Transaction A: Civil Engineering
Vol. 17, No. 2, pp. 131–140

SCIENTIA IRANICA

Rotation as a General Operation for Configuration Processing

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Abstract. *In this paper, a graph theoretical method is used to introduce three basic operations of configuration processing; rotation, translation and reflection. The relationships between translation and rotation, and reflection and rotation, are established. Examples are then constructed using three different procedures of configuration processing. From the comparison and discussion of the results, it is concluded that rotation can be considered as a general operation of configuration processing.*

Keywords: *Configuration processing; Graph; Rotation; Translation; Reflection.*

INTRODUCTION

For a large system, configuration processing is one of the most tedious and time-consuming parts of the analysis. Different methods have been proposed for configuration processing and data generation. Nooshin [1,2] developed a mathematical tool, so-called Formex algebra, for configuration processing (see also [3,4]). Behravesh et al. [5] employed set theory and showed that some concepts of set algebra can be used to build up a general method for describing the interconnection patterns of structural systems, and a graph theoretical method has been developed by Kaveh [6,7]. In all these methods, a sub-model is expressed in algebraic forms and then functions are used to produce the entire model. The basic functions include translation, rotation, reflection, and projection, or a combination of these functions.

In all the above mentioned methods, neither the sub-models nor the operations to be selected are unique. A model can be generated using a translation function. The same model may be obtained by pure rotation or pure reflection. One can also use a combination of these operations in a hybrid form to generate a model.

Now, the question is whether there is a general operation which can be used to generate any required configurations. The answer seems to be positive, and rotation may be considered as the right choice.

In this paper, three basic operations of configuration processing, based on the graph theory, including rotation, translation and reflection, are first introduced. Then, the relationships between translation and rotation, and reflection and rotation, are established. Finally, two examples are constructed and different procedures of configuration processing are applied to form this model. From comparison and discussion of the results, rotation is concluded as a general operation of configuration processing.

Here, the operations that will change the shape of the graph are excluded, i.e. the operations discussed in this paper correspond only to the rigid body movements of the graphs.

CLASSIC OPERATION FUNCTIONS

Coordinate System and Definitions from Graph Theory

In graph theory, a configuration consists of a finite set of points (nodes) and a finite set of lines (members) together with a relation of incidence, which associates two points with each line. The configuration of a structure describes the connectivity of the model.

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Received 21 October 2009; accepted 30 January 2010

Whereas there is no conception of length, an integer coordinate will be sufficient for the description of a configuration. Such a system may be expressed in a one-, two- or three-dimensional coordinate system. Herein, the two-dimensional integer coordinate system (I_1, I_2) is employed and the generalization is straightforward.

A node is denoted by a two-dimensional vector $N := \{i_1, i_2\}$, $N \in \mathbf{N}^2$ with $i_1 \in \mathbf{N}$ and $i_2 \in \mathbf{N}$. A member always consists of two nodes and is defined as $M := \{N_1, N_2\}$, $M \in \mathbf{N}^2 \times \mathbf{N}^2$. If a graph contains m members, it will be written as $G = \{M_1, M_2, \cdots, M_m\}$, $G \in (\mathbf{N}^2 \times \mathbf{N}^2)^m$. For convenience, the sub-graph and base-graph should also be defined. Sub-graph G_i is a component of graph G, and $G_i \subseteq G$, $i \in \{1, 2, \cdots, k\} \wedge k \leq m \wedge k \in \mathbf{N}^+$. A graph can be expressed as:

$$G = \sum_{i}^{k} G_i. \tag{1}$$

The base-graph, S_i, is a special sub-graph and it is the basic component of sub-graph G_i; $S_i \subseteq G_i$ and:

$$G_i = f(S_i), \tag{2}$$

where $f(R)$ is the abstract operation function. The operation functions will be introduced in the following sections. Thus, selection of the base-graph and the quality of the base-graph are very important for configuration processing. The coordinate system and notations are illustrated in Figure 1.

Translation Function

If a graph is moved from one position to another, and the lines joining each initial and final nodes of the graph are a set of parallel straight lines, the operation is called translation. The 'translation' function is denoted by *Tran* and is defined as:

$$\begin{aligned}
&\mathit{Tran}\,(S,Q) : (\mathbf{N}^2 \times \mathbf{N}^2)^n \times \mathbf{N}^2 \rightarrow (\mathbf{N}^2 \times \mathbf{N}^2)^n,\\
&S = \{\{\{i_{ij1}, i_{ij2}\}\}\} \in (\mathbf{N}^2 \times \mathbf{N}^2)^n,\\
&Q = \{q_1, q_2\} \in \mathbf{N}^2 \rightarrow \{\{\{i_{ij1} + q_1, i_{ij2} + q_2\}\}\},\\
&i \in \{1, 2, \cdots, n\}, j \in \{1, 2\}.
\end{aligned} \tag{3}$$

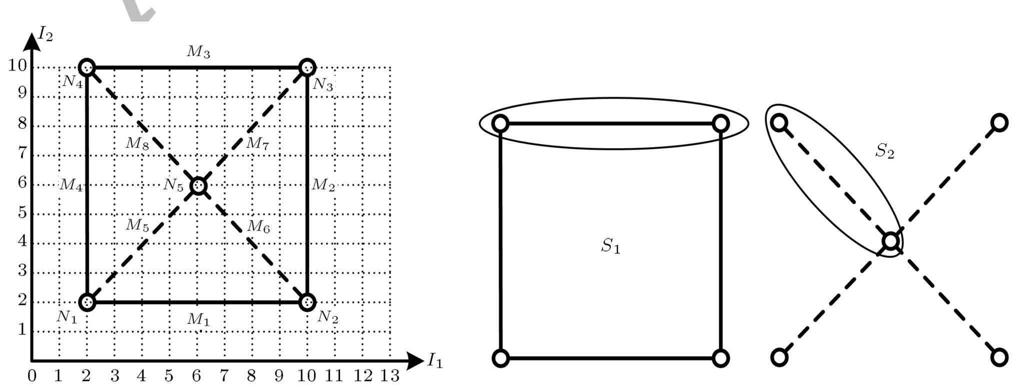
In Equation 3, n is the number of members in the graph. A translation operation is displayed in Figure 2. The dashed line indicates the graph before operation, and the solid line implies the graph after operation.

Rotation Function

For a two-dimensional graph, the operation of rotation makes the graph rotate around a center, O. For a three-dimensional graph, the rotation will take place around an axis. Herein, only the two-dimensional rotation is considered, which is denoted by *Rot*. This function is defined as:

$$\begin{aligned}
&\mathit{Rot}\,(S,O,\beta) : (\mathbf{N}^2 \times \mathbf{N}^2)^n \times \mathbf{N}^2 \times \{0, \pm\frac{\pi}{2}, \pm\pi\}\\
&\qquad \rightarrow (\mathbf{N}^2 \times \mathbf{N}^2)^n,\\
&S = \{\{\{i_{ij1}, i_{ij2}\}\}\} \in (\mathbf{N}^2 \times \mathbf{N}^2)^n,\\
&O = \{i_{O1}, i_{O2}\} \in \mathbf{N}^2, \quad \beta \in \left\{0, \pm\frac{\pi}{2}, \pm\pi\right\},\\
&\mapsto \left\{\left\{\left\{ \begin{array}{r} i_{O1} - i_{O1}\cos(\beta) + i_{O2}\sin(\beta)\\ +i_{ij1}\cos(\beta) - i_{ij2}\sin(\beta)\\ i_{O2} - i_{O2}\cos(\beta) - i_{O1}\sin(\beta)\\ +i_{ij2}\cos(\beta) + i_{ij1}\sin(\beta) \end{array} \right\}^T \right\}\right\},\\
&i \in \{1, 2, \cdots, n\}, \qquad j \in \{1, 2\}.
\end{aligned} \tag{4}$$

In Equation 4, $\beta \in \mathbf{N}$ represents the rotation angle. Since the integer coordinate is adopted, for the



(a) Graph G (b) Sub-graph G_1 (c) Sub-graph G_2

Figure 1. Node, member, and graph system.

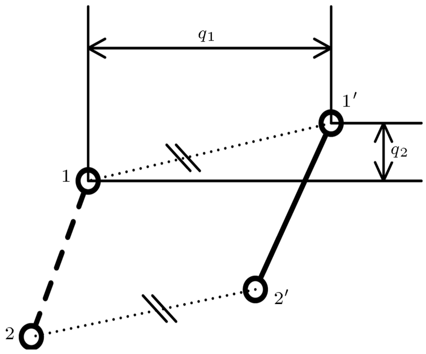


Figure 2. Translation of a sub-graph.

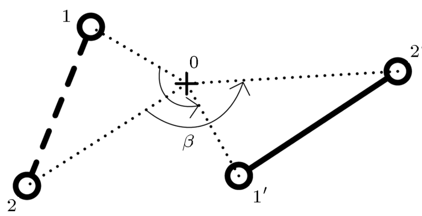


Figure 3. Rotation of a subgraph.

time being, the magnitude of β can only be one of $0, \pm\pi/2, \pm\pi$. The positive direction of rotation is anticlockwise. Figure 3 shows the rotation operation acting on a subgraph.

Reflection Function

In mathematics, a reflection is a map that transforms an object into its mirror image, thus, it can also be called mirror operation. We use *Ref* to denote this operation, and the definition of this operation is given as:

$$
\begin{aligned}
&\mathit{Ref}\,(S,h,a) : (\mathbf{N}^2\times\mathbf{N}^2)^n \times \mathbf{N}^2 \times \{1,2\} \\
&\qquad\qquad \to (\mathbf{N}^2\times\mathbf{N}^2)^n, \\
&S = \{\{\{i_{ij1}, i_{ij2}\}\}\} \in (\mathbf{N}^2\times\mathbf{N}^2)^n, \qquad a\in\mathbf{N}, \\
&h\in\{1,2\} \to \begin{cases} \text{if } h=1, \{\{\{2a-i_{ij1}, i_{ij2}\}\}\} \\ \text{if } h=2, \{\{\{i_{ij1}, 2a-i_{ij2}\}\}\} \end{cases}, \\
&i\in\{1,2,\cdots,n\}, \qquad j\in\{1,2\}.
\end{aligned} \tag{5}
$$

In Equation 5, h denotes the direction of reflection, and a is the reference axis of reflection (Figure 4). Here, a is allowed to have a non-integer value, while $2a$ should be an integer.

GENERAL PROPERTY OF ROTATION

Configuration processing can be performed by pure translation, pure rotation, pure reflection, or combinations of these three operations, and a good choice

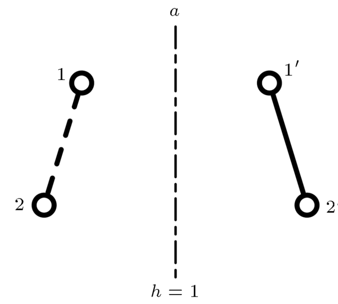


Figure 4. Reflection of a subgraph.

of operation depends on the configuration itself and the selection of the sub-graphs. However, one may want to find a general operation which can produce any configuration. Hence, the question is which one of the operations is superior: translation, rotation or reflection? In this paper, it is shown that rotation can be considered as a general operation of the three operations; translation and reflection can be expressed in the form of rotation but not vice versa.

Translation in the Form of Rotation

Translation and rotation have different properties. Translation moves a body "parallel", while rotation rotates a body, keeping a point fixed in plane and a line fixed in space. Nevertheless, if we are only concerned with the initial state and final state, a translation movement of a rigid body could be realized as a rotation. The replaced operation is called rotation-translation. Figure 5 illustrates rotation-translation which is actually a two-step rotation. First, rotation is performed on the member around R_1 through π and, then, rotated around R_2 through π again. R_1 is the center of the parallelogram composed of graphs 12 and $1''2''$, and R_2 is the mid-point of graph $1''2''$.

In mathematical language, rotation-translation can be expressed as:

$$
\begin{aligned}
&\mathit{Tran}\,(S,Q) = \mathit{Rot}\,(\mathit{Rot}\,(S,R_1,\pi),R_2,\pi), \\
&R_1 : \text{center of } S \text{ and } \mathit{Tran}\,(S,Q), \\
&R_2 : \text{mid-point of } \mathit{Rot}\,(S,R_1,\pi).
\end{aligned} \tag{6}
$$

In addition, rotation-translation has no reverse operation, which means there is no way to realize rotation by finite steps of translation.

Reflection in the Form of Rotation

Reflection is a kind of rotation, whereas the rotation center and the rotation angle are restricted. In Figure 6, it is shown that the two-dimensional reflection is a rotation around the reflection axis through π in the three-dimensional coordinate system.

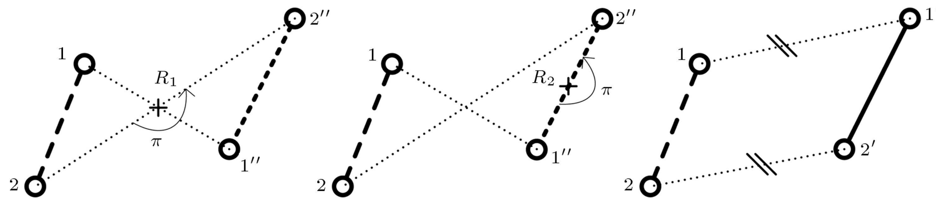


Figure 5. Illustration of translation in the form of rotation.

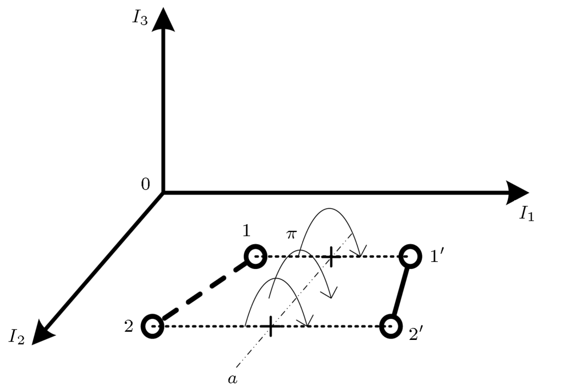


Figure 6. Illustration of reflection in the form of rotation.

Indeed, the rotation is superior to the other operations and can be considered a general operation for configuration processing.

EXAMPLES

In this section, two examples are considered to illustrate the concepts presented in the previous section.

Example I

A graph model of a planar truss selected from [1] is displayed in Figure 7.

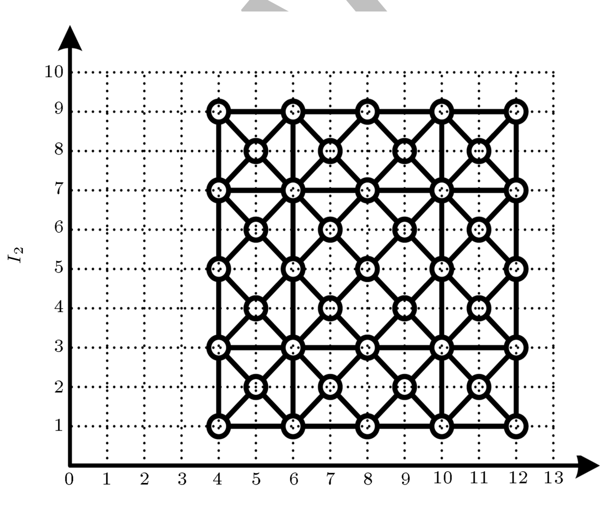


Figure 7. Graph model G of Example I.

In the following sections, three different procedures consisting of pure translation, pure reflection and pure rotation are performed for the configuration processing of G, and a comparison and discussion are provided.

Procedure 1: Pure Translation

Figure 8 provides a detailed procedure for the generation of graph G. Three base-graphs S_1, S_2 and S_3, are chosen to generate sub-graph G_1, G_2 and G_3, respectively. There is only 1 member in S_1 and S_2, whereas S_3 is comprised of 4 members. The translation function is called 4 times to generate G_1 by S_1, G_2 by S_2 and G_3 by S_3. Thus, $G = G_1 \cup G_2 \cup G_3$ is generated upon 12 times calling of the translation function.

Procedure 2: Pure Reflection

In this procedure, three base-graphs, S_1, S_2 and S_3, are chosen to generate the sub-graphs G_1, G_2 and G_3, respectively. There is only 1 member in each base-graph, which is simpler than the procedure of pure translation. The reflection function is used 4 times to generate S_1 formed by G_1, 4 times to generate G_2 formed by S_2, and 6 times to generate G_3 formed by S_3 (Figure 9). Thus, $G = G_1 \cup G_2 \cup G_3$ is generated upon 14 times calling of the reflection function.

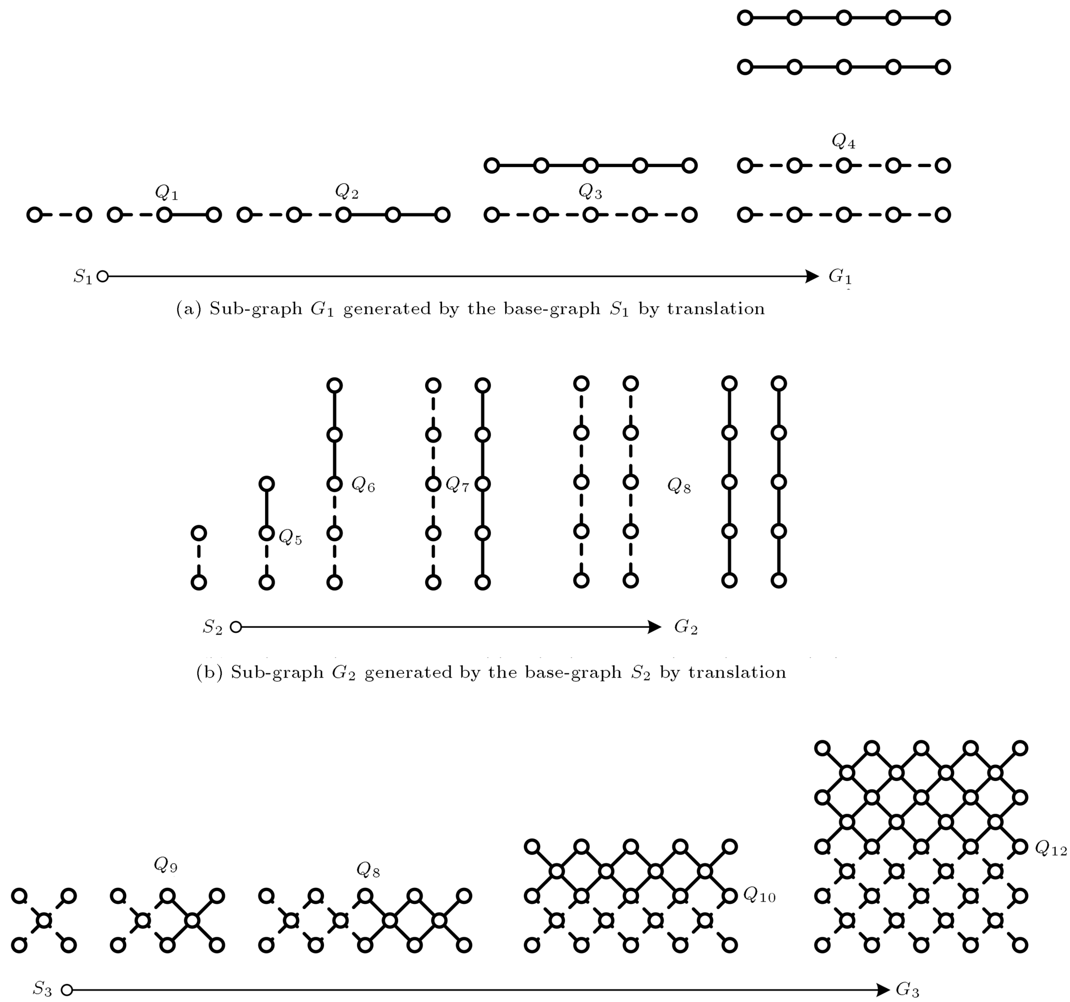
Procedure 3: Pure Rotation

The procedure of pure rotation is shown in Figure 10. Only two base-graphs, S_1 and S_2, are needed to generate the sub-graphs G_1 and G_2, respectively, and each base-graph consists of only 1 member. The total number of steps of calling the rotation function is 11. Thus, $G = G_1 \cup G_2 \cup G_3$ is generated upon 11 times calling of the rotation function.

Comparison and Discussion

From Table 1, it is easy to find out that the number of base-graphs, the number of calling the function and the procedure of pure rotation are superior to pure translation and pure reflection.

In this example, the base-graphs in the procedure of pure rotation are necessarily the simplest ones. These are denoted by $\tilde{S}_1$ and $\tilde{S}_2$, and the shapes of $\tilde{S}_1$ and $\tilde{S}_2$ are illustrated in Figure 10. It is not possible to use the translation operation to generate the entire



(a) Sub-graph G_1 generated by the base-graph S_1 by translation

(b) Sub-graph G_2 generated by the base-graph S_2 by translation

(c) Sub-graph G_3 generated by the base-graph S_3 by translation

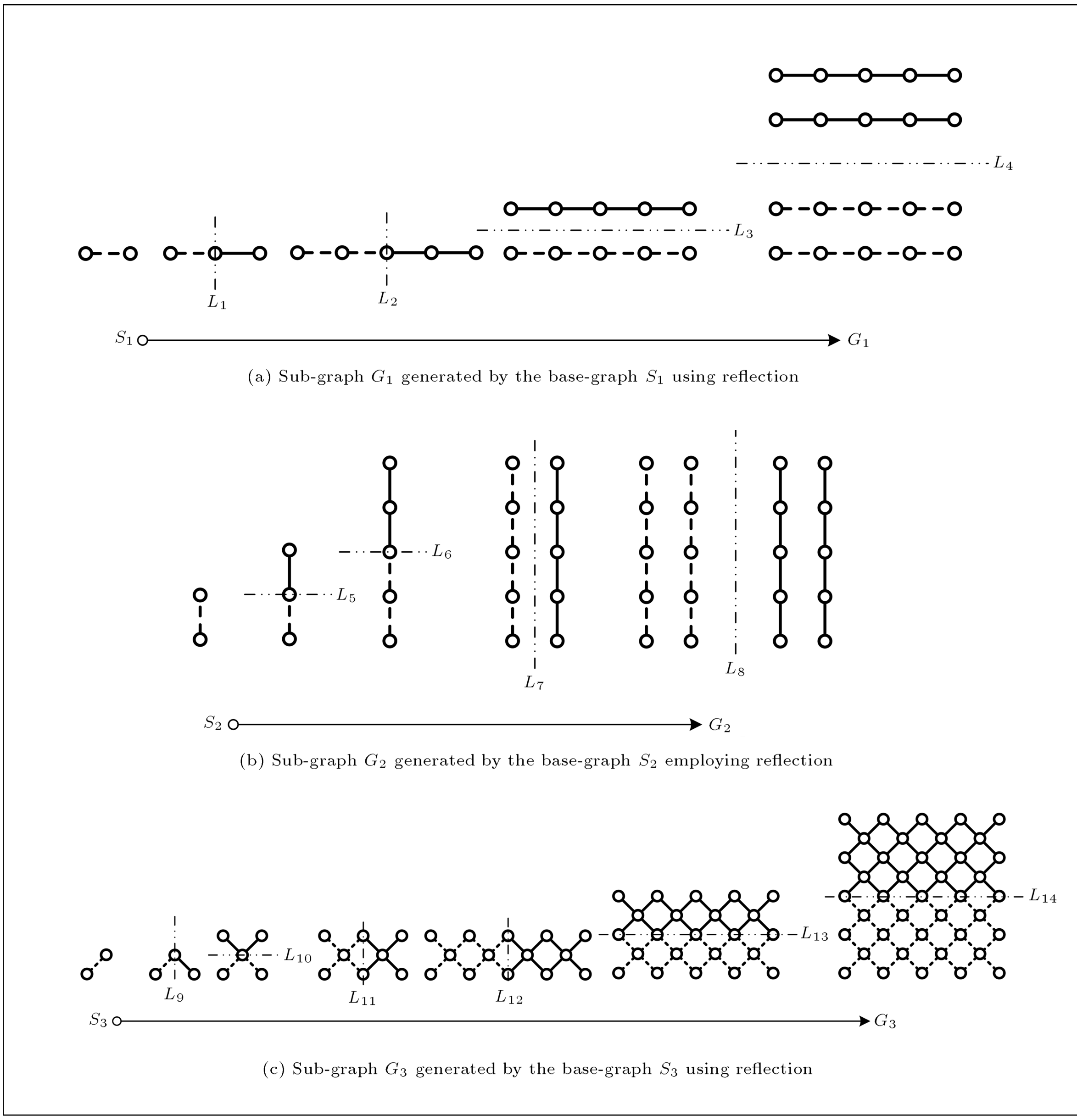
Figure 8. A planar truss generated by pure translation.

Table 1. Statistical data of three procedures.

Procedure	Number of Base-Graphs	Total Number of Members in Base-Graphs	Number of Calling the Function
Pure Translation	3	6	12
Pure Reflection	3	3	14
Pure Rotation	2	2	11

graph by $\tilde{S}_1$ and $\tilde{S}_2$; hence, we should increase the complexity of the base-graph to fulfill the procedure of pure translation. This is the reason for the numbers in the row corresponding to pure translation in Table 1 being larger.

Although it needs 14 times calling of the reflection function to generate the entire graph, which is more than that of pure translation, the sub-graphs in pure reflection are much simpler than those of pure translation. Furthermore, if we introduce a reflection axis with a slope of $\pi/4$ or $-\pi/4$, the entire graph can also be produced by pure reflection of $\tilde{S}_1$, $\tilde{S}_2$. The procedure is



(a) Sub-graph G_1 generated by the base-graph S_1 using reflection

(b) Sub-graph G_2 generated by the base-graph S_2 employing reflection

(c) Sub-graph G_3 generated by the base-graph S_3 using reflection

Figure 9. A planar truss generated by pure reflection.

shown in Figure 11. The reason for the higher number of steps of reflection is because the reflection is a kind of restricted rotation, which cannot rotate arbitrarily. Hence, the cost of the restriction is higher operation steps.

Example II

A graph model of a second truss is shown in Figure 12.

This example consists of two parts: four triangular sub-graphs and one rectangular sub-graph. As discussed previously, the reflection is a special kind of rotation, therefore, only the procedures of pure translation and pure rotation are provided in the following.

Pure Translation

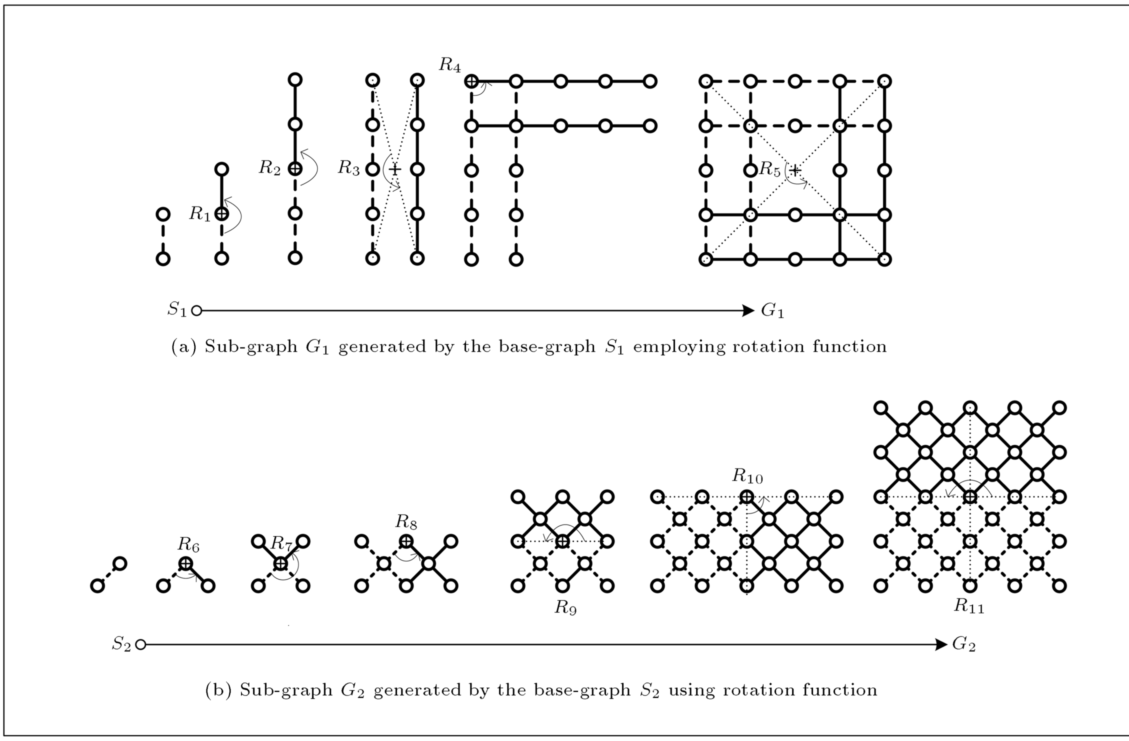
The steps of the formation of the model by pure translation are illustrated in Figure 13.

Pure Rotation

The steps of the formation of the model by pure rotation are illustrated in Figure 14.

Discussion

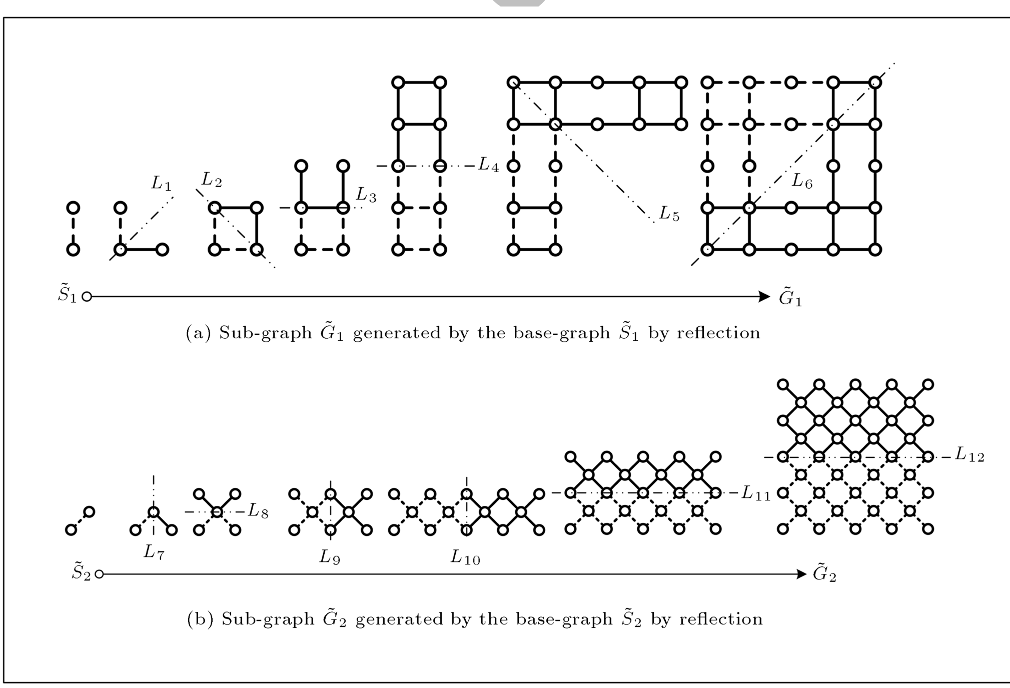
As shown above, the number of calling functions in the procedure of pure translation is far more than that of



(a) Sub-graph G_1 generated by the base-graph S_1 employing rotation function

(b) Sub-graph G_2 generated by the base-graph S_2 using rotation function

Figure 10. A planar truss generated by pure rotation.



(a) Sub-graph $\tilde{G}_1$ generated by the base-graph $\tilde{S}_1$ by reflection

(b) Sub-graph $\tilde{G}_2$ generated by the base-graph $\tilde{S}_2$ by reflection

Figure 11. A planar truss generated by pure reflection using improved reflection function.

pure rotation, as is the number of selected base-graphs. For the complex graph, it is very difficult to generate the model using pure translation. For some graphs, it is even hard to find the base-graphs, if one wants to use pure translation.

CONCLUSIONS

Three basic operation functions of configuration processing are reproduced in the paper, and rotation is found to be an intrinsic operation. Two comprehensive

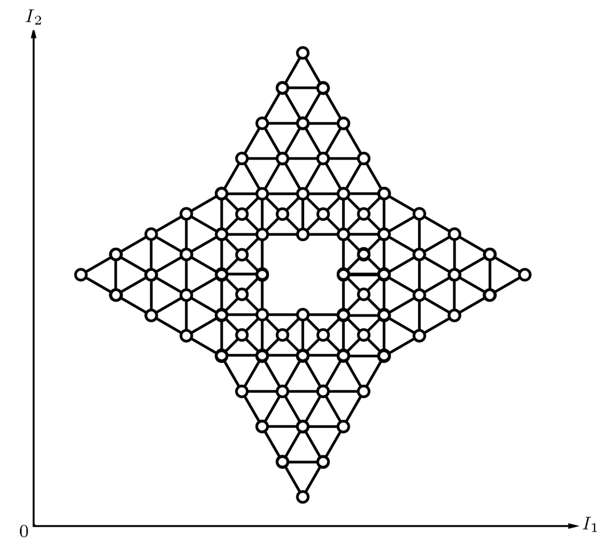


Figure 12. Graph model G of Example II.

examples are given for explaining the general property of the rotation. The conclusions from theoretical analysis and example problems are derived as follows:

- Most structural configurations can be processed via three basic operations: rotation, translation and reflection.

- Rotation is the most general operation among the three basic operations.

- Translation can be realized by a two-step rotation, but the reverse is not true.

- Reflection is a kind of restricted rotation.

- Rotation is the simplest and most efficient procedure to generate a given structural model.

Though only a planar model is considered as an example, many space structures can be generated by transforming the generated planar models. This can be achieved by means of simple functions.

Though the examples presented in this paper are selected from two-dimensional models, the ideas are quite general and applicable to three-dimensional models. Some three-dimensional models can also be formed using two-dimensional ones and by using appropriate geometric transformations.

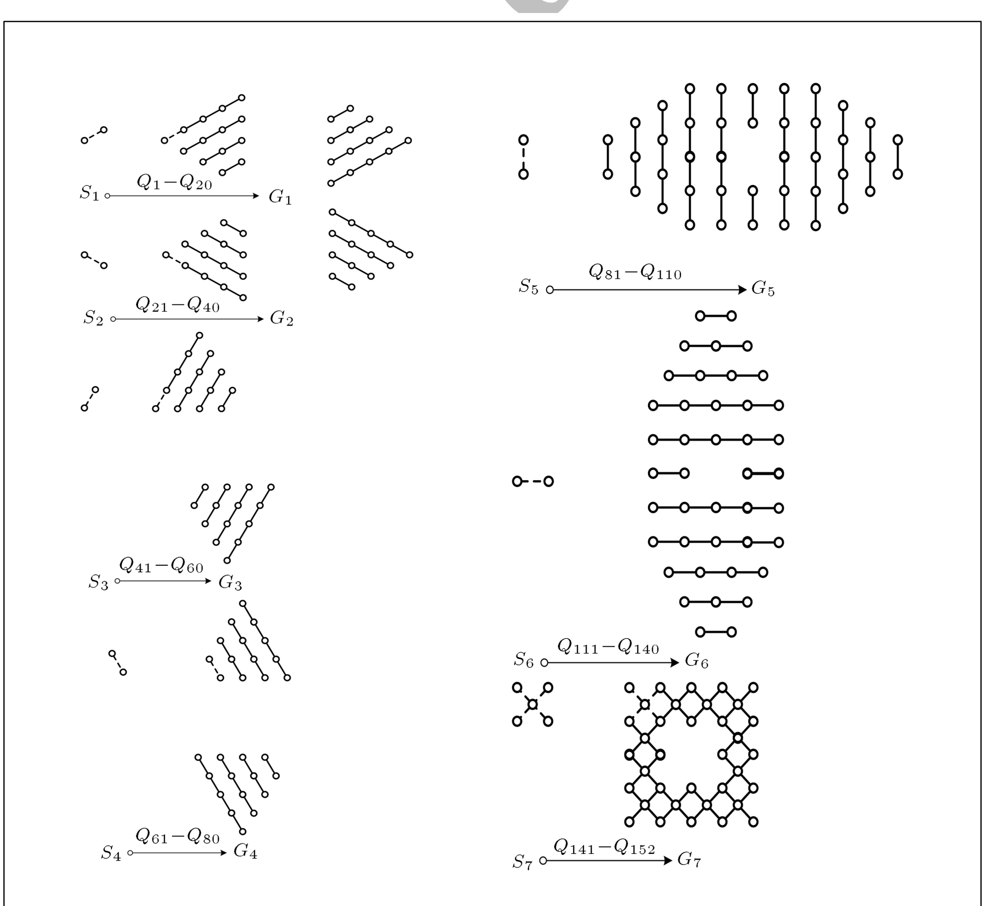


Figure 13. Steps of the generation of the model by pure translation.

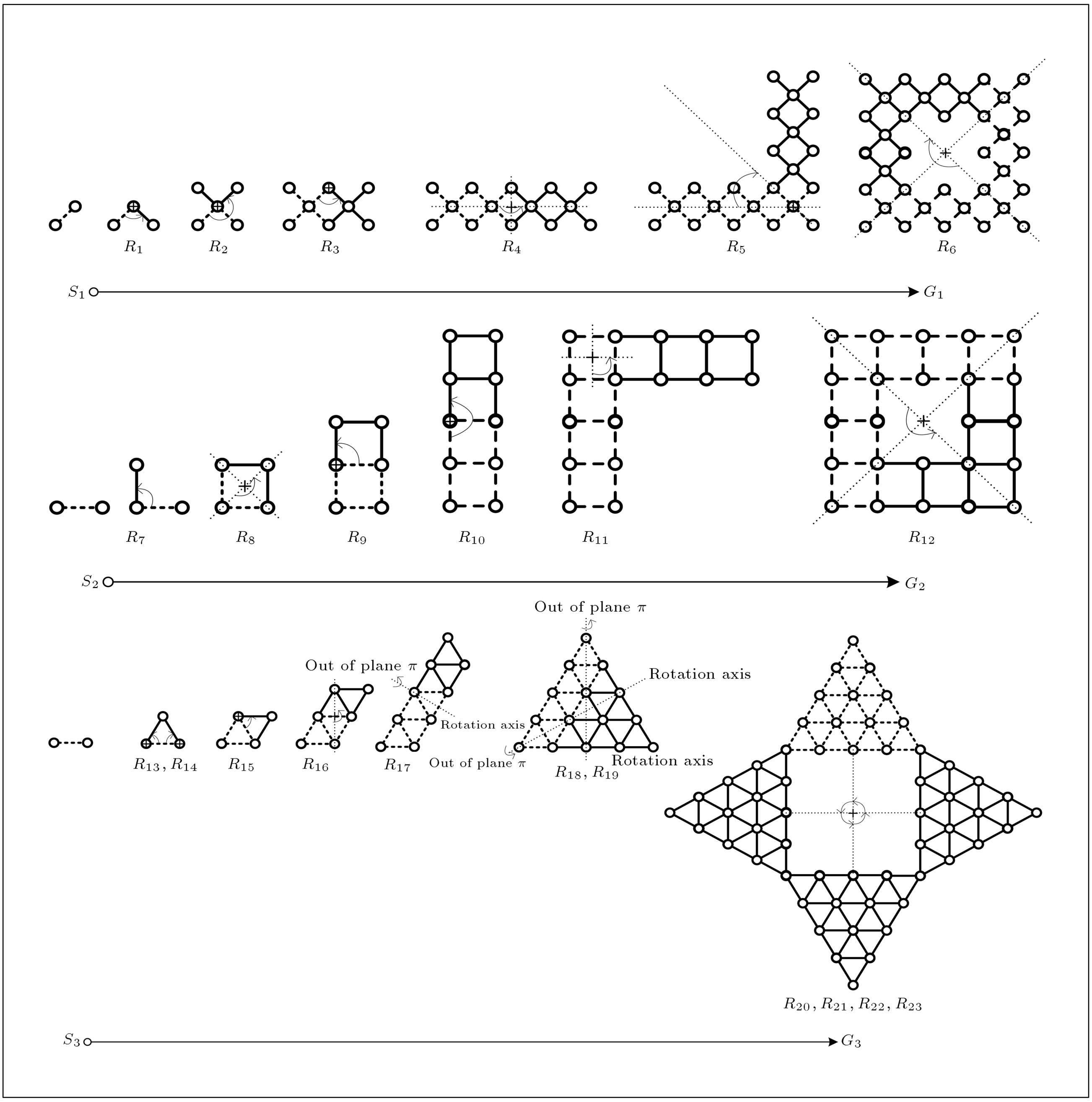


Figure 14. Steps of the generation of the model by pure rotation.

ACKNOWLEDGMENT

The first author is grateful to Iran National Science foundation for the support.

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BIOGRAPHIES

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