Scientia Iranica, Vol. 14, No. 5, pp 512 518
© Sharif University of Technology, October 2007

SCIENTIA
IRANICA

Research Note

Effect of Quantum Interference on the Absorption Spectrum of a Four-Level Atom in Photonic Crystals

S. Roshan Entezar[1] **and H. Tajalli**[*]

In this paper, the probe absorption spectrum of a four-level atom in photonic crystals has been investigated. A model is used, which assumes the transitions from the two upper levels to one of the lower levels coupled to the modes of the modified reservoir (photonic band gap) and transitions from the two upper levels to the other lower level coupled to the free vacuum modes. The effect of quantum interference in spontaneous emissions from the two upper levels to the lower levels on the absorption spectrum of an atom is investigated in detail. Most interestingly, it is shown that the atom becomes transparent to a probe laser field coupled to the free space transition only in situations where complete quantum interference takes place in both the free space transitions and the modified reservoir transitions.

INTRODUCTION

The control of spontaneous emission and probe absorption of an atom will lead to a number of potential applications in high-precision measurements, quantum computation, teleportation, quantum information theory and lasing without population inversion. Therefore, the study of spontaneous emission and probe absorption has attracted much attention in recent years. It is well known that the spontaneous emission and probe absorption of an atom depend, not only on the properties of the excited atomic system, but also on the nature of the surrounding environment [1]. The quantum interference of spontaneous emission and probe absorption is widely studied. A classic example is the V-type three-level system, consisting of two close excited levels and one ground state in the same environment reservoirs [2]. A quantum interference phenomena will occur if the dipole moments of the two arms of a transition are not orthogonal. The interference in such a system arises from indistinguishable spontaneous emission pathways. The interference, vacuum-induced or additionally driven by external coherent laser fields, is widely studied and leads to many remarkable phenomena, including very narrow absorption and fluorescence spectra [3], population trapping in a degenerate system [4], fluorescence quenching in the free space [5], phase-dependent line shapes [6,7], lasing without inversion [8] and etc. [9]. From the viewpoint of the surrounding environment of atoms, Photonic Band Gap (PBG) structures have different Densities Of State (DOS) compared with a free-space vacuum field [1,10,11]. The study of quantum and nonlinear optical phenomena for types of atoms embedded in such PBG materials leads to the prediction of many interesting effects: For example, localization of light [12,13], photon-atom bound states [1,12], suppression and even complete cancellation of spontaneous emission [14,15], enhancement of spontaneous emission [16,17], electromagnetically induced transparency and other phenomena [18].

Many papers have reported on the population involution and distribution in upper levels [1,12] and the spectral components in the spontaneous emission fields [16,17] of an atom embedded in a photonic crystal. Of particular interest, Knight and his coworkers [18] investigated the transparency phenomena of an atom in the Λ-type model, where one transition interacts with free vacuum modes and another interacts with modes near the edges of the photonic band gap. Focusing on the quantum interference of an atom without external driving coherent laser fields, Zhu [3] independently reported the quantum interference of a spontaneous emission spectrum and the probe absorption spectrum of an atom coupled to vacuum space reservoirs. The quantum interference in the spontaneous emission spectrum of an atom embedded in a double-band photonic crystal was reported by Zhang and his co-workers [19-21].

---

[1] *Research Institute for Applied Physics and Astronomy, Tabriz University, Tabriz, I.R. Iran.*

[*] *Corresponding Author, Research Institute for Applied Physics and Astronomy, Tabriz University, Tabriz, I.R. Iran.*

In this paper, the model of a four-level atom in a double-band photonic crystal is adopted. The situations, where transitions from two upper levels to one of the lower levels coupled to the free vacuum modes, and the transitions from two upper levels to the other lower level coupled to the modes, which are isotropic PBG and anisotropic PBG, respectively, are studied. The probe absorption spectrum with the transitions coupled to free vacuum modes is also investigated and the effect of quantum interference on the absorption spectrum is studied in detail. Then, the equations for describing the probe absorption spectrum of the four-level atom in a photonic crystal are deduced. After that, the general calculated results and analysis are presented. And finally, major conclusions are summarized.

EQUATIONS FOR THE PROBE ABSORPTION SPECTRUM

Consider a single four-level atom placed inside a PBG. The two excited states are labeled $|3\rangle$ and $|2\rangle$ and the two lower states of the atom $|1\rangle$ and $|0\rangle$, are as shown in Figure 1. The transitions $|3\rangle \rightarrow |1\rangle$ and $|2\rangle \rightarrow |1\rangle$ are taken to be near resonant with a modified reservoir (the non-Markovian reservoir), while the transitions $|3\rangle \rightarrow |0\rangle$ and $|2\rangle \rightarrow |0\rangle$ are assumed to be occurring in free space (the Markovian reservoir). The probe absorption spectrum of these latter transitions is of central interest in this section. In the configuration shown in Figure 1, the upper levels, $|3\rangle$ and $|2\rangle$, are of the same symmetry.

The four-level atom is initially prepared in the lower level, $|0\rangle$. At time $t = 0$, this atom starts to interact with a probe field of frequency ω_p. The dynamics of the system can be described using the Schrödinger equation. Then, the wave function of the system at time t can be expressed in terms of the state vectors, as follows:

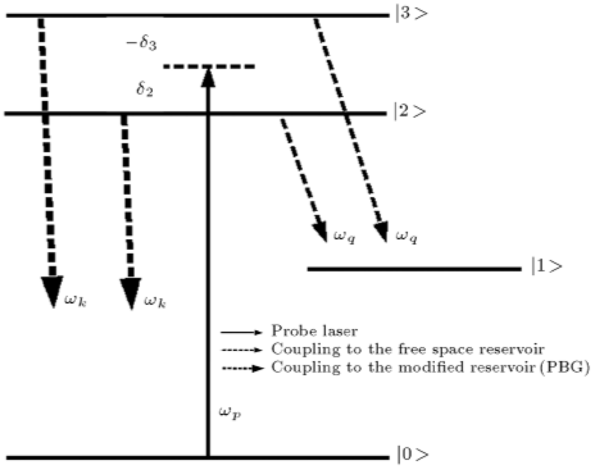


Figure 1. The schematic diagram of a four-level atomic system.

$$|\psi(t)\rangle = a_0(t)\,|0,\{0\}\rangle + a_2(t)e^{-i\delta_2 t}\,|2,\{0\}\rangle + a_3(t)e^{-i\delta_3 t}\,|3,\{0\}\rangle + \sum_{ke} a_{ke}(t)\,|0,\{ke\}\rangle + \sum_{qe} a_{qe}(t)\,|1,\{qe\}\rangle\,, \tag{1}$$

where k and q denote the momentum vectors of the emitted photons and e denotes the polarization of the emitted photons, $\delta_2 = \omega_p - \omega_{20}$ and $\delta_3 = \omega_p - \omega_{30}$. The function, $a_j(t)$ $(j = 0, 2, 3)$, gives the probability amplitude to find the atom in the excited state, $|j\rangle$, and the photon reservoir in the vacuum state. On the other hand, $a_{ke}(t)$ and $a_{qe}(t)$ give the probability amplitude to find the atom in the lower state, $|0\rangle$ and $|1\rangle$, and a single photon of wavevector k and q and polarization e in the photon reservoir, respectively.

The Hamiltonian, describing the dynamics of this system in the interaction picture and the rotating wave approximation, can be written as follows:

$$\begin{aligned} H_I = (&\Omega_2 e^{i\delta_2 t}\,|0\rangle\langle 2| + \Omega_3 e^{i\delta_3 t}\,|0\rangle\langle 3| \\ &+ \sum_{ke} g^{20}_{ke} e^{-i\delta_k t}\,|2\rangle\langle 0|\,\hat{b}_{ke} + \sum_{ke} g^{30}_{ke} e^{-i\delta'_k t}\,|3\rangle\langle 0|\,\hat{b}_{ke} \\ &+ \sum_{qe} g^{21}_{qe} e^{-i\delta_q t}\,|2\rangle\langle 1|\,\hat{b}_{qe} \\ &+ \sum_{qe} g^{31}_{qe} e^{-i\delta'_q t}\,|3\rangle\langle 1|\,\hat{b}_{qe} + H.C). \end{aligned} \tag{2}$$

Here, first and second terms describe the coupling of the probe field to the atom in transitions $|0\rangle \rightarrow |2\rangle$ and $|0\rangle \rightarrow |3\rangle$, respectively, and the next two terms express the interaction of the atom with the free vacuum reservoir in transitions $|2\rangle \rightarrow |0\rangle$ and $|3\rangle \rightarrow |0\rangle$, respectively. Finally, the last two terms explain the interaction of the atom with the free vacuum reservoir in transitions $|2\rangle \rightarrow |1\rangle$ and $|3\rangle \rightarrow |1\rangle$, respectively. Ω_2 and Ω_3 are the Rabi frequencies, which are considered real for convenience in this problem; $\delta_k = \omega_k - \omega_{20}$ $(\delta'_k = \omega_k - \omega_{30})$ represents the detuning of the radiation mode frequency, ω_k, from the atomic transition frequency, $\omega_{20}(\omega_{30})$, and $\delta_q = \omega_q - \omega_{21}(\delta'_q = \omega_q - \omega_{31})$ represents the detuning of the radiation mode frequency, ω_q, from the atomic transition frequency, $\omega_{21}(\omega_{31})$. $g^{ij}_{\lambda e}$ is the frequency-dependent coupling constant between the atomic transition, $|i\rangle \rightarrow |j\rangle$, and the mode, $\{\lambda e\}$, of the radiation field. More precisely:

$$g^{ij}_{\lambda e} = \frac{\omega_{ij}d_{ij}}{\hbar}\left(\frac{\hbar}{2\varepsilon_0\omega_\lambda V}\right)^{1/2}\overrightarrow{e}_{\lambda e}.\overrightarrow{d}_{ij}. \qquad (3)$$

Here, $\overrightarrow{d}_{ij}$ is the atomic dipole moment unit vector for the transition, $|i\rangle \rightarrow |j\rangle$, $\overrightarrow{e}_{\lambda e}$ is the polarization unit vector of the radiation fields, V is the sample volume and ε_0 is the Coulomb constant.

This Hamiltonian is substituted into the Schrödinger equation and the following set of equations is obtained:

$$i\dot{a}_0(t) = \Omega_2 a_2(t) + \Omega_3 a_3(t), \qquad (4)$$

$$i\frac{d}{dt}(a_2(t)e^{-i\delta_2 t}) = \Omega_2 e^{-i\delta_2 t}a_0(t) + \sum_{ke} g^{20}_{ke}e^{-i\delta_k t}a_{ke}(t) + \sum_{qe} g^{21}_{qe}e^{-i\delta_q t}a_{qe}(t), \qquad (5)$$

$$i\frac{d}{dt}(a_2(t)e^{-i\delta_3 t}) = \Omega_3 e^{-i\delta_3 t}a_0(t) + \sum_{ke} g^{30}_{ke}e^{-i\delta'_k t}a_{ke}(t) + \sum_{qe} g^{31}_{qe}e^{-i\delta'_q t}a_{qe}(t), \qquad (6)$$

$$i\dot{a}_{ke}(t) = g^{02}_k e^{i(\delta_k - \delta_2)t}a_2(t) + g^{03}_k e^{i(\delta'_k - \delta_3)t}a_3(t), \qquad (7)$$

$$i\dot{a}_{qe}(t) = g^{12}_q e^{i(\delta_q - \delta_2)t}a_2(t) + g^{13}_q e^{i(\delta'_q - \delta_3)t}a_3(t). \qquad (8)$$

By formal time integration of Equations 7 and 8 and by eliminating $a_{qe}(t)$ and $a_{ke}(t)$ from Equations 5 and 6, one obtains the following:

$$\dot{a}_2(t) = i\delta_2 a_2(t) - i\Omega_2 a_0(t) - \frac{\gamma_{20}}{2}a_2(t) - \frac{\eta}{2}\sqrt{\gamma_{20}\gamma_{30}}a_3(t) - \int_0^t a_2(t')e^{i\delta_2(t-t')}K_{22}(t-t')dt' - \int_0^t a_3(t')e^{i\delta_3(t-t')}K_{23}(t-t')dt', \qquad (9)$$

$$\dot{a}_3(t) = i\delta_3 a_3(t) - i\Omega_3 a_0(t) - \frac{\eta}{2}\sqrt{\gamma_{20}\gamma_{30}}a_2(t) - \frac{\gamma_{30}}{2}a_3(t) - \int_0^t a_2(t')e^{i\delta_2(t-t')}K_{32}(t-t')dt' - \int_0^t a_3(t')e^{i\delta_3(t-t')}K_{33}(t-t')dt'. \qquad (10)$$

Here;

$$K_{ij}(t-t') = \sum_{qe} g^{i1}_{qe}g^{1j}_{qe}e^{-i(\omega_q - \omega_{j1})(t-t')}, \qquad (11)$$

is the delayed Green function, where $(i,j = 2,3)$ γ_{i0} is the effective decay rate for the transition from the upper level, $|i\rangle\,(i = 2,3)$, to the lower level, $|0\rangle$. $\eta = (3/8\pi)\int(\cos\alpha_{20}\cos\alpha_{30} + \cos\beta_{20}\cos\beta_{30})d\Omega$, where $\{\alpha_{i0},\beta_{i0},\theta_{i0}\}(i = 2,3)$ are the directional angles of the dipole moment unit vector, $\overrightarrow{d}_{i0}$, in a coordinate system defined by the unit victors $\{\overrightarrow{e}_{k1},\overrightarrow{e}_{k2},\overrightarrow{k}\}$ and $d\Omega$ is the solid angle. So, $\eta = 1.0$, when $\overrightarrow{d}_{20}.\overrightarrow{d}_{30} = \pm 1$ and $\eta = 0$, when $\overrightarrow{d}_{20}.\overrightarrow{d}_{30} = 0$; otherwise, $0 < \eta < 1.0$. In obtaining Equations 9 and 10, the following is used (see [1]):

$$\sum_{ke} g^{i0}_{ke}g^{j0}_{ke}e^{-i(\omega_k - \omega_{j0})(t-t')} = \frac{\eta_{ij}}{2}\sqrt{\gamma_{i0}\gamma_{j0}}\delta(t-t'), \qquad (12)$$

where, $\eta_{ij} = \delta_{ij} + \eta(1 - \delta_{ij})(i,j = 2,3)$.

The delayed Green function, $K_{ij}(t-t')$, can be expressed in the following forms [22]:

$$K_{ij}(t-t') = \frac{p_{ij}}{2}\sqrt{\beta^{3/2}_{i1}\beta^{3/2}_{j1}}\left\{\frac{e^{i[\delta^j_{g2}(t-t') - \pi/4]}}{\sqrt{\pi(t-t')}} + \frac{e^{i[\delta^j_{g1}(t-t') + \pi/4]}}{\sqrt{\pi(t-t')}}\right\}, \qquad (13)$$

$$K_{ij}(t-t') = \frac{p_{ij}}{2}\sqrt{\alpha_{i1}\alpha_{j1}}\left\{\frac{e^{i[\delta^j_{g2}(t-t') + \pi/4]}}{\sqrt{4\pi(t-t')^3}} + \frac{e^{i[\delta^j_{g1}(t-t') - \pi/4]}}{\sqrt{4\pi(t-t')^3}}\right\}, \qquad (14)$$

for isotropic PBG and anisotropic PBG reservoirs, respectively, with $\delta^j_{gi} = \omega_{j1} - \omega_{gi}$ $(i = 1,2$ and $j = 2,3)$. Here, $p_{ij} = \delta_{ij} + p(1 - \delta_{ij})$ $(i,j = 2,3)$ and $p = (3/8\pi)\int(\cos\alpha_{21}\cos\alpha_{31} + \cos\beta_{21}\cos\beta_{31})d\Omega$, where $\{\alpha_{i1},\beta_{i1},\theta_{i1}\}$ $(i = 2,3)$ are the directional angles of the dipole moment unit vector, $\overrightarrow{d}_{i1}$, in a coordinate system defined by the unit victors $\{\overrightarrow{e}_{k1},\overrightarrow{e}_{k2},\overrightarrow{k}\}$. So, $p = 1.0$, when $\overrightarrow{d}_{21}.\overrightarrow{d}_{31} = \pm 1$ and $p = 0$, when $\overrightarrow{d}_{21}.\overrightarrow{d}_{31} = 0$; otherwise, $0 < p < 1.0$. The definitions of the parameters are, as follows:

$$\beta^{3/2}_{i1} = \frac{1}{2\pi\varepsilon_0}\frac{\omega^2_{i1}d^2_{i1}\omega^{3/2}_{gi}}{3\hbar c^3}, \qquad (15)$$

$$\alpha_{i1} = \frac{1}{2\pi\varepsilon_0}\frac{\omega^2_{i1}d^2_{i1}\omega^{1/2}_{gi}}{3\hbar c^3}. \qquad (16)$$

The aim here is to investigate the absorption and dispersion properties of the system for a weak probe laser field. The equation of motion for the electric field amplitude, $E(z,t)$, is given by [23],

$$\left(\frac{\partial}{\partial z} + \frac{1}{v_g}\frac{\partial}{\partial t}\right)E(z,t) = -i\frac{\omega}{2c}\chi(\delta)E(z,t), \qquad (17)$$

where $\chi(\delta)$ is the steady state linear susceptibility of the medium and $v_g = c/[1 + (\omega/2)(\partial \mathrm{Re}(\chi)/\partial\omega)]$ is the group velocity of the laser pulse with the derivative of the real part of the susceptibility being evaluated at the carrier frequency. Here, the definition of δ is $\delta = \delta_3 + 0.5\omega_{32} = \delta_2 - 0.5\omega_{32} = \omega_p - 0.5(\omega_{20} + \omega_{30})$. Since the transitions $|0\rangle \rightarrow |2\rangle$ and $|0\rangle \rightarrow |3\rangle$ are treated as occurring in free space, the steady state linear susceptibility is given by [24]:

$$\chi(\delta) = -4\pi N\left(\frac{|d_{02}|^2}{\Omega_2}a_0(t \rightarrow \infty)a_2^*(t \rightarrow \infty)\right.$$
$$\left.+\frac{|d_{03}|^2}{\Omega_3}a_0(t \rightarrow \infty)a_3^*(t \rightarrow \infty)\right), \tag{18}$$

with N being the atomic density. The solutions of Equations 9 and 10 are obtained by means of a perturbation theory [24-26]. It is assumed that the interaction between the probe laser and the atom is very weak ($\Omega_2, \Omega_3 << \beta, \alpha$), so that $a_0(t) \approx 1$ for all times. With the use of the Laplace transform, one obtains, from Equations 9 and 10,

$$s\tilde{a}_2(s) = -i\frac{\Omega_2 D(s) - \Omega_3 B(s))}{A(s)D(s) - B(s)C(s)}, \tag{19}$$

$$s\tilde{a}_3(s) = -i\frac{\Omega_2 C(s) + \Omega_3 A(s))}{A(s)D(s) - B(s)C(s)}, \tag{20}$$

where:

$$A(s) = s - i\delta_2 + \widetilde{K}_{22}(s - i\delta_2) + \gamma_{20}/2, \tag{21}$$

$$B(s) = \widetilde{K}_{23}(s - i\delta_3) + \eta\sqrt{\gamma_{20}\gamma_{30}}/2, \tag{22}$$

$$C(s) = \widetilde{K}_{32}(s - i\delta_2) + \eta\sqrt{\gamma_{20}\gamma_{30}}/2, \tag{23}$$

$$D(s) = s - i\delta_3 + \widetilde{K}_{33}(s - i\delta_3) + \gamma_{30}/2, \tag{24}$$

here, $\tilde{a}_i(s) = L[a_i(t)]$, $\widetilde{K}_{ij}(s) = L[K_{ij}(t)]$ $(i, j = 2, 3)$, s is the Laplace variable and:

$$\widetilde{K}_{ij}(s) = \frac{p_{ij}}{2}\sqrt{\beta_{i1}^{3/2}\beta_{j1}^{3/2}}\left(\frac{i}{\sqrt{is + \delta_{g1}^j}} + \frac{1}{\sqrt{is + \delta_{g2}^j}}\right), \tag{25}$$

$$\widetilde{K}_{ij}(s) = \frac{p_{ij}}{2}\sqrt{\alpha_{i1}\alpha_{j1}}\left(-i\sqrt{is + \delta_{g1}^j} + \sqrt{is + \delta_{g2}^j}\right), \tag{26}$$

for double-band isotropic PBG and double-band anisotropic PBG reservoirs, respectively.

Therefore, one can easily obtain, using the final value theorem, the long time behavior of the probability amplitude,

$$a_2(t \rightarrow \infty) = -i\frac{\Omega_2 D(0) - \Omega_3 B(0))}{A(0)D(0) - B(0)C(0)}, \tag{27}$$

$$a_3(t \rightarrow \infty) = -i\frac{\Omega_2 C(0) + \Omega_3 A(0))}{A(0)D(0) - B(0)C(0)}. \tag{28}$$

Equations 18, 27 and 28, obtained above are used and the absorption spectrum for several parameters of the system are calculated.

RESULTS AND DISCUSSION

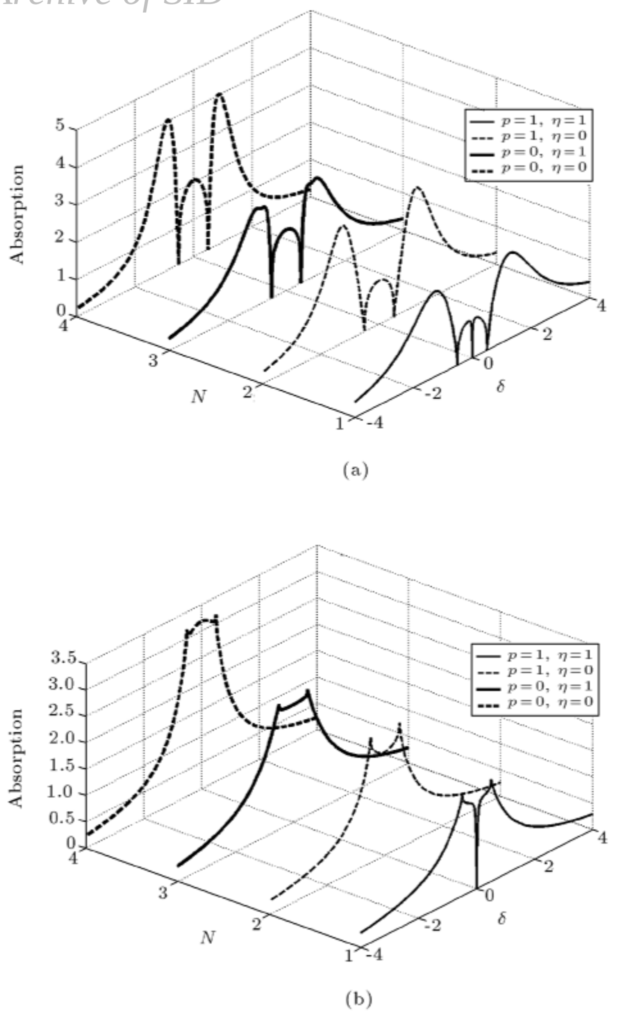
In the model shown in Figure 1, the transitions from the two upper levels to the two lower levels, $|1\rangle$ and $|0\rangle$, are coupled, respectively, by the free vacuum modes and PBG modes, leading to two possible types of quantum interference. The first is ascribed to atomic transitions coupled to the PBG modes and the second arises from atomic transitions coupled to the free vacuum modes. As far as it is known, there are no reports on the effects of the two types of quantum interference on the probe absorption spectrum. The following definitions are used:

$$\delta_{g1} = \delta_{g1}^2 + 0.5\omega_{32} = \delta_{g1}^3 - 0.5\omega_{32}$$
$$= 0.5(\omega_{21} + \omega_{31}) - \omega_{g1}, \tag{29}$$

$$\delta_{g2} = \delta_{g2}^2 + 0.5\omega_{32} = \delta_{g2}^3 - 0.5\omega_{32}$$
$$= 0.5(\omega_{21} + \omega_{31}) - \omega_{g2}. \tag{30}$$

The spectrum of $-\mathrm{Im}[\chi(\delta)]$, which is usually considered to be the absorption spectrum, is shown in Figure 2 as a function of the detuning δ of the probe field for the cases of the transitions $|3\rangle \rightarrow |1\rangle$ and $|2\rangle \rightarrow |1\rangle$ coupled to the isotropic PBG reservoir and the anisotropic PBG reservoir, respectively. The plots at the positions $N = 1, 2, 3, 4$ correspond to $(p = 1, \eta = 1)$, $(p = 1, \eta = 0)$, $(p = 0, \eta = 1)$ and $(p = 0, \eta = 0)$, respectively. Here, symmetric values of parameters (i.e., same detunings $\delta_{g1} = -\delta_{g2} = 0.5$ and same coupling constants $\beta_{21} = \beta_{31} = 1.0$ and $\alpha_{21} = \alpha_{31} = 1.0$) are employed.

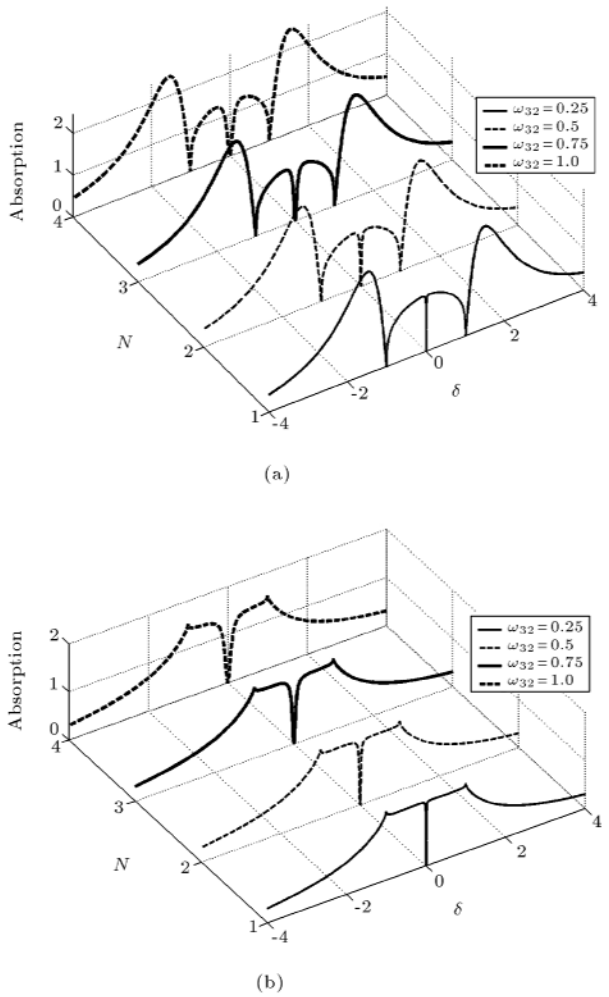
Figure 2 shows that, altogether, three zeros appear at position $N = 1$ and two zeros appear at positions $N = 2, 3, 4$ in the case of the isotropic PBG reservoir (Figure 2a). But there exists only one zero at position $N = 1$ and it is not seen at other positions ($N = 2, 3, 4$) in the cases of the anisotropic PBG reservoir (Figure 2b). In Figure 2, the zeros at the center of $\delta = 0$ at position $N = 1$, in both cases of the isotropic PBG reservoir and the anisotropic PBG reservoir, are due to the quantum interference



(a)

(b)

Figure 2. The absorption $\mathrm{Im}(\chi)$ spectrum (in arbitrary units) as a function of detuning δ of probe field, for different reservoirs; a) Double-band isotropic PBG reservoir and $\beta_{21} = \beta_{31} = 1.0$ and b) double-band anisotropic PBG reservoir and $\alpha_{21} = \alpha_{31} = 1.0$. The parameters used are $\delta_{g1} = \quad \delta_{g2} = 0.5$, $a_0(0) = 1.0$ and $\omega_{32} = 0.5$. All parameters in this paper are in units of γ_{20}.



(a)

(b)

Figure 3. Same as Figure 2 for $\delta_{g1} = \quad \delta_{g2} = 1.0$ and $(p = 1, \eta = 1)$.

of the transitions between ($|3\rangle \to |0\rangle$, $|2\rangle \to |0\rangle$) and ($|3\rangle \to |1\rangle$, $|2\rangle \to |1\rangle$). The two dark lines at symmetric sides of $\delta = 0$ in Figure 2a originate from the two singularities in the Laplace transform of the delayed Green's function, as shown in Equation 25, which is equivalent to the DOS of the isotropic PBG modes.

In order to investigate the effect of ω_{32} (the frequency distance between the two upper levels) on the absorption spectrum, the absorption spectrum was plotted as a function of detuning δ, in the case of $p = 1, \eta = 1$, as shown in Figure 3. Here, the parameters used are $\delta_{g1} = \quad \delta_{g2} = 1.0$ and $a_0(0) = 1.0$. The plots at the positions $N = 1, 2, 3, 4$ correspond to $\omega_{32} = 0.25$, $\omega_{32} = 0.5$, $\omega_{32} = 0.75$ and $\omega_{32} = 1.0$, respectively. It is seen that, by increasing ω_{32}, the transparency window around $\delta = 0$ becomes wider.

In Figure 4, the absorption spectrum was plotted as a function of detuning δ, for different widths of PBG, in the case of $p = 1$ and $\eta = 1$. Here, the parameters used are $\omega_{32} = 0.5$ and $a_0(0) = 1.0$. The plots at the positions $N = 1, 2, 3, 4$ correspond to $\delta_{g1} = \quad \delta_{g2} = 0.25$, $\delta_{g1} = \quad \delta_{g2} = 0.5$, $\delta_{g1} = \quad \delta_{g2} = 1.0$ and $\delta_{g1} = \quad \delta_{g2} = 5.0$, respectively. It is easily seen that, by increasing the width of the PBG, the probe absorption spectrum becomes similar to the absorption spectrum of a three-level atom placed in the free space reservoir. The reason for this behavior is that the spontaneous emissions of the transitions $|3\rangle \to |1\rangle$, $|2\rangle \to |1\rangle$ are suppressed by PBG. This behavior is shown in Figure 4 at position $N = 4$.

CONCLUSION

The absorption spectrum of a four-level atom embedded in a double-band photonic crystal has been investigated. The transitions from the two upper levels to one of the lower levels was assumed to interact with

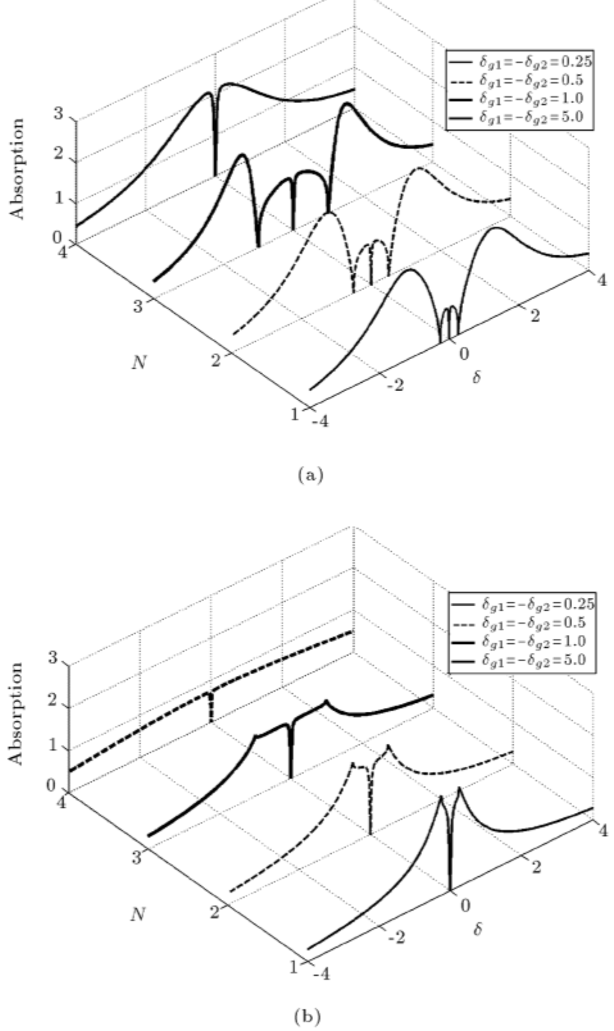


Figure 4. Same as Figure 2 for $\omega_{32} = 0.5$ and $(p = 1, \eta = 1)$.

free vacuum modes, and the transitions from the two upper levels to the other lower level were assumed to interact with anisotropic PBG modes. The absorption spectrum for the transitions coupled to the free vacuum modes is studied. Most interestingly, it is shown that there are zero absorptions in the absorption spectrum, only in the case of complete quantum interferences of the transitions between $(|3\rangle \rightarrow |0\rangle, |2\rangle \rightarrow |0\rangle)$ and $(|3\rangle \rightarrow |1\rangle, |2\rangle \rightarrow |1\rangle)$. It is shown that the number of zeros is different for a double-band isotropic PBG reservoir and a double-band anisotropic PBG reservoir. This difference is due to the singularities in the Laplace transform of the delayed Green's function, in the case of a double-band isotropic PBG reservoir.

REFERENCES

1. Lambropoulos, P. et al. "Fundamental quantum optics in structured reservoirs", *Rep. Prog. Phys.*, **63**(4), pp 455-503 (2000).
2. Zhou, P. and Swain, S. "Quantum interference in probe absorption: Narrow resonances, transparency, and gain without population inversion", *Phys. Rev. Lett.*, **78**, pp 832-835 (1997).
3. Zhu, S.Y. et al. "Spontaneous emission from a three-level atom", *Phys. Rev. A.*, **52**, pp 710-716 (1995).
4. Radmore, P.M. and Knight, P.L. "Population trapping and dispersion in a three-level system", *J. Phys. B.*, **15**, pp 561-573 (1982).
5. Lee, H. et al. "Quenching of spontaneous emission via quantum interference", *Phys. Rev. A.*, **55**, pp 4454-4465 (1997).
6. Paspalakis, E. and Knight. P.L. "Phase control of spontaneous emission", *Phys. Rev. Lett.*, **81**, pp 293-296 (1998).
7. Paspalakis, E. et al. "Transparency induced via decay interference", *Phys. Rev. Lett.*, **82**, pp 2079-2082 (1999).
8. Scully, M.O. et al. "Degenerate quantum-beat laser: Lasing without inversion and inversion without lasing", *Phys. Rev. Lett.*, **62**, pp 2813-2816 (1989).
9. Agarwal, G.S. and Menon, S. "Quantum interferences and the question of thermodynamic equilibrium", *Phys. Rev. A.*, **63**, pp 023818 (2001).
10. Yablonovitch, E. "Inhibited spontaneous emission in solid-state physics and electronics", *Phys. Rev. Lett.*, **58**, pp 2059-2062 (1987).
11. John, S. "Strong localization of photons in certain disordered dielectric superlattices", *Phys. Rev. Lett.*, **58**, pp 2486-2489 (1987).
12. John, S. and Quang, T. "Spontaneous emission near the edge of a photonic band gap", *Phys. Rev. A.*, **50**, pp 1764-1769 (1995).
13. John, S. and Wang, J. "Quantum optics of localized light in a photonic band gap", *Phys. Rev. B.*, **43**, pp 12772-12789 (1991).
14. Kofman, A.G. et al. "Spontaneous and induced atomic decay in photonic band structures", *J. Mod. Optic.*, **41**(2), pp 353-384 (1994).
15. Bay, S. et al. "Atom-atom interaction in strongly modified reservoirs", *Phys. Rev. A.*, **55**, pp 1485-1496 (1997).
16. Zhu, S.Y. et al. "Quantum interference effects in spontaneous emission from an atom embedded in a photonic band gap structure", *Phys. Rev. Lett.*, **79**, pp 205-208 (1997).
17. Zhu, S.Y. et al. "Spontaneous radiation and lamb shift in three-dimensional photonic crystals", *Phys. Rev. Lett.*, **84**, pp 2136-2139 (2000).
18. Paspalakis, E. et al. "Transparency near a photonic band edge", *Phys. Rev. A.*, **60**, pp R33-R36 (1999).
19. Zhang, H.Z. et al. "Quantum interference in spontaneous emission of an atom embedded in a double-band photonic crystal", *Phys. Rev. A.*, **65**, 063802 (2002).

20. Zhang, H.Z. et al. “Spontaneous emission spectrum from a V-type three-level atom in a double-band photonic crystal”, *J. Opt. B: Quantum Semiclass. Opt.*, **4**, pp 300-307 (2002).
21. Zhang, H.Z. et al. “Spontaneous emission from a double V-type four-level atom in a double-band photonic crystal”, *J. Mod. Opt.*, **50**, pp 1649-1662 (2003).
22. Woldeyohannes, M. and John, S. “Coherent control of spontaneous emission near a photonic band edge”, *J. Opt. B: Quantum Semiclass. Opt.*, **5**, pp R43-R82 (2003).
23. Harris, S.E. et al. “Dispersive properties of electromagnetically induced transparency”, *Phys. Rev. A.*, **46**, pp R29-R32 (1992).
24. Scully, M.O. and Zubairy, M.S. “Quantum optics”, in *Quantum Coherence and Interference Effects*, Cambridge University Press, Cambridge (1997).
25. Harris, S.E. et al. “Nonlinear optical processes using electromagnetically induced transparency”, *Phys. Rev. Lett.*, **64**, pp 1107-1110 (1990).
26. Imamoglu, A. “Interference of radiatively broadened resonances”, *Phys. Rev. A.*, **40**, pp 2835-2838 (1989).