

# Analytical Approach for Flow over an Oblique Weir

A.R. Kabiri-Samani<sup>1</sup>

**Abstract.** *Flow discharge over an oblique weir is greater than that over a straight or plain weir for the same water head due to its extra length with respect to the channel width or fully extended plain weir. In this study, a new theoretical approach is used for the hydraulics of oblique weirs. The main objective is to investigate the effect of different hydraulic and geometric properties of the flow and the weir on the flow deflection angle and discharge coefficients for free and submerged flow over oblique weirs. This approach is based on energy, momentum and continuity equations. For improving the performance of this kind of weir, one approach is to increase the flow deflection until it is perpendicular to the oblique weir for maximum use of the weir length. The submerged guide vanes have also been used and investigated theoretically. The data for calibration of the models are taken from Borghei et al. (2003). It is shown that by employing guide vanes, for some cases, the discharge coefficient can be increased up to 33%. Finally, new relations were developed for practical purposes.*

**Keywords:** *Oblique weir; Analytical model; Flow deflection angle; Discharge coefficient; Guide vane.*

## INTRODUCTION

Sharp crested weirs have been used extensively for various purposes, such as flow measurement, flow diversion and discharge control, in hydraulics, the environment and in irrigation and chemical engineering projects. As an example, Vaseli and Monadjemi [1] introduced a model in which storm-water runoff is captured and stored behind a small dam with an overflow weir to waste excess flows. They have presented a relationship which defines the reclaimable water as a function of weir properties. Due to the application, weirs would be situated normal (plain or straight weir) [2,3] or lateral (side weir) [4] to the flow direction in the channel.

For a plain weir placed normal to the flow direction, the general stage-discharge ( $H - Q$ ) formula is:

$$Q = \frac{2}{3} C_d \sqrt{2g} L H^{1.5}, \quad (1)$$

where  $C_d$  is the discharge coefficient to be found

experimentally,  $L$  is the weir length and  $g$  is the gravitational acceleration [2].

There is extensive knowledge on normal sharp crested weirs. Kandaswamy and Rouse [3] studied simple rectangular weirs experimentally, and Swamee [5] analyzed their experimental results and presented a relation for the discharge coefficient due to the weir height and flow head. Kindsvater and Carter [6] studied the effect of viscosity and surface tension on the equations of flow motion. The effect of weir contraction and upstream head is investigated by [5,7]. Others have studied different aspects of flow over weirs such as air concentration in the water flow above the weir [8].

Also, depending on the downstream depth, weirs can be used as submerged or free as shown in Figure 1. Weir submergence occurs when the tail-water rises over the weir crest causing an increase in the upstream approaching head for a given discharge relative to a free-discharge condition. The advantage of a submerged condition would be a smaller energy loss in the system, but the disadvantage is a higher head at both sides of the weir. Also, for a free condition, only an upstream head would be needed for calculating the discharge while in the case of a submerged weir, upstream and downstream water heads would be needed [9].

1. Department of Civil Engineering, Isfahan University of Technology, Isfahan, P.O. Box 84156, Iran. E-mail: akabiri@cc.iut.ac.ir.com

Received 2 June 2009; received in revised form 8 January 2010; accepted 8 February 2010

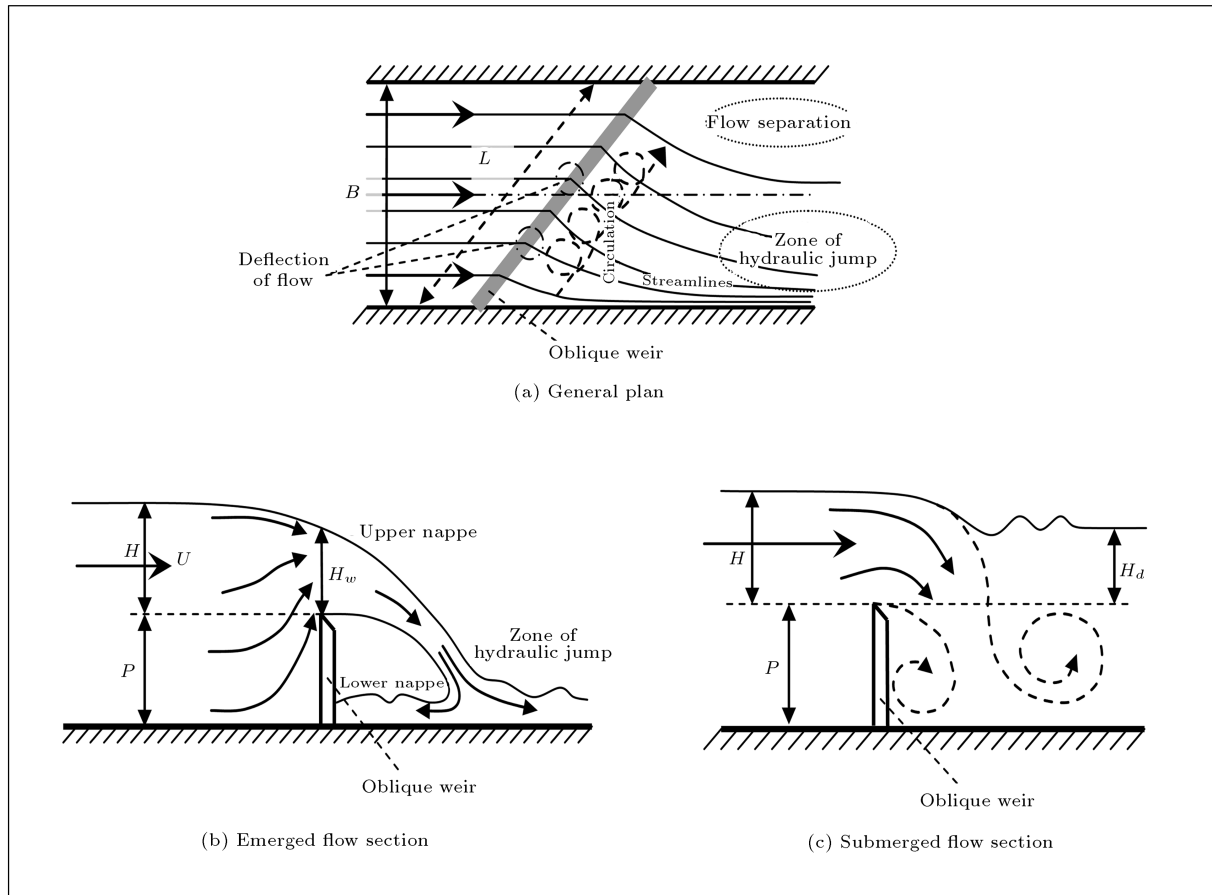


Figure 1. Different aspects of flow over oblique weirs.

For a submerged plain weir, the general relation for discharge due to submergence ( $Q_s$ ), relative to the discharge due to free flow ( $Q$ ) is given as  $Q_s = K_s Q$ . Wu and Rajaratnam [9] and Brater and King [3] introduced the coefficient  $K_s$  for a normal rectangular weir with free flow.

An efficient way to increase the discharge with a limited upstream water head, with a constant channel width and less energy loss, would be an oblique weir (Figure 1). For restricted channel width, the weir is placed obliquely to the flow and, hence, the effective weir length is increased beyond the channel width, which decreases the water head for a certain discharge flowing in the channel or increases the flow discharge for a given water head. This yields to an increase in the efficiency of the weir [10]. Although, due to the immense use of plain weirs, the hydraulic behavior of this kind of weir has been studied for a long time, few studies have been done on weirs placed obliquely to the flow.

Aichel [11] presented a new relation for the discharge coefficient of a round-crested skew weir compared with that of a normal weir. Aichel introduced an equation to relate the discharge for plain and oblique weirs with different angles as:

$$\frac{q}{q_n} = 1 - \frac{H}{P} \delta, \quad (2)$$

where  $q$  is the discharge per unit width of the oblique weir,  $q_n$  is the discharge per unit width of a plain weir and  $\delta$  is a coefficient depending on the oblique angle. Another approach to the study of flow over oblique weirs was used by [12] to examine the influence of the obliqueness of the weir on the flow. Ganapthy et al. [13] established design graphs for the discharge coefficient and the head for different skew angles. Borghei et al. [10] used experimental results to show the relation between discharge coefficient ( $C_d$  in Equation 2) and  $H/P$  for oblique weirs as:

$$C_d = \left( 0.701 - 0.121 \frac{B}{L} \right) + \left( 2.229 \frac{B}{L} - 1.663 \right) \frac{H}{P}, \quad (3)$$

where  $B$  is the channel width, which is less than the oblique weir length ( $L$ ). They showed that as the length of the weir relative to the channel width (or oblique angle) increases, the discharge coefficient decreases for the same  $H/P$ .

Also, for a submerged oblique weir, Borghei et al. [10] presented the result for the coefficient to the

discharge for the submerged weir as:

$$K_s = \left[ \left( 0.008 \frac{L}{B} + 0.985 \right) + \left( 0.161 \frac{L}{B} - 0.479 \right) \left( \frac{H_d}{H} \right)^3 \right]^2, \quad (4)$$

where  $K_s$  is the discharge coefficient. Prakash and Shivapur [14] investigated inclined weirs having an inclination with respect to the vertical plane, experimentally. They developed an expression for discharge as a function of the angle of inclination using a new approach. Noori and Chilmeran [15] studied the characteristics of free flow over normal and oblique weirs with semicircular crests, experimentally. They have constructed and tested 48 weirs. They showed that, for normal weirs, the discharge coefficient ( $C_{dw}$ ) increases with the increase of  $H/P$ . In the case of oblique weirs, the value of  $C_{dw}$  decreases with the increase of  $H/P$  values. Normal weirs of semicircular crests perform better than those of sharp crested weirs for all values of weir height and crest radius tested in this study. Weirs of small oblique angles give a high discharge magnification factor and high performance.

Tuyen [16] investigated flow over oblique weirs experimentally in a shallow flume under various flow conditions. Three different types of impermeable weir including a sharp-crested weir, a rectangular broad-crested weir (both placed 45 degrees obliquely to the flow direction) and a dike-form weir with both upstream and downstream slopes of 1:4 were investigated. He compared the discharge coefficient and its relations to other flow and geometry parameters obtained from this research with the available knowledge on oblique weirs [10,12,17]. Borghei et al. [18] employed an Incomplete Self-Similarity (ISS) concept to develop the equations from existing experimental results of the flow over an oblique rectangular sharp-crested weir for both free and submerged flow. They obtained a more accurate stage-discharge relationship based on application of the dimensional analysis and the ISS methodology than that of [10]. Tuyen [19] has performed a laboratory investigation on the flow over oblique weirs including the behavior and hydraulic characteristic of the flow. He concluded that by increasing the oblique angle of the weir, the discharge coefficient will slightly decrease, while the discharge capacity of the weir will increase.

The main objective of this study is an analytical investigation of flow behavior over oblique weirs due to different oblique angles under emerged and submerged conditions. Also, to improve the flow characteristics, the effect of employing submerged guide vanes, placed at the upstream face of the oblique weir, will be presented based on an analytical model.

## ANALYTICAL MODELS

Because of the complex nature of the flow over oblique weirs, i.e. being 3-D, it would be very difficult to obtain a clear expression for the flow as a function of all relevant parameters. Hence, simplified models combined with experimental results can be used as good tools for prediction.

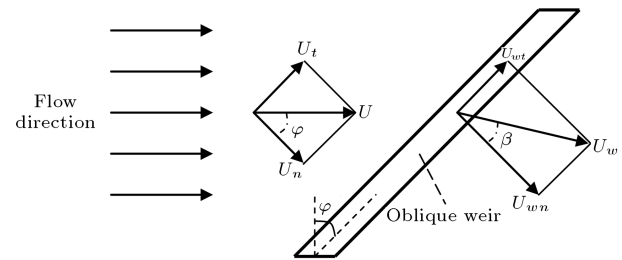
Present analytical models of flow are based on energy conservation for the upstream and downstream of the weir, momentum conservation and continuity equations. These models are based on the following assumptions:

- The pressure upstream and over the weir is hydrostatic,
- Water flows at a certain initial velocity ( $U$ ) upstream of the weir,
- All the water in the channel is flowing over the weir,
- The water surface is horizontal,
- The weir is placed vertical to the channel bottom.

### Deflection Angle of Emerged Flow over Oblique Weir

The flow direction passing over the oblique weir is assumed to have an arbitrary angle ( $\beta$ ) between the approach and the normal direction to the weir (Figure 2). By increasing the oblique angle, the effective length of the weir increases significantly, whereas the discharge coefficient,  $C_d$ , slightly decreases with the actual length of the weir [10]. The combination of the two effects increases the discharge capacity of the oblique weir.

The components of the approaching upstream velocity ( $U$ ), parallel and perpendicular to the weir, are  $U_t$  and  $U_n$  (Figure 2), while the components of the velocity over the weir ( $U_w$ ), parallel and perpendicular to the weir, are  $U_{wt}$  and  $U_{wn}$ , respectively. For a plain weir, the flow streamlines are perpendicular to the weir crest, thus,  $\beta = 0$  and, then,  $U_{wt} = 0$ . However, for an oblique weir ( $\varphi > 0^\circ$ ), the value of  $\beta$  can be predicted using the energy and continuity equations. Taking the weir crest as the reference level, the ideal flow energy



**Figure 2.** Flow velocity vectors in upstream channel and in the vicinity of oblique weir.

for the weir reads:

$$H + \frac{U^2}{2g} = H_w + \frac{U_w^2}{2g}, \quad (5)$$

where  $H$  and  $H_w$  are the upstream and over weir flow depths, respectively,  $U$  and  $U_w$  are the upstream and over weir flow velocities, respectively. Also, since the continuity equation always holds true for the flow, thus:

$$U_n(H + P)L = U_{wn}H_wL, \quad (6)$$

where  $n$  denotes the normal component of velocity,  $P$  is the weir height and  $L$  is the length of the oblique weir (or  $L = B/\cos\varphi$ ). According to Figure 2, the two main velocities ( $U$  and  $U_w$ ) are:

$$U^2 = U_n^2 + U_t^2, \quad U_w^2 = U_{wn}^2 + U_{wt}^2, \quad (7)$$

where  $t$  denotes the parallel component of the velocity. Experimental study showed that the variation of the velocity component perpendicular to the weir contributes greatly to the variation of the total velocity, while the velocity components parallel to the weir play a lesser role [20]. Also, the parallel component of velocity changes very slightly, since the acceleration force caused by gravitation only acts on the velocity component perpendicular to the weir crest. As the velocity component parallel to the weir crest is constant everywhere, before and over the weir crest, i.e.  $U_t = U_{wt}$ , together with the continuity equation in Equation 5 would be:

$$H + \frac{U_n^2}{2g} = H_w + \frac{U_{wn}^2}{2g}, \quad (8)$$

then:

$$\frac{1}{2g}U_n^2 \left( \frac{(H + P)^2}{H_w^2} - 1 \right) + (H_w - H) = 0. \quad (9)$$

For a given  $H_w$ , the value of  $U_{wn}$  can be calculated using Equation 6, and  $U_n$  can be found from Equation 9. Also, it can be seen that:

$$U = \frac{U_n}{\cos\varphi}, \quad H = \frac{Q}{BU} - P. \quad (10)$$

Then,  $U_w$  can be calculated from Equation 5. Finally, knowing the values of  $U_{wn}$  (Equation 6) and  $U_w$  (Equation 5), the oblique angle of the flow streamlines above the weir is found as:

$$\beta = \cos^{-1} \left( \frac{U_{wn}}{U_w} \right). \quad (11)$$

By repeating the above stated process for a certain range of  $H_w$ , the dependency of the flow deflection angle ( $\beta$ ) on the upstream Froude number ( $Fr$ ) and upstream water head to weir height ratio ( $H/P$ ) can be found for any flow discharge ( $Q$ ) and weir oblique angle ( $\varphi$ ).

### Discharge Coefficient for Emerged Flow over Oblique Weir

The main forces governing the flow over a weir are gravity and inertia; usually, other effects like viscous and surface tension are of minor importance as with open channel flow situations. For a steady and ideal flow, the Bernoulli equation, for the two sections of upstream and on the weir taking the weir crest as datum would be written as Equation 9. Also, the continuity equation for the ideal discharge ( $Q_i$ ) can be written as Equation 6. Substituting and combining Equations 6 and 9 would give:

$$H + \frac{Q_i^2}{2g(H + P)^2L^2} = H_w + \frac{Q_i^2}{2gH_w^2L^2}, \quad (12)$$

or:

$$\frac{Q_i^2}{2gL^2} \left( \frac{1}{H_w^2} - \frac{1}{(H + P)^2} \right) = H - H_w, \quad (13)$$

then:

$$\frac{Q_i^2}{2gL^2} = \frac{(H - H_w)H_w^2(H + P)^2}{(H + P)^2 - H_w^2}. \quad (14)$$

An experimentally found discharge coefficient,  $C_d$ , is then introduced to account for the actual discharge ( $Q_a$ ) as:

$$C_d = \frac{Q_a}{Q_i} = \frac{Q_a}{\sqrt{2gLH_w(H + P)} \sqrt{\frac{(H - H_w)}{(H + P)^2 - H_w^2}}}. \quad (15)$$

Equation 18 can be used for the sub-critical and free flow over an oblique weir.

### Discharge Coefficient for Submerged Flow over Oblique Weir

For submerged flow, the momentum equation is used instead of the energy equation on the analytical model. The momentum equation for two sections, upstream and downstream of the weir in  $+x$  orientation, would be:

$$\bar{P}A - \bar{P}_dA_d - F = \rho Q_s(U_d - U), \quad (16)$$

where  $Q_s$  is the discharge for the submerged condition,  $\bar{P}$  is the pressure,  $A$  is the flow area and subscript  $d$  is for the downstream condition. Assuming that the external force affecting the flow control volume throughout passing the weir as a hydrostatic force acts on the weir, it could be defined as:

$$F = \frac{1}{2}\gamma(H^2 - H_d^2)\frac{B}{\cos\varphi}. \quad (17)$$

By substituting the amount of each term in Equation 16, it becomes:

$$\frac{1}{2}\gamma(H+P)^2B - \frac{1}{2}\gamma(H_d+P)^2B - \frac{1}{2}\gamma(H^2 - H_d^2)\frac{B}{\cos\varphi} = \rho Q_s(U_d - U). \quad (18)$$

Combining the above stated equation with the continuity equation and simplifying it yields to:

$$Q_s = \frac{1}{2}B\sqrt{2g} \times \sqrt{\left[(H+H_d)\left(1 - \frac{1}{\cos\varphi}\right) + 2P\right] \times [(H_d+P)(H+P)]}. \quad (19)$$

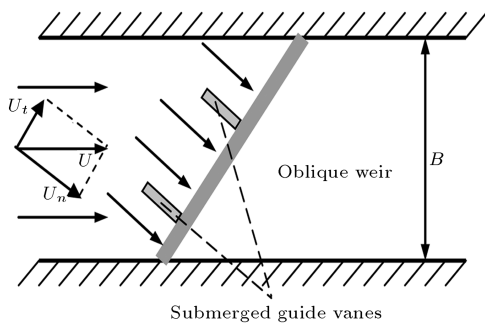
Due to the simplification assumption, the coefficient  $C_{ds}$  should be introduced to Equation 19 as:

$$C_{ds} = Q_s / \left(\frac{1}{2}B\sqrt{2g}\right) \times \sqrt{\left[(H+H_d)\left(1 - \frac{1}{\cos\varphi}\right) + 2P\right] \times [(H_d+P)(H+P)]}. \quad (20)$$

For submerged flow over oblique weirs, the discharge coefficient depends on the downstream head as well. As no explicit relation exists between the discharge and water head for the case of submerged flow, this study will use Equation 19 as the main objective to introduce a straight relation for determining the discharge coefficient of the submerged flow.

### Effect of Guide Vanes

As mentioned, streamlines over an oblique weir tend to deflect when they are passing over the weir but not when perpendicular to the weir plate. By using submerged guide vanes at the upstream face of the weir, the direction of flow can be controlled (Figure 3).



Guide vanes consist of thin plates, when placed normal to the oblique weir, then, the component of the velocity over the weir will be maximum and almost at right angles to the weir (or  $\beta = 0$ ). Hence, the flow discharge and, of course, the discharge coefficient will be increased. For this case, the energy and continuity equation can be written in the form of Equations 5 and 6, respectively. Combining these equations gives:

$$\frac{Q_i^2}{2gL^2} \left( \frac{1}{H_w^2} - \frac{1}{(H+P)^2 \cos^2\varphi} \right) = H - H_w. \quad (21)$$

Simplifying this equation yields to:

$$Q_i = H_w(H+P)\sqrt{2gL \cos\varphi} \times \sqrt{\frac{H - H_w}{(H+P)^2 \cos^2\varphi - H_w^2}}. \quad (22)$$

Again, due to the simplification and ideal flow assumption, coefficient  $C_{dv}$  has been employed to simulate the actual condition.

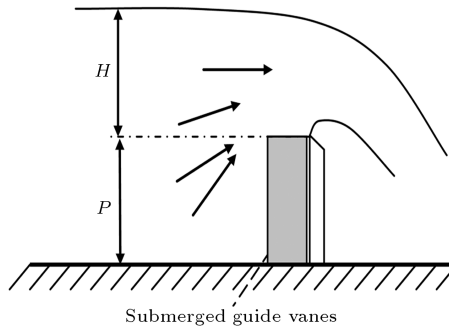
$$C_{dv} = Q_i / H_w(H+P)\sqrt{2gL \cos\varphi} \times \sqrt{\frac{H - H_w}{(H+P)^2 \cos^2\varphi - H_w^2}}. \quad (23)$$

### ERROR FUNCTION

The SPSS mathematical program and trial and error methodology were used to find the best relations for estimating the unknown parameters of the analytical derived equations. The error functions, Normalized Root Mean Square (NRMSE) [21] and Weighted Quadratic Deviation (WQD) [22], expressed by Equations 24 and 25 were also used:

$$\text{NRMSE} = \sqrt{\frac{\sum(F(x) - f(x))^2}{\sum(f(x) - \bar{f})^2}}, \quad (24)$$

$$\text{WQD} = \frac{\sqrt{\sum(F(x)f(x)(F(x) - f(x))^2)}}{\sum(F(x)f(x))}, \quad (25)$$



**Figure 3.** Schematic submerged guide vane plates at the upstream face of the weir.

where  $F(x)$  is the estimated amount;  $f(x)$  is the measured data and  $\bar{f}$  is the average of the measured data. The error functions, NRMSE and WQD, must be smallest in order to have the best relation among all parameters. Hence, applying the SPSS program (the nonlinear regression analysis), relations have been found to estimate the characteristics of flow over an oblique sharp-crested weir for both free and submerged flow. No doubt due to different combinations of the variables many functions can be introduced for each trial and error turn, but one's judgment can achieve a more accurate result.

## RESULTS

### Results of the Deflection Angle

The predicted angle of deflection ( $\beta$ ) from theoretical analysis is shown in Figures 4 and 5. Figure 4 shows the relation between  $\beta$  and  $H/P$ . As shown, by increasing  $H/P$ , the deflection angle increases, which is expected due to less influence of the weir on higher water depth values. Also, for a certain  $H/P$ , increasing  $L/B$  will increase  $\beta$ . Also, in this figure, the experimental data of [16] for different  $H/P$  and  $\varphi = 45^\circ$  or  $L/B = 1.4$  is shown. The correlation of the measured data with the predictions from theoretical analysis is shown. Figure 5 shows the dependency of  $\beta$  on the upstream Froude number (Fr). As seen, for a constant  $L/B$ , an increasing Froude number tends to increase in deflection angle and, similarly, for a constant Froude number, it causes an increase in  $\beta$ . Sensitivity analysis showed that the most important and relevant parameters affecting  $\beta$  are  $H/P$ ,  $\cos \varphi$  and Fr. As a result, Equation 26 with the minimum error functions of NRMSE = 0.076 and WQD

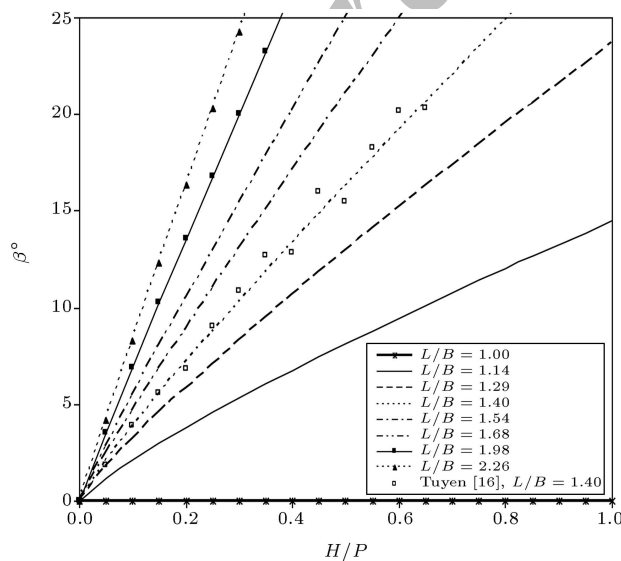


Figure 4. Relation between deflection angle and  $H/P$ .

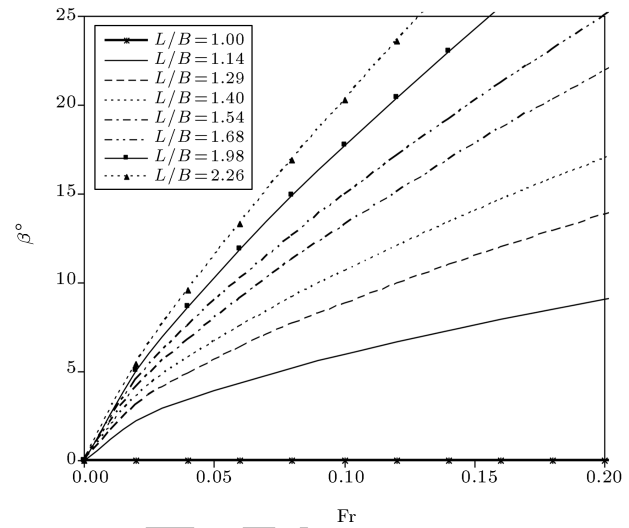


Figure 5. Relation between  $\beta$  and the upstream Froude number (Fr).

= 0.005 is achieved.

$$\beta^\circ = 1.8 \frac{\left(\frac{H}{P}\right)^{3.6}}{(\sin^{-1}(\cos \varphi))^3 \text{Fr}^{2.2}} \quad (26)$$

Figure 6 shows the comparison between the estimated and analytical deflection angle for the obtained equation (Equation 26). In this figure, the ranges of  $\pm 5\%$  variation bounds are also shown. As almost all the data points lie within  $\pm 5\%$  tolerance, it means that this equation is in good agreement with the theoretical results.

### The Results of Free Flow Discharge Coefficient

In order to check the results of the analytical model, the data from the experimental study of [10] have

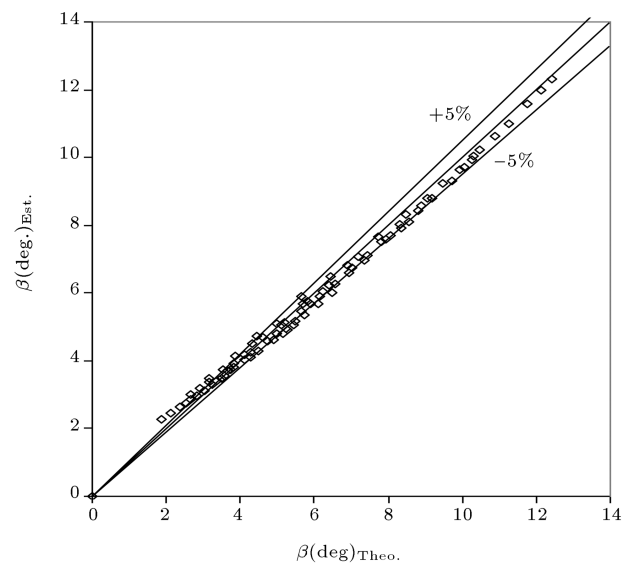


Figure 6. Comparison between estimated and analytic deflection angle.

been used. As the most important dependent flow parameter is the discharge coefficient; representing the flow rate over the weir reflected from the head losses over the weir, the discharge coefficient for the emerged flow ( $C_d$ ) will be presented with the flow conditions. For this purpose, by employing different parameters, many mathematical equations were used and the smallest error functions were checked. It is shown that  $C_d$  depends only on  $L/B$ . This was checked by the SPSS program and sensitivity analysis, and the following polynomial relation (Equation 27) has been found between  $C_d$  and  $L/B$  as the best fit (Figure 7). As seen,  $C_d$  decreased slightly due to the increasing of  $L/B$ .

$$C_d = -0.014 \left( \frac{L}{B} \right)^2 - 0.002 \left( \frac{L}{B} \right) + 0.946. \quad (27)$$

Figure 8 shows the comparison between the experimental discharge coefficient ( $C_d$ ) and the estimation from Equation 27. The  $\pm 2.5\%$  variation between the results shows that for free flow over an oblique weir, Equation 15 is a good tool to predict the actual discharge.

### The Results of Submerged Flow Discharge Coefficient

For the submerged flow condition, the sensitivity analysis shows that  $C_{ds}$  is related to the relative upstream water head (the ratio between upstream water depth and the weir height,  $H/P$ ) as well as the ratio between the downstream and upstream water head from the weir crest level. The relation between  $C_{ds}$  and  $H/P$  for different  $L/B$  is shown in Figure 9, and the dependency of  $C_{ds}$  on  $H_d/H$  is shown in Figure 10. For a certain  $L/B$ ,  $C_{ds}$  increases with increasing  $H/P$  and  $H_d/H$ .

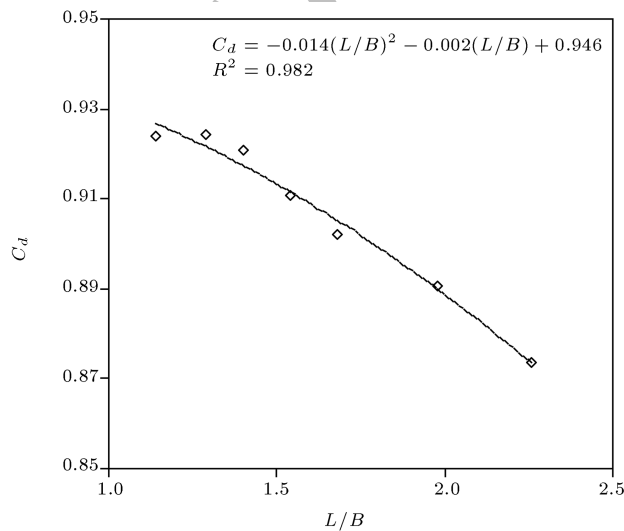


Figure 7. Relation between  $C_d$  and  $L/B$ .

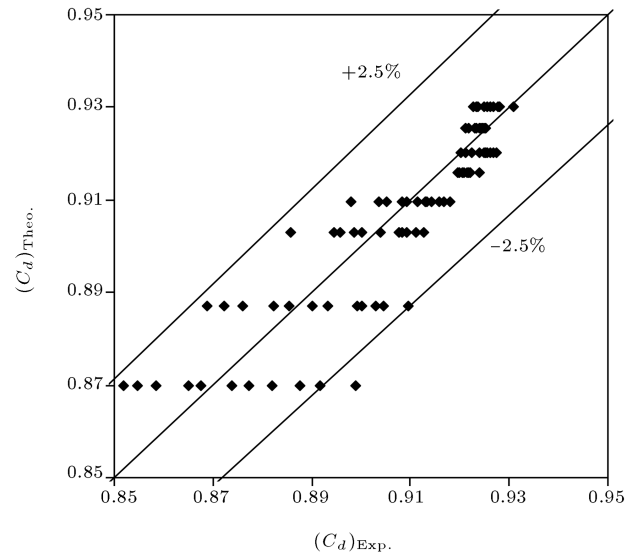


Figure 8. Comparison between experimental discharge coefficient and estimated values (Equation 27).

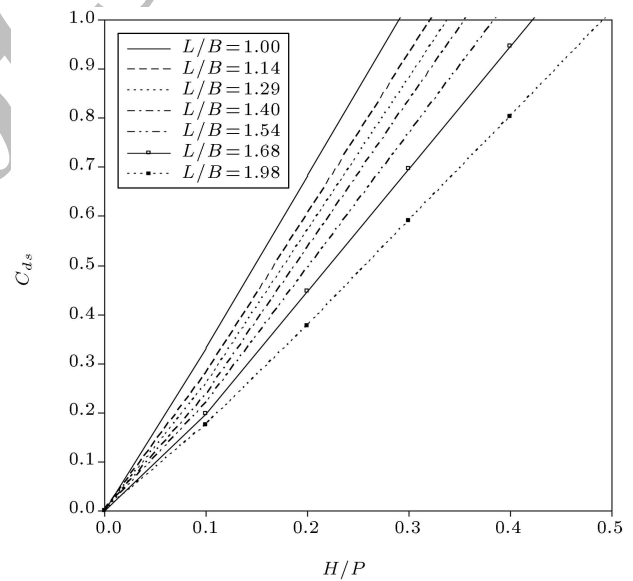
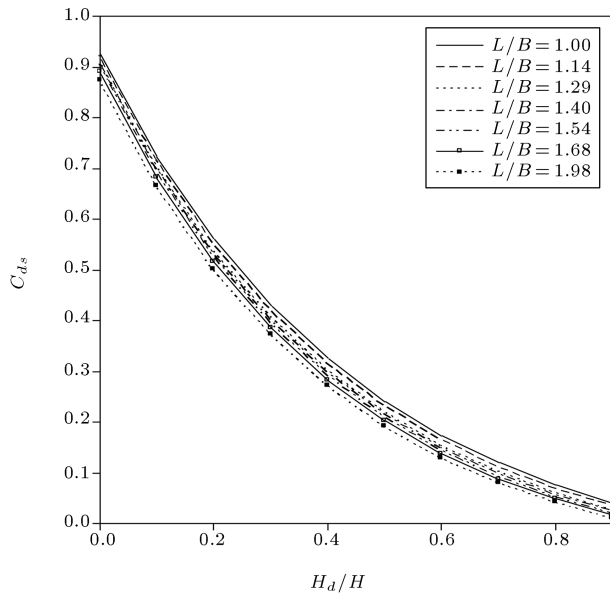


Figure 9. Relation between  $C_{ds}$  and  $H/P$  for different  $L/B$ .

Again, using the same methodology for analysis as mentioned in earlier cases, the relation for  $C_{ds}$  is obtained. Applying the SPSS program with a trial and error procedure, Equation 28 is obtained, having NRMSE = 0.067 and WQD = 0.003.

$$C_{ds} = 2.3 \left( \frac{L}{B} \right)^{0.86} \left( \frac{H}{P} \right)^{1.09} \left( \frac{H_d}{H} \right)^{0.05}. \quad (28)$$

The values of error functions are small, thus, Equation 28 is a good tool to predict the discharge coefficient of submerged flow over an oblique weir. The comparison between the estimated results (Equation 28) and experimental results is shown in Figure 11. The good

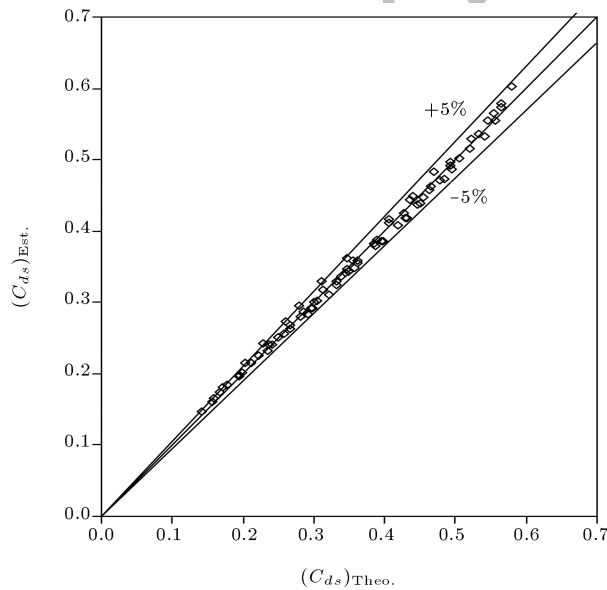


**Figure 10.** Dependency of  $C_{ds}$  on  $H_d/H$  for different  $L/B$ .

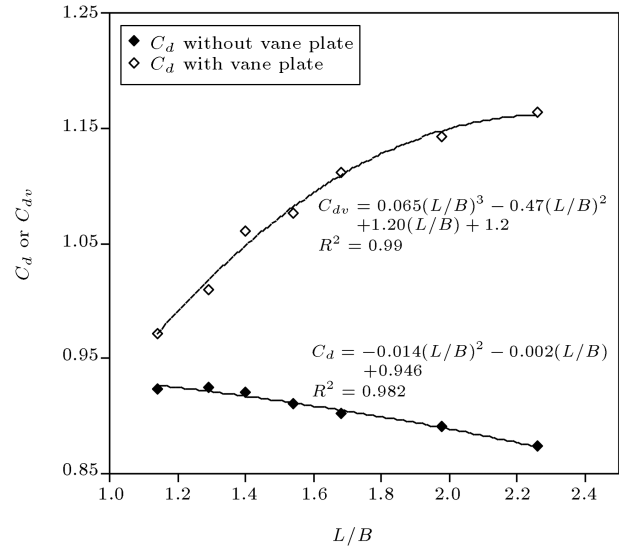
agreement between the results can be observed. As seen, for both cases of free and submerged flow, the discharge coefficient appears very sensitive to  $L/B$ .

### Effect of Guide Vanes

Finally, the results of using submerged guide vanes at the upstream face of the oblique weir for the free flow situation are presented. As mentioned before, the guide vanes are placed perpendicular to the upstream face of the weir. Figure 12 shows the dependency of  $C_d$  (without the vanes) and  $C_{dv}$  (with the guide vanes)



**Figure 11.** Comparison between the estimated results (Equation 28) and experimental results.

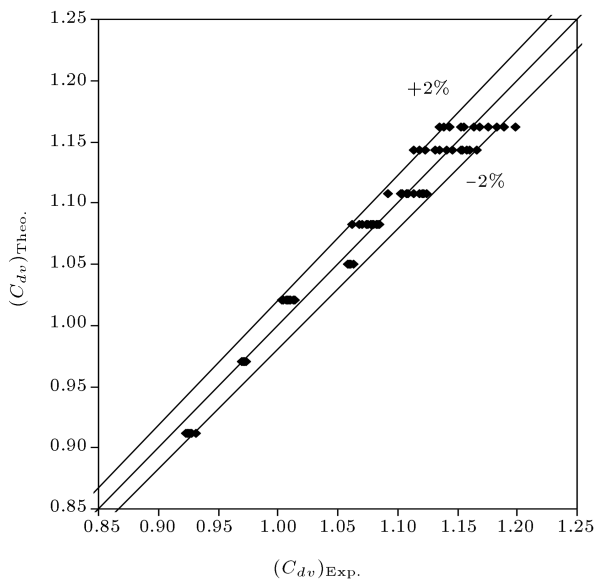


**Figure 12.** Relation between  $C_d$  and  $L/B$  for both using and not using guide vanes.

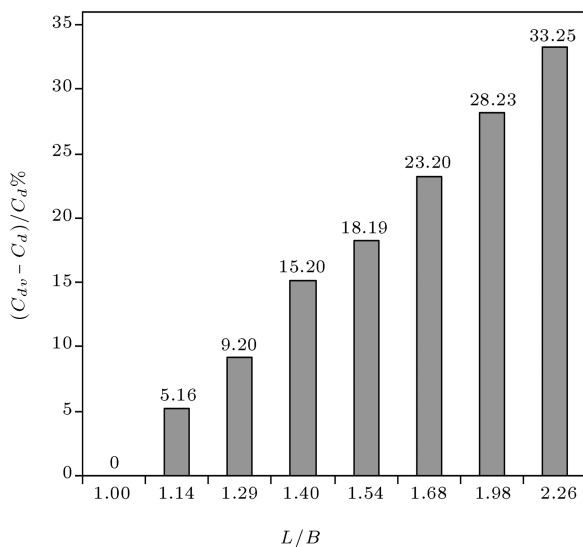
on  $L/B$  for both cases. It is illustrated in the figure that in spite of decreasing  $C_d$  with increasing  $L/B$ , the value of  $C_{dv}$  increases. The reason is that by employing these devices,  $U_{wt}$  becomes zero, and the perpendicular component of the flow velocity becomes greater. Hence, the streamlines tend to become perpendicular to the weir and at the same time accelerate passing over the weir crest. By increasing the flow acceleration, the water flow jet be thrown far away from the downstream of the weir. Simultaneously, increasing the weir length decreases the effects of stagnation regions significantly. As a result, the discharge capacity increases and, for the same water head, the discharge coefficient increases. On the other hand, by using a group of vanes at the upstream face of the oblique weir and omitting the tangential velocity component, the oblique weir flow behaves the same as a normal weir whose coefficient is increasing by  $L/B$ . Also, it is seen that the discharge coefficient for this case, similar to  $C_d$ , relates only to  $L/B$  and can be presented as:

$$C_{dv} = 0.065 \left( \frac{L}{B} \right)^3 - 0.47 \left( \frac{L}{B} \right)^2 + 1.20 \left( \frac{L}{B} \right) + 0.12. \quad (29)$$

Figure 13 shows the comparison between the experimental discharge coefficient ( $C_{dv}$ ) and the estimation from Equation 29 with submerged guide vanes. The majority of the points are within  $\pm 2\%$  bound, which shows a very good result for this kind of theoretical work. Hence, this equation can be applied simply to the discharge equation of free flow over an oblique weir together with the guide vanes (Equation 23). Figure 14 illustrates the percentage of normalized difference between  $C_d$  and  $C_{dv}$  ( $|C_{dv} - C_d|/C_d\%$ ) due to employing submerged guide vane plates for different



**Figure 13.** Comparison between experimental data and estimated discharge coefficient for the case of using submerged guide vanes.



**Figure 14.** Influence of guide vanes on increasing  $C_d$ .

$L/B$  values. As seen, for higher values of  $L/B$ , the discharge coefficient increases up to 33%. This means that the idea of using guide vanes for increasing the discharge coefficient is valuable. There is no doubt that many more studies, especially experimental studies, are needed to evaluate the optimized number and arrangements of guide vanes.

## CONCLUSIONS

In this paper, the results of analytical approaches were presented to establish relationships for the angle of flow deflection and the stage-discharge for both free

and submerged flow conditions. Also, this paper is showing the effect of submerged guide vanes at the upstream face of a rectangular sharp-crested oblique weir. The results include the equations obtained for estimating the flow characteristics, such as  $\beta$ , for free flow (Equation 26), the discharge coefficient for free and submerged flow (Equations 27 and 28, respectively), and the optimized discharge coefficient, due to employment of guide vanes (Equation 29) with the smallest error values.

Conclusions drawn from the analysis are:

- By increasing  $H/P$ , the deflection angle increases; also, for a constant  $H/P$ , increasing the ratio  $L/B$  will increase  $\beta$ ;
- For a constant  $L/B$ , an increasing Froude number tends to increase the deflection angle;
- For a certain  $L/B$ ,  $C_{ds}$  increases, while the relative upstream water head increases;
- Although the discharge coefficient for the case of a weir without a vane ( $C_d$ ) decreases slightly, the discharge coefficient of flow over the weir using guide vane plates increases with increasing  $L/B$ ;
- By using guide vanes, the discharge coefficient increases up to 33%.

## ACKNOWLEDGMENTS

The author wishes to express his special thanks to Prof. S.M. Borghei, professor of Civil Engineering, Sharif University of Technology, for his interest and constructive comments on this paper. Also, the financial support provided by the Chaharmahale Bakhtiari regional water company, Iran, is acknowledged.

## NOMENCLATURE

$A$	the upstream flow area
$A_d$	the downstream flow area
$B$	the channel width
$C_d$	the discharge coefficient
$C_{ds}$	the submerged flow discharge coefficient
$C_{dv}$	the emerged discharge coefficient with vane plates
$C_s$	the relative discharge defined as $C_s = Q_s/Q$
$F(x)$	the estimated amount
$H$	the upstream water head above the weir crest
$H_d$	the downstream water head above the weir crest
$H_w$	over weir flow depth

$K_s$	coefficient
$L$	the weir length
$P$	the weir height relative to upstream channel bed elevation
$\bar{P}$	pressure
$Q$	the flow discharge
$Q_s$	the submerged flow discharge
$U$	the upstream velocity
$U_d$	the downstream velocity
$U_w$	the velocity over weir crest
$U_n$	the normal component of upstream velocity
$U_{wn}$	the normal component of velocity over weir crest
$U_{te}$	the parallel component of upstream velocity
$U_{wt}$	the parallel component of velocity over weir crest
$f$	function
$f(x)$	the measured data
$\bar{f}$	the average of the measured data
$g$	gravitational acceleration
$n$	constant which depends on the weir geometry
$q$	the discharge per unit width of oblique weir
$q_n$	discharge per unit width of plain weir
$\beta$	estimated deflection angle
$\delta$	coefficient, depending on oblique angle
$\varphi$	oblique angle

## REFERENCES

- Vaseli, N. and Monadjemi, P. "Reclamation potential of urban stormwater runoff in Iran", *Scientia Iranica*, **12**(4), pp. 338-347 (2005).
- Brater, E.F. and King, H.W., *Handbook of Hydraulics*, 6th Ed., McGraw Hill Book Co., New York, N.Y. (1976).
- Kandaswamy, P.K. and Rouse, H. "Characteristics of flow over terminal weirs and sills", *Journal of Hydraulics Engineering*, **83**(4), pp. 286-298 (1957).
- Ghodsian, M. "Flow over triangular side weir", *Scientia Iranica*, **11**(1-2), pp. 114-120 (2004).
- Swamee, P.K. "Generalised rectangular weir equation", *Journal of Hydraulics Engineering, ASCE*, **104**(8), pp. 945-949 (1988).
- Kindsvater, C.E. and Carter, R.W. "Discharge characteristic of rectangular thin plate weirs", *Journal of Hydraulics Engineering, ASCE*, **83**(6), pp. 1-36 (1957).
- Ramamurthy, A.S., Tim, U.S. and Rao, M.J. "Flow over sharp-crested plate weirs", *Journal of Irrigation and Drainage Engineering, ASCE*, **113**(2), pp. 163-172 (1987).
- Jalili, M.R. and Zarrati, A.R. "Development and calibration of a resistivity probe for measurement of air concentration and bubble count in high-speed air-water flows", *Scientia Iranica*, **11**(4), pp. 312-319 (2004).
- Wu, S. and Rajaratnam, N. "Submerged flow regimes of rectangular sharp-crested weirs", *Journal of Hydraulics Engineering, ASCE*, **122**(7), pp. 412-414 (1996).
- Borghei, S.M., Vatannia, Z., Ghodsian, M. and Jalili, M.R. "Oblique rectangular sharp-crested weir", *Proceedings of the Institution of Civil Engrg. (ICE), Water and Maritime Engineering*, **156**, WM2, pp. 185-191 (2003).
- Aichel, O.G. "Abflusszahlen für schiefe weher. (discharge ratios for oblique weirs)", *Z. VDI*, **95**(1), pp. 26-27 (1953).
- De-Vries, M., *Oblique Weirs*, Report WL, Delft Hydraulics, In Dutch (1959).
- Ganapathy, K.T., Rajaratnam, N. and Ramanathan, V. "Discharge characteristics of oblique anicuts", *The New Irrigation Era., Irrigation Branch*, Public Works Department, Madras, India, **9**(1), pp. 7-12 (1964).
- Prakash, M.N. and Shivapur, A.V. "Discharge characteristics for flow through inclined triangular notch-weir", *Hydrology Journal*, **27**(1), pp. 971-981 (2004).
- Noori, B.M.A. and Chilmeran, T.A.H. "Characteristics of flow over normal and oblique weirs with semicircular crests", *AL-Rafidain Engineering Journal*, **13**(1), pp. 49-61 (2005).
- Tuyen, N.B. "Flow over oblique weirs", MSc. Thesis, TUDelft (2006).
- Wols, B.A. "Scheefaangestroomde overlatten", MSc. Report, TUDelft (2006).
- Borghei, S.M., Kabiri-Samani, A.R. and Nekoei, N. "Oblique weir equation using incomplete self-similarity", *Canadian Journal of Civil Engineering, CSCE*, **33**(10), pp. 1241-1250 (2006).
- Tuyen, N.B. "Influences of the oblique obstacles to the flow", *Japan-Vietnam Estuary Workshop*, Hochiminh, Vietnam, pp. 80-85 (2007).
- Kabiri-Samani, A.R. and Ansari, A. "Flow over sharp crested weirs", *Proc. 33rd IAHR Congress*, Vancouver, Canada, pp. 1022-1029 (2009).
- Gonzalez, J., Rojas, I., Poameres, H. and Orteag, J. "A new clustering technique for function approximation", *IEEE Transactions on Neural Networks*, **13**(1), pp. 132-142 (2002).
- Petit, L., Blanpain, O. and Al-hajjar, J. "Sensitivity analysis of Muskingum model to the inaccuracy of data describing the urban drainage networks", *Proceedings of the 3rd International Conference on Hydro informatics, Copenhagen, Denmark*, Balkema, Rotterdam, pp. 1137-1144 (1998).

## BIOGRAPHY

**A.R. Kabiri-Samani** completed his PhD in civil engineering at Sharif University of Technology in the field of hydraulics. He has been faculty affiliate as Assistant Professor in the civil engineering department of Isfahan University of Technology since 2006 until the present. Since 2008, he is the graduate program coordinator of the department.

Dr Kabiri received the 2008 “Elite National Foundation” for being ranked the highest of all PhD graduated students in the civil engineering department of Sharif University of Technology and the 2006 “Tavakkoli Award” due to his applicable research in two-phase flow mechanics. In 2003, he also received an

award for being among the top 10 Education at Sharif University of Technology.

He has authored more than 40 scientific publications, including 10 papers in scientific journals, 30 conference papers and 4 technical reports. Under his guidance, six MS students have also completed their graduate degrees in hydraulic engineering.

Dr Kabiri serves on the review board of the Journal of Fluids Engineering (ASME), the Canadian Journal of Chemical Engineering (CSChE), the National Journal of Hydraulics (Iranian Hydraulic Association) and the National Journal of Water and Wastewater (Isfahan Water and Wastewater Consulting Engineers Co.). He is also a member of seven professional societies, including; IAHR, EWRI, IHA, etc.

Archive of SID