

# Second Harmonic Generation (SHG) Measurement Affected by Electric Field

A. Amjadi\*, P. Rajai<sup>1</sup> and H. Latifi<sup>2</sup>

A type II (eoe: extraordinary-ordinary-extraordinary) phase matching condition in a 2nd Harmonic Generation of Nd-YAG laser under a high electric field in a uniaxial KDP crystal has been measured. Type II phase matching takes place when the two mixing waves are of orthogonal polarizations (extraordinary-ordinary) and the 2nd harmonic radiation corresponds to an extraordinary wave. By applying an electric field along the  $X(Y)$  axes of the crystal, a new phase-matching angle was measured under the influence of different electric fields. The deflection angle of the SHG beam was measured as a function of the applied voltage. An optical collector will focus the 1st harmonic and a fraction of its 2nd harmonic generation used in surgical operations.

## INTRODUCTION

Nd:YAG laser and its 2nd harmonic generation have many applications in surgery. As the two wavelengths have different biological effects on the tissue, in some operations, a surgeon prefers to have a combination of both lasers under manipulation. In order to change the ratio of the two lasers, an optical collector (simply a lens ( $f$ ) and a grating) is located at a  $2f$  distance between the KDP crystal and the operation area. The optical axes of the optical collector is aligned on the Nd:YAG laser, therefore, its power intensity on the operation area remains constant, while, by applying a voltage to the KDP crystal, the 2nd harmonic generation angle ( $\theta_m$ ) changes and its power intensity passing through the grating will change. By this technique, a system has been developed, whereby one can simply change the ratio of the 2nd harmonic over the 1st harmonic generation by applying an electric field to the KDP Crystal.

When an electromagnetic (EM) wave propagates through a medium, the total electric field, which contains all of the frequency components of the input

wave, acts on each particle of the medium. The resultant polarization generated by the input EM wave can be written as:

$$P \propto \chi_1 E + \chi_2 E^2 + \chi_3 E^3 + \dots, \quad (1)$$

where,  $E$  stands for the electric field of the propagated wave in the crystal,  $\chi_1$  is linear susceptibility and  $\chi_2$  is the 2nd order nonlinear polarizability, etc [1].

In certain classes of crystal, in addition to linear response, an electric field produces a polarization proportional to the square of the field. One application of this phenomenon is SHG, in which part of the energy of an optical wave of frequency,  $\omega$ , propagated through a crystal is converted to a wave at  $2\omega$ . The first experiment in this field was performed in 1961 by Franken et al. [2]. Lack of inversion symmetry is a prerequisite for SHG.

The nonlinear response can give rise to an exchange of energy between a number of EM fields of different frequencies. Particularly, all SHG materials are birefringent crystals. When the SHG is under consideration, the  $i$ th component of the total charge polarization,  $\mathbf{P}$ , in a birefringent medium contains two contributions:

$$P_i = P_i^{(\omega)} + P_i^{(2\omega)}, \quad i = 1, 2, 3,$$

where:

$$P_i^{(\omega)} = \epsilon_0 \chi_{ij}^{(\omega)} E_j^{(\omega)}, \quad i, j = 1, 2, 3,$$

and:

$$P_i^{(2\omega)} = \epsilon_0 \chi_{ijk}^{(2\omega)} E_j^{(\omega)} E_k^{(\omega)}, \quad i, j, k = 1, 2, 3.$$

\*. Corresponding Author, Department of Physics, Sharif University of Technology, P.O. Box: 11155-9161, Tehran, I.R. Iran.

1. Sharif Applied Physics Research Center, Sharif University of Technology, P.O. Box: 11155-9161, Tehran, I.R. Iran.

2. Laser and Plasma Research Institute, Shahid Beheshti University, P.O. Box 19835-63113, Tehran, I.R. Iran.

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The first term of  $P_i$ ,  $P_i^{(\omega)}$ , accounts for the linear parts of the response of the medium. The second term,  $P_i^{(2\omega)}$ , is quadratic in the electric field and introduces the third-rank tensor,  $\chi_{ijk}^{(2\omega)}$ . The superscripts  $(\omega)$  and  $(2\omega)$  are now necessary to distinguish the frequencies at which the respective quantities must be evaluated. Thus, two sinusoidal electric field components at frequency  $\omega$  acting in combination exert a resultant containing the double frequency  $2\omega$  (and a dc term). The susceptibility,  $\chi_{ijk}$ , must be evaluated at the combination frequency [3].

## THEORY

SHG is the limiting case of the three-frequency interaction where two of the frequencies,  $\omega_1$  and  $\omega_2$ , are equal and  $\omega_3 = 2\omega_1$ . If one assumes that the amount of power lost by the input ( $\omega_1$ ) beam (by conversion to  $2\omega_1$ ) is negligible, so that  $\frac{dE_{1i}}{dz} \approx 0$ , which is valid for the majority of the experimental situations and the medium is transparent at  $\omega_3$  ( $\sigma_3 = 0$ ), also assuming no 2nd harmonic input ( $E_{3j}(0) = 0$ ), one may obtain an expression for the 2nd harmonic power output,  $P^{(2\omega)}$ , efficiency at the end of a crystal of length  $L$ , as [4]:

$$\frac{P^{(2\omega)}}{P^{(\omega)}} = 8 \left( \frac{\mu_0}{\varepsilon_0} \right)^{3/2} \frac{\omega^2 (d'_{ijk})^2 L^2}{n^3} \left( \frac{P^{(\omega)}}{\text{Area}} \right) \frac{\sin^2 \frac{\Delta K L}{2}}{\left( \frac{\Delta K L}{2} \right)^2},$$

where  $d'_{ijk}$  is the nonlinear susceptibility tensor and  $\Delta K = K_3^j - K_1^i - K_2^k = K^{(\omega_3)} - K^{(\omega_1)} - K^{(\omega_2)}$ . Using  $K^{(\omega)} = \omega \sqrt{\mu \varepsilon_0} n^{(\omega)}$ , where  $n^{(\omega)}$  is the variable refractive index according to frequency, one can rewrite it as:

$$\Delta K = \frac{\omega}{c} (n^{(\omega_1)} + n^{(\omega_2)} - 2n^{(\omega_3)}).$$

A prerequisite for efficient SHG is that  $\Delta K = 0$ , which is called a Phase Matching Condition. Using  $\omega_1 = \omega_2 = \omega$  and  $\omega_3 = 2\omega$ , and, also, taking advantage of the natural birefringence of anisotropic crystals to satisfy this requirement [5,6], one may obtain  $n^{(2\omega)} = n^{(\omega)}$ . So, the indices of refraction at the fundamental and 2nd harmonic frequencies must be equal. In normally dispersive materials, the index of the ordinary wave or the extraordinary wave along a given direction increases with  $\omega$  [7]. It is impossible to satisfy this equation when both the  $\omega$  and  $2\omega$  beams are of the same type - that is, when both are extraordinary or ordinary. One can, however, under certain circumstances, satisfy this condition by using two waves of a different type: One ordinary and one extraordinary. In this study, the type II (eoe) phase matching is desired. Type II phase matching takes place when the two mixing waves are of orthogonal polarizations (extraordinary-ordinary) and the 2nd harmonic radiation corresponds to an extraordinary wave.

If the fundamental beam contains both ordinary and extraordinary components and produces the 2nd harmonic along an extraordinary direction, there exists the angle,  $\theta_m$ , between the propagation direction and the crystal optics ( $z$ ) axis, which satisfies the type II phase matching condition. In this case, one will have:

$$n_o(\omega) = n_e(\omega, \theta_m) = 2n_e(2\omega, \theta_m).$$

Consider the dependence of the index of refraction of the extraordinary wave in a uniaxial crystal on the angle  $\theta$  between the propagation direction and the crystal optics ( $z$ ) axis, which is given by:

$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2(\theta)}{n_o^2} + \frac{\sin^2(\theta)}{n_e^2},$$

where  $n_o$  and  $n_e$  are the ordinary and extraordinary refractive indices, respectively. By substituting the values of  $n_e(\omega, \theta)$  and  $n_e(2\omega, \theta)$  into the phase matching equation, one will find an expression for  $\theta_m$ , as follows [8]:

$$\left[ \frac{\cos^2(\theta_m)}{(n_o^{2\omega})^2} + \frac{\sin^2(\theta_m)}{(n_e^{2\omega})^2} \right]^{1/2} = \frac{1}{2} \left\{ n_o^\omega + \left[ \frac{\cos^2(\theta_m)}{(n_o^\omega)^2} + \frac{\sin^2(\theta_m)}{(n_e^\omega)^2} \right]^{1/2} \right\}.$$

## EXPERIMENTAL SETUP

KDP crystal was grown in a temperature controlled bath and the two copper electrodes (1mm in diameter) were placed inside the bath, while the crystal was growing. The distance between the two electrodes was  $4 \text{ mm} \pm 0.1 \text{ mm}$  and SHG was measured in this region.

The KDP crystal, with the electrodes was fixed on a tunable holder, so that the angle  $\theta_m$  could be tuned very precisely (Figure 1).

To measure the deflection angle more accurately, the output beam was reflected twice; so, for small angles,  $\Delta\theta = 4\frac{d}{D}$  (Figure 2).

A laser beam was focused on the region between the two electrodes inside the crystal. By applying a DC high voltage on the electrodes, the SHG was studied. The intensity variation of the beam on the active medium could be calculated numerically [9]. The deflection angle of the SHG beam was measured as a function of applied voltage.

The laser beam was focused on the region between two electrodes in the crystal after passing through a pin hole. The SHG output, after being reflected by mirrors, M1 and M2, was focused on a tape measure ( $T$ ) to measure  $\theta_m$ . A variable high voltage power supply was used to apply voltage to the KDP crystal.

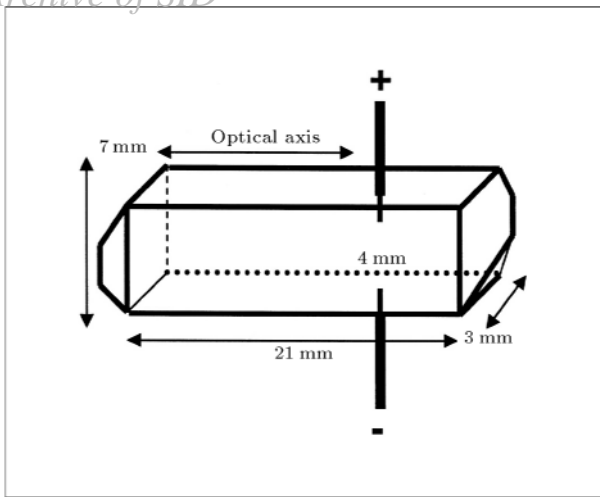


Figure 1. KDP crystal with electrodes.

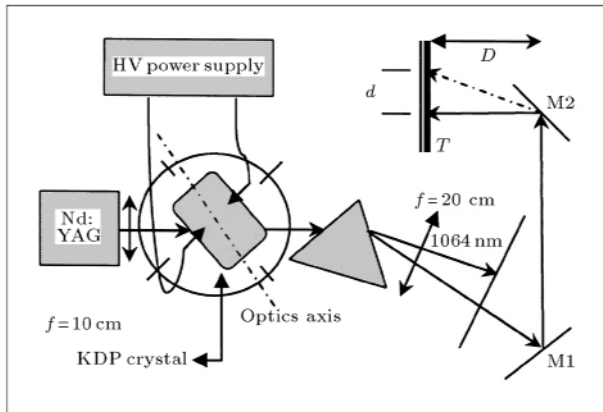


Figure 2. Experimental setup.

## DISCUSSION

The expression for  $\theta_m$ , which, when obtained, is relatively complicated for negative uniaxial crystals, can be calculated, approximately, by [1]:

$$\tan^2 \theta_m^{eoe} \cong \frac{1}{W} \frac{U}{R},$$

where:

$$U = \frac{(A+B)^2}{C^2}, \quad W = \frac{(A+B)^2}{F^2},$$

$$R = \frac{(A+B)^2}{(D+B)^2}, \quad A = B = \frac{n_o^\omega}{\lambda^\omega},$$

$$C = \frac{n_o^{2\omega}}{\lambda^{2\omega}}, \quad D = \frac{n_e^\omega}{\lambda^\omega}, \quad F = \frac{n_e^{2\omega}}{\lambda^{2\omega}}.$$

If the fundamental beam derived from the following Nd:YAG laser ( $\lambda = 1064 \text{ nm}$ ), according to the dispersion

relations (Sellmaier equations) for the KDP crystal, which, at  $T = 24.8^\circ\text{C}$ , are [10]:

$$n_o^2 = 2.259276 + \frac{0.01008956}{\lambda^2} + \frac{13.00522\lambda^2}{\lambda^2 400},$$

and:

$$n_e^2 = 2.132668 + \frac{0.008637494}{\lambda^2} + \frac{3.2279924\lambda^2}{\lambda^2 400},$$

with  $\lambda$  given in  $\mu\text{m}$ , the refractive indices are  $n_o^{(\omega)} = 1.4938$ ,  $n_e^{(\omega)} = 1.4599$ ,  $n_o^{(2\omega)} = 1.5123$  and  $n_e^{(2\omega)} = 1.4705$ . After placing the parameter values, one may find that  $\theta_m^{eoe} = 59.9^\circ$ .

In the experiment, first, the crystal angle was tuned for maximum 2nd harmonic intensity. Under matching conditions and, after observing SHG on the KDP crystals, the perturbation of the phase matching condition in the SHG of the 1064nm laser beam in the KDP crystal was tested.

## RESULT AND CONCLUSION

By applying an electric field up to 37.5 KV/cm along the X(Y) axes of the crystal, a decrease in 2nd harmonic intensity was observed after the optical collector. A new phase-matching angle was measured under the influence of different electric fields.

The measured deflected angle of the SHG beam was plotted as a function of the applied electric field for the KDP crystal (Figure 3).

The data shows a linear change on  $\theta_m$  as a function of the electric field with the slope of  $0.0266 \pm 0.003$  (Degree.cm/KV).

The maximum electric field, which could be applied to the crystal, was measured to be 37.5 KV/cm.

It was several times experienced that application of an electric field above 37.5 KV/cm would crack the crystal.

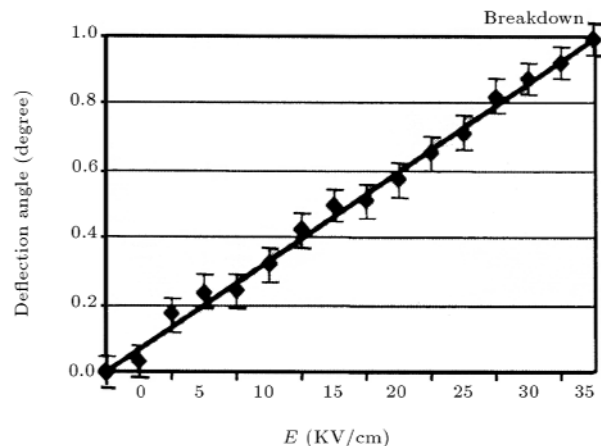


Figure 3. Deflection angle as a function of electric field.

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