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Behaviour of Plate Girders Subjected to Combined Bending and Shear Loading

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Theoretical predictions regarding the ultimate resistance of slender plate girders to applied shear loading, based on existing theories and formulas, show close correlation with the test data presented. When a plate girder is subjected to a bending moment in addition to shear, the determination of the ultimate resistance becomes more complex. Herein, an interaction formula for the ultimate resistance of slender plate girders to combined bending and shear loading is proposed, which shows satisfactory correlation with the available theories and which is acceptable for practical purposes. The proposed interaction equation covers web panel aspect ratios, b_w/d_w, from 1 to 2 and slender ratios, d_w/t_w, from 150 to 300.

INTRODUCTION

Slender steel plate girders are used in a variety of structural engineering applications, owing to their high strength-to-weight ratios and post-buckling reserves of strength. The web panels of plate girders may be subjected to shearing forces, bending moments and a combination of such loading. For example, end panels are subjected to shear loading and central panels are subjected to bending, while intermediate panels are subjected to combined bending and shear loading.

During the past four decades, numerous tests have been performed on slender plate girders to provide a better understanding of the modes of failure and the influence of geometric and material parameters on their ultimate resistance. Previous experimental and theoretical studies of slender web panels subjected to pure shear loading have been reviewed by Porter et al. [1], Evans [2] and Lee and Yoo [3]. Numerous tests have indicated that failure occurs in a typical shear sway mode, which is characterized by the formation of large inclined plastic shear buckles in the web and plastic hinges in the flanges. Rockey et al. [4] and Evans [2] have studied bending and the interaction between bending and shear loading. They proposed a procedure for quantifying the interaction between bending and shear, in terms of the moment at first yield, and the yield shear strength of the web. It seems that the proposed procedure by Rockey et al. and Evans is more complex and is not suitable for practical purposes. Herein, an interaction formula for the ultimate resistance of slender plate girders to combined shear loading and bending is proposed, which shows satisfactory correlation with the available theories and which is acceptable for practical purposes.

ULTIMATE SHEAR RESISTANCE

Theoretical predictions of the ultimate shear resistance of the test girders, V_u, were made, in accordance with the tension field theory developed by Porter et al. [1] and Evans [2]. For a web panel having width of b_w, depth of d_w, thickness of t_w and similar top and bottom flanges (Figure 1), the ultimate shear resistance, V_u, is given by:

$$V_u = \tau_{cr} d_w t_w + \sigma_t^y \sin^2\theta (d_w \cot\theta \quad b_w) t_w + 4 d_w t_w \sin_\theta \sqrt{(\sigma_{ow} M_p^* \sigma_t^y)}, \qquad (1)$$

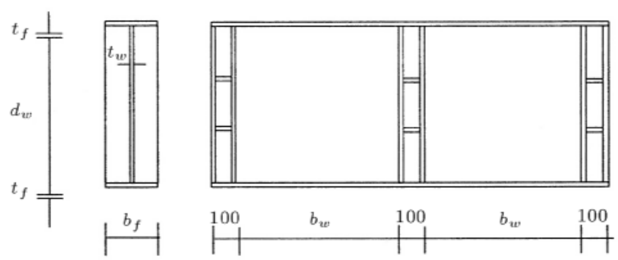

Figure 1. Details of test girders.

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Table 1. Details of test plate girders.

Girder	b_w mm	d_w mm	t_w mm	b_f mm	t_f mm	σ_{ow} N/mm^2	σ_{of} N/mm^2
PG1	600	600	4.1	200	12.5	343	257
PG2	900	900	3.1	300	10.2	285	254
PG3	900	600	3.2	200	10.1	282	264
PG4	1000	500	1.9	200	9.9	250	293

where τ_{cr} is the critical shear stress of an assumed, simply supported web plate given by:

$$\tau_{cr} = K\frac{\pi^2 E}{12(1 \quad \nu^2)}\left(\frac{t_w}{d_w}\right)^2, \tag{2}$$

where K is the buckling coefficient given by:

$$K = 5.34 + 4\left(\frac{d_w}{b_w}\right)^2 \quad \text{when} \quad \frac{b_w}{d_w} \geq 1, \tag{3a}$$

$$K = 5.35\left(\frac{d_w}{b_w}\right)^2 + 4 \quad \text{when} \quad \frac{b_w}{d_w} \leq 1. \tag{3b}$$

σ_t^y is the web tension field membrane stress defined by the following equation:

$$\frac{\sigma_t^y}{\sigma_{ow}} = \sqrt{\left(1 \quad \left(\frac{\tau_{cr}}{\tau_{ow}}\right)^2\left(1 \quad \frac{3}{4}\sin^2 2\theta\right)\right)}$$
$$\frac{\sqrt{3}}{2}\frac{\tau_{cr}}{\tau_{ow}}\sin 2\theta, \tag{4}$$

where τ_{ow} is the shear yield stress of the web given by:

$$\tau_{ow} = \frac{\sigma_{ow}}{\sqrt{3}}. \tag{5}$$

θ is the inclination of the web tension field, assumed, approximately, to be two thirds of the inclination of the web panel diagonal, i.e.:

$$\theta = \frac{2}{3}\tan^{\quad 1}\left(\frac{d_w}{b_w}\right), \tag{6}$$

where σ_{ow} is the yield stress of the web and M_p^* is a non-dimensional flange strength parameter defined as follows:

$$M_p^* = \frac{M_{pf}}{d_w^2 t_w \sigma_{ow}}. \tag{7}$$

M_{pf} is the fully plastic moment of the flange, which for a rectangular flange having width of b_f, thickness of t_f and yield stress of σ_{of} is given by:

$$M_{pf} = 0.25 b_f t_f^2 \sigma_{of}. \tag{8}$$

A series of tests has been conducted on short span, welded plate girders, illustrated in Figure 1 [5]. The dimensions of the test girders, denoted PG1 to PG4, and material yield strengths are presented in Table 1. All the girders were simply supported and loaded in an Avery hydraulic testing machine under deflection control.

Theoretical ultimate resistances of the test girders to shear loading, V_u, which are determined in accordance with Equation 1, are compared with the maximum experimental results, V_{ex}, shown in Table 2. As can be seen, there is close correlation between V_u and V_{ex}, so any one of them can be used in the next calculations.

ULTIMATE BENDING RESISTANCE

Based on an experimental study, Cooper [4] proposed the following empirical equation for determining the ultimate bending resistance of slender plate girder web panels, M_u:

$$\frac{M_u}{M_y} = 1 \quad 0.0005\frac{A_w}{A_f}\left[\frac{d_w}{t_w} \quad 5.7\sqrt{\frac{E}{\sigma_{of}}}\right], \tag{9}$$

where M_y is the moment at first yield of the extreme fibres of the compression flange, A_w and A_f are the cross-section areas of the web and compression flange, respectively, and E is Young's modulus.

COMBINED BENDING AND SHEAR LOADING

When a plate girder is subjected to a bending moment in addition to shear, the determination of the ultimate resistance becomes more complex. Rockey et al. [4] and Evans [2] have studied the interaction between

Table 2. Comparison of test results with theoretical predictions.

Test Girder	V_{ex} kN	V_u kN
PG1	373	386
PG2	271	268
PG3	202	182
PG4	87	70

bending and shear loading. The interaction between these two forms of loading was presented by the type of diagram shown in Figure 2. The portion of the curve between points S and C represents the region within which the girder will fail by the development of a shear mechanism. The vertical ordinate of point S represents the pure shear resistance given by Equation 1. This shear resistance is reduced gradually by the presence of an increasing bending moment.

Beyond point C, where the applied bending moment is high, failure occurs in the flanges, either by the yielding of the flange or by the inward or lateral buckling of the compression flange. The following empirical relationship has been developed for locating point C [1].

$$\frac{V_c}{V_{yw}} = \frac{\tau_{cr}}{\tau_{ow}} + \left(\frac{\sigma_t^y}{\sigma_{ow}}\right) \sin\left(\frac{4\theta_d}{3}\right)$$

$$\left[0.554 + \frac{36.8 M_{pf}}{M_F}\right]\left[2 \quad \left(\frac{b_w}{d_w}\right)^{1/8}\right], \qquad (10)$$

$$V_{yw} = \tau_{ow} d_w t_w, \qquad (11)$$

where θ_d is the inclination of the web panel diagonal and M_F is the plastic moment resistance of the flanges about the neutral axis of the girder, given by:

$$M_F = b_f t_f \sigma_{of}(d_w + t_f). \qquad (12)$$

Equation 10 defines the vertical ordinate of point C and, in conjunction with the horizontal coordinate value, M_F, enables the position of point C to be located.

If a plate girder is subjected to a bending moment in excess of M_F, it will fail in a bending mode. If sufficient lateral support is provided to ensure that lateral buckling does not occur, a thin walled plate girder will fail by an inward collapse of the compression flange. This will occur when the applied bending moment, M_u, is close to that moment, M_y, which produces a yielding of the extreme fibres of the compression flange.

Equation 9 defines the position of point D on the horizontal axis of the interaction diagram. M_y is the moment required to produce yield in the extreme fibres of the flange, while the corresponding stresses in the web are below yield. Consequently, the web can resist a certain amount of coexistent shear loading. This shear is defined by the ordinate of point B, lying vertically above point D. The ordinate, V_B, is given by [1]:

$$\frac{V_B}{V_C} = \sqrt{\left(\frac{M_p \quad M_u}{M_{pw}}\right)}, \qquad (13)$$

where M_p is the fully plastic moment resistance of the complete cross-section and M_{pw} is the fully plastic moment of the web given by:

$$M_{pw} = 0.25 t_w d_w^2 \sigma_{ow}. \qquad (14)$$

Equation 13 defines the position of the final point on the interaction diagram. The portion, CB, of the curve can be assumed to be either parabolic or linear. In this way, the complete diagram can be drawn.

PROPOSED INTERACTION EQUATION

A summary of the calculations required to draw the interaction curves for test girders PG1 to PG4 is represented in Table 3 and the completed diagrams are shown in Figures 3 to 6.

The method proposed by Rockey et al. [4] and Evans [2] for quantifying the interaction between bending and shear involves sufficient complexity to

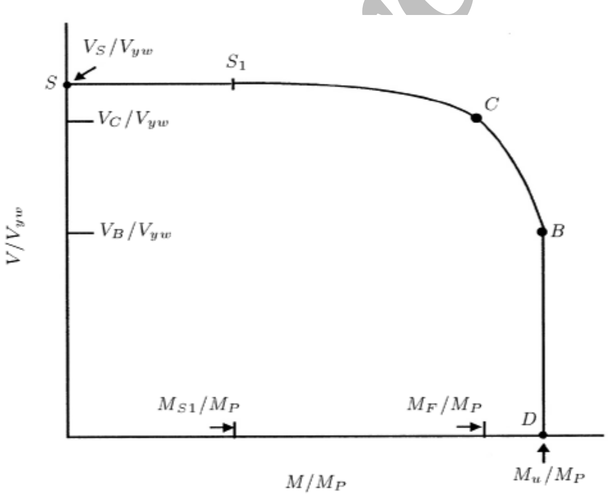

Figure 2. Interaction diagram for shear and bending of plate girders.

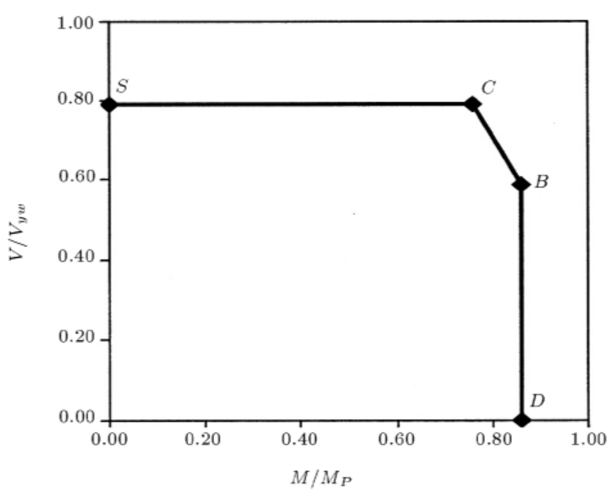

Figure 3. Interaction diagram between shear and bending for girder PG1.

Table 3. Calculations to draw interaction curve between shear and bending.

Girder	V_{yw} kN	$\frac{V_S}{V_{yw}}$	$\frac{V_C}{V_{yw}}$	$\frac{V_B}{V_{yw}}$	M_P kN.m	$\frac{M_F}{M_P}$	$\frac{M_u}{M_P}$
PG1	487	0.79	0.79	0.59	5200	0.76	0.86
PG2	460	0.58	0.58	0.42	8863	0.80	0.85
PG3	313	0.58	0.58	0.45	4065	0.80	0.88
PG4	137	0.50	0.50	0.42	3255	0.91	0.93

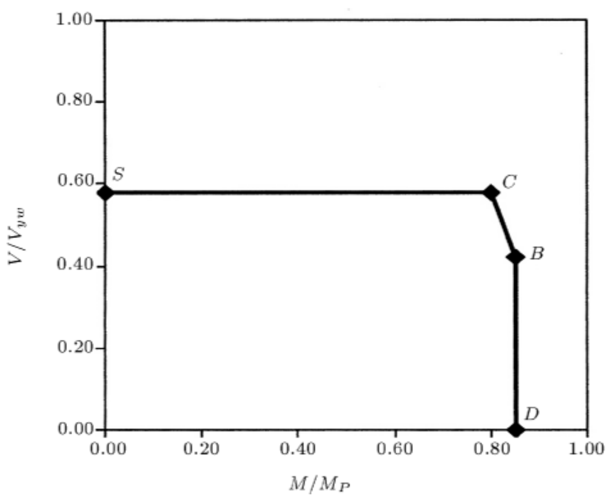

Figure 4. Interaction diagram between shear and bending for girder PG2.

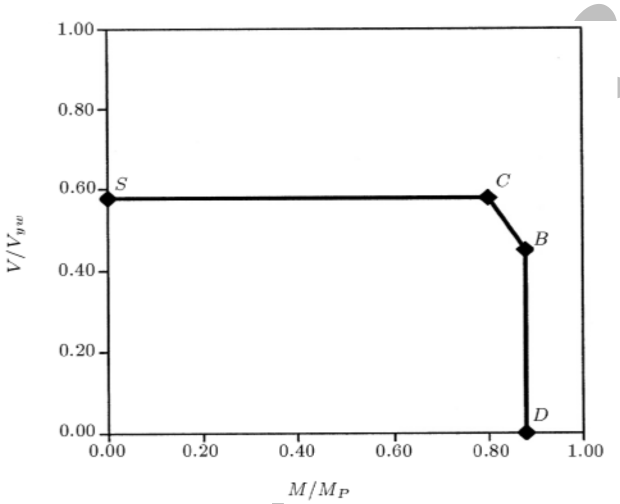

Figure 5. Interaction diagram between shear and bending for girder PG3.

discourage use of the method. Much of the complexity is associated with the need to determine the ordinates of points to plot the interaction diagram.

To deduce a formula for the interaction between shear loading and bending, the results presented in Table 3 are normalised, with respect to V_u (or V_{ex}) and M_u, instead of V_{yw} and M_p and represented in Table 4 and Figure 7. As can be seen, a conservative interaction formula for combined shear and bending is,

Table 4. Calculations to draw proposed interaction curve between shear and bending.

Girder	$\frac{V_C}{V_u}$	$\frac{M_F}{M_u}$
PG1	1.00	0.88
PG2	1.00	0.93
PG3	1.00	0.91
PG4	0.96	0.97

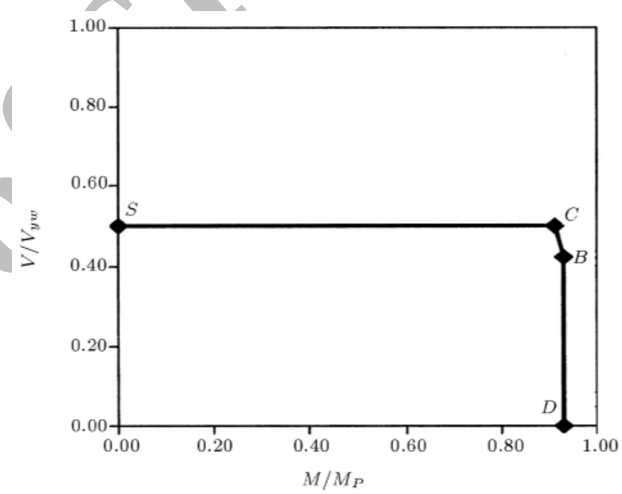

Figure 6. Interaction diagram between shear and bending for girder PG4.

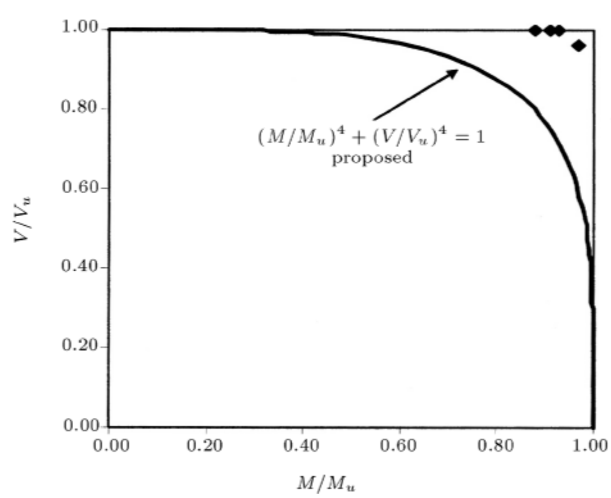

Figure 7. Proposed interaction diagram between bending and shear.

as follows:

$$\left(\frac{V}{V_u}\right)^{4.0} + \left(\frac{M}{M_u}\right)^{4.0} = 1, \quad (15)$$

where V is the shear resistance of the plate girder in the presence of an applied bending moment, $M.V_u$ and M_u are defined by Equations 1 and 9, respectively.

DISCUSSION AND CONCLUSIONS

A series of tests have been conducted on short span, welded plate girders to investigate their ultimate resistance to shear loading. Failure of the test girders occurred in a typical shear failure mode, characterised by the formation of large inclined plastic shear buckles in the web and plastic hinges in the flanges. Theoretical predictions of the ultimate resistance of slender plate girders to applied shear loading, based on existing theories and formulas, show close correlation with the test data presented herein, so, any of them may be used in the calculations.

If a plate girder is subjected to a bending moment, it fails in a bending mode. If sufficient lateral support is provided to ensure that lateral buckling does not occur, a thin walled plate girder fails by inward collapse of the compression flange. This will occur when the applied bending moment is close to the moment that produces yielding of the extreme fibres of the compression flange.

When a plate girder is subjected to a bending moment in addition to shear, determination of the ultimate resistance becomes more complex. In this paper, an interaction formula for the ultimate resistance of slender plate girders to combined loading is proposed, which shows satisfactory correlation with the available theories data and which is suitable for practical purposes. The proposed interaction equation covers web panel aspect ratios, b_w/d_w, from 1 to 2 and slender ratios, d_w/t_w, from 150 to 300.

NOMENCLATURE

A_f	flange cross section area
A_w	web cross section area
b_f	width of flange
b_w	clear width of web between stiffeners
d_w	depth of web
E	Young's modulus
K	buckling coefficient
M	moment resistance in the presence of shear
M_F	plastic moment resistance provided by the flanges
M_p	fully plastic moment resistance of the complete cross-section
M_{pf}	plastic moment resistance of the flanges
M_{pw}	plastic moment of the web
M_u	ultimate bending resistance
M_y	moment, which produces yielding of the extreme fibres of the compression flange
t_f	thickness of flange
t_w	thickness of web
V	shear resistance in the presence of bending
V_{ex}	experimental ultimate shear resistance
V_u	ultimate shear resistance
V_{yw}	shear yield resistance of the web
θ	inclination of web tension field
θ_d	inclination of web panel diagonal
σ_{of}	yield stress of flange
σ_{ow}	yield stress of web
σ_t^y	web tension field membrane stress
τ_{cr}	critical shear stress of web
τ_{ow}	shear yield stress of web

REFERENCES

1. Porter, D.M., Rockey, K.C. and Evans, H.R. "The collapse behaviour of plate girders loaded in shear", *The Structural Engineer*, **53**(8), pp 313-325 (1975).
2. Evans, H.R. "Longitudinally and transversely reinforced plate girders", *Plated Structures, Stability and Strength*, R. Narayanan, Ed., Elsevier, Applied Science Publishers, London, UK (1983).
3. Lee, S.C. and Yoo, C.H. "Strength of plate girder web panels under pure shear", *Journal of the Structural Engineering Division of the ASCE*, **124**(2), pp 184-194 (1998).
4. Rockey, K.C., Evans, H.R. and Porter, D.M. "A design method for predicting the collapse behaviour of plate girders", *Proc. Instn. Civ. Engrs., Part 2*, **65**, pp 85-112 (1978).
5. Shahabian, F. and Roberts, T.M. "Combined shear and patch loading of plate girders", *Journal of Structural Engineering, ASCE*, **126**(3), pp 316- 321(2000).