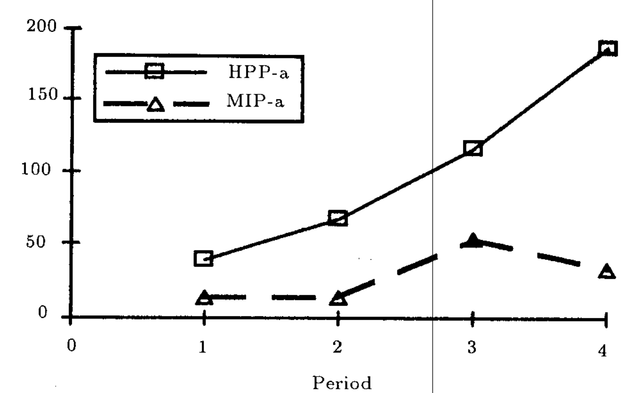

Figure 4a. Total no. of shortage (Comparison 3).

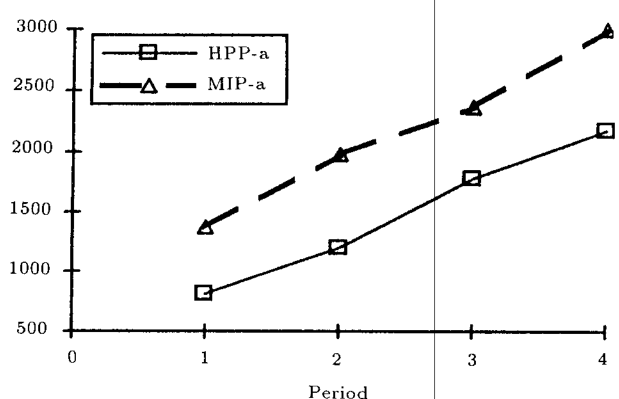

Figure 4b. Total no. of overage (Comparison 3).

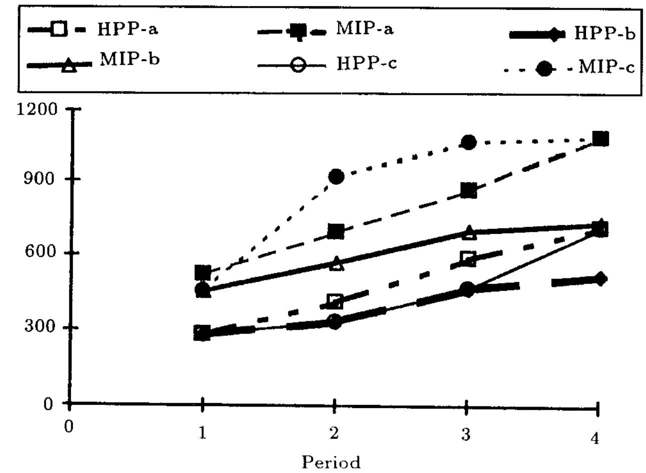

Figure 5. Total cost of shortage and overage inventory.

that HPP approaches are a suitable alternative to solving the aggregate production problems, even in the presence of the set-up cost and stochastic demands.

Figure 5 shows a general comparison in terms of total inventory cost among all of the HPP and MIP models under consideration. In both approaches, formulation b has superior performance.

Comparison 4

The last analysis is to consider the effect of adjusted set-up costs on two decision making criteria i.e., shortage and overage number of product families. The same DPP model is used but the objective function is changed into a simple one, i.e., minimizing the total set-up cost as the 'Unadjusted DPP' (U-DPP) model and solving the problem in hand for 3 versions of APP. Figures 6a and 6b show that the 'Adjusted DPP' (A-DPP) model has a strongly superior performance of the U-DPP model, both in terms of number of shortage and number of overage inventory and, therefore, total inventory costs.

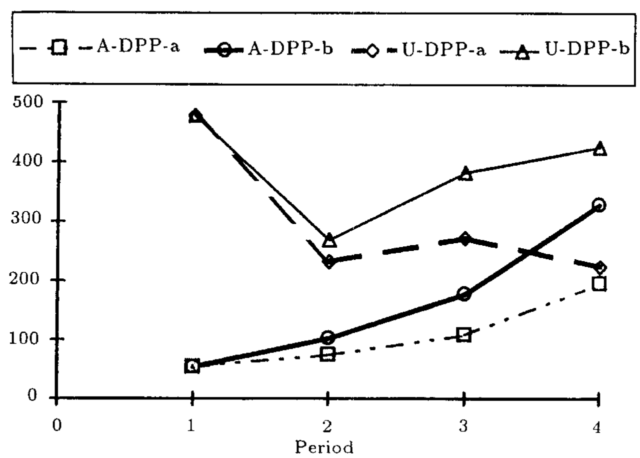

Figure 6a. Total no. of shortage (Comparison 4).

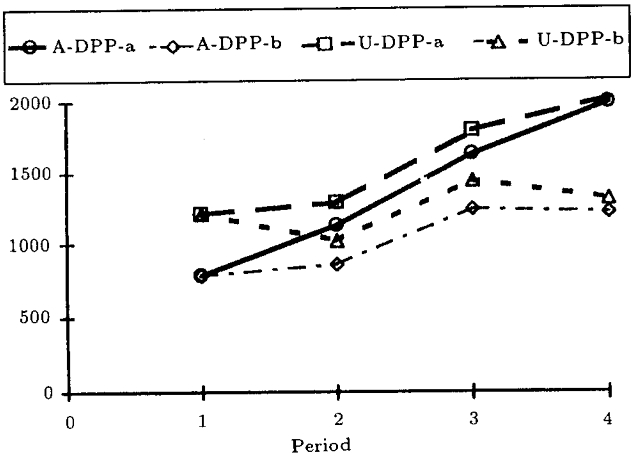

Figure 6b. Total no. of overage (Comparison 4).

It should be noted that realized performance of all models in all experiments have a shortage/demand ratio of less than 5% on average for all families in each period. The greatest number for this ratio is 4%, which is associated to period 4 of unadjusted DPP solution for APP-b formulation. For the ratio overage/demand, the worst case is related to a MIP model, with 3.8% for period 4. This ratio for A-DPP is less than 1% and for U-DPP is 2.5%.

CONCLUSION

In this paper, a multi-objective HPP model has been developed and implemented under uncertainty. The model has two levels: Product type and product family. A type consists of several families with different set-up costs. It was assumed that all demand values have a normal probability distribution for which their means and variances for types are known, in advance, for all periods during the planning horizon and are available for families at the beginning of each period of disaggregation.

Using chance-constrained goal programming as a method of solving the multi-objective APP for the first level of HPP, three formulations were developed for the APP model, for which a service level constraint was taken into account to prevent the production plan having an undesirable number of backorders. It should be noted that other model variants for goal programming could also be applied, which is a matter for further research.

To disaggregate the higher level results, a modified set-up cost was proposed to develop the objective function for the DPP model, i.e., the new cost function of the original set-up cost and the expected shortage cost for each family. The backorder cost in the model can be interpreted as the management priority factor for each family.

The results of our implementation were compared with some alternative formulations of APP solved by goal programming and mixed-integer programming. Finally, to investigate the impact of the proposed modification to DPP, the results of two adjusted and unadjusted DPP models were compared with each other. The results demonstrate a realistic performance of the HPP approach as a tool for production planning problems and, also, confirm the superior performance of the proposed DPP model.

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APPENDIX A

For the variance of type demand, it is assumed that the coefficient of variation ($c = \sigma/\mu$) has been increased by time as follows:

$$c_{it} = \frac{2+t}{70}\mu_{it}, \tag{A1}$$

where c_{it} is the variation coefficient of type demand i in period t and μ_{it} is the demand mean of type i in period t.

Also the family demand mean is calculated as follows:

$$\mu_{ij} = \lambda_{ij}.\mu_i, \tag{A2}$$

$$\sum_j \lambda_{ij} = 1, \tag{A3}$$

where μ_{ij} is the mean of family demand j in the product type i and λ_{ij} is the proportion of family demand j to the total demand of its type i.

Finally, the demand variance for each family may be calculated as follows:

$$\sigma_{ij} = c_{ij}.\mu_{ij}, \tag{A4}$$

$$c_{ij} = c_i/\sqrt{\sum \lambda_{ij}^2},$$

which are obtained in regard to the following equations:

$$\sigma_j^2 = \sum_j \sigma_{ij}^2,$$

$$\mu_i = \sum_j \mu_{ij},$$

$$c_j = \frac{\sigma_i}{\mu_i}.$$

APPENDIX B

A typical computer output of the programs is shown in Tables B1 to B4.

Table B1. A sample of output for APP-a resulted from preemptive GP.

The Results Related to the First Level of HPP Solved by Preemptive GP							
Aggregate Production Plan Over a 4-Month Planning Horizon							
Type Name	**Period**	**Demand Mean**	**Demand Standard Deviation**	**Planned Production**	**Safety Stock ($\alpha = 0.95$)**	**Cumulat. Ext. Inv. ($\alpha = 0.50$)**	**Cumulat. Sho. Inv. ($\alpha = 0.50$)**
PT1	1	5000.00	214.29	5352.46	352.46	352.46	0.00
PT1	2	4000.00	228.57	4162.88	162.88	515.33	0.00
PT1	3	6000.00	428.57	6337.86	357.86	873.20	0.00
PT1	4	4000.00	342.86	4166.27	166.27	1039.47	0.00
PT2	1	6000.00	257.14	6422.95	422.95	422.95	0.00
PT2	2	5000.00	285.71	5209.29	209.29	632.24	0.00
PT2	3	4000.00	321.43	4691.92	191.92	824.16	0.00
PT2	4	4000.00	342.86	4174.47	174.47	998.63	0.00

Table B2. Required workforce time for APP-a solution.

Anticipated Resource Utilization Over the Planning Horizon					
Period	**Available Reg. Hours**	**Required Reg. Hours**	**Available Over. Hours**	**Required Over. Hours**	**Total Reg.**
1	700.00	700.00	200.00	156.39	856.39
2	700.00	679.00	200.00	0.00	676.75
3	700.00	700.00	200.00	170.38	870.38
4	700.00	625.00	200.00	0.00	625.35

Table B3. Statistical results after 20 iterations of adjusted-DPP for APP-a.

Statistical Results are of HPP for All Families						
Average After 20.00 Simulation Runs						
A P P : a D P P : A						
Period	**Total Real Demand**	**Total No. of Short.**	**Cost of Overage**	**Total No. Overage**	**Cost of Invent.**	**Total Cost**
1	11045.14	-53.34	11.63	785.59	268.80	280.42
2	9048.94	-74.89	19.10	1132.19	392.21	411.34
3	10577.03	-108.56	27.29	1638.33	546.53	573.80
4	8067.50	-195.20	49.96	2000.08	694.39	744.35

Table B4. A sample of adjusted-DPP output for APP-a related to random generator with seed no. equal to 20.

Prediction Results of Family Option Seed = 20							
Family's Name	**Demand Mean**	**Demand Stan. Dev.**	**Planned* Production**	**Service Level**	**Expected Shortage**	**Real Demand**	**Actual End. Inv.**
Period 1							
Type 1 family 1	3000.00	142.64	3218.52	0.94	3.88	2801.65	416.87
Type 1 family 2	2000.00	95.09	2134.94	0.92	3.34	1933.70	181.24
Type 2 family 1	1200.00	88.74	1358.68	0.99	0.19	1148.22	210.46
Type 2 family 2	1800.00	100.11	1938.67	0.92	3.79	1785.52	153.15
Type 2 family 3	3000.00	166.85	3126.60	0.78	21.56	3056.73	69.87
Period 2							
Type 1 family 1	2400.00	152.15	2325.21	0.99	0.65	2801.65	533.65
Type 1 family 2	1600.00	101.43	1838.67	1.00	0.00	1550.61	469.30
Type 2 family 1	1000.00	74.16	1061.74	1.00	0.00	942.47	329.73
Type 2 family 2	1500.00	111.24	1539.54	0.96	1.88	1483.91	208.78
Type 2 family 3	2500.00	185.39	2609.01	0.83	16.51	2563.04	115.84
Period 3							
Type 1 family 1	3600.00	285.27	4035.31	1.00	0.35	3203.31	1358.65
Type 1 family2	2400.00	190.18	2321.55	0.97	0.00	2307.39	483.46
Type 2 family 1	900.00	83.43	927.82	1.00	0.00	835.28	422.27
Type 2 family 2	1350.00	125.14	1396.17	0.89	0.96	1331.89	273.05
Type 2 family 3	2250.00	208.57	2368.93	0.87	13.60	2320.92	163.86
Period 4							
Type 1 family 1	2400.00	228.22	1040.52	1.00	0.02	2082.65	343.52
Type 1 family 2	1600.00	152.15	3126.75	1.00	0.00	1525.91	2084.30
Type 2 family 1	800.00	88.99	802.74	1.00	0.00	730.96	494.04
Type 2 family 2	1200.00	133.49	1249.93	0.99	0.34	1180.69	342.26
Type 2 family 3	2000.00	222.48	2122.81	0.90	10.38	2075.64	211.02

*: indicates optimal solution.