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Resource-Constrained Project Scheduling Problem with Flexible Work Profiles: A Genetic Algorithm Approach

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Abstract. *This paper deals with the resource-constrained project scheduling problem with flexible work profiles. In this problem, a project contains activities interrelated by finish-start-type precedence constraints with a time lag of zero. In many real-life projects, however, it often occurs that only one renewable bottleneck resource is available and that activities do not have a fixed prespecified duration and associated resource requirement, but a total work content, which essentially indicates how much work has to be performed on them. Based on this work content, all feasible work profiles have to be specified for the activities, each characterized by a fixed duration and a resource requirement profile. The task of the problem is to find the optimum work profile and start time of each activity in order to minimize the project makespan. We propose a procedure to find all feasible work profiles of each activity and we use a genetic algorithm with a new crossover operator to schedule the activities. Computational results on a randomly generated problem set are presented.*

Keywords: *Project scheduling; Heuristic; Genetic algorithm.*

INTRODUCTION

This research considers the Resource Constrained Project Scheduling Problem with Flexible Work Profiles (RCPSP_FWP). This problem was proposed by Kolisch [1] in the field of pharmaceutical R&D projects and deals with the lead optimization phase of pharmaceutical research where a number of leads (molecules as a basis for potential drugs) are processed by different departments in order to optimize their biochemical characteristics. The RCPSP_FWP can be stated as follows. A single project consists of a set of activities that are interrelated by finish-start-type precedence relations with a time lag of zero. There is a single constrained renewable resource (laboratory) in which all activities have to be processed. For each activity, instead of a fixed duration and resource requirement, the total work content is given, which essentially indicates how much work has to be performed. In other words, activity duration and resource usage at each time period of their execution are unknown. The way each activity is processed, i.e. its work profile, is not predetermined but limited by the following five constraints:

a) No preemption is allowed.
b) The resource usage of each activity in a processing period has to be within a resource-dependent interval.
c) The summation over used resource units for all periods of execution of each activity should be equal to the work content of the activity.
d) During processing, the resource usage of each activity has to be constant for a resource-dependent period.

Each work profile for an activity is considered as an activity mode and all feasible work profiles for an activity constitute its set of modes.

The constraints (a)-(d) assure that the processing of activities is done in an efficient way by reducing the number of setups due to constraints (a) and (d) and by forbidding the case of too few or to many resources assigned to an activity within one period

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by constraint (b). Since the number of feasible work profiles of each activity in the general form and on the basis of conditions (a)-(d) may be very large, we limit ourselves to only smooth (SM), Non-Increasing (NI), Non-Decreasing (ND) or triangular (TR) types of figure for work profiles. Figures 1 to 4 illustrate some possible work profiles of an activity with work content of 15.

The RCPSP_FWP is a different version of the well-known Resource Constrained Project Scheduling Problem (RCPSP) in which a single renewable resource is available and activity duration and resource usage to a single renewable resource are known constants. Also, the RCPSP_FWP is a generalization of the discrete time/resource trade-offs (DTRTP), introduced by De Reyck et al. [2]. In the DTRTP, only smooth work profiles are considered for each activity, and constraints (b), (c) and (d) are ignored.

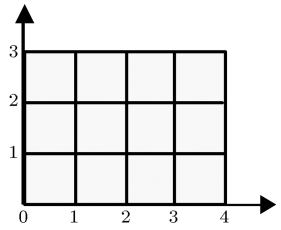


Figure 1. Smooth work profile.

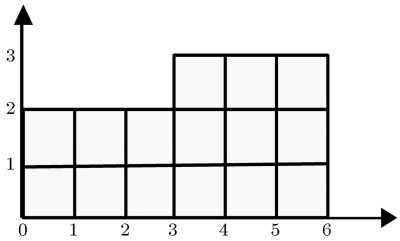


Figure 2. Non-decreasing work profile.

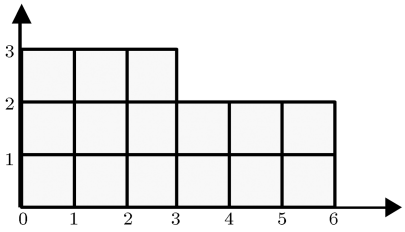


Figure 3. Non-increasing work profile.

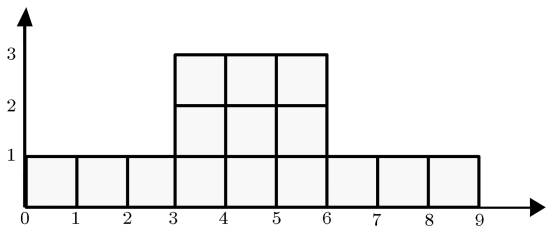


Figure 4. Triangular work profile.

To the best of our knowledge, literature on the RCPSP_FWP, as defined in this paper, is virtually void. Research efforts have been concentrated on two related problems, RCPSP and DTRTP. Salewski et al. [3], Brucker et al. [4], Kolisch and Padman [5] and Kis [6] include some noteworthy literature survey research for the RCPSP. Also, numerous exact and heuristic procedures are presented in the literature for the RCPSP. Chapter 6 of the project scheduling research handbook of Demeulemeester and Herroelen [7] gives an extensive literature overview of the RCPSP and, hence it is not repeated here. In addition, Kolisch and Hartmann [8] present a classification and performance evaluation of different heuristic and metaheuristic algorithms for the RCPSP. De Reyck et al. [2] present several heuristic procedures for the DTRTP based on the Tabu Search (TS) and some local searches. These heuristic procedures are based on the decomposition of the problem into a mode assignment phase and a resource-constrained project scheduling phase with fixed mode assignments. Demeulemeester et al. [9] have developed a Branch-and-Bound algorithm (B&B) for the DTRTP based on the concept of activity-mode combinations, i.e. subsets of activities executed in a specific mode. Also, Ranjbar et al. [10] present an efficient hybrid metaheuristic, based on the scatter search for the DTRTP and RCPSP.

The NP-hardness of the RCPSP was shown by Blazewicz et al. [11] as a generalization of the job shop scheduling problem. Furthermore, Demeulemeester et al. [9] show that the DTRTP, as a generalization of the parallel machine problem, is strongly NP-hard. Therefore, the RCPSP-FWP as a generalization of the DTRTP is strongly NP-hard.

Since the RCPSP_FWP is a very complex problem, we use a Genetic Algorithm (GA) as our solution approach. Two famous GAs applied to project scheduling problems are Mori and Tseng [12] and Hartmann [13].

The contribution of this paper is twofold. First, we present a linear model for the RCPSP_FWP and also an enumeration scheme to generate feasible work profiles. Second, we propose a metaheuristic solution procedure based on GA to solve RCPSP_FWP.

The outline of the paper is as follows. First, the problem formulation and notation are described and generation of feasible work profiles of a single activity is discussed. After that, schedule representation and the generation scheme are explained and the general structure of the genetic algorithms and its operators are described. In the section following these computational results are reported. Finally, overall conclusions and suggestions for future research are drawn.

PROBLEM FORMULATION AND NOTATION

The objective of the RCPSP_FWP is to schedule each activity of a project in one of its defined modes subject to precedence and resource constraints, in order to minimize the project makespan. The parameters used are defined in Table 1.

The dummy activities, 0 and $n+1$, represent the start and completion of the project, respectively, with $w_o = w_{n+1} = 0$. The RCPSP_FWP can be formulated by introducing the following decision variables:

$$s_{it} = \begin{cases} 1; & \text{if activity } i \text{ is started at time instant } t \\ 0; & \text{otherwise} \end{cases}$$

$$f_{it} = \begin{cases} 1; & \text{if activity } i \text{ is finished at time instant } t \\ 0; & \text{otherwise} \end{cases}$$

$$h_{it} = \begin{cases} 1; & \text{if } r_{i(t-1)} \neq r_{i(t)} \\ 0; & \text{otherwise} \end{cases}$$

In the following, we represent an MIP formulation for the RCPSP_FWP.

$$\min Z = \sum_{t=ef_{n+1}}^{lf_{n+1}} t f_{(n+1)t}, \tag{1}$$

subject to:

$$\sum_{t=es_i}^{ls_i} s_{it} = 1; \qquad i \in N, \tag{2}$$

$$\sum_{t=ef_i}^{lf_i} f_{it} = 1; \qquad i \in N, \tag{3}$$

Table 1. Notation.

$N = \{0, \cdots, n, n+1\}$	Set of activities with index i
$E = \{(i,j); i,j \in N\}$	Set of precedence relations
P_i	Set of all predecessors of activity i; $i \in N$
S_i	Set of all successors of activity i; $i \in N$
a	Constant availability of the single renewable resource
w_i	Work content of activity i; $i \in N$
$\underline{r}_i, \bar{r}_i$	Lower and upper bounds of resource usage of activity i in each time period; $i \in N$
$\underline{t}$	Minimum number of consecutive periods where all activities should have a constant resource usage
d_i	Duration of activity i; $i \in N$
$\underline{d}_i = \left\lceil \frac{w_i}{\bar{r}_i} \right\rceil$	Lower bound for duration of activity i; $i \in N$
$\bar{d}_i = \left\lfloor \frac{w_i}{\underline{r}_i} \right\rfloor$	Upper bound for duration of activity i; $i \in N$
$d_i^{\min}$	Minimum possible duration for a feasible work profile of activity i; $i \in N$
$d'^{\min}_i$	Minimum possible duration for an infeasible work profile of activity i with minimum violation; $i \in N$
$T = \sum_{i=1}^{n} \bar{d}_i$	An upper bound on the project makespan
$es_i = \max\left\{\left(\frac{\sum_{j \in P_i} w_j}{a}\right), \left(\max_{j \in P_i}\left\{es_j + d_j^{\min}\right\}\right)\right\}$	Earliest start time of activity i; $i \in N$
$ef_i = es_i + \underline{d}_i$	Earliest finish time of activity i; $i \in N$
$lf_i = \min\left\{\left(T - \frac{\sum_{j \in S_i} w_j}{a}\right), \left(T - \min_{j \in S_i}\{ls_j\}\right)\right\}$	Latest finish time of activity i; $i \in N$
$ls_i = lf_i - \underline{d}_i$	Latest start time of activity i; $i \in N$
$(t) = (t-1, t]$	Time period (t); $t = 1, 2, \cdots, T$
$r_{i(t)}$	Resource usage of activity i at time period t; $i \in N$, $t = es_i + 1, \cdots, lf_i$
$wp_i = (r_{i(t)}); t = es_i + 1, \cdots, lf_i$	Work profile of activity i; $i \in N$

$$\sum_{t=es_i+1}^{lf_i} r_{i(t)} = w_i; \qquad i \in N, \tag{4}$$

$$\sum_{t=ef_i}^{lf_i} tf_{it} \leq \sum_{t=es_j}^{ls_j} ts_{jt}; \qquad \forall (i,j) \in E, \tag{5}$$

$$\sum_{i=1}^{n} r_{i(t)} \leq a; \qquad t = 1, \cdots, T, \tag{6}$$

$$r_{i(0)} = 0; \qquad i \in N, \tag{7}$$

$$\sum_{\tau=t}^{t+\underline{t}-1} h_{i\tau} \leq 1; \qquad i \in N, t = es_i + 1, \cdots, lf_i, \tag{8}$$

$$r_{i(t+1)} \leq \bar{r}_i(r_{i(t)} + s_{it}); \qquad i \in N, t = 1, \cdots, T, \tag{9}$$

$$r_{i(t)} \leq \bar{r}_i(r_{i(t+1)} + f_{it}); \qquad i \in N, t = 1, \cdots, T, \tag{10}$$

$$r_{i(t+1)} - r_{i(t)} \leq \bar{r}_i.h_{i,t+1}; \quad i \in N, t = 1, \cdots, T, \tag{11}$$

$$r_{i(t)} - r_{i(t+1)} \leq \bar{r}_i.h_{i,t+1}; \quad i \in N, t = 1, \cdots, T, \tag{12}$$

$$s_{it}, f_{it}, h_{it} \in \{0,1\}; \qquad i \in N, t = 1, \cdots, T, \tag{13}$$

$$r_{i(t)} \in \{0\} \cup \{\underline{r}_i, \underline{r}_i + 1, \cdots, \bar{r}_i\};$$

$$i \in N, t = 1, \cdots, T. \tag{14}$$

Objective Function 1 minimizes the project makespan. Constraints 2 and 3 ensure that exactly one start and finish time are assigned to each activity, respectively. Constraint 4 assures that used resource units by each activity are equal to the activity work content. Constraints 5 and 6 indicate the precedence and resource constraints, respectively. Constraint 7 indicates that the resource is available after time instant zero. Constraint 8 corresponds to the constraints (d) of the work profile constraints. Also, Constraints 9-12 ensure that every change in the level of resource usage for each activity is less than its resource usage upper bound. Finally, Constraints 13 and 14 show the value domains of the decision variables.

ASSIGNMENT OF FLEXIBLE WORK PROFILES

In this section, we consider the problem of generating feasible work profiles of a single activity within the RCPSP_FWP. We first propose a model that generates the work profile with minimum durations. Next, we present an enumeration procedure for generating the NI, ND and SM work profiles. We further discuss the generation of TR work profiles based on NI and ND work profiles.

Generation of Work Profiles with Minimum Duration

Based on the defined notations in Table 1 and the definition of the two following decision variables, we present a model for finding a feasible work profile with minimum duration for each single activity, $i \in N$.

$x_{ri} =$ number of periods with a resource usage of r units during execution of activity $i \in N (r = \underline{r}_i, \underline{r}_i + 1, \cdots, \bar{r}_i)$,

$$y_{ri} = \begin{cases} 1; & \text{if there is at least one period with resource usage of } r \text{ units during execution of activity } i \\ 0; & \text{otherwise} \end{cases}$$

$$d_i^{\min} = \min \sum_{r=\underline{r}_i}^{\bar{r}_i} x_{ri}, \tag{15}$$

subject to:

$$\sum_{r=\underline{r}_i}^{\bar{r}_i} r x_{ri} = w_i, \tag{16}$$

$$\underline{t} y_{ri} \leq x_{ri} \leq \bar{d}_i y_{ri}, \tag{17}$$

$$x_{ri} \in \{0, 1, \cdots\}, \qquad r \in [\underline{r}_i, \bar{r}_i], \tag{18}$$

$$y_{ri} \in \{0,1\}, \qquad r \in [\underline{r}_i, \bar{r}_i]. \tag{19}$$

Objective Function 15 minimizes the number of periods where the activity has a positive resource usage and, thus the duration of the activity. Constraint 16 takes care that the total used resource units equals the activity work content. Constraint 17 couple x_{ri}-variable with the y_{ri}-variable; if $y_{ri} = 0$, then x_{ri} is set to zero, otherwise, if $y_{ri} = 1$, $\underline{t} \leq x_{ri} \leq \bar{d}_i$. Finally, Constraints 18 and 19 show the value domains of the decision variables. We call Constraints 16 to 19 the Feasible Work Profile Space (FWPS_i) for activity i. If $\text{FWPS}_i = \emptyset$, there is not any feasible work profile for activity i. In this case, we consider the work profiles with minimum violation.

Generation of Work Profiles with Minimum Violation

In the case there is no feasible work profile for an activity, we consider only one work profile for the activity which has a minimum violation from work profile constraints. To this purpose, we alter the previous model in order to find an infeasible work

profile that minimizes the number of periods that are short of the minimum number of periods with constant duration $\underline{t}$. We define decision variable $v_{ri} \geq 0$ as the number of periods with resource usage r short of the minimum number of periods regarding to activity i. Note that v_{ri} is formally defined as $v_{ri} = \max(0, \underline{t} - x_{ri})$ and the new model is as follows:

$$d_i'^{\min} = \min \sum_{r=\underline{r}_i}^{\bar{r}_i} v_{ri}, \tag{20}$$

subject to:

$$\sum_{r=\underline{r}_i}^{\bar{r}_i} r x_{ri} = w_i, \tag{21}$$

$$\underline{t} y_{ri} - v_{ri} \leq x_{ri} \leq \bar{d}_i y_{ri}, \qquad r \in [\underline{r}_i, \bar{r}_i], \tag{22}$$

$$x_{ri} \in \{0, 1, \cdots\}, \qquad r \in [\underline{r}_i, \bar{r}_i], \tag{23}$$

$$y_{ri} \in \{0, 1\}, \qquad r \in [\underline{r}_i, \bar{r}_i], \tag{24}$$

$$v_{ri} \geq 0, \qquad r \in [\underline{r}_i, \bar{r}_i]. \tag{25}$$

Objective Function 20 minimizes the number of periods that are short of the minimum number of periods with a constant resource usage. Altered Constraint 22 sets the variable, v_{ri}. Other constraints are taken from the previous model.

An Enumeration Procedure for the ND Work Profiles

In order to enumerate all ND work profiles of an activity, i, we use an enumeration tree in which level $l (l = 1, \cdots, \bar{r}_i - \underline{r}_i + 1)$ corresponds to all possible values of x_{ri}, defined as the number of periods with a resource usage of $r = l - 1 + \underline{r}_i$ units. In each node of level l, $\hat{w}_i$ indicates the remaining work content of activity i, i.e. the work content of activity i minus the total assigned resource units in nodes located on the path from the start node ($l = 0$) to the current node. At each level, $l > 0$, there are $\bar{t} - \underline{t} + 1$ possible descendants for a parent node, located at level $l - 1$ where $\bar{t} = \lfloor \hat{w}_i / r \rfloor$. These descendants are generated by assigning different values to x_{ri} from set $\{0, \underline{t}, \underline{t}+1, \cdots, \bar{t}\}$. The remaining work content is updated as $\hat{w}_i := \hat{w}_i - r x_{ri}$ after assignment. We define a block in a work profile as a number of consecutive periods with equal levels of resource usage and $\mathbf{x}_i = (x_{\underline{r}_i}, x_{\underline{r}_i+1}, \cdots, x_{\bar{r}_i})$ as the block representation of the work profile of activity i in which x_r represents the length of the block with a resource usage of r units where $r \in [\underline{r}_i, \bar{r}_i]$. In each node of the enumeration tree wherein we face one of the following conditions, we fathom the node.

1. Finding a feasible work profile, i.e. $\hat{w}_i = 0$. In this case, the feasible work profile is obtained by following the path between the start node and the current node.
2. Finding a positive remaining work content that is not enough for the extension of more blocks, i.e. in a level, $l = r + 1 - \underline{r}_i$, $0 < \hat{w}_i < \underline{t}(r + 1)$.
3. Reaching an infeasible work profile. If in each node of level $l = r + 1 - \underline{r}_i$ we replace $\hat{w}_i$ with w_i and $\underline{r}_i$ with r in the model for FWPS_i, and we find $\text{FWPS}_i = \emptyset$, then all descendants of this node will be infeasible.
4. Reaching the last level of the enumeration tree, i.e. $l = \bar{r}_i - \underline{r}_i + 1$.

To generate all NI work profiles, we correspond each NI work profile $\mathbf{x}_i = (x_{\underline{r}_i}, x_{\underline{r}_i+1}, \cdots, x_{\bar{r}_i})$ to the ND work profile $\mathbf{x} = (x_{\bar{r}_i}, x_{\bar{r}_i-1}, x_{\underline{r}_i})$.

Generation and Enumeration of the TR Work Profiles

All feasible TR work profiles of each activity can be generated from its ND or NI work profiles. It should be noted that ND and NI work profiles are two special types of TR work profile. Also, SM work profiles are an especial type of ND and NI work profiles. Thus, we can conclude SM⊆NI⊆TR and SM⊆ND⊆TR. Each ND work profile has a unique corresponding NI work profile and possibly some other TR work profiles constituting a set called a *neighborhood*. To generate all feasible TR neighbors of a feasible ND work profile, we do as follows. Assume the feasible ND work profile includes $\bar{b}$ different resource usage levels, as $\mathbf{x} = (x_{r_1}, x_{r_2}, \cdots, x_{r_{\bar{b}}})$, where $\underline{r} \leq r_1 < r_2 < \cdots < r_{\bar{b}} \leq \bar{r}$ and block $r_{\bar{b}}$ is the vertex of each generated TR work profile. We may use block r_b for $1 \leq b < \bar{b}$ in both left and right legs of each generated TR work profile with durations $x^l_{r_b}$ and $x^r_{r_b}$, respectively. If the duration of a single block, r_b, is $x_{r_b} \geq 2\underline{t}$, we can distribute x_{r_b} between $x^l_{r_b}$ and $x^r_{r_b}$. The number of different ways of this work is equal to the number of common solutions of the following equations:

$$x^l_{r_b} + x^r_{r_b} = x_{r_b}, \tag{26}$$

$$x^l_{r_b}, x^r_{r_b} \in \{0, \underline{t}, \cdots, x_{r_b}\}. \tag{27}$$

If $x_{r_b} \geq 2\underline{t}$, the number of solutions of Equations 26 and 27 is equal to $x_{r_b} - 2\underline{t} + 3$, including:

$$\{(x^l_{r_b} = 0, x^r_{r_b} = x_{r_b}), (x^l_{r_b} = \underline{t}, x^r_{r_b} = x_{r_b} - \underline{t}),$$

$$(x^l_{r_b} = \underline{t} + 1, x^r_{r_b} = x_{r_b} - \underline{t} - 1), \cdots,$$

$$(x^l_{r_b} = x_{r_b} - \underline{t}, x^r_{r_b} = \underline{t}), (x^l_{r_b} = x_{r_b}, x^r_{r_b} = 0)\},$$

and, if $x_{r_b} < 2\underline{t}$, we will have only two possible values for $x^l_{r_b}$ and $x^r_{r_b}$, i.e.:

$$\{(x^l_{r_b} = x_{r_b}, x^l_{r_b} = 0), (x^l_{r_b} = 0, x^l_{r_b} = x_{r_b})\}.$$

If we define function f for integer numbers as:

$$f(a) = \begin{cases} a; & a \geq 0 \\ -1; & a < 0 \end{cases}$$

we generally have $f(x_{r_b} - 2\underline{t}) + 3$ different solutions for Equations 26 and 27. Thus, the number of total different feasible TR work profiles that can be obtained from a feasible ND work profile, $\mathbf{x} = (x_{r_1}, x_{r_2}, \cdots, x_{r_b})$, is equal to $\prod_{b=1}^{\bar{b}-1}(f(x_{r_b} - 2\underline{t}) + 3)$.

It can be seen that the FWPS for each activity is constituted of different neighborhoods. At the core of each neighborhood, there are two feasible work profiles, NI and ND, and other neighbors are possibly other feasible TR work profiles. Each feasible TR work profile can be obtained from only one core and, hence, is located in only one neighborhood. Therefore, FWPS is partitioned by this neighborhood definition. Figure 5 shows one ND, one NI and two TR work profiles that constitute a neighborhood.

Upper and Lower Bounds of the Project Makespan

Regarding Equations 15 to 19, we obtain the upper bound UB for the project makespan as UB= $\sum_{i=1}^{n} d_i^{\min}$. The project makespan lower bound LB is the maximization of a critical path-based lower bound LB_0 and a resource-based lower bound LB_r. The lower bound LB_0 is obtained by calculating the critical path in the activity network, where each activity, i, is assigned its shortest feasible work profile with duration $d_i^{\min}$, taking into account the resource availability, a. The resource-based lower bound LB_r is computed as:

$$\mathrm{LB}_r = \lfloor \sum_{i=1}^{n} w_i / a \rfloor,$$

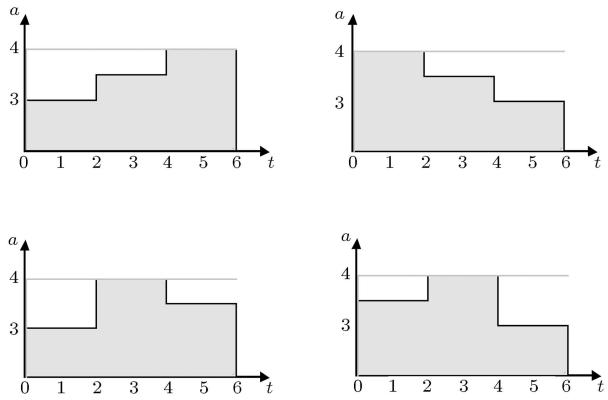


Figure 5. An example of four neighbor work profiles.

where $\lfloor x \rfloor$ denotes the largest integer equal to, or less than, x.

SCHEDULE REPRESENTATION AND GENERATION SCHEME

Our constructive heuristic algorithm uses a schedule representation to encode a project schedule and a Schedule Generation Scheme (SGS) to translate the schedule representation to a real schedule. The SGS determines how a feasible schedule is constructed by assigning starting times to the activities whereby the relative priority and activity work profiles are determined by the schedule representation in our algorithm. There are various approaches for both the representation and generation of a schedule.

We represent a schedule, S, of the RCPSP_FWP by a double list $(\boldsymbol{\rho}, \mathbf{wp})$. The first list is a vector, $\boldsymbol{\rho} = (\rho_1, \cdots, \rho_n)$, such that positive real number $\rho_i \in \mathbf{R}^+$ represents the priority value of activity i. This list was introduced by Kolisch and Hartmann [14] as Random Key (RK) representation for the RCPSP. The second list is a work profile list $\mathbf{wp} = (wp_1, \cdots, wp_n)$ where wp_i represents the work profile chosen for activity i. We utilize a modified version of the RK representation developed by Debels et al. [4] in which the Topological Order (TO) concept is used in the construction of the RK, which we denote as TORK. Each TORK is constructed by scheduling the activities using a SGS, obtaining schedule S and considering priority values as $\rho_i = s_i(S); \forall i \in N$ where $s_i(S)$ indicates the start time of activity i in schedule S. For a detailed discussion of the advantages of the topological order representation, we refer to [15].

Besides different schedule representations, there also exist three different types of SGSs in the literature: the Serial SGS (SSGS), the parallel SGS and the bidirectional SGS [16]. As it is sometimes impossible to reach an optimal solution with parallel and bidirectional SGSs, we opt for serial SGS. At each iteration of the SSGS, the activity with the highest priority is chosen and assigned the first possible starting time such that no precedence or resource constraint is violated. Although any generated schedule of the RCPSP by the SSGS belongs to the set of active schedules [17], it does not necessarily hold for the RCPSP_FWP.

Figure 6 illustrates an example project with five non-dummy activities and one renewable resource with $a = 6$ and $\underline{t} = 2$. The schedule of Figure 7 is obtained, based on the random priority list, $\boldsymbol{\rho} = (1, 2, 3, 4, 5)$, and SSGS. The corresponding TORK is $\boldsymbol{\rho} = (0, 0, 8, 8, 15)$.

GENETIC ALGORITHM

We propose a genetic algorithm to schedule activities in a RCPSP_FWP. The general structure of our GA

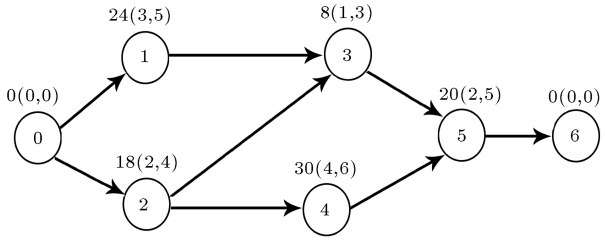
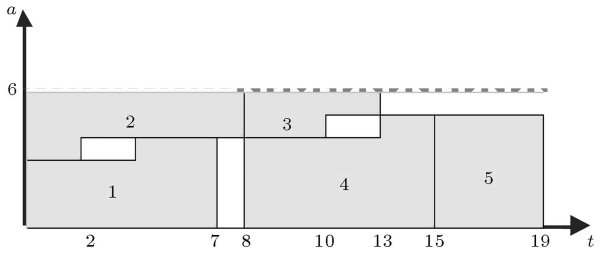


Figure 6. Example project.



TORK: $\rho = (0, 0, 8, 8, 15)$

Figure 7. A schedule and its TORK.

1) Build an initial population P of direct schedules
while (termination criterion is not satisfied) **do**
  2) ***NewP***$=\varnothing$
  for [$e = 1$, to $psize$] **do**
    3) Select a mate randomly for element e
    4) Apply crossover operator to build S_s and S_d
    5) Apply mutation operator to S_s and S_d with probability p_{mut}
    6) Apply SGS to both S_s and S_d
    7) Select S_c and
    8) Apply local search operator to S_c with probability p_{ls}
    9) Add S_c to the ***NewP***
  end for
  10) ***P = NewP***
end while

Figure 8. General structure of GA.

is shown in Figure 8. The procedure starts with the random generation of an initial population, P, with size of $psize$ of schedules. In order to generate each element, $e = (\boldsymbol{\rho}, \mathbf{wp})$, of initial population, P, we generate a random priority list, $\boldsymbol{\rho}$, randomly select a work profile for each activity from its set of feasible work profiles and decode element, e, to a real schedule using SSGS. Then, the procedure is followed by an iterative process that continues until the termination criterion is satisfied. The iterative process consecutively applies crossover, mutation and local search operators in order to generate new solutions with higher quality. In the second step, the $NewP$, representing a new population, is initialized as an empty set and will include the generated schedules in each iteration. In order to construct the new elements, each element, $e = 1, \cdots, psize$ of P, considered as the father schedule (S_f), is mated randomly with element c ($c \neq e$ and $1 \leq c \leq psize$) of P, considered as the mother schedule (S_m), to constitute a couple. Each couple generates two children schedules considered as the son schedule (S_s) and the daughter schedule (S_d). The children generation process is performed using two operators, i.e. crossover and mutation where mutation is applied with probability p_{mut}. The child with the better makespan obtained using SSGS is chosen as the selected child schedule (S_c). To improve S_c, a local search procedure is applied to it with probability p_{ls}. Then, P is updated by transferring all elements of $NewP$ to it.

Crossover

After the selection of parents, a crossover operation combines the S_f and S_m to generate S_s and S_d. We use a two-point crossover operator, based on the idea of replacement of weak sub-schedules with better sub-schedules, in which the crossover points are determined, based on the resource profile for the given schedule. The crossover point, cp_1, is defined as an activity's start time and the crossover point, cp_2, is defined as an activity's finish time. All activities executed totally between these two crossover points are called a sub-schedule. We define the Resource Utilization Ratio (RUR) as follows. Let $A[t_1, t_2] = \{i | s_i \geq t_1 \text{ and } f_i \leq t_2\}$ as the set of activities that are executed totally in interval $[t_1, t_2]$. The RUR for schedule S in interval $[t_1, t_2]$ is defined as $\text{RUR}^S[t_1, t_2] = \sum_{t=t_1+1}^{t_2} \sum_{i \in A[t_1,t_2]} (r_{it}/a)$. In order to determine crossover points, we look for sub-schedules including at least $\lfloor n/3 \rfloor$ and, at most, $\lceil 2n/3 \rceil$ activities giving the maximum RUR. Assume $cp_1 = s_i$ and $cp_2 = f_j$, where $i, j \in \{1, \cdots, n\}$ and subject to the two following conditions:

I) $s_j \geq s_i$,

II) $f_j > f_i$ or $f_j = f_i$ where $j \neq i$.

For all real activities i and j, we consider all possible intervals $\lfloor t_1 = s_i, t_2 = f_j \rfloor$ regarding Conditions I and II as the candidates for the cross points. The two points, t_1^* and t_2^*, are considered as the crossover points, cp_1 and cp_2, respectively, for which the maximum resource utilization ratio is obtained. In other words, the cross points are determined based on the following condition:

$$\text{RUR}^S[t_1^*, t_2^*] \geq \text{RUR}^S[t_1, t_2],$$

$$\forall t_1 = s_i,$$

and:

$$t_2 = f_j, \qquad i, j = 1, \cdots, n,$$

and holding Conditions I and II.

Each activity, i, for which $s_i \geq cp_1$ and $f_i \leq cp_2$, constitutes the subset N_{sub}. In the following, we describe how the son schedule, S_s, is built from S_f and S_m. First, we determine $cp_1(S_f)$ and $cp_2(S_f)$ using the above mentioned two-point crossover operator. To generate the vectors, $\boldsymbol{\rho}_s$ and $\mathbf{X}_s$, of the S_s, for every activity, $i \in N$, we define the following combination rule. If in schedule S_f activity $i \in N_{\text{sub}}$, let $\rho_{si} = \rho_{fi}$ and $wp_{si} = wp_{fi}$. Otherwise, if $\rho_{fi} < cp_1(S_f)$, let $\rho_{si} = \rho_{mi} - b$ and $wp_{si} = wp_{mi}$ and if $\rho_{fi} > cp_1(S_f)$, let $\rho_{si} = \rho_{mi} + b$ and $wp_{si} = wp_{mi}$ where b is a very big constant.

This combination rule allows the crossover operator to place the good sub-schedules with the high resource utilization of the S_f in S_s and replace the weaker sub-schedules of the S_f with the lower resource utilization by corresponding sub-schedules of the S_m. The big constant b is used in order to prevent the relative priority structure between the activities of a case being mixed with the priority values of activities of another case in the combination rule. By applying a SSGS on the obtained $\boldsymbol{\rho}_s$ and $\mathbf{wp}_s$, S_s is obtained. If we assume $S_c = S_s$, by obtaining the TORK corresponding to S_c and applying the SSGS to it, the generation of S_c is finished. By exchanging the role of S_f and S_m in generation of the child schedule, S_d can be easily obtained. Based on the lower makespan, S_s or S_d is selected as the child schedule, S_c. Schedules represented in Figures 9 to 11 are related to the example network of Figure 6 and illustrate the crossover operator in which son schedule S_s is considered as the selected child schedule S_c.

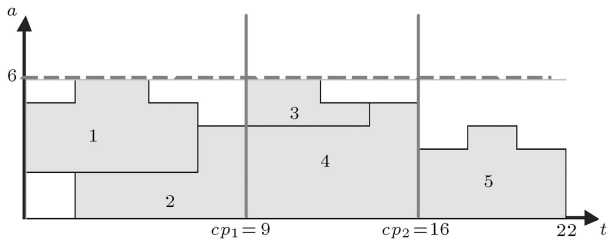


Figure 9. Father schedule (S_f).

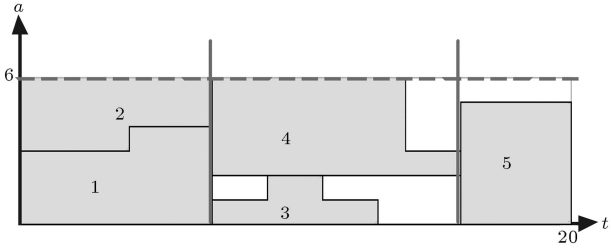


Figure 10. Mother schedule (S_m).

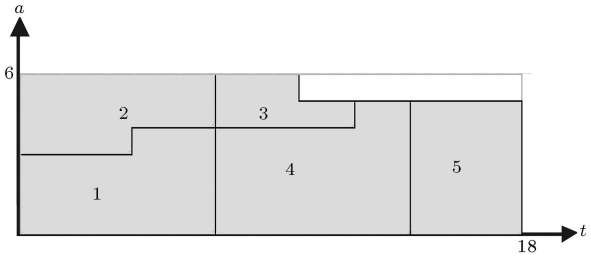


Figure 11. Child schedule (S_c).

Mutation

After applying the crossover operator, each generated schedule is muted with a probability of p_{mut} using a mutation operator that works as follows. First, it selects, randomly, nrm out of n non-dummy activities as the chosen activities for mutation. Then, for each selected activity i, it chooses, randomly, a feasible work profile from the non-neighbors work profiles of its current work profile, wp_i. Finally, for the selected activity i, it changes the priority value as $\rho_i = \rho_i + r_i$, where $r_i \in [1/3f, 2/3f]$ is a random integer.

Local Search

A local search procedure is applied over each generated schedule with probability P_{ls}, described by a Pseudo-code in Figure 12. In this procedure, given a schedule S, $f(S)$ denote the makespan of schedule S, *Improvement* is a Boolean variable and SA indicates the set of selected activities for a change in the work profile. This procedure tries to decrease $f(S)$ by alternating the work profile of each activity in the neighborhood of the activity work profile.

This procedure includes a *While* loop, in which all activities are selected based on their priority values, re-

1) Build the priority list $\boldsymbol{\rho} = (\rho_1, \cdots, \rho_n)$ based on schedule S where $\rho_i = s_i$.
2) Let *Improvement* = *true*.
While (*improvement* = *true*) **do**
  3) Let *improvement* = *false* and $SA = \varnothing$
  for (i = 1 to n) **do**
    4) Select activity j for which $\rho_j = \min(\rho_1, \cdots, \rho_n)$ and $j \neq SA$.
    5) Add activity j to SA.
    6) Change the work profile of activity j in its defined neighborhood, apply SSGS for each change in work profile and select the best one resulting in schedule S'.
    7) If $f(S') < f(S)$, let $S = (S')$ ($\boldsymbol{\rho} = \boldsymbol{\rho'}.\boldsymbol{wp} = \boldsymbol{wp'}$), $f(S') = f(S)$ and *improvement*=*true*.
  and for
end while
8) If (*improvement* =*true*) go to 1, otherwise stop.

Figure 12. Local search procedure.

spectively. In the main step (Step 6) of this procedure, for each selected activity j, all neighbors of its current work profile are replaced one by one as the work profile of activity j, while the other activities' work profiles and priority values are not changed. Each generated schedule in Step 6 is evaluated using its makespan and, finally, the best work profile is selected for activity j, resulting in schedule S'. If $f(S') < f(S)$, schedule S' is replaced with schedule S and *Improvement* is changed to *true*. In the last step, the procedure is stopped if no improvement is obtained; otherwise, it goes to the first step and updates the priority values based on the best found schedule of the loop.

COMPUTATIONAL RESULTS

Benchmark Problem Set

We have coded the procedure in Visual C++ 6.0 and performed all computational experiments on a PC Pentium IV 3GHz processor with 1024 MB of internal memory. We have validated the procedure on two sets of instances developed by Kolisch [1]; one with 30 non-dummy activities and one with 60 non-dummy activities (additionally, each instance has a dummy start and a dummy end activity, each with a duration of zero and a work content of zero). These instances have been generated by transforming a subset of instances from the 30- and 60-activity sets of the instances as available in the PSPLIB library (http://www.bwl.uni-kiel.de/bwlinstitute/Prod/psplib/datasm.htm. Only one renewable resource is considered in the generation of these instances.

Each instance is characterized by the following four parameters: *rurf*, t min, *rsf* and NC, which are detailed below.

a) *rurf* stands for the resource usage range factor. It determines for each activity i the range of $[\underline{r}_i, \bar{r}_i]$ for the per period resource usage of this activity. $rurf \in [0,1]$ where 0 defines a range with $\underline{r}_i = \bar{r}_i$ and 1 defines a large range. In these two sets of instances, *rurf* is set to 0.1 and 0.3, respectively. Based on the *rurf* value, for each activity i, $\underline{r}_i$ and $\bar{r}_i$ are calculated as $\underline{r}_i = \lfloor w_i(1 - rurf) \rfloor$ and $\bar{r}_i = \lceil w_i(1 + rurf) \rceil$.

b) t min stands for the minimum number of consecutive periods where activity i has to have a constant resource usage. t min has been set to 1 and 3 for all activities, respectively.

c) *rsf* stands for the resource strength factor. *rsf* determines the scarcity of the resources. For $rsf = 0$, the resources will be as scarce as possible to allow a feasible solution. For $rsf = 1$, the amount of capacity is such that the resource constraints are not binding any more. In our problem instance, *rsf* has been set to 0.2 and 0.5, respectively.

d) NC stands for the network complexity. It measures the average number of non-redundant arcs per activity in the network. NC has been set to 1.5, 1.8 and 2.1, respectively.

Using a full factorial design, $2^*2^*2^*3 = 24$ instances have been generated with 30 and with 60 activities, respectively. Also, for each activity i, the work content, w_i, is a random integer from interval $[1, 100]$.

Parameter Settings

To test our procedure, we predefine the settings of the parameters of the genetic algorithm. The number of children, *nrc*, is fixed at 2, the number of mutated activities in the mutation step is set equal to $n/10$. Using fine tuning, we obtained the values of p_{mut} and p_{ls} parameters as $p_{\text{mut}} = 0.04$, $p_{ls} = 0.1$. Also, the fined tuned values of $psize$ related to the stop condition are considered as three CPU-time-limits 1, 10 and 50 seconds, and the size of the problem-instances are shown in Table 2.

Number of Feasible Work Profiles

Table 3 indicates the average number of feasible NI, ND and TR work profiles for each activity based on the different values of parameters t and *rurf*. It can be seen that the number of feasible work profiles is an increasing function of $1/\underline{t}$ and *rurf*.

Detailed Computational Results

The performance of the GA for the RCPSP_FWP on the 24 instances is summarized in Table 4. We report the total sum of the makespan of each obtained solution

Table 2. Tuned values of $psize$.

Instances Set	CPU-Time-Limit (s)		
	1	10	50
J30	50	110	425
J60	30	75	400

Table 3. Average number of feasible work profiles.

	Type of Work Profile		
Values of Parameter	**SM**	**ND (NI)**	**TR**
$\underline{t} = 1$, $rurf = 0.1$	4.83	36.04	294.08
$\underline{t} = 1$, $rurf = 0.3$	6.15	50.12	381.42
$\underline{t} = 3$, $rurf = 0.1$	1.56	6.33	9.58
$\underline{t} = 3$, $rurf = 0.3$	2.62	10.16	15.72

Table 4. Detailed computational results.

Problem Set	CPU-Time Limit (s)	Data Set	
		J30	J60
Sum of the makespans	5	1410	2310
	10	1319	2221
	50	1027	1783
Average deviation from the LB	5	35.03%	38.61%
	10	32.74%	37.11%
	50	25.49%	29.75%

and the average deviation from the lower bound LB, described previously.

Table 5 shows the effect of the work profile types. The results are presented as the average deviation from the LB for 1, 10 and 50 seconds CPU-time-limit and for four types of work profile: SM, ND, NI and TR. The results related to TR and SM work profile are the best and the worst, respectively. This result is in line with our expectations because SM⊆NI⊆TR and SM⊆ND⊆TR.

Impact of the Mutation and the Local Search

Figures 13 and 14 indicate the impact of the mutation and the local search on the RCPSP_FWP results by changing p_{mut} and p_{ls}, respectively. The results are obtained based on the average percent deviation from the lower bound LB when the allowed CPU-time limit is 10 seconds. The results reveal that the best values for the probability of applying these two operators on each generated schedule are as $p_{\text{mut}} = 0.04$ and $p_{ls} = 0.1$.

Table 5. Effect of the work profile types.

	CPU-Time Limit (s)		
Work Profile	5	10	50
SM	44.16%	42.87%	41.59%
ND	37.64%	36.28%	29.47%
NI	37.69%	36.27%	29.45%
TR	35.22%	35.06%	26.48%

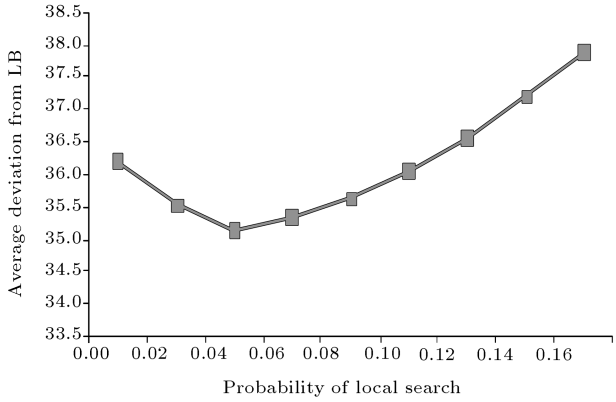


Figure 13. Impact of the mutation operator.

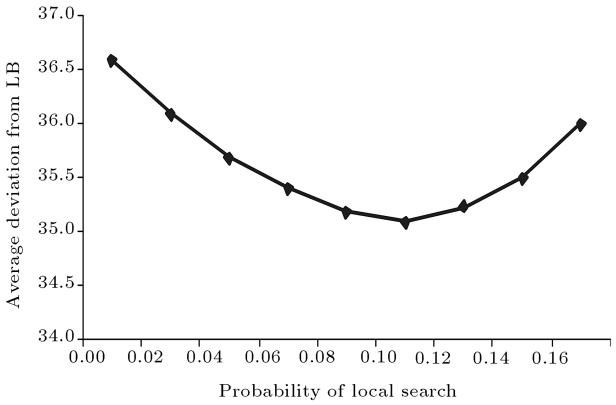


Figure 14. Impact of the local search operator.

CONCLUSION

In this paper, we introduced the Resource-Constrained Project Scheduling Problem with a Flexible Work Profile (RCPSP_FWP). We developed a linear model for the problem, an enumeration procedure for generation of feasible work profiles and a metaheuristic, based on the Genetic Algorithm (GA), for solving the problem. As the crossover operator, the GA uses a two-point crossover based on the idea of the replacement of weak sub-schedules with better ones. To that purpose, we used the resource utilization ratio to evaluate different sub-schedules. We also developed a local search incorporated with GA to improve the solutions' quality. We performed detailed computational results on two problem sets, J30 and J60, each including 24 problem instances.

Our future research efforts will focus on two extensions. First, we focus on developing an exact solution approach for the RCPSP_FWP and also better metaheuristics. Second, we focus on considering work profiles in general forms and developing a procedure to generate them.

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BIOGRAPHIES

Mohammad Ranjbar was born on January 9th, 1978 in Mashad, Iran. He obtained his BS and PhD from the Department of Industrial Engineering at Sharif University of Technology in 2000 and 2007, respectively, receiving his MS from the Faculty of Engineering at Tehran University. From October 2005 to March 2006, he was a visiting PhD student at the Department of Operations Management of the London Business School. In February 2008, he joined Ferdowsi University in Mashhad and, presently, is assistant professor of Industrial Engineering at this university. Dr. Ranjbar is the author of six papers published in international journals, one of which was presented at an international conference.

Ferydoon Kianfar was born on May 17th, 1942 in Tehran, Iran, later receiving his BS in Mechanical Engineering from Tehran Polytechnic. He obtained his MS and PhD in Industrial Engineering from Illinois Institute of Technology (IIT) in Chicago, USA, where, from 1969 to 1970, he was employed as an assistant professor. Since 1970 he has been working at Sharif University of Technology in Tehran, Iran, and is now a full professor of Industrial Engineering.