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Research Note

Contribution of the Solar Magnetic Field on Gravitational Moments

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Studies of the solar magnetic field is one of the key method of explaining important phenomena found in the Sun. In order to determine the contribution of the magnetic field on the solar outer shape, here, the dynamo model of the Babcock-Leighton type [1] is considered. This model explains that the surface eruptions of the toroidal magnetic field, such as the eruptions of the flux tubes, are the source of the poloidal field, whereas, generation of the toroidal field takes place in a thin, deep seated layer, called the Generating Layer (GL), at the bottom of the Solar Convection Zone (SCZ). To calculate the indicating quantity of the solar shape, i.e. the gravitational moments, J_n, several methods can be used: Stellar equations combined with a differential rotation model, inversion techniques applied to helioseismology and based on the Von Zeipel theorem, the theory of figures of the Sun [2]. In this paper, this last theory was used, but adding the magnetic field contribution. Different estimates were obtained for the successive J_n ($n = 2, 4, 6, 8$), in terms of different values of B_{cr} (the critical field), where the maximum value of the toroidal magnetic field in the GL is $1.5 \times B_{cr}$.

INTRODUCTION

The nature of sunspots and of the 22-year cycle has been a subject of speculation during the six decades since the discovery of magnetism in sunspots and of the polarity rules governing successive cycles [3]. An early idea held that sunspots are vortex tubes similar to terrestrial tornadoes, and that the magnetic field is somehow produced by this vortex motion. Now, it is obvious that the magnetic field is the primary quantity, and that most of the properties normally associated with sunspots result from the presence of strong magnetic fields in the Sun's outer layers. Ideas about the cause of the solar cycle have also undergone considerable changes. The periodicity has been imputed to the circulation of meridional mass currents in the Sun's outer convective layers, to the propagation of hydro magnetic waves through the Sun, or to torsional oscillations of the outer layers. In this paper, the authors have been interested to study the effect of such a magnetic field on the parameters explaining perturbation of the solar gravitational potential due to rotation. In three first sections, the relationship between solar outer shape and gravitational moments of the Sun considering the gravitational and rotational potentials will be explained. Then, separately, in the fourth section, components of the magnetic field inside the Sun will be presented in order to add its contribution on the gravitational moments and, finally, in the fifth section, calculations and results are presented.

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SOLAR OUTER SHAPE

If the Sun is described as a sphere, then, the gravitational and pressure gradient forces are in hydrodynamic equilibrium. But due to the non-homogeneous mass distribution and differential rotation inside the Sun, its outer shape turns out to be distorted in latitude. However, in order to determine the solar outer shape, one needs to define an apparent physical surface at a given wavelength. On the other hand, the Sun has an extended atmosphere and it is not so simple to consider the upper limit of its photosphere. Rozelot and Lefebvre [4] have defined the free surface of the Sun as a level, where a given physical parameter, like the temperature, the density and the pressure, etc. is constant. This free surface does not coincide with an ellipsoid of revolution: The true figure is far more complex and shows "asphericities" (measured by "shape coefficients"). To the first order, this shape lies between the spherical and ellipsoidal shape, which is a figure called "spheroid"; this could be recognized by the oblateness parameter, ε. Since determination of the above mentioned parameters are difficult, the most simple approach is to define the solar shape as an equipotential surface, with respect to the total potential (gravitational, rotational and magnetic). This last potential is concluded from the magnetic field inside the Sun, between solar radiation and convection zones, as explained in the following sections. Using the Chandrasekhar approach [5] for the slowly rotating stars, one can obtain, in principle, the gravitational moments of the Sun.

GRAVITATIONAL AND ROTATIONAL POTENTIALS

A mass distribution in space is considered to calculate the gravitational potential. The integration is taken at a point, (x, y, z), over a mass element, dm:

$$V(x,y,z) = G\int\int\int dm/l, \qquad (1)$$

where G is the gravitation constant and l=$((x \;\; x\prime)^2 + (y \;\; y\prime)^2 + (z \;\; z\prime)^2)^{\frac{1}{2}}$ (the "prime" designs target points). Following Rozelot and Lefebvre [4], the Laplace equation of the potential in the spherical coordinates gives the following solution:

$$V(r,\theta,\phi) = \sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\sum_{m=0}^{\infty}\Bigg[A_{nm}P_{nm}(cos(\theta)cos(m\phi) + B_{nm}P_{nm}(cos(\theta))sin(m\phi)\Bigg], \qquad (2)$$

where r is the vector radius and P_{nm} are the associated Legendre functions of degree n for a given order, m. For $n = 0$, the potential reduces to GM/r, so Equation 2 becomes:

$$V(r,\theta,\phi) = \quad GM/r\Bigg[1 + \sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\left(\frac{a}{r}\right)^n[P_{nm}(\cos(\theta))[C_{nm}\cos(m\phi) + S_{nm}\sin(m\phi)]\Bigg], \qquad (3)$$

where a is the equatorial radius of the body and C_{nm} and S_{nm} are related to the coefficients A_{nm} and B_{nm}. The rotational symmetry of the Sun implies that $C_{nm} = 0$ and $S_{nm} = 0$, if $m \neq 0$. Defining $C_{n0} = \quad J_n$ as the zonal harmonic coefficients of degree n of the potential, the expansion of Equation 3 reduces to:

$$V(r,\theta,\phi) = \quad GM/r\left[1 \quad \sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^n J_nP_n(\cos(\theta))\right], \qquad (4)$$

where J_n are the gravitational moments and $P_n(\cos\theta)$ are the Legendre polynomials of degree n. In ascending order, J_n measure the distortions that affect the shape of the body under rotation.

The centrifugal potential related to the above mentioned mass distribution with angular velocity, ω, is as follows:

$$V_{rot} = \quad \int_0^R R\omega^2 dR, \qquad R = r\sin(\theta). \qquad (5)$$

The magnetic field of the Sun and its potential will be explained in the following sections.

GRAVITATIONAL MOMENT

As mentioned above, if one considers the pressure equilibrium at the solar surface, the shape of the Sun will be determined as the shape of a surface where its gravitational potential is constant. Bearing in mind the Von Zeipel law, it turns out that the other parameters, such as density, temperature, pressure and the effective potential (the sum of all potentials), are also constant.

In order to obtain the gravitational moments, one needs to compute the total effective potential containing the gravitational, rotational and magnetic ones. To order $n = 2$, the gravity potential is as follows:

$$\Phi_1 = \quad GM/r\left[1 \quad \left(\frac{a}{r}\right)^2 J_2P_2(\cos(\theta))\right]. \qquad (6)$$

If the Sun is considered an ellipsoid of equatorial (R_{eq}) and polar (R_{pol}) radii, by using the ellipsoidal equations, the solar radius is written as follows:

$$r = R_{eq}\left(1 \quad \frac{1}{3}f \quad \frac{2}{3}fP_2\right), \qquad (7)$$

where $f = \frac{R_{eq} \;\; R_{pol}}{R_{sp}}$ is the flattening after some reductions and bearing in mind that $R_{sp} = (R_{eq}^2 R_{pol})^{\frac{1}{3}}$ (which is the radius of the best sphere passing through the equatorial and polar radii determined by $P_2 = 0$ in Equation 7). Thus, for $R_{sp} = R_{eq}(1 \quad 1/3f)$, one gets:

$$\frac{1}{r} = \frac{1}{R_{sp}}(1 + \frac{2}{3}fP_2) + O(f^2). \qquad (8)$$

where $O(f^2)$ are the higher orders of f. From these considerations, now, for the gravity potential, one has:

$$\Phi_1 = \quad GM/R_{sp}\left[1 + (\frac{2}{3}f \quad J_2)P_2)\right]. \qquad (9)$$

To the same order, the rotational potential is as follows:

$$\Phi_2 = -1/3\omega^2 R_{sp}^2(1 - P_2). \tag{10}$$

So,

$$\Phi_{tot} = \Phi_1 + \Phi_2 = -GM/R_{sp}\left[\left(1+\frac{1}{3}m\right) + \left(\frac{2}{3}f - J_2 - \frac{1}{3}m\right)P_2\right], \tag{11}$$

where $m = \frac{\omega^2 R_{sp}^3}{GM}$. If the solar ellipsoid is a surface limit, Φ_{tot} must be constant on it, i.e., the effective potential becomes independent of the θ (polar angle). Thus:

$$\Phi_{tot} = -GM/R_{sp}\left(1+\frac{1}{3}m\right) = \Phi_0 = \text{Const.}, \tag{12}$$

therefore, $\frac{2}{3}f - J_2 - \frac{1}{3}m) = 0$ and:

$$J_2 = \frac{2}{3}f - \frac{1}{3}m. \tag{13}$$

The values of m, ω and f being known, one can find to first order the gravitational moment, J_2. However, one can see that, in the solar case, $(2/3)f$ is of the order of $(1/3)m$ and, so J_2 is nearly equal to 0. In spite of the fact that this theory could be invalidate, as m is taken as a constant both in latitudes and in depth, it is straightforward to see that J_2 will remain of the order of 10^{-7}. All other theories lead to the same result. Nevertheless, an accurate value of J_2 is useful to compute dynamical effects, like light deflection in the vicinity of the Sun or in planetary ephemeris. Conversely, a precise dynamical estimate of J_2 might be crucial to constrain solar density and rotation models. Accordingly, the authors wished to see if the magnetic field could influence Equation 13.

SOLAR MAGNETIC FIELD

It is straightforward that the origin of solar surface magnetic fields is the occurrence of magnetic flux tubes inside the Sun. However, these flux tubes are produced in a thin layer at the bottom of the Solar Convection Zone (SCZ), which is called the Generating Layer (GL) [6]. The interior magnetic structure from the GL to the surface is relevant, because it is responsible for the appearance of solar magnetic features, such as sunspots and the faculae. In order to verify the contribution of the solar magnetic field on the gravitational moments, the same model used by Durney [7] has been applied. This dynamo model is of the Babcock-Leighton type [8,9]. If one considers the spherical symmetric coordinate system for the Sun and the magnetic field, $\vec{B} = (B_r, B_p, B_\Phi)$, the toroidal field, B_Φ, is generated in GL, by a shear in the angular velocity acting on the poloidal field, $B_p(= \nabla \times [0,0,A_\Phi])$. Axial symmetry is assumed and $\nabla\Omega$ in the GL is such that, within this layer, a transition to uniform rotation takes place. The choice of angular velocity Ω in the GL was based on this assumption that, within this layer, all thin shells with thickness dr have the same angular momentum, regardless of their radius, r. This assumption leads to an angular velocity that [1]:

- Increases inwards for $\theta < 63.4°$,
- Is constant with r for $\theta = 63.4°$,
- Decreases inwards for $\theta > 63.4°$,

in agreement with helioseismic observations [10]. Note that such an estimate for θ is obtained when sorting the observed diameters of the Sun by heliographic latitude bins [11]. If for GL and for a certain value of the polar angle, θ, $|B_\Phi|$ exceeds a critical field, B_{cr}, then, the eruption of a flux tube occurs [12]. This flux tube (assuming radial rise) generates, when reaching the surface, a Bipolar Magnetic Region (BMR) with fluxes Φ_p and Φ_f for the preceding and following spots, respectively. The ensemble of individual successive eruptions acts as the source term for the poloidal field (no mean field equation is used to approximate the source term). This field (with opposite polarity), generated in the surface layers, is mainly transported by the meridional motions (and by diffusion) to the GL, reversing the poloidal field. The shear in ω acts on a reversed poloidal field, then, generates the new toroidal field resulting in the oscillatory behavior of the large scale solar magnetic field.

The meridional motions, U, are the superpositions of a one-cell velocity field, which rises at the equator and sinks at the poles and of a two-cell circulation, which rises at the equator and poles and sinks at mid latitudes. The toroidal field and its potential, A_Φ, were expanded in terms of Legendre polynomials and the coupled partial differential equations (in time and radial coordinates), represented by the coefficients in these expansions, were solved by a finite difference method.

Now, the theory of the model is considered as mentioned above. The useful equation, which should be solved in the Dynamo model, is the induction equation:

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left\{\vec{U}\times\vec{B} - \eta\nabla\times\vec{B}\right\} + \nabla \times \left(0,0,\frac{\partial S}{\partial t}\right), \tag{14}$$

where η is the diffusivity; S the source term, due to the flux eruption and U the meridional motions. Therefore, one can obtain the following equations [7,12]:

$$\frac{\partial A_\Phi}{\partial t}=U_rB_\theta \quad B_rU_\theta+\eta\left[\nabla^2 \quad \frac{1}{(r\sin\theta)^2}\right]A_\Phi+\frac{\partial S}{\partial t}, \tag{15}$$

$$\frac{\partial B_\Phi}{\partial t}=\frac{\acute{\rho(r)}}{\rho(r)}U_rB_\Phi \quad r\sin\theta\vec{B}.\nabla\Omega$$

$$\vec{U}.\nabla\left(\frac{B_\Phi}{r\sin\theta}\right)+\eta\left[\nabla^2 \quad \frac{1}{(r\sin\theta)^2}\right]B_\Phi, \tag{16}$$

$$B_r=\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\Phi\sin\theta), \qquad B_\theta= \quad \frac{1}{r}\frac{\partial}{\partial r}(rA_\Phi), \tag{17}$$

where $\acute{\rho(r)}$ is originated from $\nabla\times(\vec{U}\times\vec{B})= \quad \vec{B}\nabla.\vec{U}+(\vec{B}.\nabla)\vec{U} \quad (\vec{U}.\nabla)\vec{B}$ and from the continuity equation, $\nabla.\vec{U}= \quad \frac{1}{\rho}\left(\frac{d\rho}{dr}\right)U_r$. Temporal integration of the above partial differential equations gives the following:

$$A_\Phi=a_\Phi\frac{\sin\theta}{r}, \qquad B_\Phi=b_\Phi\frac{\sin\theta\cos(\theta)}{r},$$

$$S=s\frac{\sin\theta}{r}. \tag{18}$$

CALCULATION

Toroidal Magnetic Field and its Contribution on J_n

In this section, the toroidal magnetic potential will be calculated, using the same model inferred above, i.e. the Babcock-Leighton type of the dynamo model. In the numerical calculations, the BMR (in the previous section) is replaced by its equivalent axisymmetrical magnetic ring doublet and the following initial conditions [1] are considered:

- A weak polar magnetic field,
- $|B_\phi|_{\max}=1.5\times B_{cr}$ (in the GL).

In Leighton's model, there exists a relationship between the time derivative of the radial component and the latitudinal derivative of the toroidal one, which means:

$$\frac{\partial\vec{B}_r}{\partial t}=\begin{cases}\frac{Ha}{2\pi R^2\tau}\frac{\partial(B_\Phi\sin\gamma)}{\partial\mu} & |B_\Phi|>B_{cr}\\ 0 & |B_\Phi|<B_{cr}\end{cases}, \tag{19}$$

where $H\ll R$ is the thickness of a thin shear layer in the outer part of the Sun; τ is a time constant, for which the amount of field erupted within time dt is proportional to $|B_\Phi|dt/\tau$; γ is the tilt angle formed by the magnetic axis of a bipolar magnetic region with the east-west line; a is a free parameter, which enters in the angular velocity equation: $\Omega=\Omega_s+(a+\beta\sin^n\theta)\frac{R \quad r}{H}$, where β and n are also free parameters; r is the variable radius and Ω_s is the observed angular velocity of the differential rotation at the solar surface, which is, approximately, $\Omega_s=18\sin^2\theta$ rad/yr with respect to the polar regions [13]. Hence and considering the above initial conditions, the radial or toroidal magnetic field can be written as follows:

$$B_r=\sum_l c_l(2l+1)^{1/2}P_l(\cos\theta), \tag{20}$$

where P_l are normalized to unity, when $\theta=0$ and c_l are the expansion coefficients given by:

$$c_l=1/2\int_0^\pi B_r(2l+1)^{1/2}P_l(\cos\theta)\sin\theta d\theta. \tag{21}$$

Using Equation 20, the derived magnetic potential is $\vec{A}=(0,0,A_\phi)$. Considering $\vec{B}=\nabla\times\vec{A}$ and $B_r=\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\phi)$, after integration of this relationship and by replacing the result in Equation 20, one gets:

$$A_\phi=\frac{r}{\sin\theta}\sum_l c_l(2l+1)^{1/2}\int P_l(\cos\theta)d(\cos\theta). \tag{22}$$

In order to apply this equation in calculating J_n, the expansion coefficients should be found as follows:

$$c_l=3/4B_{cr}(2l+1)^{1/2}\int_0^\pi P_l(\cos\theta)\sin\theta d\theta. \tag{23}$$

However, the interval $[0,\pi]$ related to the poles is the boundary condition for which, for $\theta=0$, or π, one has a zero value for vector potential A and magnetic field B (meaning that, at the poles, the mean field and the mean current density have finite values). Now, the boundary conditions are discussed. To avoid a null value of c_l in this interval, one may use the royal belt latitudes, where magnetic features occur and are confined during the activity cycle. The "butterfly" pattern shows a typical, $[\quad\pi/3,\pi/3]$, latitude interval. Checking by computation, subsequently, the validity of this shortening integration on the results, no significant departure from the final result was found. The plot on the left shows the latitude of sunspot occurrence versus time (in years). Sunspots are typically confined to an equatorial belt between altitude of -35 degrees south and +35 degrees north. At the beginning of a new solar cycle, sunspots tend to form at high latitudes, but as the cycle reaches a maximum (large numbers of sunspots), the spots form at lower latitudes. Near the minimum of the cycle, sunspots appear even closer to the equator and, as a new cycle starts again, sunspots again appear at high latitudes. This recurrent behavior of sunspots gives rise to the "butterfly" pattern shown and was first discovered by Edward Maunder in 1904. The reason for this sunspot migration pattern is unknown. Understanding this pattern would tell us something about how the Sun's internal magnetic field is generated. To investigate the contribution of A_ϕ in J_n, first, the $n = 2$ order of J_n may be considered. Let us denote by $\zeta = \sin\theta_{er}$ the colatitude, where the maximum magnetic field, $|B_\phi|$, reaches its critical value, B_{cr}. Putting Equation 8 into Equation 22, one gets:

$$A_\phi = \left(\frac{9\sqrt{3}}{16}B_{cr}/\zeta\right) R_{sp}\left(1 \quad \frac{2}{3}fP_2\right), \tag{24}$$

by $\quad \vec{\mu}.\vec{B}$, where μ is the magnetic moment and B the magnetic field. Numerical calculations will be performed in IS, so that μ is in Amp.m^2 and B in $Tesla$. According to Durney [7], a layer of thickness, $H = 1.9\times10^7$ m (at the bottom of the solar convection zone where the toroidal field is amplified), is considered. The mesh consists of 101×101 points in the r- and θ-directions. Therefore, $\Delta r = (R_0 \quad R_c)/100 = 1.9\times10^6$ m ($R_0 = 6.9\times10^8$ m is the upper boundary of the SCZ and $R_c = 5\times10^8$ m is the lower boundary, i.e. the GL) and $\Delta\theta = \pi/200$ rad. However, detailed computations are carried out for the Bipolar Magnetic Region (BMR) replaced by a magnetic ring doublet (say, the sunspot) at the surface with angular separation $\chi = 4\cos\theta_{er}\Delta\theta$ ($\theta_{er} = 45°$, considered the latitude for the eruption) and thickness $h = 0.2\times10^6$ m below the surface.

Bearing in mind the definition of the magnetic potential energy, $\Phi_{mag} = \Phi_3 = \quad \vec{\mu}\cdot\vec{B}$, where μ is the magnetic moment (per unit mass) and B the magnetic field and by considering a Bipolar Magnetic Region (BMR) on the surface of the Sun (as mentioned before and according to Durney [7]), it turns out that $\Phi_{mag} = \quad \mu A_\phi/R_{sp}$. Hence:

$$\Phi_3 = \quad \mu\left(\frac{9\sqrt{3}}{16}B_{cr}/\zeta\right)\left(1 \quad \frac{2}{3}fP_2\right), \tag{25}$$

is the magnetic field (that is the radial component of the magnetic field times the distance, which here is considered equal to R_{sp}). Now, using Equations 9, 10 and 25, Φ_{tot} is written as follows:

$$\Phi_{tot} = \quad GM/R_{sp}\left[\left(1+\frac{1}{3}m+\acute{m}\right) + \left(\frac{2}{3}f \quad J_2 \quad \frac{1}{3}m \quad \frac{2}{3}\acute{m}f\right)P_2\right], \tag{26}$$

where m demonstrates the magnetic potential contribution; $\acute{m} = \mu/M_{BMR}(\frac{9\sqrt{3}}{16})B_{cr}/\zeta(\frac{R_{sp}}{GM})$ (M_{BMR} is the mass contained in the fractional part of the region BMR and is directly obtained from the known density for h). A constant equipotential level is defined in the following:

$$\Phi_{tot} = \quad GM/R_{sp}\left[\left(1+\frac{1}{3}m+\acute{m}\right)\right] = \Phi_0 = \text{Const.}, \tag{27}$$

which implies that the coefficients of P_2 must vanish as follows:

$$J_2 = \frac{2}{3}f(1 \quad \acute{m}) \quad \frac{1}{3}m. \tag{28}$$

This (first order) determination of the gravitational moment in the presence of the magnetic field can be, thus, (for the first time) compared with Equation 13. Values of the parameters being known, J_2, can be deduced. In the next section, the upper orders of J_n for $n = 4$, 6 and 8, have been calculated.

RESULTS

The numerical calculations are performed in IS while m and $\acute{m}$ are dimensionless. The following values of the parameters were used to compute J_n:

$M_\odot = 1.9891\times10^{30}$ kg,

$G = 6.67259\times10 \quad ^{11}\text{m}^3\ \text{kg} \quad ^1\text{s} \quad ^2$,

$R_{sp} = 6.95989299\times10^8$ m,

$f = 1.06\times10 \quad ^5$.

$\mu_\odot = 5\times10^{29}$ Am^2 was taken for the total solar magnetic moment. Following the same procedure as in [7], a thin layer, under the surface of thickness $h = 2\times10^5$ m at a colatitude θ_{er} can be considered, where a Bipolar Magnetic Field (BMR) occurred. The magnetic moment per unit mass can be estimated as $\mu = 1.4\times10^{26}$ Am^2/Kg. The critical value of the magnetic field, B_{cr}, is 0.1 T (in Table 1 runs were also made with $B_{cr} = 0$ and 0.3 T). In the integration process of Equation 23, one can estimate the c_l for different values of the boundaries and, then, to extrapolate to the $[0,\pi]$ interval. As seen before, changing these boundaries affects the coefficients in Equation 22, but on physical grounds, the latitude interval of appearance and the storage of magnetic features is the most likely $[\quad \pi/3, \pi/3]$. $[\quad \pi/2, \pi/2]$. Then, one can check the influence of different θ_{er} ($\pi/6, \pi/4$ and $\pi/3$) in computing the magnetic potential. Results show that the order of magnitude of the J_n ($10 \quad ^7$) is unaffected as the estimates ranges from 2.55 to 2.74 only. Therefore, $\pi/4$ is kept in the computations.

Results are listed in Table 1. One can see that for the critical value of magnetic field, $B_{cr} = 10 \quad ^1$ T, J_2 is equal to $\quad 2.613.10 \quad ^7$, instead of J_2 being equal

Table 1. Calculated values of J_n for $n = 2$, 4, 6 and 8 and taking into account the magnetic field.

$\boldsymbol{B_{cr} = 0}$	$J_2 = \;\; 2.300 \times 10^{\;\;7}$, $J_4 = 6.2924 \times 10^{\;\;7}$, $J_6 = \;\; 1.416 \times 10^{\;\;8}$, $J_8 = 5.049 \times 10^{\;\;13}$
$\boldsymbol{B_{cr} = 0.1}$ **Tesla**	$J_2 = \;\; 2.613 \times 10^{\;\;7}$, $J_4 = 6.2922 \times 10^{\;\;7}$, $J_6 = \;\; 1.4164 \times 10^{\;\;8}$, $J_8 = 5.049 \times 10^{\;\;13}$
$\boldsymbol{B_{cr} = 0.3}$ **Tesla**	$J_2 = \;\; 3.24 \times 10^{\;\;7}$, $J_4 = 6.2911 \times 10^{\;\;7}$, $J_6 = \;\; 1.4164 \times 10^{\;\;8}$, $J_8 = 5.049 \times 10^{\;\;13}$

to $\;\; 2.300.10^{\;\;7}$ without the magnetic field ($\acute{m} = 0$). If both of them have the same order of magnitude, the presence of the magnetic field increases the absolute value of 13%.

CONCLUSION

In this study, the contribution of the solar magnetic field to solar gravitational moments has been estimated, using the dynamo model of the Babcock-Leighton type. The gravitational moments (J_n) sorted in ascending order ($n = 2, 4, 6$ and 8) show a measure of the successive deviations that affect the outer shape of the rotating Sun, i.e., determine the departure of the solar material contents from a spherical distribution. Comparing the J_n's results for different B_{cr} and by introducing the concept of "magnetic potential" in the total effective potential ($\Phi_{tot} = \Phi_{\text{gravity}} + \Phi_{\text{rotation}}$) it has been shown that the magnetic field increases J_2 and slightly decreases J_4 and the other gravitational moments remain constant. It is worthy to note that J_2 remains of the same order of magnitude as J_4. The magnetic phenomena, which occur at the surface of the Sun, such as sunspots and faculae, are the relevant candidates for affecting the solar shape. These features, indeed, originate from far inside the Sun, where the flux tubes are formed in the lower layers of the solar convection zone and may explain the phenomena.

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