Scientia Iranica, Vol. 13, No. 1, pp 91 105
© Sharif University of Technology, January 2006

SCIENTIA
IRANICA

Research Note

Ranking Analysis and Modeling of State Run Universities

M.A.S. Monfared*, **N.S. Gharneh**[1] **and S.N. Mirkhani**[2]

State evaluation of universities is important for administrators to serve as a basis for performance monitoring, policy formulations and distribution of funds. In this paper, a formal methodology was proposed to build a rational basis for evaluation and ranking of Iranian state run universities. In the first step, the entropy method from MCDM is applied and tested using real data from 47 state universities. Poor results of entropy led to the development of a new nonlinear programming formulation. This new model was then reduced to an interactive linear programming formulation in order to solve the problem. Real data is used to test and validate our methodology.

INTRODUCTION

The university system has historically been developed with a commitment to academic independence of the sort expressed by Von Humboldt in early nineteenth-century Europe. 'Pursuit of truth' came to be seen as a sufficient safeguard of quality outcome in higher education. However, changes such as 'massification' and subsequent blurring of the divide between university and vocational education have led to increased demands for accountability and effectiveness.

In addition, the severe financial and supply constraints currently experienced by many universities have further highlighted the need for performance measurement and careful and thorough assessment of the quality of services provided. Hence, developing appropriate performance measures and indicators are needed to provide a basis for strategic planning, policy formulating, measuring operational effectiveness, monitoring processes and operations and demonstrating the value obtained by users, in order to bolster intelligent decision making and evidence based management [1,2].

State evaluation of universities is important for administrators to establish a basis for distribution of funds and for strategic planning. At the university level a ranking system that is designed purposefully can help the rectors' council to evaluate performances across schools and departments and to formulate policies for improvement. Managers and consultants in the private sector are concerned with quality assessment of universities for recruiting competent graduates. Applicants, on the other hand, can take advantage of the results of university rankings for the choice of university.

In spite of the importance of university ranking and evaluation, only few developments have been reported in the literature, as considered below.

The Ministry of Science and Research of the state of North-Rhine Westphalia, as reported by Fandel and Gal [3,4], considered two distinct classes of measures for teaching and research evaluations. The procedure that is accepted by all universities is a three-level decision process with multiple objectives. The three decision levels are the ministry, the rectors' council and the individual universities.

Criteria for a performance- and success-oriented distribution of funds in teaching are the proportion of academic personnel employed, the proportion of students in the first 4 semesters and the proportion of graduates. Criteria for assessing successful research are the proportion of outside funds and the proportion of PhDs. Given these criteria, the solution process consists of agreeing on the weights for the criteria. Here, the ministry prefers the proportion of students in the first 4 semesters and the proportion of graduates. On the other hand, at the rectors' council, weights are sought that do not cause redistribution of the budget for teaching and research to deviate too much from

*. *Corresponding Author, Department of Engineering, Alzahra University, P.O. Box 1993891176, Tehran, I.R. Iran.*
1. *Department of Industrial Engineering, University of Amir Kabir, Tehran, I.R. Iran.*
2. *Department of Engineering, Alzahra University, Tehran, I.R. Iran.*

the actual distribution of funds among universities. Each individual university, however, is interested in weights that lead to its receiving the lion's share of the redistribution budget. The conflicting situations make the model structure vitally important.

The University of Pittsburgh has developed an On-line Student Survey System (OS3) to facilitate EC 2000 assessment and cross-institutional benchmarking [5]. OS3 allows multiple engineering schools to conduct customized, routine program evaluations using web-based surveys specifically designed to meet EC 2000 objectives. Since its inception, seven engineering schools have adopted OS3.

Differences across UK universities in 1993 life as is sciences students' degree performance were investigated by Bratti using individual-level data from the universities' statistical record [6]. Differences across universities are analyzed by specifying and estimating a subject-specific educational production function. Even after including a wide range of controls for the quality of students, significant differences emerge across universities in students' degree performance. A two-stage estimation procedure was applied to find evidence that a large part of 'university effects' cannot be explained by the kind of institutional inputs commonly used in the literature on school quality. Finally, comparison was made between the unadjusted rankings of universities, based on the proportion of 'good' (first and upper second class honors) degrees awarded and that based on the estimated probability of a 'good' degree obtained from the micro econometric model. Significant differences were found between the two indicators of the universities' performance.

Another development reported in literature by Weeks [7] is a benchmarking system that compares the ways in which university teachers are prepared for their teaching role. This was developed for the Queensland University of Technology. Therefore, as university evaluation is a complex task, it is not useful to build a ranking and evaluation model that aims to furnish all different motivations and objectives at different levels.

In this research, the development of a model for state level evaluation of Iranian universities is considered. Here, a university is characterized by different performance measures and productivity indicators. The developed ranking model benefits from these measures and indicators and proposes a grade for each university, as will be discussed in this paper.

This paper is organized as in the following. First, an understanding of the ranking and evaluation problem in Iranian state universities is developed. Then, different approaches for modeling this problem are investigated and it is shown that the conventional entropy method from MADM literature cannot sufficiently model this problem. After that, a new interactive mathematical programming model is proposed and implemented which leads to superior results. Finally, a conclusion will be presented by a considering ways to improve the modeling approaches.

STATE OF THE PROBLEM

The metrics used here are based on the definitions put forward by the National Iranian Productivity Organization (NIPO) [8]. Accordingly, a productivity indicator measures the ratio of outputs against inputs, while a performance measure describes the working nature of the university system, regardless of its immediate outcomes. Five out of twenty metrics are classified as productivity indicators, which are denoted by $i4, i6, i7, i9$ and $i10$, as illustrated in Table 1. The rest are classified as performance measures, which are denoted by $i1$ $i3, i5, i8$ and $i11$ $i20$, as illustrated in Table 1.

The Office for Productivity and Administrative Developments (OPAD) of the Ministry of Science has taken regular data from 47 universities all around the country, starting from 1997 [8]. A sample data for the year 1999 is exhibited in Table 2 for $i1$ to $i10$. Table 3 contains data for $i11$ to $i20$ for the same year of 1999.

Using these metrics, a committee of experts from the OPAD has currently classified 47 state universities into three grades, namely grade one universities (G1), grade two universities (G2) and grade three universities (G3), as illustrated in Figure 1.

In fact, this analysis was not exclusively based on numerical data, but, also, influenced by past performance and reputation. Unless there is a sound model to support the ranking scheme, it is understandable that publicity regarding current ad-hoc rankings could lead to administrative disputes. It is for this reason that the performed analysis by OPAD was undertaken informally and for internal consumption only.

The major issue with the present grading scheme, as illustrated in Figure 1, is that it is subjected to human judgment. The problem of interest to this paper is how to exploit the rationale behind this grading scheme and present it in a more systematic and quantitative based fashion. Hence, the problem in this undertaking is to develop a university ranking formal

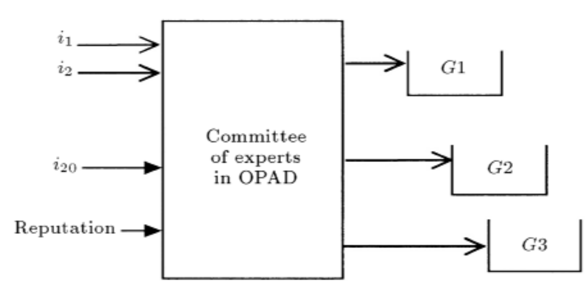

Figure 1. The present grading scheme.

Table 1. Productivity indicators and performance measures.

Metrics (i)	Definition	Type
1	Average number of faculty members per each major (undergraduate level)	Performance
2	Books per student	Performance
3	Invited faculty members to local faculty members	Performance
4	Lecture hours per year per faculty	Productivity
5	Staffs per student	Performance
6	Budget allocated for procurement of material for teaching and research to total budget	productivity
7	Budget allocated to labs and workshops to total budget	Productivity
8	Budget allocated to books and periodicals to total budget	Performance
9	Budget allocated for research activities to total budget	Productivity
10	Budget allocated for sabbatical to total budget	Productivity
11	Faculty receivable due to extra lecturing hours to total faculty receivable	Performance
12	Total budget per student	Performance
13	Extra income made by contracts to total budget	Performance
14	Students per faculty	Performance
15	Number of assistant professors to faculty members	Performance
16	Number of associate professors to faculty members	Performance
17	Number of full professors to faculty members	Performance
18	Number of lecturers to faculty members	Performance
19	Renovation budget to total budget	Performance
20	Professors*4+associate professors*3+assistant professors*2+ lecturers to faculty members	Performance

model that employs numerical metric data as input and generates university grades as output, as illustrated in Figure 2.

The advantages of this formal modeling are many. The first advantage is that a formal model can explain the outcome decisions using a systematic and algorithmic procedure. This is where the current ad-hoc model is based on an experts view with no explanation power. The second advantage of a formal model is that a university that is rated G2 or G3 can back-track its metrics and find out exactly where the weaknesses are. The third advantage of a formal model is that it is possible to perform inconsistency analysis. Inconsistency refers here to the difference between a grade assigned by an experts' committee and the grade suggested by the model. Inconsistency analysis helps human experts and, also, the model builder to work on their differences and, hence, improve the decision model. A fourth advantage of a formal model is that it provides a mechanism for upgrading and downgrading the status of the universities according to the changes that take place in their metrics.

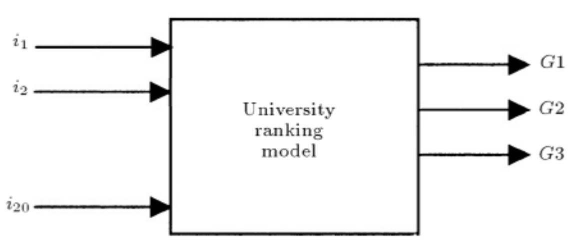

Figure 2. The proposed modeling logic.

PROBLEM MODELING

Modeling Techniques

Techniques for modeling the ranking problem are essentially considered in the framework of the Multiple Criteria Decision Making (MCDM) discipline [9]. Techniques include distance minimizing, linear programming, goal programming, Linear Programming for Multidimensional Analysis of Preference (LINMAP), weighted least square, eigenvalue, entropy, data min-

Table 2. Data for $i = 1$ to $i = 10$ (1999).

$(J)^\dagger$	University Name	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$
1	Isfahan University of Technology	8.1	9.127	34	46	14.4	2.34	8.97	4	7.14	0.8
2	Esfahan	6.7	17	17	43.1	6.4	4.02	7.74	3.78	4.24	0.57
3	Shiraz	11.1	20.61	44	64.41	11.1	4.53	5.39	6.26	5.33	0.52
4	Allame-Tabatabaei	10.9	*	127	32.4	4.8	2.09	0.26	0.78	2.14	0.26
5	Khajeh Nasir University of Technology	16.9	11.92	31	32.5	9.9	0.52	10.08	4.65	3.6	0.26
6	Tarbiyat Modares	2.0	29	138	44.4	12	1.89	4.87	5.4	8.5	*
7	Tehran	11.8	48.78	51	10.95	16.2	4.73	10.1	4.3	3.46	0.82
8	Shahid Beheshti	13.0	31.38	110	51	8.5	3.98	5.69	3.79	4	0.57
9	Amir Kabir University of Technology	8.0	17.77	69	68.76	8.9	3.48	11.07	1.58	8.55	1.49
10	Science and Technology of Iran	23.7	16.77	78	32.86	9.6	5.56	7.65	4.27	12.33	0.84
11	Sharif University of Technology	*	*	*	*	12	2.11	6.65	2.22	17.62	0.89
12	Zanjan University of Graduate Studies	*	124.6	73	22.04	34.6	0.73	9.04	9.22	21.03	1.71
13	Ferdosi Mashhad	9.2	22.4	102	58.51	9.8	3.46	9.03	3.31	4.74	1.2
14	Tabriz	4.5	31	41	28.8	8.7	5.04	5.45	4.06	2.22	1.74
15	Yazd	5.5	5.5	26	48.2	5.3	1.22	6.2	4.15	6.84	0.62
16	Shahid Bahonar	5.0	10	50	37.5	5.8	3.54	7.52	1.73	5.2	0.76
17	Alzahra	10.3	28.72	70	38.22	8.8	5.12	7.69	7.43	2.6	0.37
18	Art	6.4	13.87	167	23.33	8.6	5.01	5.98	3.22	4.86	0.79
19	Tehran Teacher Education University	22.8	21.92	21	17.66	8.4	1.85	8.51	1.62	3.76	0.46
20	Shahid Chamran	6.8	26	22	37.9	10.5	2.67	4.98	2.76	3.47	0.18
21	Gilan	14.4	12.57	173	10.79	9.5	3.72	3.67	3.03	2.74	0.38
22	Mazandaran	8.0	35	46	55.8	7.5	2.3	5.51	3.24	1.68	0.27
23	Sistan-Baloochestan	6.7	8.02	15	30.69	5.8	2	4.78	2.69	2.87	0.27
24	Shahrekord	10.0	5.5	20	5.5	3.6	1.7	5.76	2.88	5.76	0.19
25	Kordestan	5.3	15.1	14	15.1	4.2	2.17	4.85	1.85	2.62	0.18
26	Razi	5.0	11.22	30	11.22	5.2	3.18	1.95	3.18	4.5	0.21
27	Ielam	6.3	7.2	51	7.2	3.7	5.96	7.62	2.8	2.78	0
28	Booali Sina	3.7	18.635	4	18.635	5.4	0.82	3.19	0.88	3.15	0.72
29	Lorestan	4.5	12	16	12	5	2.2	12.74	1.36	2.6	0.29
30	Persian Gulf	6.8	3.4	78	3.4	3.2	1.72	6.07	3.42	3.74	0.79
31	Yasooj	8.3	12.02	90	12.02	5.8	1.64	3.16	0.93	2.05	0
32	Valiasr Rafsanjan	4.4	7.6	23	7.6	3.8	4.91	4.15	0.75	6.02	0
33	Hormozgan	5.2	6	25	6	2.8	1.98	9.63	1.32	5.78	0.79
34	Arak	4.3	15.75	27	15.75	33	1.18	4.87	1.94	3.41	0.27
35	Imam Khomeini International University	8.4	15.6	55	15.6	47.6	2.71	5.4	2.44	3.83	0
36	Sahand University of Technology	9.3	15.48	80	15.48	33.3	2.68	6.94	3.85	4.79	0.07
37	Oroomiye	7.0	12	3	12	12.1	3.63	4.57	2.1	2.45	2.36
38	Zanjan	6.0	25.55	21	25.55	10.8	0.9	6.59	2.91	6.03	1.32
39	Mohaghegh Ardebili	8.7	15	50	15	5.3	2.59	8.5	2.99	2.99	0.48
40	Semnan	9.0	18	2.4	18	5.3	2.63	11.56	2.06	2.88	0.25
41	Kashan	4.3	45	88	45	5	8.04	2.22	1.17	1.5	0
42	Shahrood	7.5	9.2	75	9.2	4.6	0.78	12.71	0.78	4.97	0
43	Gorgan	7.2	8.2	45	8.2	5.6	1.73	6.37	2.86	3.14	0.99
44	Damghan	7.8	*	6.3	*	3.7	1.61	14.08	1.61	2.41	0
45	Sabzevar Teacher Education University	3.4	14.1	35	14.1	6	1.59	5.33	1.16	2.82	0
46	Birjand	11.4	16.56	4	16.56	8.6	4.07	4.88	2.44	2.62	0.21
47	Tabriz Teacher Education University	*	*	*	*	*	3.42	5.27	3.21	2.16	0.14

$\dagger$: University index, *: Missing data,

Table 3. Data for $i = 11$ to $i = 20$ (1999).

$J^{\dagger}$	$i = 11$	$i = 12$	$i = 13$	$i = 14$	$i = 15$	$i = 16$	$i = 17$	$i = 18$	$i = 19$	$i = 20$
1	3.79	5586.4	12.71	19.517	48.9	15.2	3.3	32	23	1.88
2	4.82	3869.4	18.68	23.716	34.5	10.9	3.3	50.6	21	1.66
3	6.56	6507.3	19	23.785	38.6	9.7	10.2	40.2	23	1.87
4	4.47	4817.1	25.29	20.058	44.2	10.6	1.1	58.9	21	1.72
5	0.59	5949.8	16.05	*	0.559	2.7	0.5	36.4	20	1.58
6	6.67	11754.3	14.45	10.461	65.3	10.8	2.1	21.8	30	1.93
7	4.62	9145.1	18	14.314	42.3	13.8	1.17	32.2	20	2.05
8	2.77	6890.8	19.91	15.065	43.1	15.6	5.7	23.7	22	1.89
9	7.68	8437.9	12.41	17.137	50.3	9.1	4.8	34.8	33.4	1.82
10	7.94	7679.0	26.07	22.142	55.3	8.6	3.8	32	19	1.84
11	3.43	5146.1	10.97	27.149	48	18.9	13.9	18	*	2.26
12	4.91	67631.8	13.68	4.05	85	5	10	*	66	2.25
13	4.7	5304.2	15.92	19.396	32	11	6	46.1	29	1.67
14	8.55	8658.9	18.22	14.693	37.2	13	5	42.9	32	1.76
15	6.58	6411.7	25.48	12.575	24.3	10	0.7	72.3	60	1.27
16	4.64	4039.0	17.12	23.219	32.9	5.2	1	58	15	1.44
17	2.97	6466.6	16.69	12.576	42.7	5.2	1	49.7	31	1.55
18	7.17	5632.5	14.58	*	14	4	*	78	59	1.18
19	4.62	8469.2	24.79	10.461	65.3	9.3	8.2	46.3	23	1.75
20	5.28	5374.0	10.9	21.451	2.9	6.2	3.7	56.3	33	1.48
21	4.27	6399.7	16.33	15.347	35.3	2.4	*	59.5	52	1.37
22	5.64	7342.9	24.87	13.578	483	3	1.4	44.9	70	1.56
23	7.66	3906.1	*	31.685	15.2	1.1	0.2	83.5	16	1.16
24	2.82	2962.6	2.88	23.447	40	*	*	59.3	55	1.39
25	5.36	5062.3	16.54	*	15.6	0.7	*	83	32	1.16
26	3.93	4804.5	22.23	18.824	36.8	0.8	0.4	56.3	42	1.34
27	5.81	4542.3	12.98	24.154	11.5	*	*	85.6	55	1.09
28	7.3	4295.2	18.1	23.959	31.1	4.1	*	60.2	44	1.35
29	1.95	2840.2	8.14	0.081	25.9	*	*	73.2	81	1.25
30	3.75	6204.9	36.83	22.725	23.5	*	*	76.5	62	1.24
31	8.37	4038.8	9.95	26.62	18	*	*	82	66	1.18
32	21.91	1907.7	16.69	31.273	15.9	*	*	84.1	*	1.16
33	5.2	2877.4	15.83	50.673	30.8	1.9	*	67.3	14	1.35
34	8.97	5535.6	25.12	24.463	33.7	2.1	1.1	58.9	21	1.37
35	2.35	4589.5	11.61	19.685	47.6	0.7	*	5.17	83	1.49
36	4.2	5921.0	3.03	13.194	33.3	*	*	66.7	148	1.33
37	0.39	6815.8	14.4	16.792	33.9	5.2	2.1	57.4	35	1.49
38	0.34	6685.1	4.61	15.962	33.8	*	0.8	64.6	56	1.35
39	0.37	4524.2	27.7	18.058	23.3	1	*	74.8	24	1.24
40	5.08	4895.8	21.25	22.991	21.3	*	*	75	48	1.18
41	4.78	4435.0	12.4	23.114	35.1	*	*	62.3	16	1.04
42	7.05	3729.7	16.49	19.069	19.3	0.7	*	73.1	27	1.14
43	3.67	3646.5	16.97	30.888	36.3	1.5	15	60.4	50	1.44
44	3.33	2017.9	*	26.213	31.9	*	*	68.1	68	1.32
45	5.58	4021.3	18.76	19.281	16.9	*	*	83.1	94	1.17
46	3.95	4815.4	21.8	16.121	19.2	0.5	0.5	74.7	28	1.17
47	*	*	21.39	*	*	*	*	*	*	1.23

†: University index, *: Missing data

ing, Data Envelopment Analysis (DEA) and neural networks [10-15]. In the area of university ranking, however, only a few developments have been reported in the literature [10,14], as considered in the following.

Sarrico and Dyson [10] reported on the process of performance measurement and ranking evaluation for the University of Warwick using the Boston Consulting Group (BCG) matrix and DEA technique. Post and Spronk [14] reported on the development of a procedure for performance benchmarking for UK university departments that extends the Data Envelopment Analysis (DEA) technique, to incorporate the interactive decision procedure, such as Interactive Multiple Goal Programming (IMGP). The resulting procedure is called an Interactive Data Envelopment Analysis (IDEA). It is a decision support tool that helps decision makers to select performance benchmarks that are both feasible and desirable and to identify benchmark partners that may be helpful in uncovering ways for achieving the selected performance standards. The IDEA concepts and characteristics were illustrated by means of the real data of UK university departments.

Analysis of Modeling Techniques for University Ranking Problem

Theoretically speaking, the different techniques mentioned above can be applied to the authors ranking problem. Practically speaking, however, the successful application of these techniques are limited by the nature of the problem and the available data. The problem, as defined in the previous section, can be considered as a Multi Attributes Decision Making (MADM) problem. In this problem, a decision matrix is defined with m alternatives and n metrics (indicators, attributes or criteria). Alternatives are denoted by A_j, where $j = 1, 2, \cdots, m$ and metrics are denoted by X_i, where $i = 1, 2, \cdots, n$. Hence, r_{ij} in this matrix defines the value obtained by the alternative, i, from the metric, j, as illustrated in Table 4 (real data for r_{ij} are reported in Tables 2 and 3). In this case, an alternative defines a university and a metric defines an indicator (i.e., productivity indicator or performance measure, as illustrated in Tables 2 and 3 in terms of $i = 1$ to $i = 20$).

Techniques such as entropy, LINMAP, weighted least square and eigenvalue can be applied to the MADM problem. In any case, alternatives, A_i, can be ranked, which is a desirable feature in analyzing the university ranking problem. The Eigenvalue method requires that the decision maker provides data regarding the pair-wise comparison of alternatives and the pair-wise comparison of metrics. However, for the problem described here, no such data is available and, hence, method cannot be applied.

Table 4. Matrix definition of a MADM problem.

	X_1	X_2	$\cdots$	X_n
A_1	r_{11}	r_{12}	$\cdots$	r_{1n}
A_2	r_{21}	r_{22}	$\cdots$	r_{2n}
$\cdots$				
$\cdots$				
A_m	r_{m1}	r_{m2}	$\cdots$	r_{mn}

In LINMAP, a linear program is used to define an ideal case and, then, alternatives are ranked, according to their proximity to the ideal case, using Euclidean distance. In addition, weights for indicators are also estimated, which is desirable. However, LINMAP requires the Decision Maker (DM) to perform a pairwise comparison on alternatives. Again, no such data is available here and, hence, the method cannot be applied. Similar difficulty arises when the weighted least square method is applied.

The DEA technique, although very promising in handling a problem such as the presented technique, is not applicable. This is because, in the present situation, one cannot sharply distinguish between the input variables and the output variables, which are vital in DEA. In a sense, neither the productivity indicators nor the performance measures, as illustrated in Table 1, could be fully distinguished as outputs and inputs, respectively. Refining the definitions of performance measures and productivity indicators and structuring them in a hierarchical fashion can encourage modeling using DEA approaches. Accordingly, within the framework of MADM techniques and considering the way measures are defined, it is the entropy technique which can be examined.

Modeling University Ranking Problem Using Entropy

This section reports on the development of a formal model using the entropy method for ranking Iranian universities.

For each r_{ij} defined in the presented decision matrix (Table 4), one can estimate the P_{ij} i.e., the probability of an event called alternative j and indicator i, as:

$$P_{ij} = r_{ij} / \sum_{i=1}^{n} r_{ij} \quad \forall\, i, j.$$

In a sense, P_{ij} is the normalized value for r_{ij} and $\sum_{i=1}^{n} P_{ij} = 1$. By defining P_{ij}, one can now move to define entropy. Entropy is a concept in physical science, information theory and, even now, in social science that was originally proposed by Shannon [16-18]. It measures the amount of uncertainty existent in the content of information in a message denoted by E_i

Table 5. Excerpts from entropy calculations.

J	**Grade**	r_1	**Normalized** P_1	$P_1(\ln P_1)$	$\cdots$ $\cdots$	r_{16}	**Normalized** P_{16}	$P_{16}(\ln P_{16})$	**Total Points** (TP)
1	1	8.1	0.022	-0.084	$\cdots$	1.88	0.027	-0.098	616.2
2	1	6.7	0.018	-0.073	$\cdots$	1.66	0.024	-0.089	429.3
3	1	11.1	0.030	-0.106		1.87	0.027	-0.097	719.6
4	1	10.9	0.030	-0.105		1.72	0.025	-0.092	539.3
5	1	16.9	0.046	-0.142		1.58	0.023	-0.086	653.1
6	1	2	0.005	-0.028		1.93	0.028	-0.100	1293.9
7	1	11.8	0.032	-0.111		2.05	0.030	-0.104	1003.6
8	1	13	0.036	-0.119		1.89	0.027	-0.098	766.5
9	1	8	0.022	-0.084		1.82	0.026	-0.095	930.7
10	1	23.7	0.065	-0.177		1.84	0.027	-0.096	847.7
11	1	0	0.000	0.000		2.26	0.033	-0.111	559.7
12	1	0	0.000	0.000		2.25	0.032	-0.111	7334.7
13	1	9.2	0.025	-0.093	$\cdots$	1.67	0.024	-0.090	594.8
14	1	4.5	0.012	-0.054	$\cdots$	1.76	0.025	-0.093	949.6

Note that data for $j = 11$ and $j = 47$ are incomplete and, hence, both universities are not included in the computational phase.

and defined as in the following:

$$E_i = \quad k\sum_{i=1}^{n} P_{ij} \ln P_{ij} \quad \forall\ i.$$

Also, "ln" stands for a Neperian logarithm and k is a positive constant defined as $k = 1/\ln m$, which is a normalizing positive constant to maintain $0 \leq E_i \leq 1$, where m denotes the number of alternatives (universities). The amount of uncertainty or entropy is maximized when different indicators share equal chance, i.e. for any i that $P_{ij} = 1/n$, then:

$$k\sum_{i=1}^{n} P_{ij} \ln P_{ij}$$

$$= \quad k\left\{\frac{1}{n}\ln\frac{1}{n} + \frac{1}{n}\ln\frac{1}{n} + \cdots + \frac{1}{n}\ln\frac{1}{n}\right\}$$

$$= \quad k\left\{\frac{n}{n}\ln\frac{1}{n}\right\} = \quad k\ln\frac{1}{n}.$$

As entropy E_i obtains an appositive value between 0 and 1, it is useful to define d_i as the deviation of the ith indicator in the form of $d_i = 1 \quad E_i$ (for all i). The variable d_i is also called the degree of diversification or variety. If the DM has no reason to prefer one criterion over another, the principle of insufficient reason suggests that each one should be equally preferred. Then, the best weight set (W_i) one can expect, instead of the equal weight, is:

$$W_i = d_i / \sum_{i=1}^{n} d_i \quad \forall\ i.$$

If the DM has a prior, subjective weight, λ_j, then, this can be adapted with the help of W_j information. The new weight, W_i, is:

$$W_{i'} = [\lambda_i.W_i] / \sum_{i=1}^{n} \lambda_i.W_i \quad \forall\ i.$$

Implementation of Entropy Model

The entropy method has been implemented for the paper's case problem and the results are reported in Table 5. It should be noted, however, that indicators $i15, i16, i17$ and $i18$ are not considered in computation since their impacts are seen in the definition of $i20$, i.e. $i20 = 0.1(2^*i15 + 3^*i16 + 4^*i17 + i18)$. Hence, only 16 indicators are remained, i.e. $i1$ to $i14, i19$ and $i20$. From now on, they are renumbered as $i1$ through $i16$.

Table 6 illustrates the weight factors for $i1$ through $i16$ as estimated by the entropy method. In

Table 6. Linear weights from entropy model.

Weight	**Value**
W_1	0.0517
W_2	0.1037
W_3	0.0961
W_4	0.0773
W_5	0.0602
W_6	0.0404
W_7	0.0268
W_8	0.0419
W_9	0.0590
W_{10}	0.1323
W_{11}	0.0550
W_{12}	0.1084
W_{13}	0.0338
W_{14}	0.0437
W_{15}	0.0628
W_{16}	0.0057

Table 7. Results from the entropy model.

J	University Name	Present Grade	Total Points Obtained (TP_j)	New Grade	Mismatch Index
1	Isfahan University of Technology	1	616.2	2	1
2	Esfahan	1	429.3	3	2
3	Shiraz	1	719.6	1	0
4	Allame-Tabatabaei	1	539.3	3	2
5	Khajeh Nasir University of Technology	1	653.1	2	1
6	Tarbiyat Modares	1	1293.9	1	0
7	Tehran	1	1003.6	1	0
8	Shahid Beheshti	1	766.5	1	0
9	Amir Kabir University of Technology	1	930.7	1	0
10	Science and Technology of Iran	1	847.7	1	0
11	Sharif University of Technology	1	559.7	3	2
12	Zanjan University of Graduate Studies	1	7334.7	1	0
13	Ferdosi Mashhad	1	594.8	2	1
14	Tabriz	1	949.6	1	0
15	Yazd	2	706.0	2	0
16	Shahid Bahonar	2	449.0	3	1
17	Alzahra	2	716.0	1	1
18	Art	2	633.7	2	0
19	Tehran Teacher Education University	2	925.5	1	1
20	Shahid Chamran	2	593.2	2	0
21	Gilan	2	716.6	1	1
22	Mazandaran	2	812.6	1	1
23	Sistan-Baloochestan	2	430.4	3	1
24	Shahrekord	3	328.9	3	0
25	Kordestan	3	554.5	3	0
26	Razi	3	529.2	3	0
27	Ielam	3	503.1	3	0
28	Booali Sina	3	473.2	3	0
29	Lorestan	3	317.3	3	0
30	Persian Gulf	3	685.6	2	1
31	Yasooj	3	454.0	3	0
32	Valiasr Rafsanjan	3	213.7	3	0
33	Hormozgan	3	320.9	3	0
34	Arak	3	609.5	2	1
35	Imam Khomeini International University	3	511.4	3	0
36	Sahand University of Technology	3	661.4	2	1
37	Oroomiye	3	743.7	1	2
38	Zanjan	3	734.7	1	2
39	Mohaghegh Ardebili	3	500.7	3	0
40	Semnan	3	538.5	3	0
41	Kashan	3	499.1	3	0
42	Shahrood	3	416.3	3	0
43	Gorgan	3	404.2	3	0
44	Damghan	3	225.2	3	0
45	Sabzevar Teacher Education University	3	448.6	3	0
46	Birjand	3	528.4	3	0
47	Tabriz Teacher Education University	3	1.2	3	0
	Total Mismatch	22			

fact, $i1$ to $i14$ matched exactly metrics 1 to 14, $i15$ matched metric 19 and $i16$ matched metric 20, as illustrated in Table 1. These weights report on the relative importance of the presented performance measures and productivity indicators. As one can see, the most important factors are $i10, i12, i2$ and $i3$, among others.

However, the entropy method cannot suggest to which grade a university belongs. Using the weight factors illustrated in Table 6, one can calculate the total points obtained by any university, as illustrated in Table 7. In order to determine a university ranking, the decision maker should intervene to determine the dividing points between neighboring classes of university. By looking into Table 7, one can see that, according to entropy implementation, the total points obtained range from 213.7 for $j = 32$ (i.e., $TP_{32} = 213.7$) to 7334.7 for $j = 12$ (i.e., $TP_{12} = 7334.7$). This is an extremely wide range. Meanwhile, the DM has to make decision on departing points in order to enable grade classification G1, G2 and G3. For example, if the DM assumes the classification, as suggested in Table 8, then, a university is G1 if at least 716 points are obtained and a university is G2 if the points obtained dropped between 593 and 715. Also, a university becomes G3 if points obtained are less than 592.

Based upon the decision made for points and grades, as in Table 8, the entropy based grading scheme against the current grading scheme are reported in Table 7. A mismatch (or inconsistency) index is introduced in this table in order to measure the difference between the current grade and the proposed new grade. As one can see, there is a large amount of mismatch between the present grading scheme and the one proposed by the entropy method. This is further analyzed in Table 9.

From Table 9, one can see that only 8 out of 14 currently G1 universities can still rank as G1, 3 universities are now located in G2 and 2 universities in G3. Again, from 9 currently G2 universities, only 3 universities are ranked G2 and the rest are displaced either in G1 or G3. The situation is better with G3 universities, where 19 out of 24 currently evaluated G3 universities have obtained the same grade, as illustrated in Table 9. However, the inconsistency between the entropy proposed grading scheme and the one currently in place is very large (total mismatch index is 22 units). In general, the inconsistency indicator is 42% since 14 out of 33 universities are displaced. Because it is assumed that the grades proposed by the committee of experts are valid, the present situation of 42% inconsistency can be evaluated as a failure for the entropy based modeling of the problem.

Table 8. Points and grades.

Grade 1	$P_{j,G1} \geq 716$
Grade 2	$593 \leq P_{j,G2} \leq 715$
Grade 3	$P_{j,G3} \leq 592$

Table 9. Summary of changes in grades according to entropy model.

Grade	**Number of Universities in the Present Qualitative Based Assessment**	**Number of Universities Assessed by Entropy Model**		
		G1	**G2**	**G3**
G1	14	8	3	3
G2	9	4	3	2
G3	24	2	3	19
Total	47	14	9	24

There are other difficulties associated with an entropy based modeling of the problem. The first difficulty is that a sharp distinction between a G1 university and a G2 university is obscured when the points obtained are about the border point (i.e., 715). A similar problem arises between a G2 university and a G3 university if the points obtained are around 592. The second difficulty is that the departing points between grades, i.e., between G1-G2 and G2-G3, are now dependent upon DM, which is a deficit. The third difficulty is that the points obtained by the universities are largely dispersed. From Table 7, for instance, it is evident that the university denoted by $j = 12$ obtained 7334.7 points, while the university denoted by $j = 17$ obtained only 716.0 points, realizing the fact that both belonged to the same G1 class. Therefore, this method is neutral toward the dispersion of points.

In order to enhance the modeling efforts, it is now required to improve the methodology, where the mere nature of the problem can be adequately captured and inherent difficulties, as discussed above, can be overcome. The adequacy of the model will be measured by a low index of inconsistency. This is considered in the next section where a new methodology, using mathematical programming techniques, is developed.

DEVELOPMENT OF A NEW METHODOLOGY

The weight set, W_i, suggested by the entropy method were used in a linear combination, with indicator values, r_{ij}, to achieve the total points obtained by a university denoted as TP_j, i.e.:

$$TP_j = \sum_{i=1}^{n} W_i r_{ij} \quad \forall\ j.$$

This may cause some difficulties. First, a university from the G1 class and a university from the G2 class may both have almost similar TP_j. Second, deviation between TP_j within a class may become very large, i.e., two universities in the same class might have very different values of TP_j. To avoid these difficulties, the weight set should be developed in such a way that sharp distinction between classes becomes possible and dispersion within a class becomes limited.

In order to provide sharp distinction, one needs to provide a gap between classes, as illustrated in Figure 3. Here, any one of the G1 universities has a total point of less than $\Omega 0$ (i.e., $TP_j \leq \Omega 0$ for j belongs to G1 universities). In addition, if a university belongs to the G2 university class, then, its total points must be within the range of $\Omega 1$ to $\Omega 2$ (i.e., $\Omega 1 \leq TP_j \leq \Omega 2$ for j belongs to G2 universities). A university whose total point is greater than $\Omega 3$ is a G3 university (i.e., $TP_j \geq \Omega 3$ for j belongs to G3 universities).

For example, consider a university which has taken total points of 669 and is now in the G1 class and another university which has taken total points of 660 and is in the G2 class, as illustrated in the second column of Table 10. Here, enough distinction between G1 and G2 classes is not maintained, since two neighboring universities are different by only 9 points. This can be very disputable. This problem could be avoided if enough distinctions were introduced in the formal model as, for example, is illustrated in the third column of Table 10. In this case, both universities, with 660 and 669 points, belong to the G1 class.

In Table 10, two variables are introduced as α (i.e., $\Omega 1 \quad \Omega 0$) and β (i.e., $\Omega 3 \quad \Omega 2$), which defined the gap between neighboring classes G1-G2 and G2-G3, respectively (see also Figure 3). If the authors model can predict values for $\Omega 0$, $\Omega 1$, $\Omega 2$ and $\Omega 3$ with no intervention from the DM side, then, the second problem of the entropy based method can be resolved.

In order to manage the third problem of the entropy based model, that is, the dispersion problem discussed earlier, it can be assumed that points obtained by a university span from 0 to 1000 (recall that using the entropy model points obtained by different universities ranges from 213.7 for $j = 32$ to 7334.7 for $j = 12$). Here, again the total points obtained by a university is still estimated by $TP_j = \sum_{i=1}^{n} W_i r_{ij}$, but, the weight factors need to be managed in such a way that the total points do not exceed 1000. The following linear programming model is formulated to find weight factors (i.e., W_i) in order to accomplish our goals.

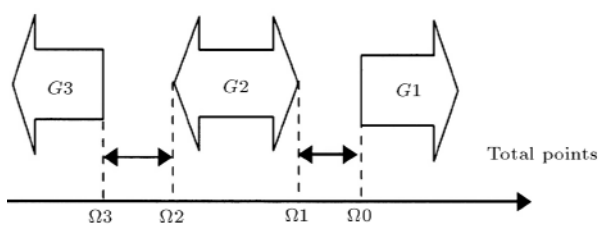

Figure 3. Providing gaps between neighboring classes.

Table 10. Ranges for G1, G2 and G3 universities.

Grade	Classification with no Distinction (i.e., $\alpha=0, \beta=0$)	Classification with Distinction (e.g., $\alpha=50, \beta=50$)
G3	0-333.3	0-300
G2	334-666.6	350-650
G1	667-1000	700-1000

Model (1)

$$\max(\alpha+\beta)$$

st

$$\sum_{i=1}^{n} W_i r_{ij} \geq \Omega 0 \quad \forall\, j = 1, 2, \cdots 13$$

(for 14 universities in G1 excluding $j = 11$),

$$\Omega 1 \leq \sum_{i=1}^{n} W_i r_{ij} \leq \Omega 2 \quad \forall\, j = 14, 16, \cdots, 22$$

(for 9 universities in G2),

$$\sum_{i=1}^{n} W_i r_{ij} \leq \Omega 3 \quad \forall\, j = 23, 25, \cdots, 45$$

(for 24 universities in G3 excluding $j = 47$),

$$\sum_{i=1}^{n} W_i = 1,$$

$$\Omega 1 \quad \Omega 0 \geq \alpha,$$

$$\Omega 3 \quad \Omega 2 \geq \beta,$$

$$W_i \geq 0 \quad \forall\, i = 1, 2, \cdots, 16,$$

$$\Omega 0, \Omega 1, \Omega 2, \Omega 3, \alpha,\ \beta \geq 0.$$

If α is considered equal to β, that is, to give equal distinction across neighboring grades, then, the objective function in Model 1 reduces to maximize α. Different or equal values for α and β solely depend on the objectives of the modeling put forward by the stakeholders.

Solving linear program (Model 1) apparently leads to computation of weights, W, and decision parameters, $\Omega 0$, $\Omega 1$, $\Omega 2$ and $\Omega 3$. The "weights" are used for the computation of total points obtained by a university. The "decision parameters" are used to make a decision regarding the rank for universities, as illustrated in Table 11.

Table 11. Evaluation of university ranking.

If	Then it is a
$\sum_{i=1}^{n} W_i r_{ij} \geq \Omega 0$	G1 university
$\Omega 1 \leq \sum_{i=1}^{n} W_i r_{ij} \leq \Omega 2$	G2 university
$\sum_{i=1}^{n} W_i r_{ij} \leq \Omega 3$	G3 university

In this procedure, it is possible that the total points obtained by a university fall into gap intervals α or β and produce vague situations. This is because $\Omega 1 \leq \sum_{i=1}^{n} W_i r_{ij} \leq \Omega 2$ and $\Omega 2 \leq \sum_{i=1}^{n} W_i r_{ij} \leq \Omega 3$ are not defined. In such cases, a further elaboration of facts and figures are needed to enable decision-making.

If the linear program (Model 1) were feasible to solve, it would become clear that all 47 universities would be graded as the experts' committee at the Ministry of Science originally graded them, i.e., with a 0% inconsistency index. In this case, the linear program (Model 1) optimally determines values for weights, W, and decision parameters, Ω's. These results could also be used to verify the situation of any new university.

However, the implementation of linear program (Model 1), using the numerical data presented in Tables 2 and 3, has led to infeasible solution space, meaning that it is not possible to determine W and Ω's such that a current university's grades remain unchanged. Neither could sharp distinction between classes be maintained. This illustrates the fact that decisions made by the experts' committee at the Ministry of Science were not fully consistent. Such inconsistency can be attributed to the fact that human judgment is used to rank universities and this judgment could be wrong. It is also possible that inconsistency can be attributed to wrong data gathered from different universities, as illustrated in Tables 2 and 3. In any case, inconsistency could be actual and unavoidable.

The implication of an infeasible linear program (Model 1) is that the present ranking for some universities must be changed. Therefore, the implementation of a linear program (Model 1) can be used as a test program signaling the existence of logical inconsistency. This, however, cannot solve our problem, since weight factors (w) and decision parameters (Ω's) are needed to be determined. This is the subject of the next subsection.

A Nonlinear Programming Model

In order to remedy the inconsistency problem illustrated above, a new model is presented. This model is a nonlinear integer-programming model, as illustrated below.

Model (2)

$$\max Z = \sum_{j=1}^{n} y_j,$$

st

$$\sum_{i=1}^{n} W_i r_{ij} \geq \Omega 0.y_j \quad \forall\ j = 1, 2, \cdots, 13$$

(for G1 universities),

$$\sum_{i=1}^{n} W_i r_{ij} \geq \Omega 1.y_j \quad \forall\ j = 14, 16, \cdots, 22$$

(for G2 universities, left tail),

$$\sum_{i=1}^{n} W_i r_{ij} \leq \Omega 2.y_j \quad \forall\ j = 23, 25, \cdots, 31$$

(for G2 universities, right tail),

$$\sum_{i=1}^{n} W_i r_{ij} \leq \Omega 3.y_j \quad \forall\ j = 32, 34, \cdots, 54$$

(for G3 universities),

$$W_i \geq 0.04 \quad \forall\ i = 1, 2, 16,$$

$$y_j \varepsilon \{0, 1\} \quad \forall\ j = 1, 2, 54,$$

$$\Omega 0, \Omega 1, \Omega 2, \Omega 3 \geq 0.$$

The model is nonlinear because $\Omega.y_j$ is of a multiplicative nature. Also, the model is integer because y_j variables are of a zero-one type. In this model, each university is located into its present grading scheme unless it causes inconsistency. If $y_j = 1$, it means that the present ranking for the jth university can be respected. If $y_j = 0$ it means that the jth university should be displaced into a different class.

The inconsistency index, i.e. defined by $[45 \sum_{i=1}^{n} y_j]$, is minimized in the authors' objective function. This leads to maximization of $\sum_{j=1}^{n} y_j$, i.e. the number of correctly evaluated and rated universities.

In this model, 54 y_j variables are introduced. This is because 45 universities are considered, including 13 G1, 9 G2 and 23 G3 universities ($J = 11$ and $J = 47$ are excluded for their incomplete data sets). Since universities in the G2 class are checked from both tails, one needs extra 9 y_j, making $45 + 9 = 54$ y_j in total.

When Model 2 is solved, all information required for decision making and analysis becomes available. The values for $\Omega 0, \Omega 1, \Omega 2$ and $\Omega 3$ provide break points for deciding on grades G1, G2 and G3, as illustrated earlier. The values for weightings, W_i, enable the evaluation of a new university or a change in the status of a present one. Finally, the values for y_j determine whether or not a university is ranked according to what is presently suggested by the experts. For example, if $y_j = 1$, then, it can be said that the jth university is correctly assessed. But, if $y_j = 0$, then, the jth university's ranking suggested by the model is different from the present one. Here, in order to find the new ranking, the total points obtained, i.e. $TP_j = \sum_{i=1}^n W_i r_{ij}$, can be estimated.

Despite the advantages of the newly developed model (Model 2), it is difficult to solve this model. This is because no solution procedure is available for Model 2. However, it is possible to reduce Model 2 into an integer program in which Ω's variables assume constant values, as described in the following procedure:

1. Set values for Ω's,
2. Solve reduced form of Model 2 and obtain W_i,
3. Find inconsistency index, if satisfactory, terminate,
4. Go to step 1 and change values.

Through experimentation, it was found that relaxing $\sum_{i=1}^n W_i = 1$, as considered in Model 1, can improve the results. Hence, this equation is not included in Model 2. It was also found that adding a lower bound on W_i could also improve the results, so that $W_i \geq 0.04$ was included in Model 2. This procedure was implemented and the results are reported in Tables 12 and 13.

Table 12. Weights estimated by the new model.

Weight	Value
W_1	0.040
W_2	0.040
W_3	0.056
W_4	0.040
W_5	0.040
W_6	3.453
W_7	1.014
W_8	0.522
W_9	1.710
W_{10}	0.040
W_{11}	1.204
W_{12}	0.006
W_{13}	0.218
W_{14}	0.768
W_{15}	0.099
W_{16}	0.040

The results illustrated in Table 13 are interesting. The total mismatch (or inconsistency) index is now reduced from the 22 units of an entropy based model to only 11 for the new model. Since the present ranking scheme is based mainly on human judgment, one can expect to have some degree of inconsistency.

CONCLUDING REMARKS

General purpose MADM techniques, such as entropy, could not effectively model public sector university ranking decision problems. Such decision problems require a new methodology to be developed, as shown in this paper, in order to capture the mere nature of the particular situation.

In the framework of an input-output analysis, a new methodology was developed and tested using real data to formalize the currently ad-hoc decision making system. The first advantage of the authors methodology is that it proposes a sound and reasonable platform for the evaluation of state run universities. Therefore, the results obtained by this formal methodology illustrate more rationality and transparency, leading to more acceptances among administrators. The second advantage of the proposed model is that it enables the systematic evaluation of new universities that have yet to be evaluated. Finally, this model can be used as a vehicle for detecting the constantly changing status of the universities and, hence, provide new evaluations. Despite the above advantages, it must be noted that finding decision parameters (Ω's) is currently very tedious. Further investigation is needed to facilitate currently interactive integer programming problem solving.

ACKNOWLEDGMENT

The data provided by the OPAD of the Ministry of Research, Science and Technology of Iran is greatly appreciated. Interesting and careful comments made by three anonymous referees have added to the readability and clarity of the paper, for which the authors are grateful.

Table 13. Results from the new model.

University Name	**Present Grade**	**New Grade**	**Mismatch Index**
Isfahan University of Technology	1	1	0
Esfahan	1	1	0
Shiraz	1	1	0
Allame Tabatabaei	1	1	0
Khajeh Nasir University of Technology	1	1	0
Tarbiyat Modares	1	1	0
Tehran	1	1	0
Shahid Beheshti	1	1	0
Amir Kabir University of Technology	1	1	0
Science and Technology of Iran	1	1	0
Sharif University of Technology	1	1	0
Zanjan University of Graduate Studies	1	1	0
Ferdosi Mashhad	1	1	0
Tabriz	1	1	0
Yazd	2	2	0
Shahid Bahonar	2	3	1
Alzahra	2	1	1
Art	2	2	0
Tarbiat-e Moalem Tehran	2	1	1
Shahid Chamran	2	2	0
Gilan	2	1	1
Mazandaran	2	1	1
Sistan-Baloochestan	2	2	0
Shahrekord	3	3	0
Kordestan	3	3	0
Razi	3	3	0
Ielam	3	3	0
Booali Sina	3	3	0
Lorestan	3	3	0
Khalij-e Fars	3	2	1
Yasooj	3	3	0
Valiasr Rafsanjan	3	3	0
Hormozgan	3	3	0
Arak	3	2	1
Imam Khomeini International University	3	3	0
Sahand University of Technology	3	2	1
Oroomiye	3	1	2
Zanjan	3	2	1
Mohaghegh Ardebili	3	3	0
Semnan	3	3	0
Kashan	3	3	0
Shahrood	3	3	0
Gorgan	3	3	0
Damghan	3	3	0
Sabzevar Teacher Education University	3	3	0
Birjand	3	3	0
Tabriz Teacher Education University	3	3	0
Total Mismatch	11		

REFERENCES

1. Penington, D. "Managing quality in higher education institutions of the 21st century: A framework for the future", *Australian Journal of Education*, **42**(3), pp 256-270 (Nov 1998).
2. Davies, J.E. "What gets measured, gets managed: Statistics and performance indicators for evidence based management", *Journal of Librarianship and Information Science*, **34**(3), pp 129-133 (Sep. 2002).
3. Fandel, G. and Gal, T. "Redistribution of funds for teaching and research among universities", *Research and Practice in Multiple Criteria Decision Making*, **487**, pp 400-408 (2000).
4. Fandel, G. and Gal, T. "Redistribution of funds for teaching and research among universities: The case of North Rhine-Westphalia", *European Journal of Operational Research*, **130**(1), pp 111-120 (Apr. 2001).
5. Hoare, R., Besterfield-Sacre, M., Ertman, D., Gerchak, J., Johnson, T., Shields, R. and Shuman, L. "Cross-institutional assessment: Development and implementation of the on-line student survey system", *Computer Applications in Engineering Education*, **10**(2), pp 88-97 (2002).
6. Bratti, M. "Does the choice of university matter? A study of the differences across UK universities in life sciences students' degree performance", *Economics of Education Review*, **21**(5), pp 431-443 (Oct 2002).
7. Weeks, P. "Benchmarking in higher education: An Australian case study", *Innovations in Education and Training International*, **37**(1), pp 59-67 (Feb. 2000).
8. *Internal Report of the Office of Productivity and Administrative Developments (OPAD)*, The Ministry of Science, Research and Technology (MSRT), Tehran, Iran (1997-2000).
9. Evangelos Triantaphyllou, *Multi-Criteria Decision Making Methods: A Comparative Study*, Kluwer Academic Publishers, Dordrecht (2000).
10. Sarrico, C.S. and Dyson, R.G. "Using DEA for planning in UK universities-an institutional perspective", *Journal of the Operational Research Society*, **51**(7), pp 789-800 (Jul. 2000).
11. Shi, Y., Yu, P.L., Zhang, D.Z. "Eliminating permanently dominated opportunities in multiple-criteria and multiple-constraint level linear-programming", *Journal of Mathematical Analysis and Applications*, **183**(3), pp 685-705 (May 1994).
12. Yeo, A.C., Smith, K.A., Willis, R.J. and Brooks, M. "A mathematical programming approach to optimise insurance premium pricing within a data mining framework", *Journal of the Operational Research Society*, **53**(11), pp 1197-1203 (Nov. 2002).
13. Zhu, J. "Imprecise data envelopment analysis (IDEA): A review and improvement with an application", *European Journal of Operational Research*, **144**(3), pp 513-529, (Feb. 2003).
14. Post, T. and Spronk, J. "Performance benchmarking using interactive data envelopment analysis", *European Journal of Operational Research*, **115**(3) pp 472-487 (Jun 16 1999).
15. Brans, J.P., Macharis, C., Kunsch, P.L., Chevalier, A. and Schwaninger, M. "Combining multicriteria decision aid and system dynamics for the control of socio-economic processes: An iterative real-time procedure", *EJOR* (Sept. 1998).
16. Shannon, C.E. "A mathematical theory of communication", *Bell Sys. Tech. Journal*, **27**, pp 623-656 (1948).
17. Jaynes, E.T. "Information theory and statistical mechanics", *Physical Review*, **106**, pp 620-630 (1957).
18. Cover, T.M. and Thomas, J.A., *Elements of Information Theory*, John Wiley and Sons Inc. New York. NY, USA (1991).